Article

Nanoscale Waveguide Beam Splitter in Quantum Technologies

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Abstract: Usually in quantum optics, the theory of large- and small-scale waveguide beam splitters is the same. In this paper it is shown that the theory of the nanoscale waveguide beamsplitter has a significant difference from a similar device, but of a larger scale. It is shown that the previously known theory of the waveguide beam splitter is a particular case of the theory presented here. The wave function at the output ports of the nanoscale beam splitter is analyzed. The results obtained are sensitive to the size of the beam splitter, the coupling parameter of the two waveguides, and the degree of nonmonochromaticity of the photons entering the first and second ports of the beam splitter. The results are important for quantum technologies using a nanosized beam splitter.

Keywords: Beam splitter, nanosize, photons, wave function, non-monochromatic photons, reflection coefficient, transmission coefficient

1. Introduction

It is well known that the waveguide beam splitter (BS) is one of the main devices in quantum technologies [1–7]. This device has a great prospect of application due to its small size. This, in turn, leads to the fact that at small sizes new phenomena can arise that are not inherent in similar devices, but on a large scale. It is usually considered that the main characteristics of a waveguide BS are the reflection coefficients R and transmission coefficients T are constant values. This means that by setting these parameters one can always obtain the required characteristics at the output ports of the BS [3,8–13]. Previously, the results obtained did not qualitatively depend on the size of the BS. Although quite obvious is the fact of a qualitative change in the properties of photons at the output ports of the BS depending on the dimensions of the BS (more precisely, the coupling region in the BS, see Fig.1). Indeed, if the size of the BS becomes comparable in order of magnitude with the wavelength of photons incident on ports 1 and 2 of the BS, then this should affect the properties of photons at the output ports of the BS. It was shown in [14,15] that using a BS based on coupled waveguides, i.e. waveguide BS Hong-Ou-Mandel (HOM) effect may not be performed even if R=T=1/2 and the photons used are identical. It was also pointed out in these papers that in the main such changes in the example of the HOM effect appear for sufficiently small waveguide BS. Further development of the theory of coupled waveguides showed that the previously known theory is not always applicable not only for the HOM effect, but also for a waveguide BS in general [16,17]. This is due to the fact that in old theories the coefficients R and T are always constant for waveguide BSs, and as shown in [16–18], this is not the case in the case of non-monochromatic photons incident on 1 and 2 BS ports. These coefficients are dependent on the frequencies of the incident photons on the ports of the BS.

In this paper we show that the properties of nonmonochromatic photons at the output ports of the BS strongly depend on the size in the case of the nanowave BS. In this case, as the size of the BS increases, the properties of photons at the output ports become constant and do not depend on its size. In the case of monochromatic photons the properties of the BS are the same regardless of its size.

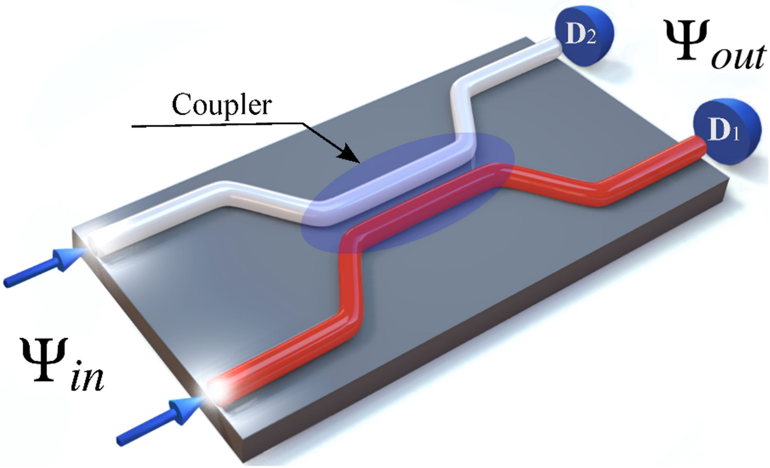

Figure 1. 3D representation of the waveguide BS. Photons (in the general case nonmonochromatic) fall on the input ports BS. At the output ports of the BS are detectors D_1, D_2 registering photons. The figure highlights the coupling region of the waveguide, where the electromagnetic fields from ports 1 and 2 overlap.

Further we will use the atomic system of units: $\hbar = 1$; $|e| = 1$; $m_e = 1$, where $\hbar$ is the Dirac constant, e is the electron charge, m_e is the electron mass.

2. Materials and Methods

Consider a waveguide BS of arbitrary size. At the input ports of such a BS fall two modes of electromagnetic field described by the wave function Ψ_{in}. At the output ports of this BS (see. Fig. 1) photons described by the wave function Ψ_{out} are registered.

It has been shown in [16–18] that photon states at output ports of a waveguide BS can be represented by

$$\Psi_{out} = \sum_{k=0}^{s_1+s_2} \int \phi(\omega_1,\omega_2) c_{k,p} |k, s_1+s_2-k\rangle d\omega_1 d\omega_2,$$

$$c_{k,p} = \sum_{n=0}^{s_1+s_2} A^{s_1,s_2}_{n,s_1+s_2-n} A^{*k,p}_{n,s_1+s_2-n} e^{-2in\,\arccos\left(\sqrt{1-R}\sin\phi\right)},$$

$$A^{k,p}_{n,m} = \frac{\mu^{k+n}\sqrt{m!n!}}{(1+\mu^2)^{\frac{n+m}{2}}\sqrt{k!p!}} P_n^{(-(1+m+n),m-k)}\left(-\frac{2+\mu^2}{\mu^2}\right),$$

$$\mu = \sqrt{1+\frac{1-R}{R}\cos^2\phi} - \cos\phi\sqrt{\frac{1-R}{R}},$$

$$R = \frac{\sin^2\left(\Omega t_{BS}/2\sqrt{1+\varepsilon^2}\right)}{(1+\varepsilon^2)};\ T = 1-R;\ \cos\phi = -\varepsilon\sqrt{\frac{R}{T}};\ \varepsilon = \frac{\omega_2-\omega_1}{\Omega}, \tag{1}$$

where $|k, s_1+s_2-k\rangle = |k\rangle|p\rangle$ is the state of the photons at the output ports of the BS; s_1 and s_2 are the input number of photons in modes 1 and 2, respectively; $\phi(\omega_1,\omega_2)$ is the joint spectral amplitude (JSA) of the two-modes wavefunction ($\int |\phi(\omega_1,\omega_2)|^2 d\omega_1 d\omega_2 = 1$); Ω is a certain frequency characterizing the BS; $t_{BS} = L/v$ is the time of interaction of photons in the BS (L is the length of the binding region in the waveguide BS, v is the speed of light propagation in the waveguide). In Eq. (1) the coefficients R and T are reflection and transmission coefficients respectively. It should be added that Eq. (1) is not only responsible for the case of non-monochromatic photons falling on the BS input ports, but also monochromatic ones. For this purpose it is sufficient to set JSA $\phi(\omega_1,\omega_2)$ to zero the parameter responsible for the spectral width. It should be added that in case of monochromatic and identical photons, the obtained expressions coincide with [1], where $R = \sin^2(Cz)$, $\phi = \pi/2$, $C = \Omega/(2\mathrm{v})$ is the coupling constant between

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neighbouring waveguides. In addition, it is worth adding that in reality there are no monochromatic photons and the analysis must be based on total Eq. (1). In addition, the more coupling in the waveguide, the higher the value of Ω and vice versa. Thus, we can adjust the coupling in the waveguide by changing Ω.

To study the output states of photons at the ports of the BS we will study two characteristics is the probability $P_{k,p}$ to detect k and p photons at 1 and 2 output ports of the BS, respectively. We will take into account, as shown in [16,17], that the number of photons is conserved, i.e. $s_1 + s_2 = k + p$. An extremely important characteristic in quantum technology and information is the quantum entanglement of photons. To characterize quantum entanglement, the Von Neumann entropy S_N will be used, which as shown in [16,17], will be determined by $S_N = -\sum_k P_{k,s_1+s_2-k} \ln\left(P_{k,s_1+s_2-k}\right)$. To find the probability $P_{k,p}$ we can use the previously known [16,17] expression

$$P_{k,s_1+s_2-k} = \int |\phi(\omega_1,\omega_2)|^2 \lambda_k(R) d\omega_1 d\omega_2, \;\; \lambda_k(R) = \left|c_{k,s_1+s_2-k}\right|^2. \quad (2)$$

3. Results

The joint spectral amplitude (JSA) $\phi(\omega_1,\omega_2)$ must be determined in order to present and analyse the results of the calculations. We will use the best known form

$$\phi_i(\omega_i) = \frac{1}{(2\pi)^{1/4}\sqrt{\sigma_i}} e^{-\frac{(\omega_i-\omega_{0i})^2}{4\sigma_i^2}}, \quad (3)$$

where ω_{0i} is the mean frequency and σ_i^2 is the dispersion. Next, we will use the $\omega_{0i}/\sigma_i \gg 1$ condition, which is applicable to most photon sources. It should be added that the form Eq. (3)is the best known function and corresponds to the distribution of photons in Fock states.

Further, we will assume that the incident photons at ports 1 and 2 of the BS are identical, i.e. $\sigma_1 = \sigma_2 = \sigma$ and $\omega_{02} = \omega_{01} = \omega_0$. The use of identical photons in quantum technologies is one of the important properties, since with such an identity quantum coherence and quantum entanglement of photons begin to appear. This is easy to show qualitatively using Eq.(1) using the expression for the reflection coefficient R. Indeed, if $\omega_2 - \omega_1 \gg \Omega$ is chosen, then the coefficient $R \ll 1$, which leads to the propagation of photons along their original waveguides and the coupled waveguide does not exhibit the properties of a BS. In this case, it is not difficult to show that the main characteristics of the electromagnetic field at the output ports of the BS will depend only on two parameters σ/Ω and L/L_{BS}, where $L_{BS} = v/\Omega$. The value of L_{BS} plays an important role in the BS, firstly, if $L \ll L_{BS}$, then the properties of the BS are not observed, i.e. photons in waveguides propagate unchanged, secondly, if $L \gtrsim L_{BS}$, then the main characteristics of photons at the output ports of the BS have a non-trivial dependence. In the case of $L \gtrsim L_{BS}$, a qualitative analysis can be carried out if we consider the reflection coefficient R in (1) and consider the photons to be identical. In this case, during the analysis, the parameter $L_{const} = L_{BS}\Omega/\sigma = v/\sigma$ will appear at which if $L \gg L_{const}$ the main characteristics of photons at the output ports of the BS do not depend on of length L. For other values of L, the dependence of the main characteristics of photons at the output ports of the BS is rather complicated. It should be added that the length L_{BS} has a simple physical meaning, it is the characteristic coupling length of the waveguide, i.e. less than this length the connection between waveguides is not observed.

Next, we present an illustration of the calculation of the probability of detecting photons at 1 output port of the BS in the case when one photon each falls on the input ports, i.e., $s_1 = 1, s_2 = 1$, see Fig. 2. Let us also present, for the same data ($s_1 = 1, s_2 = 2$), the calculation of the quantum entanglement of photons at the output of the BS Fig. 3. The calculations were performed for various cases of nonmonochromaticity of incident photons. From Figs. 2, 3 we can see that the patterns are qualitatively the same. This means that the general analysis of these regularities will be similar regardless of whether

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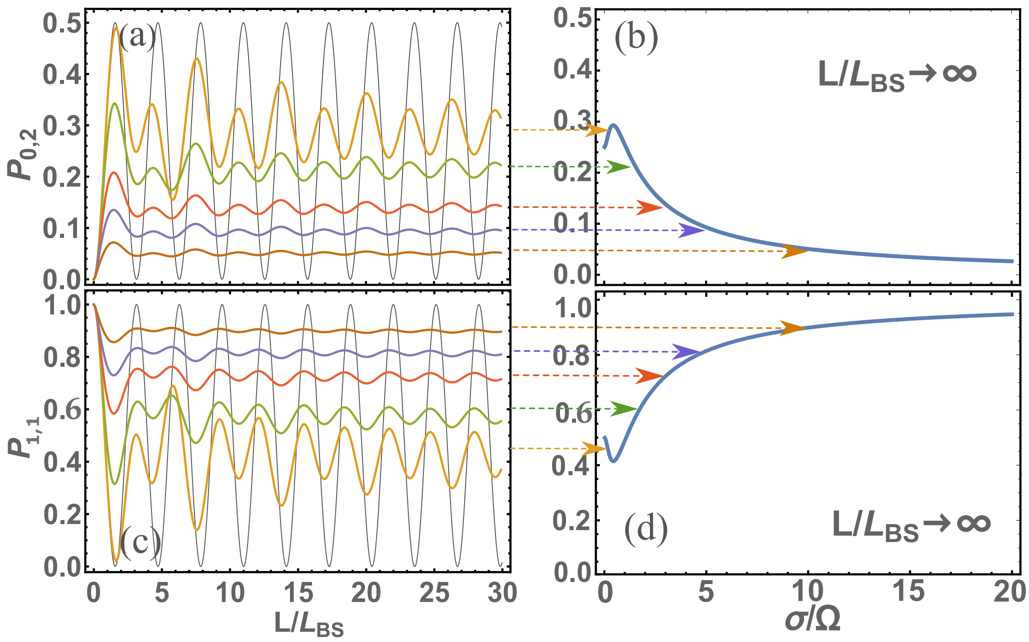

Figure 2. The calculation is presented: (a) probabilities $P_{0,2}$ of detecting 2 photons at the second detector and 0 of photons at 1 detector (with $P_{0,2} = P_{2,0}$) at different parameters σ/Ω={0,1/2,3/2,3,5,10} depending on the dimensionless BS length L/L_{BS} (top-down); (b) the same, but with larger dimensions of the beamsplitter, i.e., at $L/L_{BS} \to \infty$; (c) the same as in (a), but only for the probability $P_{1,1}$ of one photon detected at each detector; (d) the same as in (b), but only for the probability $P_{1,1}$.

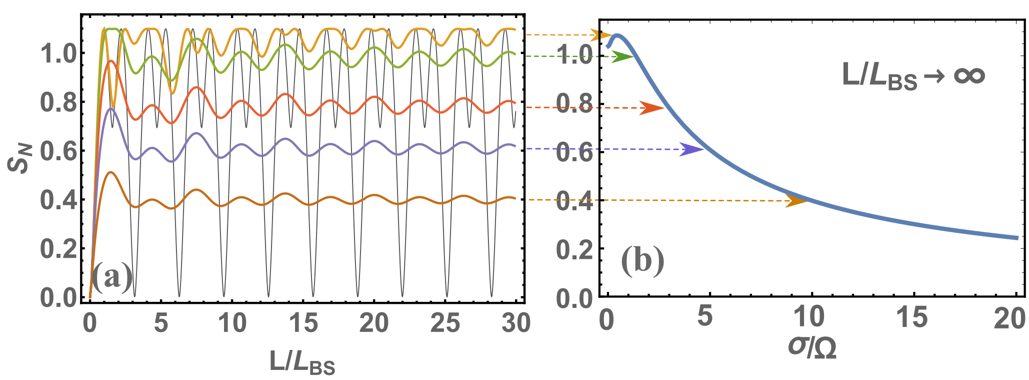

Figure 3. The calculation is presented: (a) Quantum entanglement S_N at different parameters σ/Ω={0,1/2,3/2,3,5,10} depending on the dimensionless BS length L/L_{BS} (top-down); (b) the same, but with larger dimensions of the BS, i.e., at $L/L_{BS} \to \infty$.

we consider probabilities or quantum entanglement. From the presented figures it is clearly visible that at small values of L/L_{BS} the main characteristics tend to zero, and at $L/L_{BS} \to \infty$ tend to a constant value, i.e. the main characteristics cease to depend on the light splitter length. As shown above, there is a more stringent condition for determining the transition of the basic characteristics to a constant value is $L \gg L_{BS}\Omega/\sigma = v/\sigma$. If you look at these figures, you can see that this condition is indeed satisfied. You can also see that if the photons are monochromatic, i.e., $v/\sigma \to \infty$, then $L \to \infty$. In other words, in the case of monochromatic photons, the main characteristics of photons at the output ports of the BS will never be constant and will depend on the length of the BS. Of course, in reality there is no such thing, since there are no completely monochromatic photons, which means that at a certain length of the BS the main characteristics will always be constant. Thus, we come to the definition of the main parameter $L_{const} = v/\sigma$, which determines the characteristic length of the BS, more than which the main characteristics of photons at the output ports of the BS will become constant. Typically, quantum technologies use optical photons with dispersion $\sigma \sim \{(10^{13} - 10^{14})\}$ rad/s. If we estimate the length L_{const}, we get $\sim \{10^3 - 10^4\}$nm. This is the size in which fit from units to the order of optical wavelengths. If higher-frequency radiation (e.g., ultraviolet) is used, these dimensions will be reduced and the sizes will be comparable to tens and hundreds of nanometers. If we use lower-frequency radiation (e.g. infrared), the sizes will be tens and hundreds of micrometers. Thus, when using nanoscale waveguides, the main characteristics of photons at the output ports of the BS will be sensitive to its size and will always depend on it. Moreover, these characteristics will be quite hard to predict (see Figs. 2, 3) and have not only oscillatory nature but also will change the amplitude of these characteristics. To determine these characteristics it is necessary to carry out a calculation using Eq. (1).

For a complete analysis of such BS, we need to determine the length $L_{BS} = v/\Omega$. As shown in [14,15], the frequency can be within $\Omega = \{10^{14} - 10^{17}\}$ rad/s depending on what the waveguide consists of and how the waveguides are connected together. In any case, $L_{BS} < L_{const}$, which means that there is always a region of transition from zero values of the main characteristics to values when they do not depend on the length of L (for example, see Figs. 2, 3).

4. Conclusion

Thus, it is shown that the nanosized beam splitter exhibits properties that are not manifested in large-sized BS. The boundary when the waveguide should be considered large is determined when the coupling length of the waveguide $L \gg L_{const} = v/\sigma$. Also the boundary when the coupled waveguide does not exhibit the properties of a BS is determined when $L \ll L_{BS} = v/\Omega$. The properties of the BS here meant the main characteristics of photons at its output ports are the probabilities of detecting photons at ports 1 and 2, as well as quantum entanglement. It is shown that the properties of the BS do not depend on which characteristic we consider, the probability or the quantum entanglement. All these characteristics can be calculated using Eq. (1). As an example, we performed calculations for the initial states of photons falling on the input ports of the BS at $s_1 = s_2 = 1$ (or $|1,1\rangle$). Regardless of these calculations, all conclusions are also suitable for arbitrary states $|s_1, s_2\rangle$, since a qualitative analysis can be performed in general form that leads to the same conclusions as those for calculations at $|1,1\rangle$.

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