 doi:10.20944/preprints202209.0252.v1

Article

Fundamentally Good Alternative to Normal Distribution

Kabir Bindawa Abdullahi[1*]

[1]Department of Biology, Faculty of Natural and Applied Sciences, Umaru Musa Yar'adua University, P.M.B., 2218 Katsina, Katsina State, Nigeria.
* Correspondence: kabir.abdullahi@umyu.edu.ng; kabirnamallam@gmail.com; ORCID: 0000-0002-3810-9592

Abstract: Location-and-scale transformation of a random variable underpins normal distribution, but it is however fundamentally incorrect for scale estimation such as relative dispersion. In this paper, a parametrized alternative to a normal distribution, called scaloc-normal distribution, is proposed that efficiently works and is fundamentally correct with absolute and relative dispersion estimators. The Monte Carlos simulation experiment was used to generate a total of 600,000 artificial datasets in 600 different simulation scenarios from loc-normal (normal) and scaloc-normal distributions. The absolute and relative dispersion were estimated and compared from the two distributions. The results show that scaloc-normal distribution is a good parametrized alternative to loc-normal distribution, fundamentally correct and efficient with both standard deviation and coefficient of variation. The key statistical advancement from loc-normal to scaloc-normal distribution is its fundamental correctness (i.e., scale-invariant property) with an efficient relative estimator of dispersion (i.e., coefficient of variation). Parametrically, the loc-normal and scaloc-normal distributions are very different, but both have linear transformations.

Keywords: alternative parameterization; normal distribution; dispersion estimators; location-invariance; scale-invariance; scale-and-location-invariance

1.0. Introduction

In statistics, a normal distribution (called symmetric or Gaussian distribution) belongs to a category of continues probability distribution of a real-valued, independent and identically distributed random variable. The probability density function which defines a normal distribution is parameterized by μ (equally represents the mean, median and the mode of the distribution) and σ^2 (represents the variance of the distribution) or σ (represents the standard deviation of the distribution) with a support x from a random variable [**1**]; [**2**].

Normal distributions are very important aspect of statistics, and are used especially in social and natural sciences to represent and describe the unknown distribution of real-valued, independent and identically distributed random variables. The principle of central limit theorem partly makes a normal distributions more uniquely attractive and important especially in analytic studies [**1**]; [**2**].

The mean, median and mode are the unbiased, efficient and consistent estimators of location describing a normal distribution, while variance and standard deviation are its unbiased, efficient and consistent estimators of dispersion. Another good but less utilized estimator of a normal distribution is the coefficient of variation, described as a unitless, dimensionless and standardized estimates defined by the mean and standard deviation of a normal distribution. In terms of the estimates of the estimators, variance and standard deviation are the absolute measures of dispersion characterized as location-invariant estimators, while coefficient of variation is a relative measure of dispersion characterized as scale-invariant estimator [**1**]; [**2**].

Parametrically, the additivity and scaling transformation of the mu (μ) and the variance/sigma (σ^2, σ) respectively on a random variable underpins the concept of a normal distribution. It belongs to the family of scale-and-location transformed distribution of a

Preprints (www.preprints.org) | NOT PEER-REVIEWED | Posted: 19 September 2022 doi:10.20944/preprints202209.0252.v1

continues random variables [1]; [2]. But the resultant invariance properties (i.e., location-invariant, scale-invariant or scale-and-location invariant) of its good estimators are not completely and clearly defined by the normal distribution. This is a big fundamental problem or limitation. Therefore, a better distribution based on additivity and scaling transformation of a random variable (e.g., normal distribution) should be characterized by all the invariances.

In literatures, there have been decades-old of numerous characterization, evaluation and performance studies on dispersion estimators (from the efficient as well as the robust categories of good estimators) under normal and other distributions such as the studies by [3]; [4]; [5]; [6]; [7]; [8]; [9]; [10]; [11]; [12]; [13]; [14]. Despite that, little or no attention was given to the alternative parameterization of a normal distribution so that the additivity and scaling transformations on an independent and identically distributed random variable gives to a distribution good (i.e., fundamentally correct) with any location-invariant, scale-invariant, and scale-and-location-invariant estimator. A very interesting attempt was the birth of logistic distribution, which emphasized on outliers' robustness than invariance properties [15]. The main aim of this paper is to propose parametrized alternative to a normal distribution and check its goodness and fundamental correctness with good estimators.

2.0. Background and Preliminaries

2.1. Loc-normal distribution $[\mathcal{N}^L(\mu = mu,\ \sigma = sigma)]$

. Loc-normal distribution belongs to the family of scale-and-location transformed distribution of a continues random variables [1]; [2]. The two important transformation parameters and the domain (support) are the location shift μ, sigma σ, and random variable x respectively. The resultant graph of the distribution is a linear transformation.

Supposed we have a σ scaling and μ location shift of an independent and identically distributed random variable $x = (x_1, x_2, x_3, \dots, x_n)$

$$\mathcal{N}^L = x\sigma + \mu \quad \text{(Eq.1)}$$

Let c be the coefficient of variation

If $c = \frac{\sigma}{\mu}$; then $\mu = \frac{\sigma}{c}$ and $\sigma = c\mu$

where $\mathcal{N}^L$ for loc-normal; $x, \mu \in \mathbb{R}$; $\sigma \in \mathbb{R}_{>0}$; $n \in \mathbb{N}$; σ for sigma (a scale parameter); μ for mu (a location parameter).

The random variable x has a normal distribution with a probability density function (pdf) expressed in equation 2.

$$f(x) = \frac{1}{\sqrt{2\pi\sigma}}\, exp\left\{-\frac{1}{2}\left(\frac{x-\mu}{\sigma}\right)^2\right\}, for -\infty < x < \infty. \quad \text{(Eq.2)}$$

The parameter μ and σ^2 are the mean and variance of x, respectively.

2.2. Characteristics of estimators under loc-normal distribution

I. Normal distribution belongs to the family of scale-and-location transformed distribution of a continues random variables [2]. One of the most important properties of the normal distribution is its additivity under independence [1].

II. It is a symmetrical distribution and has a zero skewness and kurtosis of its spread estimators. These estimators are very dependent on the sample size of the distribution of random variables [1]; [2].

III. The good (i.e., unbiased, efficient and consistent) estimators of location (location measures) such as the mean, median and mode are the same. These estimators, except for the median, are sensitive to outliers or contaminations and insufficient to describe a distribution because two random variables or distributions with different dispersion may have the same location estimates [13]; [14].

Preprints (www.preprints.org) | NOT PEER-REVIEWED | Posted: 19 September 2022 doi:10.20944/preprints202209.0252.v1

IV. The good estimators of dispersion (dispersion measures) such as the variance or standard deviation and coefficient variation are very related and important parameters. These estimators are sensitive to outliers or contaminations and insufficient to describe distributions because they are either location-invariant or scale-invariant and lack scale-and-location invariant property [**13**]; [**14**]. The estimates of the relative estimator of dispersion (i.e., coefficient of variation) lacks a bounded range and approaches infinity as the mean of the distribution tends to zero.

2.3. Structures of the Estimates of Location and Dispersion Estimators Under Loc-Normal Distribution

If for every location shift μ is freezes (fixes) and a set of sigma σ are rotated, then the coefficient of variation c adjusts and varies relatively (i.e., relative to the changes in μ and/or σ) (See **Fig. 3a**). If for every sigma σ is freezes and a set of location shift μ are rotated, then the mean c adjusts and varies relatively (i.e., relative to the changes in μ and/or σ) (See **Fig. 3a**).

2.4. Python code for loc-normal distribution

Here is a written python code for random number generation under loc-normal distribution using Monte Carlos simulation.

```
def locnormal_simulation(mu, sigma, sample_size, iteration):
    random.seed(111)
    multiple_list = [ ]
    for seed in np.random.randn(iteration):
        random.seed(seed)
        multiple_list.append(np.random.randn(sample_size)*sigma+mu)
    simulation = [randn for randn in (multiple_list)]
    return (simulation)
```

3.0. Propose Distribution

3.1. Scaloc-normal Distribution $[\mathcal{N}^{SL}(\nu = nu,\ \sigma = sigma)]$

Scaloc-normal distribution is a member of the family of scale-and-location transformed distribution of a continues random variables [**1**]; [**2**]. The two important transformation parameters and the domain (support) are the scaloc (i.e., scale-and-location) shift ν, sigma σ, and random variable x respectively. The resultant graph of the distribution is a linear transformation.

Supposed we have a σ scaling and ν scaloc (i.e., scale-and-location) shift of an independent and identically distributed random variable $y = (y_1, y_2, y_3, \dots, y_n)$.

$$\mathcal{N}^{SL} = \sigma(y + \nu) \quad \text{(Eq.3)}$$

where $\mathcal{N}^{SL}$ for scaloc-normal; $y, \nu \in \mathbb{R}$; $\sigma \in \mathbb{R}_{>0}$; $n \in \mathbb{N}$; σ for sigma (a scale parameter); ν for nu (a scaloc shift); $\mu = \nu\sigma$; μ is the location shift of a location parameter.

Let c be the coefficient of variation

If $\mu = \nu\sigma$ and $c = \frac{\sigma}{\mu}$

then $\nu = \frac{\mu}{\sigma}$; $\nu = \frac{1}{c}$; $c = \frac{1}{\nu}$; $\sigma = \frac{\mu}{\nu}$; $\sigma = c\mu$; $\mu = \frac{\sigma}{c}$

The random variable y has a normal distribution with a probability density function (pdf) expressed in equation 4.

$$f(y) = \frac{1}{\sqrt{2\pi\sigma}}\, exp\left\{-\frac{1}{2}\left(\frac{y - \nu\sigma}{\sigma}\right)^2\right\}, for -\infty < y < \infty. \quad \text{(Eq.4)}$$

The parameter $\nu\sigma = \mu$ and σ^2 are the mean and variance of y, respectively.

Preprints (www.preprints.org) | NOT PEER-REVIEWED | Posted: 19 September 2022 doi:10.20944/preprints202209.0252.v1

3.2. Structures of the Estimates of Location and Dispersion Estimators Under Scaloc-Normal Distribution

If for every scaloc shift ν is freezes (fixes) and a set of sigma σ are rotated, then the mean μ adjusts and varies relatively (i.e., relative to the changes in ν and/or σ) (See **Fig. 3b**). If for every sigma σ is freezes and a set of scaloc shift ν are rotated, then the mean μ adjusts and varies relatively (i.e., relative to the changes in ν and/or σ) (See **Fig. 3b**).

3.3. Python code for scaloc-normal distribution

Here is a written python code for random number generation under scaloc-normal distribution using Monte Carlos simulation.

```
def scalocnormal_simulation(nu, sigma, sample_size, iteration):
    random.seed(111)
    multiple_list = [ ]
    for seed in np.random.randn(iteration):
        random.seed(seed)
        multiple_list.append((np.random.randn(sample_size)+nu)*sigma)
    simulation = [randn for randn in (multiple_list)]
    return (simulation)
```

4.0. Methods and Simulations

Monte Carlos simulation experiment was used to generate artificial datasets for comparison of the loc-normal and scaloc-normal distributions. For each distribution, a total of 300,000 datasets in 300 different simulations using a matrix combinations of parameters were generated. The matrix combinations was guided in two scenarios: (i) $\mathcal{N}^{L}(\mu = mu, \sigma = sigma)$ or $\mathcal{N}^{SL}(\nu = nu, \sigma = sigma)$ (ii) $\mathcal{N}^{L}(\sigma = sigma, \mu = mu)$ or $\mathcal{N}^{SL}(\sigma = sigma, \nu = nu)$. The location estimators (i.e., mean, median and mode) were evaluated with the first simulation, and the dispersion estimators (i.e., standard deviation and coefficient of variation) were evaluated with the second simulation (See **Fig. 1**). For the second simulation, any mixed data point (mixed of positive and negative data points) of data points were avoided by carefully choosing the parameters such that the location parameters are relatively greater than the scale parameters for all the possible matrix combinations. The efficiency (refer to **Eq. 6**) and relative efficiency (refer to **Eq. 7**) were evaluated to check how relatively good are the dispersion estimators in the two distributions. A Bland-Altman plot was use to check the level of agreement between the loc-normal and scaloc-normal distributions in terms of skewness and kurtosis estimates for all the different same size.

For each estimates (of mean, median, mode, standard deviation, and coefficient of variation), the average or efficiency of the 1000 iterations were used in the results presentations. Python Jupyter Notebook was used to write the codes for the data generation and its in-build or imported libraries were used for the analysis.

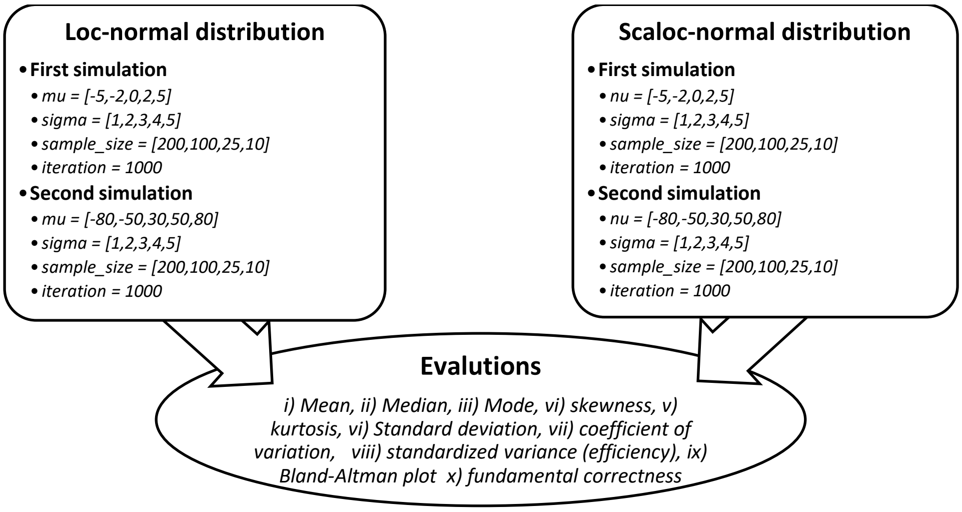


Fig. 1: Simulations parameters for random numbers generation under loc-normal and scaloc-normal distributions.

4.1. Evaluation of location, spread, dispersion and other estimators

The arithmetic mean, median, mode, skewness, kurtosis, and standard deviation were evaluated using the python in-build and imported functions. The coefficient of variation was evaluated using **Eq. 5** which was defined as a code in the python. The efficiency and relative efficiency were evaluated using a standardized variance as expressed in **Eq. 6** and **Eq. 7**.

$$Coefficient\ of\ variation\ (CV) = \frac{\sigma}{\mu} \quad \text{(Eq.5)}$$

where σ is the sigma, and μ is the arithmetic mean.

$$Efficiency\ (E) = \frac{1}{S}\sum_{i=1}^{S}\left(\frac{\sigma^2}{\mu^2}\right) \quad \text{(Eq.6)}$$

$$Relative\ efficiency\ (RE) = \frac{E_{cv}}{E_{std}} \quad \text{(Eq.7)}$$

where σ^2 is the variance of the estimates of the estimator; μ is the mean of the estimates of the estimator; $i = 1, 2, 3, \dots, S$; S was the iterations (size of the Monte Carlos simulation). If: $RE = 1$, coefficient of variation is equally efficient than standard deviation; $RE < 1$, coefficient of variation is more efficient than standard deviation; $RE > 1$, coefficient of variation is less efficient than standard deviation.

4.2. Evaluation of fundamental correctness

The fundamental correctness of the estimators under the loc-normal and scaloc-normal distribution was evaluated as fundamentally correct if the structure of the estimates of the estimator satisfies at least one of the conditions of invariances (i.e., location-invariance, scale-invariance, scale-and-location-invariance.

Let a random variable $x = (x_1, x_2, x_3, \dots, x_n)$

i. Supposed we have an a scaling of a random variable x.

Estimator E is scale-invariant (i.e., robust to scale, unitless and dimensionless) if:

$$E(x_1, x_2, x_3, \dots, x_n) = E(ax_1, ax_2, ax_3, \dots, ax_n)$$

where $x \in \mathbb{R}$; $a \in \mathbb{N}$

ii. Supposed we have a b location shift of a random variable x.

Estimator E is a location-invariant if:

$$E(x_1, x_2, x_3, \ldots, x_n) = E(x_1 + b, x_2 + b, x_3 + b, \ldots, x_n + b)$$

where $x, b \in \mathbb{R}$

iii. Supposed we have an a scaling and b location shift or scaloc shift of a random variable x.

Estimator E is scale-and-location invariant if:

$$E(x_1, x_2, x_3, \ldots, x_n) = E(ax_1 + b, ax_2 + b, ax_3 + b, \ldots, ax_n + b)$$

$$E(x_1, x_2, x_3, \ldots, x_n) = E(ax_1 + ab, ax_2 + ab, ax_3 + ab, \ldots, ax_n + ab)$$

where $x, b \in \mathbb{R}$; $a \in \mathbb{N}$

5.0. Results

5.1. Location and dispersion estimators under loc-normal and scaloc-normal distributions

The estimates location estimators (i.e., mean, median, and mode) under loc-normal and scaloc-normal distribution were evaluated with the first simulation parameters described in **Fig. 1**. The results indicate that under scaloc-normal distribution; the mean, median and mode are basically the same both in the estimates and outlooks (See **Fig. 2**). This important property of scaloc-normal distribution is analogous to that of loc-normal distribution.

The estimates dispersion estimators (i.e., standard deviation and coefficient of variation) under loc-normal and scaloc-normal distributions were evaluated with the second simulation parameters described in **Fig. 1**. The results by estimators' comparison indicates that under scaloc-normal distribution, the estimates of the standard deviation and coefficient of variation operate the location-invariance and scale-invariance property respectively (See **Fig. 3a & b**). Furthermore, the two distinct scale-invariance and location-invariance property are clearly expressed with the standard deviation and coefficient of variation respectively (See **Fig. 3a & b**). However, the results the dispersion estimators by estimators' comparison indicates that under loc-normal distribution, the estimates of the standard deviation operate the location-invariance and thus the scale-variance property, while the estimates of the coefficient of variation fails to operate neither the scale-invariance nor location-invariance property (See **Fig. 3a & b**).

Preprints (www.preprints.org) | NOT PEER-REVIEWED | Posted: 19 September 2022 doi:10.20944/preprints202209.0252.v1

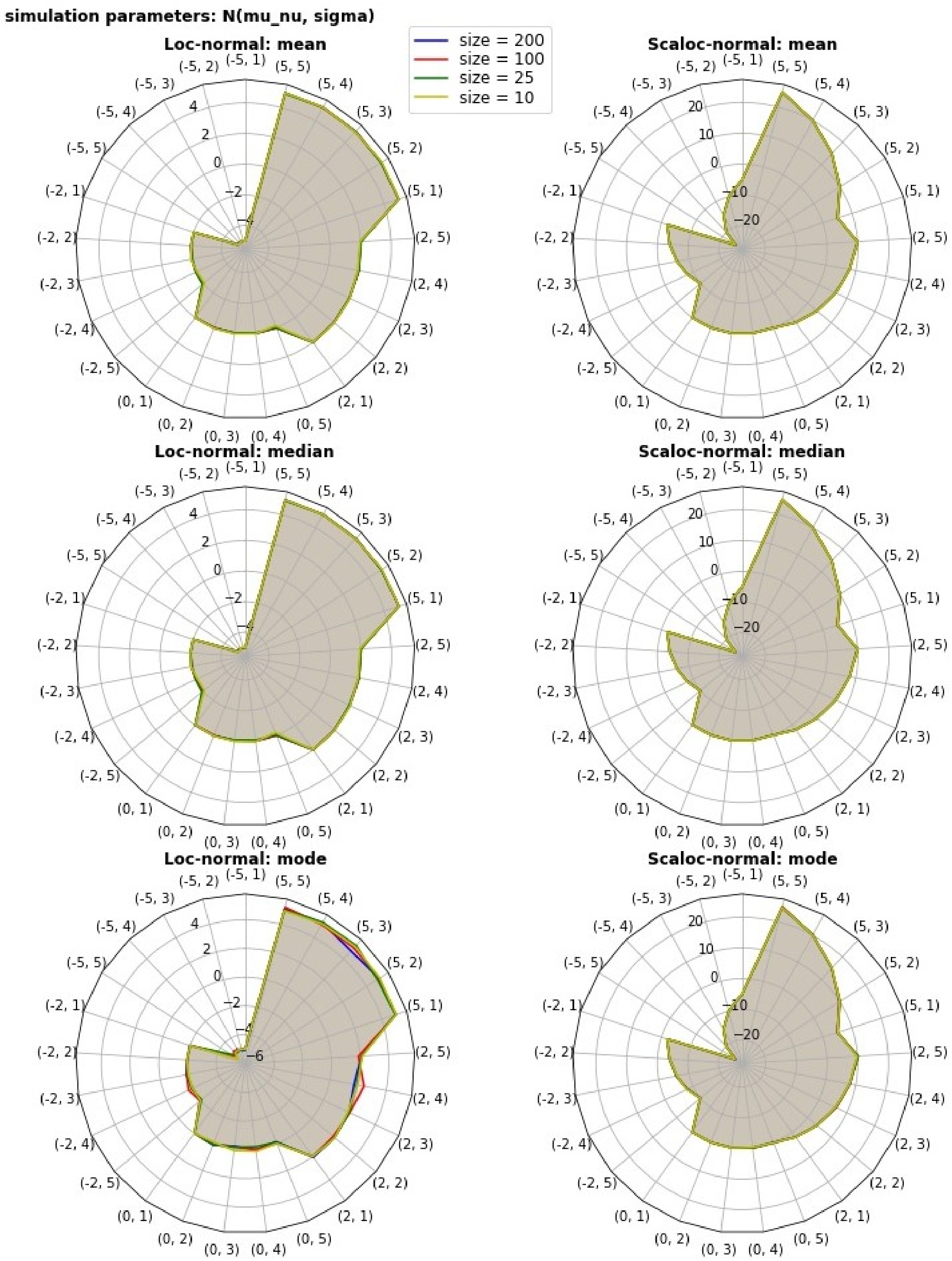


Fig. 2: Location estimators (i.e., mean, median, and mode) under loc-normal and scaloc-normal distributions.

Preprints (www.preprints.org) | NOT PEER-REVIEWED | Posted: 19 September 2022 doi:10.20944/preprints202209.0252.v1

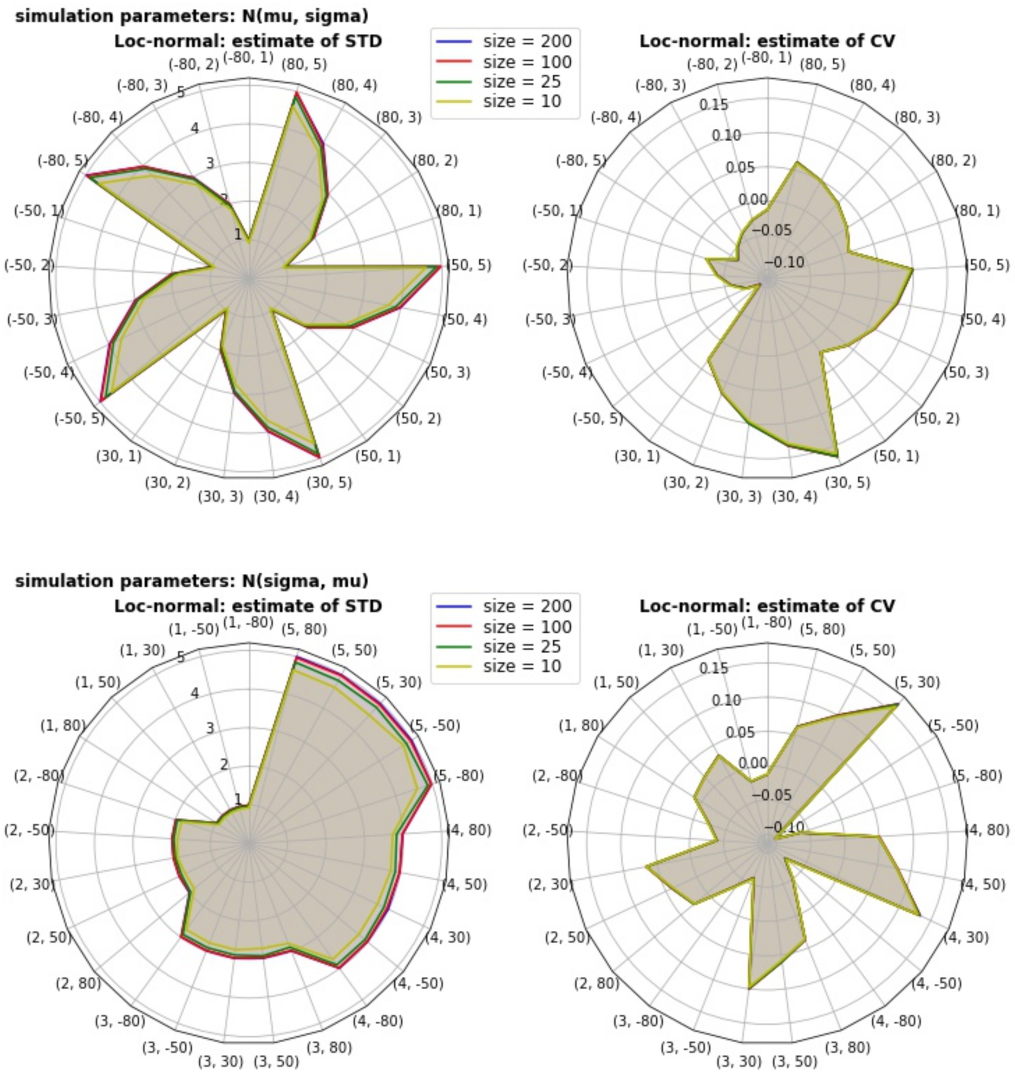
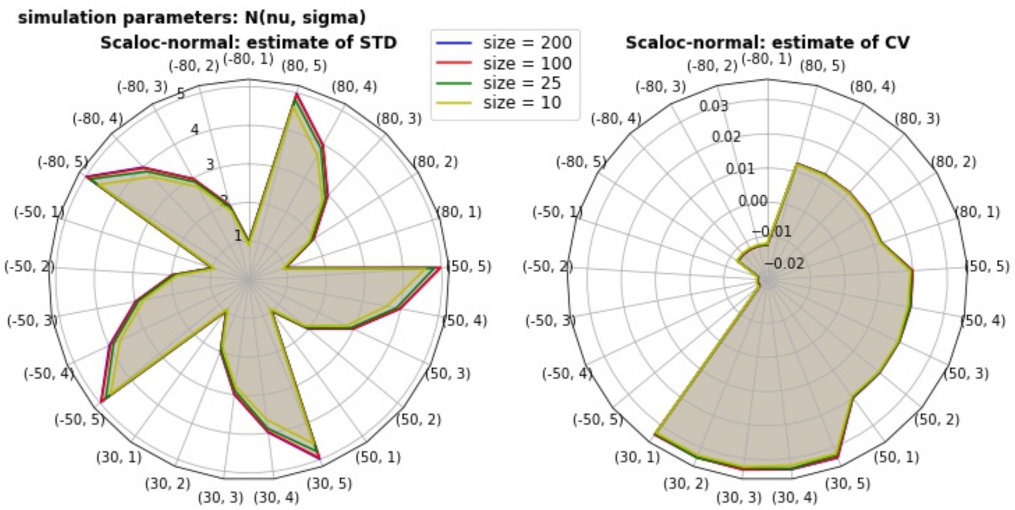


Fig. 3a: Dispersion estimators (i.e., standard deviation and coefficient of variation) under loc-normal distribution.



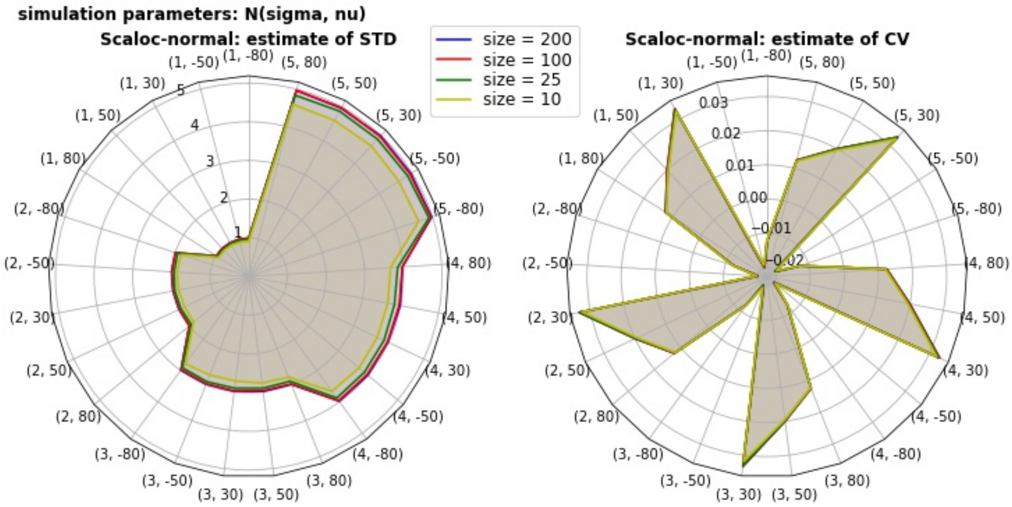


Fig. 3b: Dispersion estimators (i.e., standard deviation and coefficient of variation) under scaloc-normal distribution.

5.2. Shape estimators under loc-normal and scaloc-normal distributions

The estimates shape estimators (i.e., skewness and kurtosis) under loc-normal and scaloc-normal distributions were evaluated with the first simulation parameters described in **Fig. 1**. The results by estimates' comparison indicate that the skewness and kurtosis under loc-normal and scaloc-normal distributions are generally low (< 1.0) and also below the range marks suggested by [**16**]; [**17**]; [**18**]; [**19**]. (See **Fig. 4**). Bland-Altman plots of the skewness and kurtosis indicates very good agreement between the loc-normal and scaloc-normal distributions' shapes (See **Fig. 5a & b**).

Preprints (www.preprints.org) | NOT PEER-REVIEWED | Posted: 19 September 2022 doi:10.20944/preprints202209.0252.v1

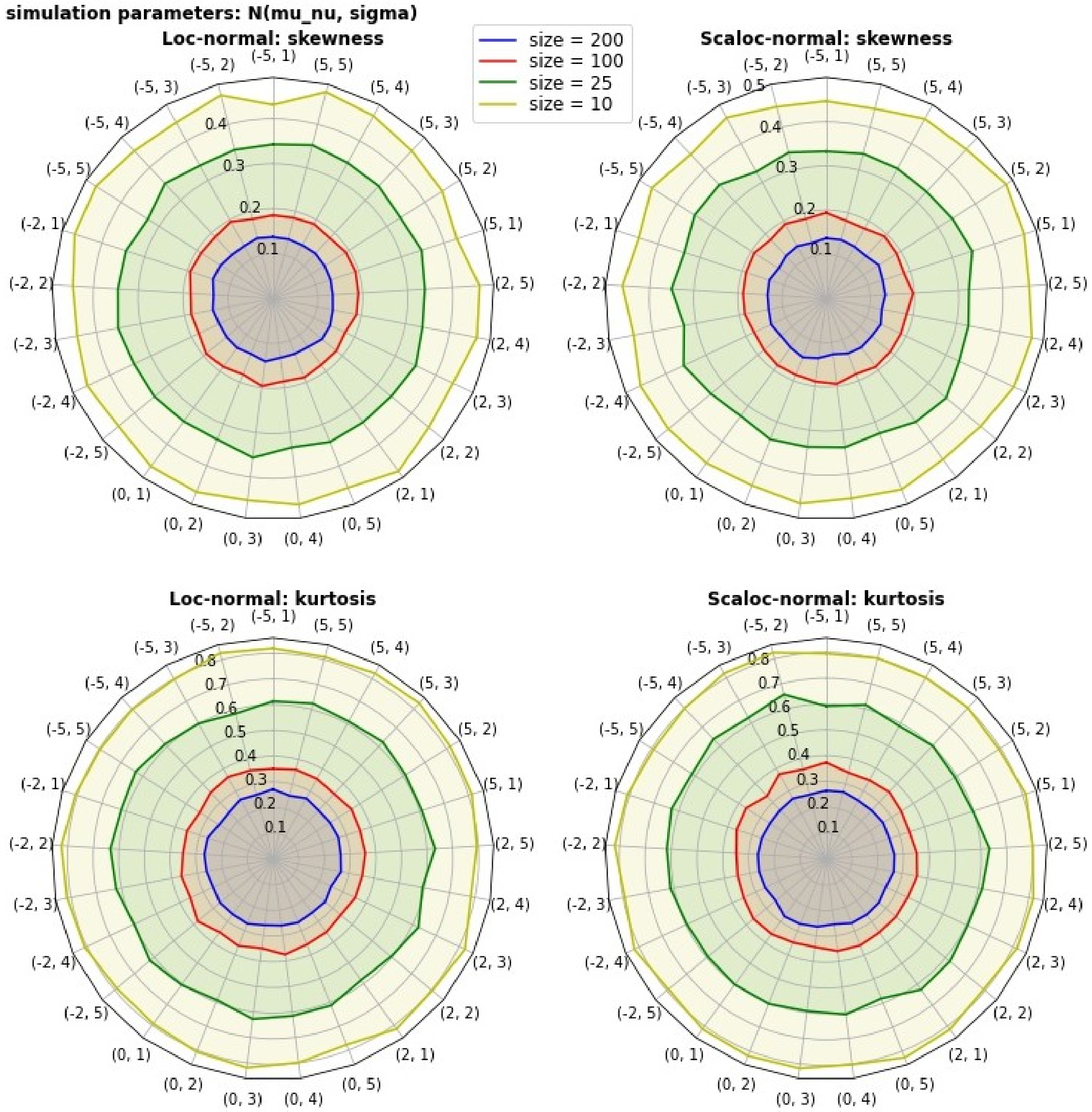


Fig. 4: Estimates of shape estimators (i.e., skewness and kurtosis) under loc-normal and scaloc-normal distributions.

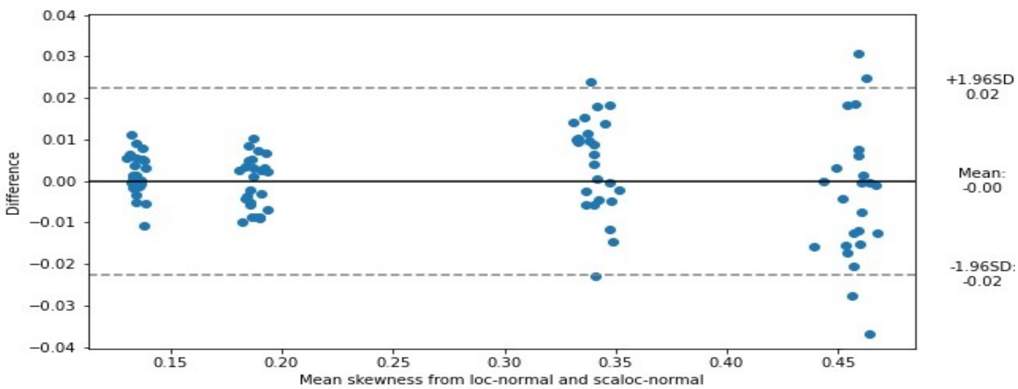


Fig. 5a: Level of agreement (Bland-Altman plots) between the estimates of skewness under loc-normal and scaloc-normal distributions.

Preprints (www.preprints.org) | NOT PEER-REVIEWED | Posted: 19 September 2022 doi:10.20944/preprints202209.0252.v1

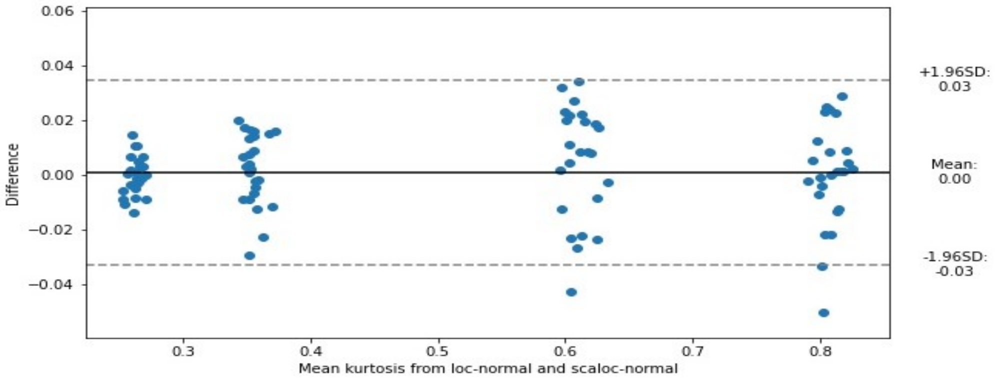
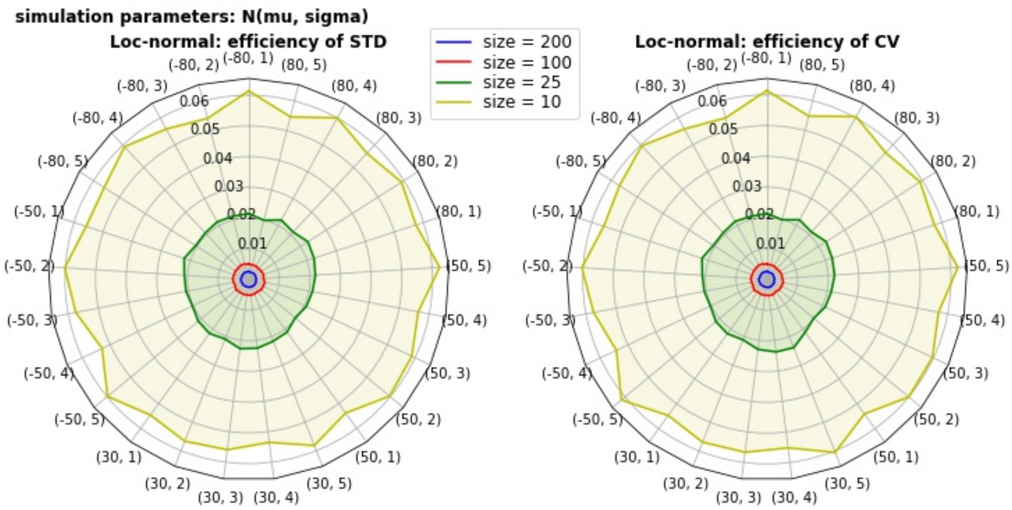


Fig. 5b: Level of agreement (Bland-Altman plots) between the estimates of kurtosis under loc-normal and scaloc-normal distributions.

5.3. Efficiency of the estimators under loc-normal and scaloc-normal distributions

The efficiency of the estimates of dispersion estimators (i.e., coefficient of variation and standard deviation) under loc-normal and scaloc-normal distributions were evaluated with the second simulation parameters described in (See **Fig. 1)**. The efficiencies of the standard deviation and coefficient of variation ranges from 0.002 to 0.06 and varies with the sample size under both the loc-normal (See **Fig. 6a)** and scaloc-normal (See **Fig. 6b)** distributions. The efficiencies are asymptotically the same under the both distributions (See **Fig. 6a & b)**.

The relative efficiency (of coefficient of variation against standard deviation) under loc-normal and scaloc-normal distributions were evaluated with the second simulation parameters described in **Fig. 1**. The results by estimates' comparison indicate that the standard deviation and coefficient of variation are almost equally efficient with estimates approximately 1.00 (See **Fig. 7**).



Preprints (www.preprints.org) | NOT PEER-REVIEWED | Posted: 19 September 2022 doi:10.20944/preprints202209.0252.v1

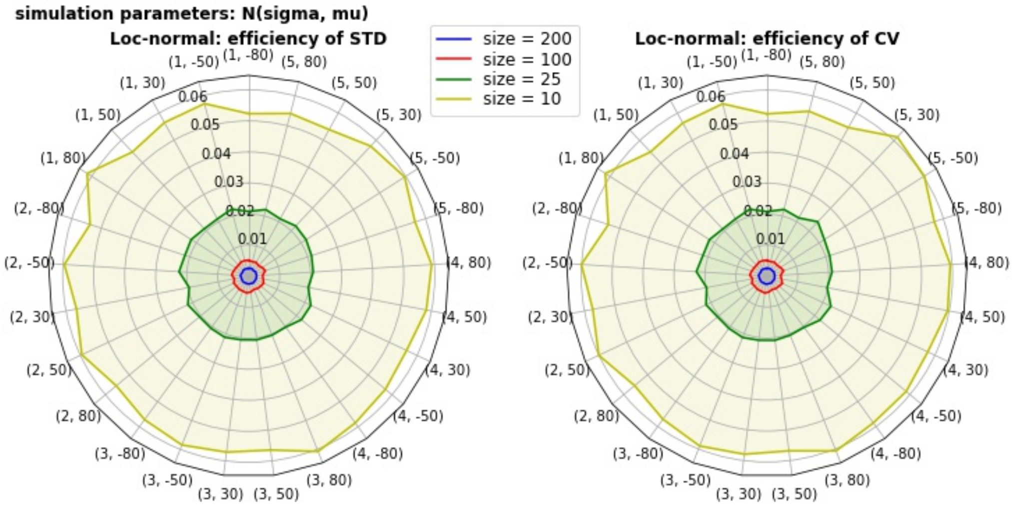


Fig. 6a: Efficiency of the estimates of dispersion estimators (i.e., coefficient of variation and standard deviation) under loc-normal distribution.

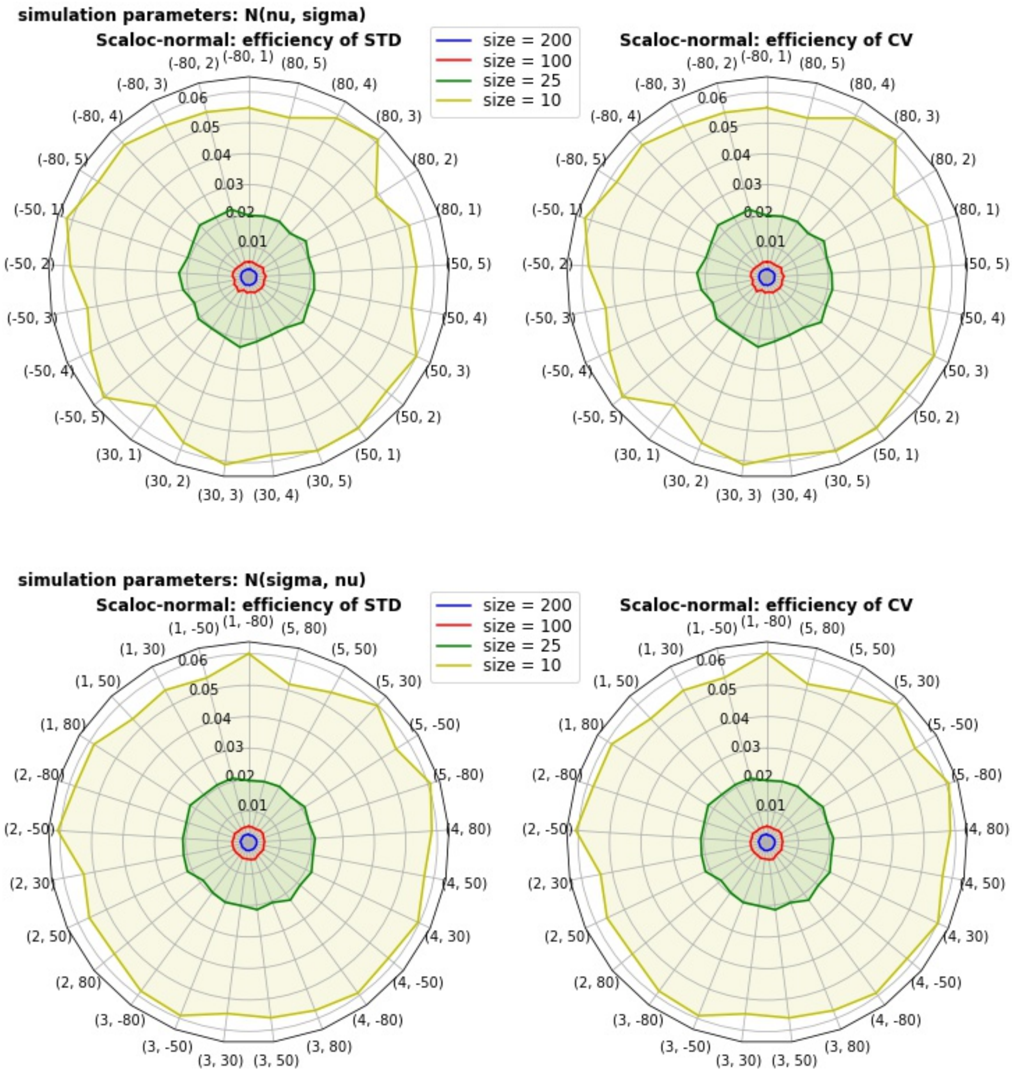


Fig. 6b: Efficiency of the estimates of dispersion estimators (i.e., coefficient of variation and standard deviation) under scaloc-normal distribution.

Preprints (www.preprints.org) | NOT PEER-REVIEWED | Posted: 19 September 2022 doi:10.20944/preprints202209.0252.v1

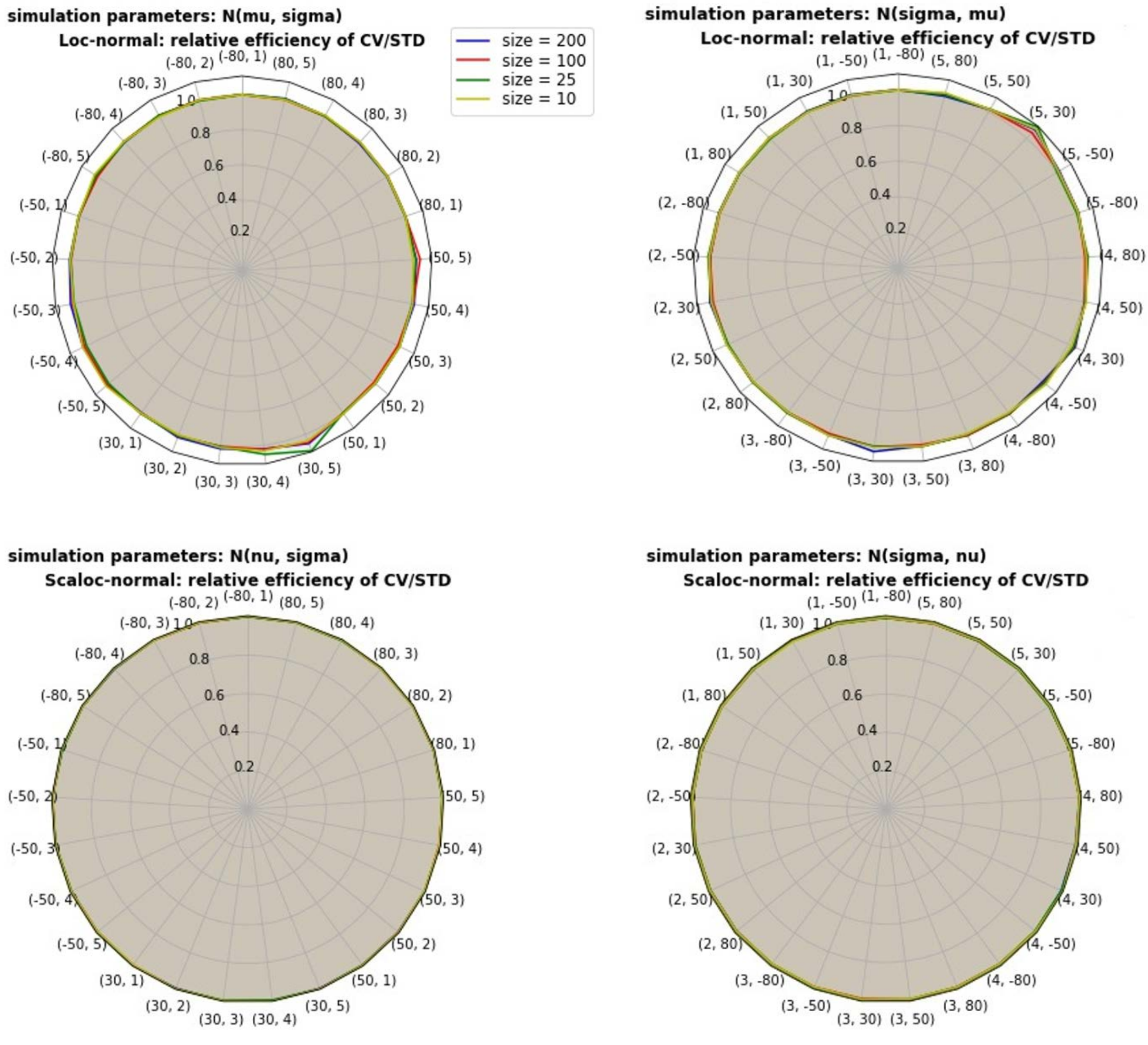


Fig. 7: Relative efficiency (of coefficient of variation against standard deviation) under loc-normal and scaloc-normal distributions.

5.4. Probability density function under loc-normal and scaloc-normal distributions

The standardized loc-normal and scaloc-normal distributions were plotted using the following standard parameters: $\mathcal{N}^L(mu = 0,\ sigma = 1)$ for loc-normal, and $\mathcal{N}^{SL}(nu = 0,\ sigma = 1)$ for scaloc-normal distributions. The probability density function of the loc-normal distribution (See **Eq. 2**) and scaloc-normal distribution (See **Eq. 4**) were used to demonstrate and compare the density fitness of the loc-normal and scaloc-normal distributions. The results by graphical comparison indicate that the probability density functions under loc-normal and scaloc-normal distributions basically fits each model and look the same (See **Fig. 8**).

Preprints (www.preprints.org) | NOT PEER-REVIEWED | Posted: 19 September 2022 doi:10.20944/preprints202209.0252.v1

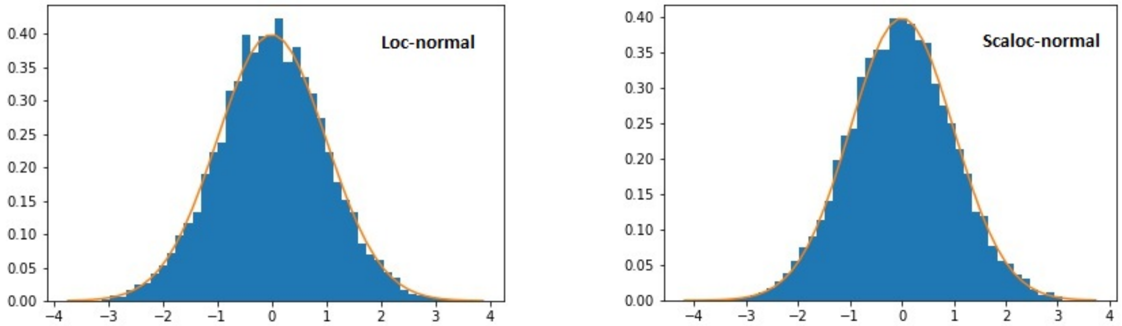


Fig 8: Standardized loc-normal and scaloc-normal distributions.

6.0. Discussion

The aim of this study was to propose a parametrized version of a statistical normal distribution that is fundamentally correct and efficient with both the absolute and relative estimator of dispersion (i.e., standard deviation and coefficient of variation respectively). The results indicated that, overall, the proposed distribution (scaloc-normal distribution) is fundamentally correct and efficient distribution with both the standard deviation and coefficient of variation, unlike the loc-nornal (normal) distribution which is only fundamentally correct and efficient distribution with the standard deviation.

Based on the parameters setting, the loc-normal (See **Eq. 1**) and scaloc-normal (See **Eq. 3**) distributions are very different, but both have linear transformation. The parametrized difference is the location shift for loc-normal and scaloc shift for scaloc-normal distributions. The additivity and scaling of the parameters on random variables make loc-normal and scaloc-normal distributions members of scaloc-transformed (i.e., a scale-and-location-transformed) distributions [**1**]; [**2**].

As presented in **Fig. 2**, the mean, median, mode, skewness and kurtosis are basically the same in both the loc-normal and scaloc-normal distributions. Based on these results, it is provided that the location estimators are the same functionally and structurally. Henceforth, the probability density function under both the distributions follow the same structure and principle (See **Fig. 8**). Therefore, the proposed distribution (i.e., scaloc-normal distribution) is a parametrized alternative to a normal distribution.

As presented in **Fig. 3b**, **Fig. 6b** and **Fig. 7** the estimates of standard deviation and coefficient of variation are fundamentally correct (i.e., operate the location-invariance, and scale-invariance properties respectively) and efficiently suitable with scaloc-normal distribution. But with loc-normal distribution, only the standard deviation is fundamentally correct (i.e., operate the location-invariance property) and efficiently suitable (See **Fig. 3a**, **Fig. 6a** and **Fig. 7**). This result is clear for what has been known that the additivity and location-invariance of a loc-normal distribution are its most important properties [**1**]. The key statistical advantage of scaloc-normal over loc-normal distribution is its goodness with both standard deviation and coefficient of variation.

Recommendations

Following the results of this paper, the following recommendations were provided in relation to which distribution suit which estimators to be used.

I. Loc-normal distribution is suitable and efficient for the mean, median, mode, and standard deviation.

II. Scaloc-normal distribution is suitable and efficient for the mean, median, mode, standard deviation, and coefficient of variation.

Supplementary material: The supplementary materials are python files consist of written in python codes.

Conflict of interest: The author declares no conflict of interest.

Funding: This research did not receive any specific grant from funding agencies in the public, commercial, or not-for-profit sectors.

References

[1] Hogg V. Robert., McKean W. Joseph., & Craig T. Allen "*Introduction to Mathematical Statistics.*" Eighth Edition, Boston: Pearson, USA, (2019).
[2] Takeuchi K. "*Some Theorems on Invariant Estimators of location. In: Contribution on Theory of Mathematical Statistics.*" Springer, Tokyo, (2020). doi.org/10.1007/978-4-431-55239-0_3
[3] Walter A. Hendricks, Kate W. Robey "The Sampling Distribution of the Coefficient of Variation." *Ann. Math. Statist.* 7(3): 129-132 (September, 1936). DOI: 10.1214/aoms/1177732503
[4] Kac M. "On the Characteristic Functions of the Distributions of Estimates of Various Deviations in Samples from a Normal Population." *Ann. Math. Statist*. 19 (2) 257 - 261, (June, 1948). doi.org/10.1214/aoms/1177730250
[5] Sumiyasu Yamamoto "On the estimation of the coefficient of variation by the ratio of two quantities in large samples," Kodai Mathematical Seminar Reports, *Kodai Math. Sem. Rep.* 4(1): 115-122 (1952). DOI: 10.2996/kmj/1138843232
[6] Dixon W. J. "Estimates of the Mean and Standard Deviation of a Normal Population." *Ann. Math. Statist*. 28 (3) 806 - 809, (September, 1957). doi.org/10.1214/aoms/1177706898
[7] Smithson M. "On relative dispersion: A new solution for some old problems". *Quality and Quantity* 16: 261-271, (1982).
[8] Sarhan A. E. "Estimation of the Mean and Standard Deviation by Order Statistics. Part III." *Ann. Math. Statist.* 26 (4) 576 - 592, (December, 1955). doi.org/10.1214/aoms/1177728418
[9] Dennis D. Boos & Cavell Brownie. "Comparing Variances and Other Measures of Dispersion." *Statist. Sci.* 19 (4) 571-578, (November 2004). doi.org/10.1214/088342304000000503
[10] Florence Newberger, Alan M. Safer, Saleem Watson. "What is "Standard" About the Standard Deviation." Missouri J. *Math. Sci*. 22 (2) 86 - 90, (May, 2010).doi.org/10.35834/mjms/1312233137
[11] Ospina R & Marmolejo-Ramos F. "Performance of Some Estimators of Relative Variability." *Front. Appl. Math. Stat.* 5:43, (2019) doi: 10.3389/fams.2019.00043
[12] Arthur G. B, Kevin W. M. "On the use of coefficient of variation as a measure of diversity." *Organizational Research Methods* 3:285-297, (2020).
[13] Wada K. "Outliers in official statistics. " *Japanese Journal of Statistics and Data Science* 3:669-691, (2020). doi: 10.1007/s42081-020-00091-y
[14] Tsamatsoulis D. "Comparing the Robustness of Statistical Estimators of Proficiency Testing Schemes for a Limited Number of Participants." *Computation* 10, 44, (2022), doi.org/10.3390/computation10030044
[15] Victor Chew. "Some Useful Alternatives to the Normal Distribution." *The American Statistician* 22 (3) 22-24, (1968). doi.org/10.1080/00031305.1968.10480473
[16] Curran P. J., West S. G., & Finch J. F. "The robustness of test statistics to nonnormality and specification error in confirmatory factor analysis." *Psychological Methods* 1 (1), 16-29. (1996).
[17] George D. & Mallery M. "*SPSS for Windows Step by Step: A Simple Guide and Reference.*" 17.0 update (10a ed.) Boston: Pearson. (2010).
[18] Byrne B. M. "*Structural equation modeling with AMOS: Basic concepts, applications, and programming*." New York: Routledge. (2010).
[19] Hair J., Black W. C., Babin B. J. & Anderson R. E. "*Multivariate data analysis*." (7th ed.). Upper Saddle River, New Jersey: Boston: Pearson. (2010).