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Inevitability and Incompleteness of the Second Law of Thermodynamics in the Expanding Universe

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Abstract: A simple line of reasoning, based on the most fundamental concepts of thermodynamics, yields some intriguing results for a better understanding of the processes occurring in the observable Universe. Gravitational mass must be continuously generated within an expanding thermodynamic system for this system to remain closed. The Second Law is a direct consequence of this production of mass. Simple expressions for the entropy and temperature of the Universe were obtained and the results agree well with observable values. Furthermore, it is demonstrated that the conservation laws within the Universe are independent of its energy density. Based on the solution for the quantum state of the Universe, it is conjectured that the Second Law is incomplete and must be complemented to a conservation law, which takes into account the growth of the amount of information within the Universe. Once the Second Law is complemented to a conservation law, the importance of mass generation within the Universe becomes well pronounced – not only gravitational effects play the role of an organising force, but also the amount of mass within the Universe defines both the amount of information within the Universe and the level of the Universe's complexity.

Keywords: Second Law of thermodynamics; Universe as a closed thermodynamic system; entropy of the Universe; temperature of the Universe; quantum state of the Universe; information within the Universe; complexity of the Universe

1. Introduction

The objective of this study is to demonstrate that a simple line of reasoning based on the most fundamental concepts of thermodynamics, such as energy conservation (the First Law), entropy, and temperature, can yield some intriguing results for a better understanding of the processes occurring on a universal scale.

The behaviour of an expanding closed thermodynamic system comprised of radiation and matter has been modelled in this work. For such a system to remain closed over its entire evolution, its total gravitational mass must increase linearly with time while its gravitational potential must remain constant. The following section demonstrates that the rate of mass creation and the gravitational potential are independent of time and uniquely specified by the gravitational constant G and the speed of light c.

The Second Law of Thermodynamics is discussed in Section 3. If the entropy of an expanding closed system is always equal to the Bekenstein-Hawking entropy of the corresponding black hole [1-2], then *the Second Law of Thermodynamics is a direct result of the ongoing mass generation* within such a system. The present value of the Universe's entropy, as calculated in this study, is in excellent agreement with the number determined by another independent approach. Moreover, it is demonstrated that the Universe's temperature varies inversely proportional to the square root of time during its entire evolution. A precise solution for the evolution of temperature has been derived and tested against the observed temperature of the cosmic microwave background (CMB).

The solution derived for the gravitational potential field of the Universe is combined with the Schrödinger equation for the probability wave function of the Universe in Section

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4. It is demonstrated that the conservation laws within the Universe are independent of its energy density. In other words, the conservation laws in the expanding Universe are independent of both the stress-energy tensor, $T_{\mu\nu}$, and the space-time metrics in Einstein's field equations.

In the same section, a non-field solution for the quantum state of the universe is obtained. From this solution, it follows that, regardless of the starting geometry of the Universe, it rapidly flattens with expansion. The non-field solution also explains why the observable Universe is slightly cooler than what was predicted in Section 3. Moreover, the non-field solution for the quantum state of the Universe suggests that our Universe may be a fully reversible thermodynamic process and that information cannot be lost within our Universe.

In Section 5, the question of incompleteness of the Second Law is discussed in more detail. It is conjectured that *the Second Law is not complete and is to be complemented to a conservation law* in such a form that, within the universe, the rate of entropy generation is to be equal to the depletion rate of the informational entropy. Once the Second Law is complemented to a conservation law in such a form, the importance of mass generation within the Universe becomes well pronounced – not only gravitational effects play the role of an organising force, but also *the amount of mass within the Universe defines both the amount of information within the Universe and the level of the Universe's complexity*.

In the concluding section of this paper, a comprehensive discussion of all the study's findings is offered.

2. Perpetual mass generation as the condition of closeness

Consider a thermodynamic system, whose total energy content, E, is in the wave-corpuscular form; that is, both the radiation (waves) and gravitational mass (corpuscles) components are always present within the system. Assume that the radiation component within the system under consideration propagates with speed c, which remains constant during the entire evolution.

For such a system to always remain closed thermodynamically, its total energy may not change with time, that is,

$$\frac{dE}{dt} = 0. \tag{1}$$

The latter condition implies that no radiation may escape the system and, hence, the Schwarzschild radius of the system must be

$$r_s(t) = \frac{2Gm(t)}{c^2}, \tag{2}$$

where G is the gravitational constant and m is the total mass of the system.

Equation (2) infers that, *for an expanding system to remain a closed thermodynamic system, its total gravitational mass, m, is to increase with time*. And since the Schwarzschild radius is $r_s(t) = ct$, Eq. (2) renders an expression for $m(t)$ as

$$m(t) = \frac{c^3 t}{2G}. \tag{3}$$

Equation (3) implies that the rate of mass production within an expanding closed thermodynamic system has to remain constant and is expressible through the fundamental constants as

$$\frac{dm}{dt} = \frac{c^3}{2G}. \tag{4}$$

The gravitational potential of the system in question is now defined as

$$\Phi = \frac{2Gm(t)}{r_s(t)} = c^2. \tag{5}$$

Equation (5) combined with Eq. (3) renders the expression for the corresponding gravitational potential field

$$U(t) = m(t)\Phi = \frac{c^5 t}{2G}. \quad (6)$$

Notice that Eq. (6) is nothing else but the expression for the total energy of the corpuscular part of the system, E_m. As can be seen, this energy grows linearly with time. Hence, for Eq. (1) to remain valid, the radiation energy of the system, E_r, must decrease with time at the same rate.

It is worth noting here also that introducing the gravitational potential is equivalent to generalizing the local Maxwell equations for the electromagnetic field to the non-local equations by defining the electromagnetic potentials of adjacent charges as electric field strength sources [3-6].

3. Second Law: entropy and temperature

The necessity of ongoing mass formation for a thermodynamic system to stay closed directly implies the growth of thermodynamic entropy within that system (Second Law). In fact, since the size of the system under consideration is always equal to its Schwarzschild radius, r_s, the amount of entropy in the system equals the Bekenstein-Hawking entropy, which is

$$S_{BH} = \pi k_B \left(\frac{r_s}{l_P}\right)^2, \quad (7)$$

where k_B is Boltzmann's constant and $l_P = \sqrt{G\hbar/c^3}$ denotes Planck's length.

Combining Eq. (7) with Eqs. (2) and (3) yields the expression for the entropy value

$$S(t) = \pi k_B \left(\frac{t}{t_P}\right)^2, \quad (8)$$

where $t_P = \sqrt{G\hbar/c^5}$ denotes Planck's time.

From Eq. (8), the rate of entropy generation within the expanding closed system with perpetual mass generation always remains positive and increases with time linearly [1], that is,

$$\frac{dS}{dt} = 2\pi k_B \frac{t}{t_P^2} > 0. \quad (9)$$

Equation (9) is essentially *a mathematical statement of the Second Law of Thermodynamics, which is not merely postulated here but is a direct consequence of the constant creation of mass* within the system under consideration.

The result rendered by Eq. (8) has been compared with the known entropy value of the observable Universe obtained independently by other methods [7]. Indeed, for the observable Universe, $t_o = 4.32 \times 10^{17}$s, Eq. (8) yields $S_o = 0.28 \times 10^{100}$J/K, which is in an excellent accord with the estimated dimensionless value $\sim 10^{123}$. Notice here that the dimensionless entropy value calculated in [7] $10^{123} \times k_B = 1.38 \times 10^{100}$J/K calculated in this study. Note also that the index 'o' refers to the observable (current) state.

From Eq. (8), it also follows that the temperature of the observable Universe decreases inversely proportional to the square root of time, because provided the initial temperature of the observable Universe was equal to Planck's temperature $T_P = \frac{1}{k_B}\sqrt{\frac{\hbar c^5}{G}} = 1.42 \times 10^{32}$K

$$\frac{T^4(t)}{T_P^4} \sim \frac{S(t_P)}{S(t)}. \quad (10)$$

The proportionality coefficient, necessary to convert expression (10) into an equality, is easily recovered from noticing that the gravitational (corpuscular) energy density of the observable Universe varies as

$$\varepsilon_m(t) = \frac{3}{8\pi G}\frac{1}{t^2}, \tag{11}$$

whereas the radiation energy density is

$$\varepsilon_r(t) = \frac{4\sigma}{c}T^4(t), \tag{12}$$

where σ is the Stefan-Boltzmann constant, and $\dot{E_r} = -\dot{E_m} = -\frac{c^5}{2G}$ must hold.

Then, the temperature evolution is given by a simple formula

$$T(t) = \frac{3}{16\pi}T_P\sqrt{\frac{t_P}{t}}. \tag{13}$$

Notice that, for the observable Universe, $t_o = 4.32 \times 10^{17}$s, Eq. (12) yields $T_o =$ 2.99 K, which is in an excellent accord with the observable temperature value of 2.725 K.

A possible reason of why the Universe is slightly cooler that what is predicted by Eq. (13) will be discussed in Section 5.

4. Conservation laws and the quantum state

The exercise presented in this section may shed some light on the fact why the temperature of the observable Universe is slightly lower than the value predicted by the model of the temperature evolution developed in the preceding section. In addition, the results of this section provide an argument in favour of the conjecture that the Second Law may not be complete.

Due to the fact that the observable Universe can be seen as a single non-relativistic particle of mass $m(t)$ in the potential field given by Eq. (6), the quantum state of the Universe is given by the Schrödinger equation

$$i\hbar\frac{\partial}{\partial t}\psi(\boldsymbol{r},t) = -\frac{\hbar^2}{2m(t)}\nabla^2\psi(\boldsymbol{r},t) + \psi(\boldsymbol{r},t)\frac{c^5 t}{2G}, \tag{14}$$

in which the potential function $\frac{c^5 t}{2G}$ is given by Eq. (6). In Eq. (14), the Schrödinger probability wave function $\psi(\boldsymbol{r},t)$ defines the quantum state of the Universe.

It is worth noting here that, in the classical interpretation of the Schrödinger equation, the spatial variable $\boldsymbol{r}$ is understood as "external" to the particle. In Eq. (14), however, the spatial variable must be treated as "internal", because the notion of "external" is not applicable in the case of a closed universe. The latter circumstance should not lead to any difficulty here, because the purpose of the following exercise is not to solve Eq. (14) explicitly, but only to demonstrate that the conservation laws within the Universe are independent of its gravitational potential.

Now, consider Eq. (14) written for the conjugate wave function, that is,

$$i\hbar\frac{\partial}{\partial t}\psi^*(\boldsymbol{r},t) = -\frac{\hbar^2}{2m(t)}\nabla^2\psi^*(\boldsymbol{r},t) + \psi^*(\boldsymbol{r},t)\frac{c^5 t}{2G}. \tag{15}$$

Multiplying Eq. (14) by ψ^* and Eq. (15) by ψ, and subtracting the resulting equations yields

$$i\hbar\left(\psi^*\frac{\partial\psi}{\partial t} + \psi\frac{\partial\psi^*}{\partial t}\right) = -\frac{\hbar^2}{2m(t)}(\psi^*\nabla^2\psi - \psi\nabla^2\psi *). \tag{16}$$

Equation (16) can be written in a compact form as

$$\frac{\partial p}{\partial t} + \nabla\cdot\mathbf{j} = 0, \tag{17}$$

where the probability density $p = \psi\psi^* = |\psi|^2$ and $\nabla\cdot\mathbf{j} = \frac{i\hbar}{2m(t)}(\psi^*\nabla^2\psi - \psi\nabla^2\psi *)$.

Equation (17) is nothing else but the most general form of the conservation law. Indeed, multiplying Equation (17) by various physical quantities yields the conservation equations corresponding to these quantities. For instance, multiplying Equation (17) by the electric charge of the particle produces the conservation equation of the electric charge, whereas multiplying Equation (17) by the particle density, ρ_m, produces the continuity equation of fluid mechanics. See the work by Landau and Lifshitz [9] for an alternate, more rigorous, derivation of Eq. (17).

Now, since the Einstein field equations

$$R_{\mu\nu} - \frac{1}{2}Rg_{\mu\nu} = \frac{8\pi G}{c^4}T_{\mu\nu} - \Lambda g_{\mu\nu}, \quad (18)$$

combined with Eq. (5) for the gravitational potential, assume the form

$$R_{\mu\nu} - \frac{1}{2}Rg_{\mu\nu} = \frac{8\pi G}{\Phi^2}T_{\mu\nu} - \Lambda g_{\mu\nu}, \quad (19)$$

it follows from Eq. (17) that the conservation laws within the expanding Universe are independent of both the stress-energy tensor, $T_{\mu\nu}$, and the space-time metrics, because Eq. (17) does not contain the gravitational potential Φ and it is the gravitational potential that the space-time characteristics.

Note that the value of the cosmological constant, Λ, related to the presence of dark energy is not discussed here. This is because, in order to reach all the results provided in this work, no dark energy assumptions are required.

From the mathematics point of view, Eq. (14) is nothing else but the diffusion (heat) equation with the imaginary diffusion coefficient, $D(t) = \frac{i\hbar}{2m(t)}$, which depends on time only. Upon introducing a new variable

$$\tau = \frac{i\hbar}{2}\int_{t_P}^{t}\frac{d\zeta}{m(\zeta)}, \quad (20)$$

Eq. (17) can be treated by the non-field method, which renders relations between the local values of the intensive properties (probability density, p, in this case) and the corresponding gradient (flux) of that intensive property (in this case, probability flux, $\mathbf{j}$) [10].

Noticing that $\nabla^2 = \frac{\partial^2}{\partial r} + \frac{\gamma}{r}\frac{\partial}{\partial r}$, where the parameter $0 \le \gamma \le 1$ characterises the domain geometry, the resulting non-field solution becomes

$$p(r,\tau) = p(r,0) + \frac{1}{\sqrt{\pi}}\int_0^{\tau}\frac{j(r,\zeta)d\zeta}{\sqrt{\tau-\zeta}} - \frac{\gamma}{r \pm r_s}\int_0^{\tau} j(r,\zeta)d\zeta, \qquad 0 \le r \le r_s, \quad (21)$$

where $\gamma = 0$ corresponds to the domain with the flat geometry (no curvature); $\gamma = \pm 1$ represents the spherical case with the convex and concave boundary, respectively; whereas $\gamma = \pm 1/2$ describes the cylinder whose boundary is either convex ($\gamma = 1/2$) or concave ($\gamma = -1/2$). The sign in front of r_s must be the same as the one of γ.

It is important to reiterate here that the non-field solution (21) relates the local values of the probability density and the corresponding probability flux. The solution is valid in all locations within the Universe including its boundary. Hence, neither the value of r nor its meaning in terms of "externality" or "internality" are unimportant. What really matters in the non-field solution is the temporal variable τ, which, by its definition given by Eq. (20), includes the gravitational effects.

As is easily seen from the non-field solution given by Eq. (21), the coefficient in front of the second integral on the right-hand side rapidly becomes negligibly small, because $r_s = ct$. This implies that, in almost every location $0 \le r \le r_s$, the expanding Universe flattens very fast, so that the solution behaves as if $\gamma = 0$.

It is important to note, however, that, according to Eq. (21), the space-time curvature must be considered for small values of τ, i.e., for typical time intervals on the order of t_P or in the vicinity of enormous masses. In all other respects, space-time can be considered flat.

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Finally, Eq. (21) also provides some clue, why the temperature of the observable Universe is slightly lower than the value predicted by the model of the temperature evolution developed in Section 3. Indeed, the difference can be explained by the presence of the second integral on the right-hand side of Eq. (21). If p represents the temperature (radiation energy density), then j in Eq. (21) corresponds to the heat flux. The minus sign in front of the integral indicates that the cooling rate of the Universe was slightly higher in the very beginning of its evolution. Hence, the resulting CMB temperature must be slightly smaller than the one predicted by Eq. (13).

5. Incompleteness of the Second Law: information and complexity

In Section 3, it has been shown that in an expanding closed thermodynamic system, the Second Law is a direct consequence of the perpetual growth of the total amount of gravitational mass. The rate of entropy generation increases with time linearly. The latter circumstance poses the question about the loss of information within the system [11].

On the other hand, in the preceding section, it has been shown that the quantum state of the Universe is uniquely determined by its initial state, $p(r,0)$ (see Eq. (21)). And, hence, information about the entire history of the Universe should be recoverable in principle.

Note also that, because the diffusivity in the Schrödinger equation, $D(t) = \frac{i\hbar}{2m(t)}$, is an imaginary value, the seemingly Gaussian kernel (Green's function of the diffusion equation), $\psi(r,t) \sim \exp\left[-\frac{r^2}{tD(t)}\right]$, in fact, produces non-decaying (actually time-independent, steady) solutions, $\psi(r,t) \sim \cos(r/l_P)^2 + i\sin(r/l_P)^2$, which indicates that the Universe must be a fully reversible thermodynamic process. This is another strong argument in favour of the fact that the Second Law is not complete and its effects on the Universe's evolution should be somehow compensated. In fact, there is some experimental evidence that the Second Law may not be complete [12-13].

The seeming contradiction described here is due to the fact that the Second Law is usually considered separately from the perpetual mass creation within the Universe. Actually, gravitational effects associated with the mass produced cannot be disregarded, because they play the role of an organising force, which provides the possibility for formation of more and more complex structures within the Universe. In fact, the more mass is created within the Universe, the more possibilities arise for formation of complex structures, for which mass corpuscles play the role of building blocks.

As has been shown in Sections 2 and 3, the processes of entropy generation and mass creation within the Universe go hand in hand. To reconcile the effects of these two processes, it is conjectured here that *the Second Law is not complete and is to be complemented to a conservation law* in the form

$$\frac{d(S+H)}{dt} = 0, \tag{22}$$

where S denotes the thermodynamic (Boltzmann's) entropy and H is the informational entropy understood in the same sense as Shannon's entropy [14].

Equation (22), once compared with Eq. (9), implies that, as the amount of the thermodynamic entropy within the Universe increases, the amount of the informational (Shannon's) entropy decreases as

$$\frac{dH}{dt} = -2\pi k_B \frac{t}{t_P^2}. \tag{23}$$

In other words, the amount of information, $I(t) = \Delta H(t) = H(t_P) - H(t) = S(t) - S(t_P)$, generated within the Universe in the course of time, is

$$I(t) = \pi k_B \left[\left(\frac{t}{t_P}\right)^2 - 1\right]. \tag{24}$$

Thus, from Eq. (8), the amount of information in the observable Universe, $t_o = 4.32 \times 10^{17}$s, should be $\sim 10^{123}$ bit.

Equation (24) implies that, in the course of its evolution, the Universe becomes more complex. Indeed, the older the Universe becomes, the longer texts (codes) are needed to describe all structures available within it – the said codes may, for example, use protons and electrons and, in general, mass generated within the Universe – as their "letters".

Indeed, if the Kolmogorov descriptive complexity, K, is used as a quantitative measure of the Universe's complexity, then

$$K(t) = \frac{m(t)}{m_P} = \frac{t}{t_P}, \tag{25}$$

where Planck's mass, m_P, is chosen as the description unit ("letter") to make the initial complexity value equal to one unit [15].

A simple comparison of Eqs. (24) and (25) shows that, within the Universe, the amount of accumulated information and complexity level of that information are not independent, but related as

$$K(t) = \sqrt{1 + \frac{I(t)}{\pi k_B}}. \tag{26}$$

To conclude this section, it is worth noting that, as can be seen, once the Second Law is complemented to a conservation law, the importance of mass generation within the Universe becomes well pronounced – not only gravitational effects play the role of an organising force, but also the amount of mass within the Universe defines both the amount of information within the Universe and the level of the Universe's complexity.

6. Discussion

For an expanding thermodynamic system, whose energy content consists of both radiation (waves) and mass (corpuscles), to remain closed at every moment of its evolution, no radiation may escape this system. Hence, the spatial size of such a system must equal its Schwarzschild radius at every time moment, or else that system cannot be treated as a fully closed thermodynamically. Because the spatial size of the system grows as ct due to the presence of the radiation (wave) component, the total gravitational mass of the system must increase with time, too. The latter is necessary to equate the size of the expanding system to its Schwarzschild radius.

In such an expanding closed thermodynamic system, the Second Law is a direct consequence of the perpetual growth of the total amount of gravitational mass.

In addition, it has been shown that, despite the perpetual growth of its gravitational mass, the geometry of the system flattens very rapidly, so that the space-time curvature is to be considered only either for characteristic time spans of the order of Planck's time or in the vicinity of huge masses.

To validate the model presented in this study, the initial (Planck's) and current (observable) states of our Universe were used as the points of reference. It has been shown that the amount of entropy within the observable Universe, predicted by the model, is close to the value estimated by another independent method.

In addition, the solution for the temperature evolution of the observable Universe has been obtained. It has been shown that the temperature change obeys a simple power law, namely: is inversely proportional to the square root of time. The temperature of the observable Universe was calculated from the solution thus obtained with the initial condition of Planck's temperature and compared with the current CMB temperature. The temperature, calculated in this study, is by 0.265 K larger than the observable CMB temperature. This difference can be explained by the presence of the second integral on the right hand side of Eq. (21). The minus sign in front of the integral indicates that the cooling rate of the Universe was slightly higher in the very beginning of its evolution. Hence, the resulting CMB temperature must be slightly smaller than the one predicted by Eq. (13).

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Furthermore, it has been shown that, although the rate of entropy generation within the Universe increases with time linearly, the quantum state of the Universe is uniquely determined by its initial state. The latter circumstance indicates that the Universe as a whole must be a fully reversible thermodynamical process, which is a strong argument in favour of the fact that the Second Law is not complete and should be complemented to a conservation law. The latter must be related to the question about the loss of information within the Universe [11].

Once the Second Law is complemented to a conservation law, the importance of mass generation within the Universe becomes well pronounced – not only gravitational effects play the role of an organising force, but also the amount of mass within the Universe defines both the amount of information within the Universe and the level of the Universe's complexity.

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