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Not peer-reviewed version

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Posted Date: 28 September 2023

doi: 10.20944/preprints202209.0151.v4

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Article

A Time Interval Only Description and Radiation

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Abstract: The time interval set approach depends on results for radiation propagation from star sources, where properties relate to sphere like propagation surfaces, finite for any realistic event and measurement. In contrast usually applied to stationary and parallel propagation is the traditional vector and one moment time coordinate kinematic approach from Newton's laws. A time interval only description necessarily has to start from its own definitions, and properties since time intervals have to be defined with time intervals. In this paper studied are time interval only set properties, commutation, addition and multiplication, and derivatives, and how to define these corresponding consistently with one moment time set properties. Time development and equilibrium within the time interval description depend on the well-known 'mean velocity theorem' and the time interval version of the Legendre transform. Radiation propagation 'away from' and gravitation 'towards' the star source are part of the one time interval description. In the second, discussion, part the time interval only set approach with only the fundamental properties is applied to radiation. Included is a derivation of star source radiation energy for both zero and non zero wave particle mass. The time interval set is not one of the usual sets encountered and the canonical property does not apply, the approach in this paper starts from commutation properties of one moment time coordinates and of time intervals and relates to Noether charges and structure constants. These are found to be equal to the radiation wave group-velocity, for photons the light velocity c , and, different from the usual derivative to one moment time, they equal specific arguments of the time interval only set derivative.

Keywords: time interval; radiation; gravitation; commutations; derivatives; propagation surface; star groups; measurement event; (a-) symmetry; time development

1. Introduction

1.1. Introduction to the time interval description

Within a description depending on time intervals only, star source radiation, with sphere like propagation and complementary wave particles, is applied as exemplary to derive general properties for the time interval only set. In contrast, the usual one moment time coordinate description applies vector kinematics, which is the traditional approach (Newton, 1686 (a)), (Goldstein, 1950), (Arnold, 1974).

Events with infinite time interval, in this paper termed Newtonian situations, the Hamiltonian H remains time independent and results are similar to the one moment time coordinate approach, where H time dependent implies a finite time interval. The time interval description includes derivatives to time intervals, commutation and equilibrium (MVT) relations and originates from (Hollestelle, 2020, 2021).

Measurement events relate to finite time intervals, and a relevant event time interval is defined, time interval Δt , to mean the reference interval for any measurement event. Radiation propagation is assumed to include a group-velocity and a propagation surface related to time interval Δt .

The possibility of a time interval only set corresponding with the one moment time coordinate set is valuable by itself. Besides this a value of the time interval only description is that it is an approach to time development, different from the Newtonian kinematic and one moment time coordinate approach, due to the application of only time intervals and of MVT equilibrium. For radiation within qm, and with the one moment time description, usually assumed are parallel plane waves and invariant energy, meaning parallel ray propagation (Sakurai, 1967). For star source

radiation, introduced are sphere like propagation surfaces and finite time intervals, and rather the term propagation than 'ray' is applied.

Measurements and uncertainty relations can be a basis for qm (Roos, 1978), implying particle properties like positions or paths can only be accessed from sequences of interactions or measurements. This philosophical discussion is not part of this paper, however measurement event description is.

Uncertainty relations continue to be part of qm if only because they relate qm subjects, like elementary particles or photons, to 'Newtonian' experimental subjects or quantities, accessible from measurements, (Roos, 1978), (Sakurai, 1967). Within the time interval description, one can derive uncertainty relations, different from the usual uncertainty relations, with 'variable' constants instead of Planck's constant, when the Hamiltonian is time dependent, however recovering these when H time independent.

Part I includes the definitions needed for the time interval only set: commutation relations, to describe time development and derivatives within the time interval set. Correspondence for one moment time coordinates with time intervals is defined from these relations. For the independent time interval only set two different correspondences are applied, paragraph 5. Several issues are being discussed: the uniqueness of the time interval linear subset, paragraph 8 and 10, and the non-linear time interval derivative, paragraph 9. The time interval only set addition, multiplication and derivative have to be defined from zero. Part II includes application to radiation: within the time interval only set, a description of star source radiation in terms of Noether charges and structure constants, terms that originate from other parts of physics however can be adequately introduced for radiation.

1.2. The time interval only description, correspondence, and the 'infinite regress' problem

The time interval and MVT equilibrium description includes one moment time quantities, since time intervals and derivatives to time intervals within its results mean boundaries are implied. To find a time interval only description, one can let one moment time quantities relate to time interval quantities through correspondence without boldly being equated one-to-one with time intervals. In this way correspondence can resolve the 'infinite regress' problem in (Hollestelle, 2020), where equilibrium is defined with the Hamiltonian being the time interval version of the Legendre transform of the Lagrangian. With one moment time quantities, like time interval boundaries, equated with time intervals this would mean, due to the derivative in the time interval version of the Legendre transform, time interval boundaries return within the definition for time interval equilibrium causing the regress. This problem is the most persistent one when introducing time intervals. The term correspondence otherwise sometimes is applied for the relation of qm concepts corresponding with experimental phenomena (Roos, 1978).

1.3. Star source radiation

The second part of this paper consists of paragraph 11 and 12. The time interval description originates from a star-source radiation approach, spherical rather than parallel. In coincidence with Curie's principle (Curie, 1894), and Newton's second law, from time interval asymmetry and time development towards symmetry, and from H time dependent towards H time independent, one can recognize change to be real or not from these properties (Hollestelle, 2020). The generating process of star-source radiation, while related to source mass, is not studied. Assumed is radiation is in 'dispersion free' motion without interaction and it's propagation surface energy remains invariant during time development, while emission energy per time interval depends on an invariant source. Meant is propagation surfaces assume a space-like limit for propagation waves, related to a finite time interval.

For both zero mass e.m. radiation waves and massive particle waves, a time interval only description of these phenomena is derived, without other assumptions. For finite and asymmetric time interval Δt , discussed are the concepts 'remote' and 'close to'. Radiation star sources are generalized to localizable or non-localizable, 'remote' or 'close', cloud like sources, paragraph 11 and

12. Results include the introduction of radiation Noether charges that can be expressed as structure constants and are equal to the propagation group-velocity.

1.4. Radiation and gravitation with opposite time development

Newton's second law relates kinematics with source related forces, applying vectors and spacetime coordinates, within a one moment time coordinate description. In this paper equilibrium is due to energy rather than action and this allows to discard the question of inertial forces.

Where the equivalence principle in GR implies gravitation energy to be equivalent with kinematic energy, the time interval description implies gravitation energy to be 'in equilibrium equivalence' with radiation wave energy, the kinetic propagation surface energy, due to star source radiation. In GR simultaneity is introduced applying a finite velocity of light. Simultaneity within the time interval description implies radiation waves emitted and 'on the way' during Δt , however only when Δt is 'simply measurable', meaning measurable within one measurement, paragraph 11. The group-velocity for non zero mass waves assumes free particle kinetic energy following the wave particle duality relation. This relation is one of the few of qm to remain with the time interval only description.

Within the usual one moment time description, like for GR, radiation propagation is away from the radiation source, where however a gravitational interaction implies movement directed towards the sources, which can be away from the line of sight which depends on the radiation source and the measurement place. With the time interval only set, one finds together in one description the opposite time development for gravitation and radiation.

2. Time interval commutation relations and derivatives and operators 'working to the right'

The time development for state vectors in qm field theory usually is introduced from commutation relations and derivatives to one moment time coordinates, the qm version of Lagrange equilibrium. For operators the concept 'working to the right' defines meaning in terms of measurement results: operators are 'working to the right' on state vectors, (Roos, 1978). In this and the following paragraphs introduced is a correspondence, to connect the one moment time set and the time interval set, applying commutation relations and derivatives including the 'working to the right' property. A tensor algebraic approach is not part of this paper.

The usual commutation, with subscript u, is defined with $[A1, A2]_u = [A1(t), A2(t) - A2(t), A1(t)]$, for two one moment time quantities A1 and A2 at the same parameter t. For the 1-dimensional one moment time set assumed are two invariant commutation quantities cn and cn' within two dependent relations, equation 1. With higher dimensional sets, one derives different numbers of quantities and relations. With multiplication . and the usual + and - indications, these relations are:

$$\begin{aligned} 1 \quad & t, cn' = \\ & cn, t \, 1/t. \\ & cn = cn'. \\ & 1/t \\ 2 \quad & [t, cn]_u = t, (cn - \\ & cn') [1/t, cn]_u = - \\ & (cn - cn'). \, 1/t \end{aligned}$$

The cn and cn' can be different from each other, and are the only commutation quantities related to the one moment time coordinate t set. The - 1 scalar multiplication is part of commutation relations within both the one moment time coordinate set and the time interval set. One can also choose for the addition inverse, paragraph 12 part V.

The commutation relations connect one moment time coordinate t and the one moment time derivative to t: d/dt. The derivative for multiplication A1. A2, meaning A1 'multiplication' A2, is d/dt [A1. A2]. Derivative d/dt is 'working to the right', on the square brackets, on every part of the multiplication within the traditional and usual first order differential calculus, (Newton, 1686 (b)), with the symmetrical second associative property for derivatives:

$$3 \quad \begin{aligned} d/dt [A1. A2] &= d/dt [A1]. A2 + A1. d/dt [A2] = 2. A1. d/dt [A2] \\ [A2] + \text{Rest Rest} &= (d/dt [A1]. A2 - A1. d/dt [A2]) \end{aligned}$$

Essentially, in the second part of this equation the multiplication disappeared and d/dt is 'working to the right' on $A1$ or $A2$ only. The interpretation of equation 3 is within the third part. While equation 3 includes only one step: d/dt 'moving to the right' of $A1$, this leaves to the left quantity $A1$, i.e. 'moving to the left' of d/dt , including 'leaving to the left' multiplication with a scalar, the scalar 2, and to the right a factor Rest. This is similar to one moment time t or $1/t$ 'moving to the right' of cn or cn' respectively in equations 1. These equations together define the properties of cn and cn' and d/dt . The derivative is discussed more generally in paragraph 9, with the introduction of a possibly nonlinear derivative 'set rule' to interpret the factor Rest, which can be different for each set.

Within the time interval description, t -quantities and Δt -quantities both occur, depending on coordinate t or time interval Δt . This allows the correspondence introduced in the following paragraphs. Assumed is that the described situation is 'remote' from H time independent, meaning time interval $\Delta t = [tb, ta]$ is asymmetric and finite such that $|tb| \ll |ta|$. In this case cn and cn' remain invariant during Δt and correspond with Δt -quantities $cn(\Delta t)$ and $cn'(\Delta t)$, where Δt -quantities are not t dependent, linear or otherwise. The indications A and M mean addition and multiplication within the time interval set, A is evaluated in paragraph 7, M in paragraph 8. Equations 11 in paragraph 5 are commutation relations for Δt -quantities $cn(\Delta t)$ and $cn'(\Delta t)$ without reference to one moment time coordinate t and part of a complete and closed time interval only description.

Time interval derivatives expressed in commutation relations

Within the one moment time description, the derivative to one moment time t , of quantity $A1(t)$, is $d/dt [A1]$, and within the time interval description and t -quantity perspective the derivative to one moment time t includes $[1/t, A1]u$, the usual commutation, equation 4, (Hollestelle, 2021).

MVT equilibrium implies linearity in t for any t -quantity $A1$ with $D^* [A1] = \pm A1. 1/t$ invariant during Δt , the $+$ or $-$ depending on $A1$ increasing or decreasing with t or $A1$ with positive or negative sign. This is part of the t -quantity perspective. The indications A and M for addition and multiplication are not applied for one moment time parameters like t or $1/t$. Within the time interval description and Δt -quantity perspective the derivative of t -quantity $A1$ to time interval Δt is defined including $[1/t, A1] | \Delta t$, the time interval commutation with $\Delta t = [tb, ta]$, equation 4.

$$4 \quad \begin{aligned} D^* [A1] &= 1/2 A [1/t. A1(t), -1. A1(t). 1/t] = [1/t, A1]u \\ D^* | \Delta t [A1] &= 1/2 A [1/tb. A1(tb), -1. A1(ta). 1/ta] = [1/t, A1] | \Delta t, \end{aligned}$$

3. Correspondence C1: invariant t -quantities and Δt -quantities

The invariant time interval derivative $D^* [A1]$ can be regarded t -quantity as well as Δt -quantity and this allows to define a correspondence. Applying time interval Δt averages $\langle \rangle | \Delta t$ for $A1$ and one moment time t , and assuming $\langle A1. 1/t \rangle | \Delta t = \langle A1 \rangle | \Delta t. 1/\langle t \rangle | \Delta t$, in terms of t -quantities only this is:

$$5 \quad A1. 1/t = \langle A1 \rangle | \Delta t. 1/\langle t \rangle | \Delta t$$

This relation is valid for any variable t -quantity $A1$ and follows from $D^* [A1] = \pm A1. 1/t$ invariant due to MVT equilibrium. Average $\langle A1 \rangle | \Delta t$ is invariant for Δt , and can be both invariant t -quantity or Δt -quantity.

Correspondence C1 depends on the definition of units for the related sets. The average of parameter t for time interval $\Delta t: \langle t \rangle | \Delta t$, depends on the (a-)symmetry of time interval Δt . Properties for time interval $\Delta t = [tb, ta]$ are defined (Hollestelle, 2020). For asymmetric $\Delta t: \langle t \rangle | \Delta t = 1/2(ta + tb) = t0$, with $t0$ the 'multiplication unit' for one moment time parameters during Δt , meaning $t0. t = t. t0 = t$. One moment time $t0$ is an indication for the time equilibrium of the interval. For the time interval set, relevant event time interval Δt is equal to the 'multiplication unit' time interval ΔU .

One defines correspondence C1 by choosing 'multiplication unit' $\langle t \rangle | \Delta t = t0$ to correspond with 'multiplication unit' $\Delta U: t0 \sim \Delta U$. Indicated with $\sim$, an average of any t -quantity corresponds

with a Δt - quantity. Equation 5 is written from t - quantity perspective, equation 6 from Δt -quantity perspective. Subscript i indicates the multiplication inverse for Δt - quantities, where $\Delta U_i = \Delta U$.

$$6 \quad A1.1/t \sim M [\Delta t3, \Delta U] = M [M [a, \Delta t], \Delta U]$$

C1, Correspondence and time interval units

For the H time dependent and asymmetrical situation, defined is the average of one moment time t for Δt equals the one moment time 'multiplication unit', $\langle t \rangle | \Delta t = t_0$, different from 0. t_0 , the 'multiplication zero' and 'addition zero'. When t_0 corresponds with Δt_0 per definition as time interval set term, and in the asymmetric case Δt_0 is equal to time interval set 'multiplication unit' ΔU , different from time interval set 'multiplication zero' ΔU_0 , one finds: $\langle t \rangle | \Delta t = t_0 \sim \Delta t_0 = \Delta U$ for asymmetric Δt .

For the H time independent and symmetrical Δt situation with $|t_b| = |t_a|$ there is $\langle t \rangle | \Delta t = t_0$, however t_0 equals $t_0 + -1.t_0 = 0$. t_0 , meaning t_0 is equal to both one moment time 'multiplication zero' and 'addition zero', and corresponds with Δt_0 equal to ΔU_0 both time interval set 'multiplication zero' and 'addition zero'. In this symmetric case $\langle t \rangle | \Delta t = t_0 = 0$. $t_0 \sim \Delta t_0 = \Delta U_0$.

Comment 1. The average $\langle t \rangle | \Delta t = t_0 \sim \Delta t_0$, meaning correspondence of one moment time t_0 with the time interval set 'multiplication unit' or 'multiplication zero' is due to H being time dependent or not. This difference of interpretation for average $\langle t \rangle | \Delta t \sim \Delta t_0$ for quantities and properties while remaining within physics is interesting by itself. A time development tending towards H time independent means average $\langle t \rangle | \Delta t = t_0$ tends towards 0. t_0 , the one moment time set 'multiplication zero', where t_0 equated also with the 'multiplication unit' needs discussion. Correspondence with Δt_0 within the time interval only set needs a similar interpretation.

Comment 2. The defining property for 'multiplication unit' ΔU is: $M [\Delta U, \Delta t1] = M [\Delta t1, \Delta U] = \Delta t1$, for the multiplication inverse $\Delta t1i$: $M [\Delta t1, \Delta t1i] = M [\Delta t1i, \Delta t1] = \Delta U$, for any time interval $\Delta t1$. One can argue, disregarding uniqueness, ΔU can be identified with relevant event time interval Δt , from $M [\Delta U, \Delta t] = \Delta t$ while also $M [\Delta t, \Delta t] = \Delta t$, a relation derived independently for addition, $A [\Delta t, \Delta t] = \Delta t$, in paragraph 7 and for M in paragraph 8. It follows $\Delta t_0 = \Delta t$ is a solution for ΔU . The discussion in paragraph 7 implies some uniqueness properties for 'addition zero' Δt .

The multiplication inverse solution $\Delta ti = \Delta t$ is well defined and belongs to the time interval set. When $M [\Delta U, \Delta t] = \Delta t$ and $M [\Delta ti, \Delta t] = \Delta U = \Delta t$ one finds Δt is a solution for Δti : $M [\Delta t, \Delta t] = \Delta U$.

Comment 3. Addition is indicated with A and one defines $A [\Delta t1, \Delta t0] = A [\Delta t0, \Delta t1] = \Delta t1$ and $A [\Delta t1, \Delta t1iv] = A [\Delta t1iv, \Delta t1] = \Delta t0$, with $\Delta t0$ time interval set 'addition zero' and $\Delta t1iv$ with subscript iv the addition inverse for any time interval $\Delta t1$. Multiplication with scalar -1 of time interval $\Delta t1$, $M [-1, \Delta t1]$, is not necessarily the addition inverse $\Delta t1iv$ for $\Delta t1$, and is not a well-defined time interval when Δt asymmetric.

4. Correspondence C2: reciprocal pairs of commutation quantities and Noether charges

The commutation quantities $cn(t)$ and $cn'(t)$ are assumed invariant t -quantities within the time interval description, meaning $cn(t) = cn$ and $cn'(t) = cn'$ during Δt with constant values within their resp. domain. Dependence on Δt is described with the positive average scalar density $D(cn, \Delta t)$. There is:

$$\langle cn(t) \rangle | \Delta t = | I^* [cn] |. 1/|\Delta t| = | \int | \Delta t dt [cn] |. 1/|\Delta t| = N. D(cn, \Delta t)$$

with $D(cn, \Delta t) = |cn\text{-domain}|. 1/|\Delta t\text{-domain}|$ an invariant scalar quantity. Constant N is defined below.

Applied is the time interval integral to one moment time t : for instance $I^* [2] = \int | \Delta t dt [2] = 2. |\Delta t|$, the domain for scalar 2 being the Δt - domain Δt itself. It is assumed that not always $D(cn, \Delta t) = 1$, due to a specific equilibrium requirement for cn and cn' .

Similar to $D(cn, \Delta t)$ defined is $D(cn', \Delta t)$ for cn' . The assumed equilibrium requirement implies $D(cn, \Delta t) + D(cn', \Delta t) = 1$, both for the same Δt . Within t -quantity perspective, and with cn and cn' the same sign, and assuming the usual one moment time addition properties it follows:

$$7 \quad cn + cn' = N. D(cn, \Delta t) + N. D(cn', \Delta t) = N$$

The t-quantities $cn(t)$ and $cn'(t)$ can be termed 'reciprocal'. The requirement exists due to cn , cn' and N , being not ordinary constants, rather physical quantities defined within the time interval MVT equilibrium description. Some arguments for the requirement, equation 7, are discussed in comment 5, and in paragraph 12, part III.

With cn and cn' correspond Δt -quantities $cn(\Delta t)$ and $cn'(\Delta t)$ and the equilibrium requirement implies $A [cn(\Delta t), cn'(\Delta t)] = \Delta N$, including $\langle N \rangle | \Delta t = N \sim \Delta N$, correspondence C1. Domain zero measure is not considered for $cn(\Delta t)$ or $cn'(\Delta t)$, since corresponding cn or cn' equal to one moment time 'multiplication zero' seems to interfere with equation 1.

Correspondence for the asymmetric situation means $\langle t \rangle | \Delta t \sim \Delta t_0 = \Delta U$: in the specific case when $\langle D(cn, \Delta t) \rangle | \Delta t$

ΔU and $cn(\Delta t) = M [\Delta N, \Delta U] = \Delta N$, it follows $\langle D(cn', \Delta t) \rangle | \Delta t \sim \Delta U_0$ and $cn'(\Delta t) = M [\Delta N, \Delta U_0] = \Delta U_0$, and the other way around, while invariant Δt -quantity ΔN remains the upper limit for $cn(\Delta t)$ or $cn'(\Delta t)$. For invariants within the one moment time description and the usual Noether charges referred is to (Noether, 1918), (De Wit, Smith, 1986). The equilibrium requirement in Δt -quantity perspective while Δt remains the variable is:

$$8 \quad A [cn(\Delta t), cn'(\Delta t)] = A [M [\Delta N, \Delta U], M [\Delta N, \Delta U_0]] = A [\Delta N, \Delta U_0] = \Delta N$$

Time interval addition A is preliminary applied for addition with ΔU and ΔU_0 , comment 3, while defined only in paragraph 7.

In support for the equilibrium requirement, one argument includes a well known theorem for time and space averages (Arnold, 1974): ΔN is an approximation for the overall time interval set average $\langle cn(\Delta t) \rangle | \text{set}$, and equal to the average $\langle cn(\Delta t) \rangle | \text{space}$, defining $cn'(\Delta t) = cn(\Delta t')$ with the same upper limits at Δt and $\Delta t'$. ΔN remains invariant for the overall time interval set, $D^* | \Delta t [\Delta N] = \Delta N$. From the similarity for A and M for time intervals, derived in paragraph 10, it follows $\Delta N = A [cn(\Delta t), cn'(\Delta t)] = M [cn(\Delta t), cn'(\Delta t)]$. $D^* | \Delta t [\Delta N] = \Delta N$ is supported by the symmetric second associative property, equation 3, for the derivative of ΔN . The derivative of the usual Noether charge NC is an invariant, and with the above theorem one can define $\Delta N \sim N = NC$.

Comment 4. The usual NC are defined within the one moment time description. Within the time interval description, Δt invariant t-quantities cn and cn' and N can be termed time parameter Noether charge NC : the invariance of the NC , paragraph 2, is required at least for the relevant event time interval Δt , to not interfere with the commutation relations equation 1, and is decisive for dispersion-free measurement event results to be equal to some Noether charge.

The Δt -quantities $cn(\Delta t)$ and $cn'(\Delta t)$ depend on the domain densities and Δt , while $\Delta N = A [cn(\Delta t), cn'(\Delta t)]$ is an invariant for the overall time interval set. ΔN can be termed overall time interval set Noether charge NC_{set} , when invariance means overall time interval set invariance.

Comment 5. The quantities $cn(\Delta t)$ and $cn'(\Delta t)$ are in principle independent. However, assuming equilibrium, in this case time interval MVT equilibrium, this means some relation for $cn(\Delta t)$ and $cn'(\Delta t) = cn(\Delta t')$ applies, the requirement to maintain equilibrium during time development. The equilibrium requirement is inferred to be universally existing, and resembles total energy $T + V$ invariance when T and V are the kinetic and potential energy within Lagrangian equilibrium.

Comment 6. The Hamiltonian remains invariant for transformations $T1$, $T2$ and $T2v$, (Hollestelle, 2020). $T1(t)$ with $T1(ta) = tb$, its opposite $T2(t)$ with $T2(ta) = -1 \cdot tb$ and its reverse $T2v$. Transformation $T1$ defines any time interval $[tb, ta]$ with $[T1(ta), ta]$, this reduces the possible tb and ta . Series of transformations $T2v$ transform Δt to another $\Delta t'$, and include a re-scaling of one moment time t , which takes care $(c \cdot q(ta))^2 \ll 1$ for transformation $T1$ to remain valid with always ta and $T1(ta) = tb$ with resp. positive and negative sign, ie future and past.

A second correspondence $C2$ depends on a specific infinitesimal Lorentz transformation iTL , where space- and time coordinates q and t transform to resp. $q' = q + qL$ and $t' = t + tL$, with qL and tL infinitesimal constant space- and time coordinates. Infinitesimal Lorentz transformation iTL is equal to transformation $T2v$, with $iTL(t) = t'$, where one moment time t and t' correspond with time interval $\Delta t = [tb, ta]$ and $\Delta t' = [tb', ta']$, and where $q' = q(ta')$ remains well-defined. The specific non-infinitesimal TL can be defined from iTL and the Hamiltonian remains invariant with TL .

Any usual Lorentz transformation TU should leave metric distances unchanged. It is argued to introduce new Lorentz transformations that leave metric surface measures unchanged since star source radiation energy, while proportional to the measure of the propagation surface, remains unchanged during propagation. The specific Lorentz transformation TL is found to be one of these Lorentz transformations, (Hollestelle, 2021).

For time interval $\Delta t = [tb, ta]$, boundary $tb = T1(ta)$ relates by definition, not regarding causality, to boundary ta with the following equation.

$$9 \quad 1/tb = -1/ta (1 - (c \cdot q(ta))^2)$$

Comment 7. The constant c with dimension similar to the multiplication inverse of q , assures scalar product $(c \cdot q)$ remains dimensionless. The factor $(c \cdot q_0)$ for $q(t) = q_0$ at $t = t_0$ is an invariant, and equal to multiplication $M[c, q_0]$ within the time interval set, where both c and q_0 remain invariant during Δt . Notice q_0 is not equal to space-like origin q_c . Constant c is not the, zero mass, photon group-velocity equal to c .

Comment 8. Two different solutions exist: $q(t) = \pm q_0 \cdot 1/t_0 \cdot t$ and $q(t) = \pm q_0 \cdot t_0 \cdot 1/t$. Due to MVT equilibrium with $D^*[q(t)] = \pm q(t) \cdot 1/t$ and $D^*[q(t)]$ independent of t during Δt , $q(t)$ necessarily is linear in t and the solution $q(t)$ linear in $1/t$ can exist only when it is the same solution, ie also linear in t .

With $t_i = 1/t$ the multiplication inverse for t , where t_i not necessarily belongs to the one moment time set, one finds: $t_i \cdot t = t \cdot t_i = t_0$, with t_0 the 'multiplication unit', which leaves different solutions t_i possible. To give meaning to t_i , it is required: solution t_i is to belong to the one moment time set, and when t_1 and t_2 belong to the one moment time set, multiplication $t_1 \cdot t_2 = t_3$ does similarly. These one moment time set properties are defined in (Hollestelle, 2020). For Δt boundaries ta and tb there is: $ta + tb = 2 \cdot t_0$ and $ta \cdot tb = 1/2 \cdot (ta^2 + tb^2)$ approaching to $ta \cdot tb = 1/2 \cdot ta^2$ for $|ta| \gg |tb|$, valid for situations with H time dependent and with Δt asymmetric. These one moment time tb and ta properties find, some, similarity with the relevant event time interval Δt properties $M[\Delta t_i, \Delta t] = M[\Delta t, \Delta t] = \Delta t = \Delta U$, comment 2.

C2, Correspondence and time interval commutations

One defines the time interval Δt - quantities $cn(\Delta t)$ and $cn'(\Delta t) = cn(TL(\Delta t))$, with $cn'(\Delta t) = cn(\Delta t')$ and $TL(\Delta t) = \Delta t'$, recall the discussion with equation 8. It follows correspondence C2: one moment time commutation pair cn and cn' , from equation 1, corresponds with time interval set commutation pair $cn(\Delta t)$ and $cn'(\Delta t)$. Correspondence C1, paragraph 3, and C2 have the same results. In equation 10, the one moment time parameter t - quantities on the left side correspond with time interval Δt - quantities on the right side.

$$10 \quad |ta| \cdot 1/|tb| = |cn'(\Delta t)| \cdot 1/|cn(\Delta t)|$$

$$ta \cdot tbi \sim M[cn'(\Delta t), cni(\Delta t)]$$

5. The time interval only set, commutation relations, and the 'infinite regress problem'

Star source approach to radiation propagation

The following description for star source radiation in terms of particular coordinates is introduced in (Hollestelle, 2020, 2021). This description is being assumed in all the following of this paper. It is meant relevant event time interval Δt is $\Delta t = [tb, ta]$ with $qa = q(ta)$ and $qb = q(tb)$, where qb indicates the star source coordinate place. The space coordinate qa , within both the one moment time and time interval description, indicates a coordinate place q at the radiation propagation sphere surface such that space coordinate origin qc is 'close' to q . The space coordinate q is a possible measurement place on the propagation surface. Space interval Δq is the line of sight from the star source at place qb to place q at the propagation surface. Interval measure $|\Delta q| = |q - qb|$ increases with propagation and time development together with a change of coordinate place q . Place q remains at the propagation surface and near qc during propagation, where place qb remains unchanged during propagation. The one moment time description due to the Δt boundaries is still included within the time interval description, in these two papers.

The time interval only set

The multiplication inverse Δt_i for Δt exists due to the relation $M[\Delta t, \Delta t] = \Delta U$, at least a solution is $\Delta t_i = \Delta t$ which is linear in Δt . Within the time interval description I^* and D^* indicate integration and differentiation to one moment time t and $I^*||\Delta t$ and $D^*||\Delta t$ integration and differentiation to time interval Δt . Due to MVT equilibrium all Δt -quantities are linear in Δt and can be reduced to time intervals, this is discussed independently in paragraph 8, with the closure theorem, and paragraph 10, the multiplication linearity theorem.

By applying correspondence C1, equation 6, and correspondence C2, equation 10, one derives for the Δt -quantity $cn(\Delta t)$ in the time interval description:

$$11 \quad I^*||\Delta t [cn(\Delta t)] = M[\Delta t, A[cn(\Delta t), -1. cn'(\Delta t)]] = A[cn(\Delta t), -1.$$

$$cn'(\Delta t)] D^*||\Delta t [cn(\Delta t)] = M[A[cn'(\Delta t), -1. cn(\Delta t)], \Delta t_i] = A$$

$$[cn'(\Delta t), -1. cn(\Delta t)]$$

Derivation of equation 11 includes the commutation versions for $D^*||\Delta t$, paragraph 2, while $I^*||\Delta t$ is defined in paragraph 4. In particular the occurrence of Δt and Δt_i is explained by the linearity in Δt for $I^*||\Delta t$, and linearity in Δt_i for $D^*||\Delta t$.

Equation 11 includes multiplication with Δt resp. Δt_i and are the time interval equivalent of equations 1 and 2: they define commutation relations for time intervals with time intervals 'working to the right'. For $cn'(\Delta t)$ similar equations can be derived.

The general result for any Δt -quantity instead of $cn(\Delta t)$ requires linearity of the time interval set which is valid due to the time interval set 'multiplication closure' theorem, or the time interval set 'multiplication linearity' theorem, derived in resp. paragraph 8 and 10. Within the general result, $cn(\Delta t)$ is linear in Δt , and any Δt -quantity can be represented linear in $cn(\Delta t)$, and exactly similar results for equation 11 are valid for all time intervals and Δt -quantities, not only for $cn(\Delta t)$ and $cn'(\Delta t)$. With equations 11 the 'infinite regress' problem for the definition of time intervals is resolved. These equations are time interval only relations, and do not require one moment time set parameter t .

From equation 11 one finds $I^*||\Delta t [cn(\Delta t)]$ and $D^*||\Delta t [cn(\Delta t)]$ to be similar, in agreement with $\Delta t_i = \Delta t$ and comment 8, except for the order of $cn(\Delta t)$ and $cn'(\Delta t)$. Since from the correspondence $t_0 \sim \Delta U = \Delta U_i$ it follows $I^*||\Delta t [cn(\Delta t)] = D^*||\Delta t [cn(\Delta t)]$, ie including the order difference, the results in terms of time intervals for integration and differentiation are the same, at least for Δt - quantities and within the time interval only description. In terms of corresponding t - quantities, the results for equation 11 seem opposed to each other.

6. The time interval only set

Commutation properties are usually defined within a one moment time description and depend on vector-like space time coordinates and quantities that can form combinations like addition or multiplication. Time development and change can be included in the one moment time description with the derivative to parameter t . The time interval description includes both t -quantities and Δt -quantities. However, for a description with time intervals only, specific new properties for addition and multiplication have to be defined, combinations for time intervals that before did not have meaning yet. In paragraph 4, originating from (Hollestelle, 2020), defined is time interval $\Delta t = [t_b, t_a]$ and its boundaries t_b and t_a within a one-dimensional time concept. A part of the past includes t_b and a part of the future includes t_a . These boundaries t_b and t_a , one moment time parameter t and the coordinate concept still remained within the time interval description. From the results of paragraph 5 one finds a time interval only set description, in paragraph 7 and 8, of time interval only addition and multiplication independent from one moment time parameters.

7. Time interval only set: time development and addition

Time development in this paper means there is the same 'time' for the cosmological universe together and no part is late or early in reference to this 'time'. Not is meant time is a zero-dimensional concept without development and not is meant reference is variable or is chosen. Rather, time development proceeds together and 'simultaneous' time measurements should give 'the same' results. A definition for 'simultaneity', in paragraph 11 and 12, starts from the discussion of star source radiation propagation. Time development with MVT time interval equilibrium means addition, however rather in a non-trivial way. Recall continuous and invariant, monotone, change within the one moment time set means at any 'time-lapse' a similar 'time-lapse' is added. This resembles in time invariant quantities like invariant Noether charges. Time development means Δ variation within the usual one moment time description, and the principle of least action, ie Lagrangian equilibrium, not to be confused with the interval indication like Δt . A second property is, time intervals do not exist outside themselves: they don't add time from outside to themselves and remain only with themselves. One finds:

12 Δt_1 'addition' Δt_1 , to the same time interval, leaves Δt_1 invariant: $A[\Delta t_1, \Delta t_1] = \Delta t_1$

Δt_1 is any time interval within the time interval only set. This confirms the interpretation of time interval only set addition with domain addition where addition of two identical domains results in the same domain.

Within the time interval only description addition is defined starting from the introduction of 'addition zero' time interval Δt_0 such that addition of Δt_0 with any other time interval Δt_1 leaves Δt_1 invariant, $A[\Delta t_0, \Delta t_1] = A[\Delta t_1, \Delta t_0] = \Delta t_1$, similar to comment 3. Within the time interval only description events and properties depend on the specific relevant event time interval Δt , with $A[\Delta t_1, \Delta t] = \Delta t_1$, comment 3.

The 'addition inverse' Δt_{iv} for the finite relevant event time interval Δt equals Δt , meaning $A[\Delta t_{iv}, \Delta t]$ equals $A[\Delta t, \Delta t] = \Delta t$. To introduce the 'addition inverse' for time interval commutation in equation 12, is possible, paragraph 12 V, while paragraph 5 applies scalar -1. Scalar multiplication -1. Δt does not equal Δt_{iv} and can not be defined from equation 9, and does not belong to the time interval set. One might argue Δt 'addition' -1. Δt equals -1. Δt from the above definitions where 'addition zero' Δt_0 equals Δt . Notice that Δt_0 is not a zero-measure time interval in the sense of domain measure equal to zero. The order within addition matters. $A[\Delta t_1, \Delta t_2]$ means addition for any two time intervals, Δt_1 'addition' Δt_2 .

8. Time interval only set: multiplication and the multiplication closure theorem

Similar to addition A one can define multiplication $M[\Delta t_1, \Delta t_2]$ for any two time intervals Δt_1 and Δt_2 , from the introduction of 'multiplication unit' ΔU , while the relevant event time interval remains Δt . Like with addition it is not clear immediately what multiplication means for the time interval only set. Several properties for M are discussed in the following, where others are already included in paragraph 3.

I. Multiplication closure. The result of multiplication is assumed to belong to the time interval only set: $M[\Delta t_1, \Delta t_2] = \Delta t_3$, a time interval, and this is termed multiplication closure for the time interval only set. This is supported by the time interval set 'multiplication unit' ΔU relation $M[\Delta U, \Delta t_1] = \Delta t_1$. The multiplication closure theorem is derived below.

II. Time development. From similar arguments for the validity of equation 12, multiplication $M[\Delta t_1, \Delta t_1] = \Delta t_1$ is valid for any time interval Δt_1 .

III. Associativity. The symmetrical first associative, 'series', property for M : $M[M[\Delta t_1, \Delta t_2], \Delta t_3] = M[\Delta t_1, M[\Delta t_2, \Delta t_3]]$ is assumed valid, the symmetrical second associative, 'parallel' or distributive, property for M : $M[\Delta t_1, M[\Delta t_2, \Delta t_3]] = M[M[\Delta t_1, \Delta t_2], M[\Delta t_1, \Delta t_3]]$ is not necessarily valid, for Δt_i any three time intervals. The first and second associative property both valid can be contradictory. The usual derivative d/dt to one moment time t of a multiplication includes the second associative property, equation 3. The derivative to a t - or Δt -quantity includes one moment time t or time interval Δt commutations, paragraph 2. In this paper derived are the associative properties for the time interval only set, associative properties for ordinary variables or sets are well known within group theory, (Jacobson, 1974).

Preliminary definition for multiplication

For time interval only set multiplication the following definition, equation 13, is feasible however preliminary, and depending on common domain. The integral $I^* || \Delta t_2$ to time interval Δt_2 is introduced, just like the usual integral I^* to one moment time t , with the integral domain equal to the Δt_2 -domain:

$$13 \quad M [\Delta t_1, \Delta t_2] = M [M [< \Delta t_1 > || \Delta t, \Delta t], \Delta t_2] = M [< \Delta t_1 > || \Delta t_2, \Delta t_2] = I^* || \Delta t_2 [\Delta t_1]$$

This equation agrees with multiplication property II, and is Δt_1 and Δt_2 order dependent. $I^* || \Delta t_2 [\Delta t_1]$ symmetrical for Δt_1 and Δt_2 is not discussed. Equation 13 can be derived assuming three time interval set properties. These seem to be definitions, however they depend on the interpretation of multiplication with shared domain. When time interval Δt_1 and Δt_2 do not extend to outside Δt , ie the common domain for Δt_1 or Δt_2 with Δt equals Δt_1 or Δt_2 , and with the domain for Δt being continuous these properties are evident. I. Since time interval Δt_1 remains invariant during Δt there is: $< \Delta t_1 > || \Delta t = \Delta t_1$. II. Averages $< \Delta t_1 > || \Delta t_2$ remain independent of time interval Δt_2 : $< \Delta t_1 > || \Delta t_2 = < \Delta t_1 > || \Delta t$ for all Δt_2 . III. The time interval average to Δt_2 and the integral to Δt_2 are related with the traditional relation $I^* || \Delta t_2 [\Delta U] = M [< \Delta U > || \Delta t_2, \Delta t_2] = M [\Delta U, \Delta t_2]$, where Δt_1 can be inserted for ΔU to preliminary derive the above definition including the integral, depending on properties I and II. There is $M [I^* || \Delta t_3 [\Delta t_1], \Delta t_3] = M [I^* || \Delta t_2 [\Delta t_1], \Delta t_2]$, in agreement with paragraph 4.

Equation 13 is valid for $\Delta t_1 = cn(\Delta t)$ or $cn'(\Delta t)$ and $\Delta t_2 = \Delta t$, paragraph 5. As a result of the preliminary definition equation 13 there are the following equations.

$$14 \quad M [\Delta t_1, \Delta t] = < \Delta t_1 > || \Delta t = I^* || \Delta t$$

$$[\Delta t_1] = \Delta t_1 \quad M [\Delta U, \Delta t] = I^* || \Delta t$$

$$[\Delta U] = \Delta t$$

$$M [\Delta U, \Delta t_i] = I^* || \Delta t_i [\Delta U] = \Delta t_i$$

Comment 9. Closed multiplication implies $M [\Delta t_1, \Delta t_2] = \Delta t_3$ is a time interval and validity of $\Delta t_3 = a \cdot \Delta t_a$, scalar a depending on time interval Δt_a , implies the existence of solution Δt_a being a time interval. Not all scalar arguments a can be allowed when $M [\Delta t_1, \Delta t_2] = a \cdot \Delta t_a = \Delta t_3$ is a proper time interval, where for all time intervals the future domain part measure exceeds the past domain part measure by definition. When Δt_a is a time interval, $-1 \cdot \Delta t_a$ is not when H is time dependent and Δt asymmetric.

For any time interval, transformation T_1 is the defining and necessary relation for one moment time parameter boundaries, comment 6. From similarity with the derivative to one moment time t , paragraph 2, Δt -quantities are expected to be linear with Δt . The closure theorem implies: Δt_3 being not only a Δt -quantity, also a time interval and linear with Δt when Δt_1 and Δt_2 are time intervals.

Closure theorem I

Time interval only multiplication for any two time intervals Δt_1 and Δt_2 , $M [\Delta t_1, \Delta t_2] = \Delta t_3$, implies result Δt_3 is a time interval and can be rewritten with a scalar multiplication $\Delta t_3 = a \cdot \Delta t_a = M [ca, \Delta t_a]$, where a is scalar and ca is some time interval, ie ca is decided by time interval only multiplication $M [ca, \Delta t_a]$.

Uniqueness property for linear dependence on Δt : when Δt_3 depends linear on Δt , there is no other $\Delta t'$ such that Δt_3 depends also linear on $\Delta t'$ independent from Δt .

Finding some time interval Δt_a and some scalar a , implies $\Delta t_3 = a \cdot \Delta t_a$ is a time interval, with reference to the constraint from comment 9. Applying the symmetrical first associative property for any time interval ca , there is $ca = M [ca, \Delta t] = M [ca, M [\Delta t_a, \Delta t_a]] = M [M [ca, \Delta t_a], \Delta t_a]$, ie there is $ca = < ca > || \Delta t_a$, invariant.

One finds with $ca = \Delta t_3$ and $\Delta t_a = \Delta t$, an existing and well defined solution for $\Delta t_3 = a \cdot \Delta t_a = M [ca, \Delta t_a]$, for any scalar a , with reference to comment 9. This solution implies, $ca = a \cdot \Delta t$ and $\Delta t_3 = a \cdot \Delta t$ both belong to the time interval only set and are linear in Δt . Once one finds one solution to be a time interval linear in Δt , one has found all linearly dependent solutions.

Other solutions for $M [\Delta t_1, \Delta t_2]$ can be found directly from $a \cdot \Delta t_a = M [ca, \Delta t_a]$ and property II, $M [\Delta t_a, \Delta t_a] = \Delta t_a$. These solutions for Δt_3 , with any Δt_a , not only $\Delta t_a = \Delta t$, are already known to exist

since the time interval set is inferred to be uniquely linear in Δt , due to being 1-dimensional, from the definition of time intervals, (Hollestelle, 2020). All these solutions are linear in Δt , with $ca = a \cdot \Delta t$ and $\Delta ta = a \cdot \Delta t$, and result $\Delta t3 = a \cdot a \cdot \Delta t$. The consideration that any time interval is invariant during Δt and linear in Δt , is discussed again in paragraph 12, part III. For the specific scalar $a = 1$ the solutions Δta and $\Delta t3 = M [\Delta t1, \Delta t2]$ are well defined and time intervals: $a \cdot \Delta ta = M [ca, \Delta ta]$ with $ca = \Delta t$. Scalar a then is the result from $ca = \Delta t$ and $\Delta ta = \Delta t3$, with reference to comment 9.

Closure theorem II

An alternative approach is the following. The relation $\Delta t3 = a \cdot \Delta ta = M [ca, \Delta ta]$ depends on scalar a having 'positive' sign. For scalar a with a 'negative' sign the scalar multiplication does not follow the requirements for comment 9. From the first associative property for $M [\Delta t1, \Delta t2]$ with $\Delta t1$ equal to $a \cdot \Delta t$, from the definition of $D^* || \Delta t$, paragraph 2, one finds $\Delta t3 = a \cdot \Delta ta = a \cdot M [\Delta t, \Delta ta] = M [a \cdot \Delta t, \Delta ta]$, equal to $\Delta t3 = M [ca, \Delta ta]$ with solution $ca = a \cdot \Delta t$, linear in Δt and from $ca = M [ca, \Delta ta]$ it follows $\Delta ta = \Delta t$. From this derivation there is $\Delta t3 = ca = a \cdot \Delta t$, ie $\Delta t3$ is itself a time interval and linear in Δt , and it follows validity for the closure theorem for any scalar a , with reference to comment 9. *Multiplication M is closed within the time interval set.*

Comment 10. The first associative property for a derivative of a multiplication seems valid if only due to the preserved order of the Δti in series, property III. More precisely, this property is related to the following commutation properties for t -quantities $A(t)$ that are linear in parameter t due to MVT equilibrium. $A(t)$ is defined from the invariant t -quantities cn and cn' with $D^* [A(t)] = cn$ and it follows from equation 1:

$$\begin{aligned} 15 \quad & t | 1. A(t) | 2. cn' = \\ & A(t) | 1. t | 2 \ 1/t | 1. \\ & A(t) | 2 = A(t) | 1. \\ & 1/t | 2. cn \end{aligned}$$

Indication |1 or |2 means the t -quantity at this place depends on parameter t with the specific value $t = t1$ or $t = t2$. Equations 15 are not definitions, rather they are derived from the relation $D^* [A(t)] = cn$, equation 1 for t -quantities cn and cn' and from MVT equilibrium within the time interval description. One finds the order of |1 and |2 is preserved when reversing the order of parameter t or $1/t$ with quantity $A(t)$ within the equations, from the specific commutation properties. Within the overall time interval set, cn and cn' correspond with the invariant Δt -quantities $cn(\Delta t)$ and $cn'(\Delta t)$, linear dependent on Δt , and this confirms the first associative property for the derivative at least for $cn(\Delta t)$ and $cn'(\Delta t)$.

9. Time interval only set: addition, derivatives, and the derivative 'set rule'

Similar with the definition for M , equation 13, proposed is a definition for $A [\Delta t1, \Delta t2]$ that includes multiplication $M [\Delta t1, \Delta t2]$ and the derivative to time interval $\Delta t2$, $D^* || \Delta t2$, from the time interval description 'addition unit' and 'addition zero'. Equations 16 and 17, where the asymmetrical second associative property is assumed for the derivative to Δt , $D^* || \Delta t$, are evidently similar with one moment time derivative d/dt , equation 3.

The one moment time derivative 'set-rule' and the asymmetric second associative property

For the one moment time derivative of a multiplication $A1 \cdot A2$, with $A1$ and $A2$ arbitrary scalar quantities, one can apply the asymmetric arguments $c1$ and $c2$: $d/dt [A1 \cdot A2] = c1 \cdot d/dt [A1] \cdot A2 + c2 \cdot A1 \cdot d/dt [A2]$, similar to equation 3, when scalars $c1 = c2 = 1$.

Assume $a \cdot A1$ with invariant scalar a , and scalar quantity $A1$. The one moment time derivative 'set-rule' does apply, when d/dt is 'moving to the right' of scalar a : $d/dt [a \cdot A1] = c1 \cdot d/dt [a] \cdot A1 + c2 \cdot a \cdot d/dt [A1] = c2 \cdot a \cdot d/dt [A1]$, equal to $a \cdot d/dt [A1]$ when $c2 = 1$. The scalar a is 'moving to the left' or released from d/dt , and in this case there is no other factor Rest remaining to the right.

The time interval only set and the time interval derivative 'set-rule'

The perspective is changed to Δt -quantities and the time interval only set. The properties: $A [a1, A1, a2, A1] = (a1 + a2) \cdot A1$, for instance with accumulating $A [1, A1, 1, A1] = A [A1, A1] = 2 \cdot A1$, where $a1$ and $a2$ are scalar numbers and $A1$ is a one moment time quantity, are usually assumed valid for the one moment time set depending on vector addition. They do not extend to the time interval set, where property $A [\Delta t1, \Delta t1] = \Delta t1$, paragraph 7, is essentially different and depends on domain addition. The time interval set derivative and addition equations 16 and 17 can be assumed, however the existence of un-decided arguments $c1$ and $c2$ is an obstacle for any application.

$$16 \quad D^* || \Delta t [M [\Delta t1, \Delta t2]] = A [c1, M [D^* || \Delta t [\Delta t1], \Delta t2], c2, M [\Delta t1, D^* || \Delta t [\Delta t2]]]$$

$$17 \quad A [a, \Delta ta, \Delta t] = D^* || \Delta t [a, \Delta ta] = A [c1, M [D^* || \Delta t [a], \Delta ta], c2, M [a, D^* || \Delta t [\Delta ta]]]$$

Comment 11. The definitions for M and A depend on the existence of a real event where the relevant event time interval is Δt . Only with $\Delta t2$ equal to Δt the properties for $\Delta t0$ and ΔU can make sense, due to the relation of averaging and measurement, where the measurement event time interval is Δt .

Preliminary definition for addition

$$18 \quad \begin{aligned} A [\Delta t1, \Delta t2] &= D^* || \Delta t2 [M [\Delta t1, \Delta t2]] \\ A [\Delta t1, \Delta t] &= D^* || \Delta t [M [\Delta t1, \Delta t]] \\ &= \Delta t1 \cdot A [\Delta t0, \Delta t] \\ &= D^* || \Delta t [M [\Delta t0, \Delta t]] = \Delta t0 \end{aligned}$$

Equation 18 is valid for $\Delta t1$ identified with Δt -quantities $cn(\Delta t)$ or $cn'(\Delta t)$ and $\Delta t2$ with Δt . Since $cn(\Delta t)$ is linear in Δt due to MVT equilibrium this supports generalization to the time interval set. The multiplication closure theorem, paragraph 8, or the multiplication linearity theorem, paragraph 10, is necessary for this generalization. A discussion of linear subsets is included in paragraph 12, part III. The first associative property is applied and canceling terms are left out. Terms cancel due to $A [\Delta t1, \Delta t] = \Delta t1$.

The time interval derivative 'set rule', equation 19 and 20, is derived from definition equation 18. Applied is, 'addition zero' $\Delta t0$ equals Δt and 'multiplication unit' ΔU similarly equals Δt , comment 2. For multiplication with scalar a brackets can be left out, writing for instance $M [a, \Delta t, \Delta t1] = a \cdot \Delta t1$. The time interval derivative 'set-rule' is such, that D^* and $D^* || \Delta t$ are 'moving to the right' of scalar a , while scalar a is 'moving to the left' and is released from derivative $D^* || \Delta t$ brackets where to the right remains factor $Rest$. This 'set rule' is non-trivial since $D^* || \Delta t [a]$, for invariant scalar a , not necessarily equals $\Delta U0$ and does not correspond with $d/dt [a]$, equal to the one moment time set 'addition zero', which itself equals the 'multiplication zero'. Differentiation implies correspondence for t -quantities with Δt -quantities only due to introducing the time interval derivative 'set rule' including a non-linear factor $Rest$.

Equation 19 and 20 are due to evaluation of A and M itself, applying derivatives $D^* || \Delta t [\Delta t1] = [1/t, \Delta t1] || \Delta t$, and $D^* [A1] = [1/t, A1]u$, for any time interval $\Delta t1$, with $\Delta t = [tb, ta]$ and one moment time quantity $A1$, paragraph 2, $D^* || \Delta t [a, \Delta t1]$ does not have to be linear in scalar a . Factors $Rest(a) | t$ and $Rest(a) || \Delta t$ are introduced for the non linear part within the derivative. The result, similar, however without any arguments $c1$ and $c2$, to the second associative property with reduction the multiplication within derivative brackets like within equation 16, includes the derivative 'moving to the right' property specific for any set and 'set rule'.

The time interval derivative 'set rule'

$$\begin{aligned}
 19 \quad D^* [a, \Delta t1] &= A [a, D^* [\Delta t1], \\
 &\text{Rest}(a) | t] \text{Rest}(a) | t = [1/tb, a]u. \\
 \Delta t &= M [D^* [a], \Delta t]
 \end{aligned}$$

$$\begin{aligned}
 20 \quad D^* | | \Delta t [a, \Delta t1] &= A [a, D^* | | \Delta t [\Delta t1], \\
 &\text{Rest}(a) | | \Delta t] \text{Rest}(a) | | \Delta t = M \\
 &[D^* | | \Delta t [a], \Delta t]
 \end{aligned}$$

With any 'set rule', scalar a is 'moving to the left', the derivative $D^* | | \Delta t [a, \Delta t1]$ does not include scalar a within $D^* | | \Delta t [\Delta t1]$, and a non-linearity factor Rest is added to the right. This agrees with the interpretation of $D^* | | \Delta t$ 'moving to the right', leaving a commutation quantity to the left like in equation 1 and 11. The factor Rest is invariant during Δt , and therefore can be considered from t -quantity or Δt -quantity perspective. $\text{Rest}(a) | | \Delta t$ is derived by applying C1, correspondence by time interval units, from paragraph 3.

The usual commutation $[1/tb, a]u$ has to be evaluated carefully. For $a = 1$ there is $[1/tb, 1]u = tbi$. $1 - 1 \cdot tbi$, with $tbi = 1/tb$, indeed part of the one moment time set. Since the specific scalar 1 is 'multiplication unit' for the scalar set, $tbi \cdot 1 - 1 \cdot tbi = tbi + (tbi)iv$ is valid only, ie equal to the one moment time 'addition zero', when the one moment time addition inverse tiv equals $-1 \cdot t$ for all t within the one moment time set. The usual commutation can be resolved by rewriting it corresponding to a time interval commutation that equals a time interval derivative, and regains commutation value 'addition zero' due to MVT equilibrium, paragraph 2. Applied is scalar a is an invariant for time interval operator t^* and the derivation includes a transformation of scalar a by multiplication with parameter t . The operator is defined with $t^* [A1] = M [D^* | | \Delta t [M [t, A1]], \Delta t] = A [t, A1]$ for any one moment time quantity $A1$, (Hollestelle, 2021). This confirms, for any scalar a : $[1/tb, a]u \sim D^* [a] | | \Delta t$ and correspondence $\text{Rest}(a) | t \sim \text{Rest}(a) | | \Delta t$.

From equation 11, with one moment time average $\langle t \rangle | | \Delta t = t0$, $D^* [a]$ equals $\langle a \rangle | | \Delta t$. $1/\langle t \rangle > | | \Delta t = a \cdot t0i$, for scalar a positive and invariant with t . Assuming H time dependent, one moment time 'multiplication unit' corresponds with time interval 'multiplication unit': $t0 \sim \Delta U$. Recall $\Delta Ui = \Delta U = \Delta t0$ and Δti equals Δt , comment 1. It follows $D^* | | \Delta t [a]$ and $\text{Rest}(a) | | \Delta t$ both equal $a \cdot \Delta U$. This means $\text{Rest}(a) | | \Delta t$ is a Δt -quantity, implying equation 20 is the time interval only derivative 'set rule'.

Comment 12. For any scalar a , $\text{Rest}(a) | t = M [a, t0i, \Delta t] \sim a \cdot M [\Delta U, \Delta t] = a \cdot \Delta U$. This is the same as $\text{Rest}(a) | | \Delta t = M [a, \Delta U, \Delta t] = a \cdot \Delta U$, equal to ΔU for $a = 1$. In this case, for $a = 1$, one finds $D^* [a, \Delta t1] = a \cdot D^* [\Delta t1]$ and $D^* | | \Delta t [a, \Delta t1] = a \cdot D^* | | \Delta t [\Delta t1]$ as it should be.

Comment 13. $\text{Rest}(a) | t = M [D^* [a], \Delta t]$, in equation 19, can be derived independently without including the second part: $[a, 1/tb]u \cdot \Delta t$. This confirms the one moment time derivative $D^* [A1] = [1/t, A1]u$, paragraph 2.

Comment 14. $M [\Delta U0, \Delta t1] = \Delta U0$, with $\Delta U0$ the time interval set 'multiplication zero' and 'addition zero', corresponding with the scalar set 'multiplication zero'. The domain measure for $\Delta U0$ is zero and $\Delta U0 = 0 \cdot \Delta t1$, for any finite $\Delta t1$ including $\Delta U0 = 0 \cdot \Delta U$. With zero domain measure, $\Delta U0$ is not a proper time interval, and resembles a one moment time parameter that does not belong to the time interval set. Time intervals with zero domain measure are not included in the time interval set.

The derivative asymmetric second associative property and arguments c1 and c2

With $c1$ and $c2$ the arguments defining the asymmetrical second associative property for derivative $D^* | | \Delta t [M [\Delta t1, \Delta t2]]$ with equation 16, and with $D^* | | \Delta t2 [\Delta t1] = \Delta t1$ for $\Delta t2$ equal to Δt , equation 18, one finds with $a = 1$:

$$21 \quad D^* | | \Delta t [M [\Delta t1, \Delta t]] = A [c1 \cdot M [\Delta t1, \Delta t], c2 \cdot M [\Delta t1, \Delta t]]$$

Within the last part, $M [\Delta t1, \Delta t] = \Delta t1$, due to $\Delta t = \Delta U$ and $\Delta t = \Delta t0$ for the time interval only set.

$$22 \quad A [\Delta t1, \Delta t] = D^* | | \Delta t [M [\Delta t1, \Delta t]] = A [c1 \cdot \Delta t1, c2 \cdot \Delta t1] = \Delta t1$$

This is the time interval only set relation equivalent to the one moment time set relation where usually $c1 = c2 = 1$. When both $c1$ and $c2$ equal scalar 1 it follows: $A [\Delta t1, \Delta t1] = \Delta t1$. This is the same

result derived from time development interpretation only, paragraph 7. Recall c_1 and c_2 are not any scalars, they have the meaning of second associative property arguments for $D^* || \Delta t [M [\Delta t_1, \Delta t_2]$, equation 16, and are possibly asymmetric and unequal. Where equation 21 depends on the second associative property for derivative $D^* || \Delta t$, instead equations 19 and 20 introduce $\text{Rest}(a) | t$ and $\text{Rest}(a) | \Delta t$, without applying associative property arguments. These are two interpretations for the derivative, one from the associative property and one from the 'moving to the right' property, introduced in paragraph 2. When both $c_1 = c_2 = 1$ and with $\Delta t_1 = \Delta t$, the time interval only set relation, $D^* || \Delta t [M [\Delta t, \Delta t]] = \Delta t$, seems to correspond and similar to the usual one moment time set relation: $d/dt [t, t] = c_1 \cdot (1, t) + c_2 \cdot (t, 1) = 2 \cdot t$, however it is not. The time interval set differs from the usual one moment time set also in this way.

10. Time interval only set: multiplication linearity theorem and time interval wave equations

Within the time interval only description, integration and derivative are implicitly defined with equations 13 and 18 in terms of time interval only set multiplication M and addition A . Equations 23 are valid for Δt_1 equal to resp. $cn(\Delta t)$ or $cn'(\Delta t)$ and $\Delta t_2 = \Delta t$ due to the results of paragraph 5. They are valid for any two time intervals Δt_1 and Δt_2 within the time interval only set due to the multiplication linearity theorem that is discussed below. This theorem is equivalent with the multiplication closure theorem, paragraph 8.

$$\begin{aligned} 23 \quad I^* || \Delta t_2 [\Delta t_1] &= M [\Delta t_1, \Delta t_2] \\ D^* || \Delta t_2 [M [\Delta t_1, \Delta t_2]] &= A [\Delta t_1, \Delta t_2] \end{aligned}$$

Comment 15. In the time interval description operators are indicated with $*$, depending on an 'operator quantity' for their interpretation. Operator quantities are indicated with $| \text{operator}^* |$, for example the time operator t^* relates to quantity $| t^* | = t$. Such operator quantities are not defined for I^* and D^* and $I^* || \Delta t_2$ and $D^* || \Delta t_2$, even while 'working to the right' like operators. Recall, paragraph 9, the operator relation $t^* [A_1] = A [t, A_1]$, for t -quantity A_1 , in agreement with the addition concept and the below theorem. Time interval operators are defined in (Hollestelle, 2021).

Since $I^* || \Delta t_2$ and $D^* || \Delta t_2$ give equal results for $\Delta t_2 = \Delta t$, equation 11, similarly M and A give equal results, at least for Δt_1 equal to $cn(\Delta t)$ or $cn'(\Delta t)$. This does not depend on the perspective: to one moment time t -quantity or Δt -quantity, since M and A are time interval multiplication and addition for both perspectives within the time interval description. From equation 13 and 18 and the discussion following 'set rule' equations 19 and 20, with $\Delta t_2 = \Delta t$ one finds $I^* || \Delta t$ and $D^* || \Delta t$ for $cn(\Delta t)$ and $cn'(\Delta t)$ correspond exactly with I^* and D^* for cn and cn' .

M and A depend on time interval properties resembling common domain and combined domain respectively. The difference, due to equation 11 including $I^* || \Delta t [cn(\Delta t)]$ and $D^* || \Delta t [cn(\Delta t)]$, depends on the order for $cn(\Delta t)$ and $cn'(\Delta t)$ and does not return in the result itself in terms of time intervals. $I^* || \Delta t$ and $D^* || \Delta t$, and M and A can have the same results, since this follows from $D^* [M] = A$ and $D^* [M] = M$, properties for $A = A [\Delta t_1, \Delta t] = \Delta t_1$ and $M = M [\Delta t_1, \Delta t] = \Delta t_1$ which are valid for any Δt_1 , due to relations $\Delta t = \Delta t_0$ and $\Delta t = \Delta U$.

Multiplication linearity theorem

Equations $I^* || \Delta t_2 [\Delta t_1] = M [\Delta t_1, \Delta t_2]$ and $D^* || \Delta t_2 [M [\Delta t_1, \Delta t_2]] = A [\Delta t_1, \Delta t_2]$ are valid for any two Δt -quantities, ie any two time intervals Δt_1 and Δt_2 from the time interval only set.

The theorem can be derived directly from the definitions for M and A however these are definitions only by inference, and depend on the multiplication linearity theorem for general validity. Instead applied are MVT equilibrium and time interval derivatives. The closure theorem, paragraph 8, is exactly similar in result, however the derivation is different.

The specific Lorentz transformation TL defines $\Delta t'$ from Δt with $cn'(\Delta t) = cn(\Delta t')$, paragraph 4. Transformation TL and the specific change Δt to $\Delta t'$ such that Hamiltonian H remains invariant is introduced in (Hollestelle, 2021). For other changes Δt_1 to Δt_2 , H can be variable. For transformation TL with Δt to $\Delta t'$, the multiplication linearity theorem is derived for $\Delta t_1 = \Delta t$ and $\Delta t_2 = \Delta t'$, part I. For all other transformations TN the linearity of Δt_2 with Δt_1 is assured, even when the linearity constants do not agree with the specific transformation TL, part II. Part III is the no-reverse case.

I. Define $cn'(\Delta t) = cn(\Delta t')$. Since due to MVT equilibrium $cn(\Delta t)$ and $cn(\Delta t')$ are linear in their respective relevant event time interval, Δt and $\Delta t'$ before and after transformation TL, and since from the definition of TL follows trivially the linear transformation Δt to $\Delta t'$, this relation can be reversed unless for instance $D^* \mid \Delta t$ for $cn(\Delta t)$ or $cn'(\Delta t)$ equals the time interval set 'multiplication zero' $\Delta U0$. In the reverse case $\Delta t'$ is linear in $cn(\Delta t)$ and similarly $\Delta t'$ is linear in Δt . This means the theorem is valid for these $\Delta t1 = \Delta t$ and $\Delta t2 = \Delta t'$.

II. For any $\Delta t2$, a nonspecific Lorentz transformation TN from $\Delta t1$ to $\Delta t2$ is a time interval addition, where H is time dependent. Addition means linear transformation, since it means MVT equilibrium time development. Similarly, when $\Delta t2$ linear in $\Delta t1$ a linear transformation does exist, this is discussed in paragraph 12, part III. The theorem is valid for any time interval $\Delta t1$ and $\Delta t2$, and for any Δt -quantities being linear in Δt , except for the no-reverse case.

An interpretation of change for Δt is necessary. For the changed situation the validity of the following is to be ensured: $M' [\Delta t1, \Delta t'] = M'' [\Delta t1, \Delta t''] = \Delta t1$, ie with the situation change the relevant event time interval $\Delta t'$ changes to $\Delta t''$ while specific properties for M and A do not change and are defined with the relevant event time interval indication Δt , one of these properties $\Delta t = \Delta U$. This agrees with star source radiation propagation, paragraph 5 and (Hollestelle, 2020). It depends on the situation present: before change $\Delta t = \Delta t'$, after change $\Delta t = \Delta t''$, where situations for $\Delta t'$ or $\Delta t''$ share properties, however not all properties, to assure change.

III. The no-reverse case

In this case a transformation TN changes $\Delta t1$ to $\Delta t2$ however cn and $cn' = cn$ remain invariant: $cn(\Delta t1) = cn(\Delta t2)$ and invariance cannot be reversed. Recall cn' relates to cn with the specific Lorentz transformation TL. Consider change $\Delta t1$ to $\Delta t2$ with distinct $\Delta t1 \neq \Delta t2$, not considering $\Delta t1 = \Delta t2$, and with $\Delta t1 = \Delta t$ before change. The time development for invariant $cn(\Delta t1)$ is $A [cn(\Delta t2), cn(\Delta t1)iv] = M [+/-1. D^* \mid \Delta t [cn(\Delta t1)], A [\Delta t2, \Delta t1iv]] = \Delta t0$, paragraph 2.

A. For $D^* \mid \Delta t [cn(\Delta t1)] = \Delta U0$, one finds $A [cn(\Delta t2), cn(\Delta t1)iv] = \Delta U0$. Then $\Delta U0 = \Delta t0$, ie 'multiplication zero' equals 'addition zero' for the time interval only set, has time interval domain measure zero and is not a proper time interval, comment 14, and $cn(\Delta t2) = \Delta U0$, for all $\Delta t2$, and is not well defined.

B. For $D^* \mid \Delta t [cn(\Delta t'')] \neq \Delta U0$, at least one of $\Delta t1$ and $\Delta t2$ equals $\Delta t = \Delta t0$ and $A [\Delta t2, \Delta t1] = A [\Delta t1, \Delta t2]$ equals $\Delta t2$ or $\Delta t1$ and similarly for $M [\Delta t1, \Delta t2]$. Say $\Delta t1 = \Delta t = \Delta t0$, the other possibility is similar.

It follows $M [+/-1. D^* \mid \Delta t [cn(\Delta t1)], \Delta t2] = \Delta t0$ for all $\Delta t2$ and $cn(\Delta t1) = \Delta U0$, for all $\Delta t1$, and is not well defined and $\Delta t0 = \Delta U0$. *This completes the derivation for the multiplication linearity theorem.*

From equations 23 for $cn(\Delta t)$ or $cn'(\Delta t)$, and the multiplication linearity theorem follow equations 24 to 27.

$$\begin{aligned}
 24 \quad & I^* \mid \Delta t [cn(\Delta t)] = A [cn(\Delta t), A [\Delta U, cn(\Delta t)]] = A [cn(\Delta t), cn(\Delta t)] \\
 & I^* \mid \Delta t [cn'(\Delta t)] = M [cn'(\Delta t), \Delta t] = A [cn'(\Delta t), A [\Delta U, cn'(\Delta t)]] = A [cn'(\Delta t), cn'(\Delta t)] \\
 & A [a. \Delta t1, \Delta t] = D^* \mid \Delta t [M [a. \Delta t1, \Delta t]] = D^* \mid \Delta t [a. I^* \mid \Delta t [\Delta t1]] = A [a. D^* \mid \Delta t [I^* \mid \Delta t [\Delta t1]], \\
 & \text{Rest}(a) \mid \Delta t]
 \end{aligned}$$

Equation 23 is valid including $\Delta t1 = cn(\Delta t)$ or $cn'(\Delta t)$. Not necessarily $cn(\Delta t)$ follows the requirement of time interval asymmetry like Δt itself for situations when H time dependent, and - 1. $cn(\Delta t)$ is valid however -1. Δt is not. Applying $I^* \mid \Delta t$ from equation 11 it follows:

$$\begin{aligned}
 25 \quad & a. cn(\Delta t) = D^* \mid \Delta t [a. I^* \mid \Delta t [cn(\Delta t)]] = A [a. D^* \mid \Delta t [I^* \mid \Delta t \\
 & [cn(\Delta t)], \text{Rest}(a) \mid \Delta t] = A [a. D^* \mid \Delta t [A [cn(\Delta t), -1. cn'(\Delta t)]], \\
 & \text{Rest}(a) \mid \Delta t]
 \end{aligned}$$

Time development is equal for $cn(\Delta t)$ and $cn'(\Delta t)$, supported by the definition $cn'(\Delta t) = cn(\Delta t')$ and linearity of $cn(\Delta t)$ in Δt . For scalar $a = 1$, $\Delta t1 = cn(\Delta t)$ and $\Delta t2 = \Delta t$ there is $\text{Rest}(a) \mid \Delta t$ is non zero and equal to $cn(\Delta t)$. Scalar a can be 'moving to the left' from $I^* \mid \Delta t$ without any non zero factor Rest , however from $D^* \mid \Delta t$ only non-linearly with 'set rule' equations 19 and 20 including $\text{Rest}(a) \mid \Delta t$. Due to property $A [\Delta t1, \Delta t1] = \Delta t1$ for any time interval $\Delta t1$ including $\Delta t1 = cn(\Delta t)$, factor $\text{Rest}(a) \mid \Delta t$ can be left out and one finds $D^* \mid \Delta t$, and $I^* \mid \Delta t$ are exactly the same.

$$26 \quad a. \text{cn}(\Delta t) = A [a. D^* | \Delta t [I^* | \Delta t [\text{cn}(\Delta t)]]],$$

Rest(a) | Δt] For $a = 1$ and with Δt -quantity $\text{cn}(\Delta t)$

linear with Δt , one finds:

$$27 \quad \text{cn}(\Delta t) = D^* | \Delta t [I^* | \Delta t [\text{cn}(\Delta t)]] = A [D^* | \Delta t [I^* | \Delta t [\text{cn}(\Delta t)]]], \text{cn}(\Delta t)$$

Time interval wave equations and structure constants

The equations 26 and 27 are similar to second order wave equations. From the discussion at the beginning of this paragraph it follows ΔN equals both addition and multiplication of the reciprocal pair of commutation quantities.

The derivative $D^* | \Delta t [\Delta N] = A [\text{cn}(\Delta t), \text{cn}'(\Delta t)] = \Delta N$, equation 11, and similarly the second derivative $D^* | \Delta t [D^* | \Delta t [\Delta N]] = \Delta N$ and equations 25 apply to ΔN . These equations are valid due to the derivatives being time interval set, MVT equilibrium, derivatives. Recall ΔN is an invariant for the overall time interval set, and ΔN , like any Δt -quantity, is part of the linearity 'subset' for $\text{cn}(\Delta t)$ or $\text{cn}'(\Delta t)$. This is also discussed in paragraph 12, part III.

From combinations of commutations, being multiplications within the generator set, one finds the set structure constants from the second derivatives, (De Wit, Smith, 1986), (Veltman, 1974). Structure constants for the time interval set depend on $D^* | \Delta t [D^* | \Delta t [C]]$ for some combination C , and from the multiplication time development property, paragraph 8 property II, multiplication within the generator set equals multiplication within the time interval set, and this means ΔN is a possible C . The second derivative invariants are Lorentz transformation TL invariants, due to the specific TL being a surface measure preserving transformation, unlike the traditional Lorentz transformations TU. Due to this the overall TL invariant ΔN can be interpreted directly to be a structure constant. The commutation constants form an n-pair and the existence of an overall equilibrium invariant, in this case $\Delta N = M [\text{cn}(\Delta t), \text{cn}'(\Delta t)]$, implies the following properties for the structure constants.

Comment 16. The dimension n of the set decides the number of structure constants within one n-pair for this set. Comment 17. For the time interval only set the n-pair is an ordinary, reciprocal, pair, due to both time and the time interval only set being one dimensional, and any time interval being linear in Δt , ie due to the closure theorem.

11. Part 2. Star-source radiation, time development, and propagation surfaces

Star source radiation, interpreted with propagation sphere surfaces, is different from radiation with stationary parallel propagation. The metric surface measure, not meaning the usual line element dependent metric, for propagation surfaces is invariant with change of relevant event time interval Δt that indicates time development within the time interval description for radiation fields (Hollestelle, 2021). Radiation is interpreted with waves or complementary wave particles of zero or non-zero mass with group-velocity $c(\Delta t)$, c for photons. An effort to describe the resulting 'wrinkled' radiation propagation surface for changing Δt is the following variation on a theorem from topology. From this description one can derive some results.

The 'wrinkled' radiation propagation surface

Propagation surface A related to radiation time interval Δt can be idealized with a regular sphere $Q(\Delta t)$, that does not fulfill the surface measure requirement. However A can be approximated with an indecomposable continuum surface P , the union of a constant number N of disjoint, open and dense, subsets P_{ci} , $i = 1$ to $N > 1$. The P_{ci} each are the limit for subsets $P_{ci}(\text{parameter})$ depending on some *subset parameter* series with an infinite limit, and the P_{ci} together are a covering space for Q , applying a construction from topology (Hocking, Young, 1961).

Similar to this is a new construction of subsets P_i , with a variable number N , and indecomposable continuum limit P : with a variable parameter w for subset width, one allows varying metric surface measure for the limit subsets P_i , the *subset parameter* series is introduced like with P_{ci} , and the P_i together remain a covering space for Q . It is inferred, when choosing a variable subset number N relative to w , which is only possible for the new subsets P_i , one can always arrange for the complete metric surface measures $m_A(\Delta t) = m_P(N, w, \Delta t) > m_Q(\Delta t)$ with the surface A being a 'wrinkled' covering space for Q . One can consider $m_A(\Delta t) = N \cdot m_P(N, w, \Delta t) > m_Q(\Delta t)$, together with decreasing parameter w the *subset parameter* increases 'sufficiently', which seems similar to the 'time lapse' variation for MVT equilibrium in terms of time intervals, paragraph 7.

Assumed is $c(\Delta t)$ remains invariant. It is argued the metric requirement for A does not apply the linear-space metric, the line element, even when the metric surface measure of a regular sphere usually relates to the metric radius measure squared. The limit parts P_i being 'linear'-like, due to decreasing w , does not imply the line element metric for them.

Comment 18. The construction of subsets to describe the propagation wave with surface P does not mean it is implied the propagation surface necessarily is divided into photon path destination parts. The discussion of propagation starting from the concept of photon paths in qm is not the subject of this paper.

For situations with Δt invariant, number parameter $N \geq 1$ still is undecided relative to parameter w . When N is independent and undecided it supports a degree of freedom. For any N , each of the parts $P_i(N, w, \Delta t)$ is dense in Q and the surface measure m_A remains equal to $m_P > m_Q$.

One interpretation is, when energy does not change with N , a change of N implies a symmetry transformation. Any $N \geq 1$ depending on w is realistic for propagation surface A , to remain a covering space for the regular sphere Q . A second interpretation is, implied is a situation with zero temperature. Several star-source radiation applications from (Hollestelle, 2021) allow for a zero temperature and at least two results are worthwhile.

Comment 19. Star-clouds, in terms of radiation

A description for star-cloud radiation, including temperature dependence for the overall star-cloud and for the star sources themselves is given in (Hollestelle, 2021). With star-cloud is meant a combination of stars grouped relatively together at a certain distance from propagation surface A , thus also at this distance from a possible measurement place q at A . Zero temperature means when, for star i with space interval Δq_i , ie place q_i relative to q and propagation surface A , and space interval ΔQ_i , ie q_i relative to average $\langle q_i \rangle$ || cloud, and $\langle \Delta Q_i \rangle$ || cloud = A || cloud ΔQ_i , meaning their average 'equals' their sum applying addition A , $\langle \Delta Q_i \rangle$ || cloud is approximately zero while the cloud source is localizable, which has implications for the symmetry of the star-cloud itself in space. Recall 1-dim. time development does not, where however 3-dim. space does, allow time translation symmetry or time inversion symmetry, for any time interval Δt . For H time dependent and finite time intervals, time development from MVT equilibrium is linear and non canonical, paragraph 10, however, within GR, Lagrangian equilibrium transformations should be non linear (Jauch, Rohrlich, 1955), (Hollestelle, 2020). When H reaches time independence with infinite time intervals, time development from MVT equilibrium means linear and canonical transformations.

Another zero temperature result is, star source i at q_i , for all i , is 'remote' from propagation surface A , where A includes place q . The concepts 'close to' and 'remote', discussed in paragraph 12 II, indicate space intervals while they are applied for measurement events defined with time intervals and propagation surfaces. The equations of state for high or low temperature equilibrium are derived in (Hollestelle, 2017), for a set of spherical surfaces with inside spikes interpreted as a concept for star-clouds, still within the one moment time description. One finds, a 'remote' localizable overall star-cloud source with radiation related to non-zero mass particles at a measurement place q' related to $\Delta t'$ will appear like an extremely 'remote' overall star -cloud source at the measurement place q'' related to $\Delta t''$ when $|\Delta t''| \gg |\Delta t'|$, with radiation related to wave particles with near to zero mass, in the infinitely 'remote' this is the zero mass limit, photons.

Simultaneous events and simultaneous distributions

Simultaneous events defined from propagation surface A need an interpretation, depending on 'timely' time intervals for radiation measurements, which is assured by the overall relevant event time interval Δt being 'timely' (Hollestelle, 2020). The term 'timely' implies: a time interval $\Delta t'$ is timely when it is itself 'simply measurable', meaning with time interval measure result $|\Delta t'|$ from one measurement within Δt . This resembles Einstein's discussion of simultaneity and 'locality' for space-time in the one moment time description (Einstein, 1923 (a)), however the differences with simultaneity for time intervals are severe due to time intervals not being translation invariant in the time interval description. Simultaneous measurement events for any $\Delta t'$ and $\Delta t''$ are possible when $\Delta t'$ and $\Delta t''$ being both timely within the relevant event time interval Δt , and Δt itself timely. With propagation surface energy is meant the complete energy, from the star source to the propagation surface, from all radiation 'on the way' while emitted during Δt , (Hollestelle, 2021).

Any distribution is assumed to be a simultaneous distribution. For the 2-dim radiation propagation surface A a distribution P_i , $i = 1$ to $N > 1$, is possible because it does not imply similarly a distribution $\Delta t_i \neq \Delta t$. Any distribution for A remains a covering space for the same Q and thus remains with the same time interval Δt , time development being the same for the complete A and Δt by definition. A distribution for the radius or line of sight for A implies a distribution for time interval Δt which is not allowed since Δt is to remain invariant, since it is the relevant event time interval for surface A. This is related to the specific Lorentz transformation TL with surface measure preserving property, paragraph 4.

'Zero mass' photons and star-source radiation energy in terms of time intervals

Assume a non-zero mass m_1 for wave particles complementary to radiation from a star-source, and introduce the mass m_2 dependent on m_A and M [m_1 , m_2]. Field energy E_e is linear with M [m_1 , m_2] and inverse linear with $|\Delta q|$, where $|c(\Delta t)| = |M [\Delta q, \Delta t]|$ is the wave group-velocity. The group-velocity in fact depends on the space time metric, and equals the 'apparent velocity' of the associated complementary wave particle, (Goldstein, 1950), (Hollestelle, 2020). In this case the space time metric is assumed diagonal and symmetric in the space coordinates, to allow the above expression for $c(\Delta t)$. This is interpreted with gravitational field energy E_e for situations without external interaction. The e.m. wave propagation surface energy, kinetic energy E_s , equals E_e , implying the propagation surface properties relate to m_2 . From the definition for $c(\Delta t)$, there is $\Delta q_i = M [\Delta t_i, c(\Delta t)] = c(\Delta t)_i$:

$$28 \quad E_s = A [1/|c(\Delta t)|, M [m_1, m_2]]$$

$$E_e = M [\Delta q_i, M [m_1, m_2]] = D^* | \Delta q [M [m_1, m_2]] = D^* | \Delta t [M [1/|c(\Delta t)|, M [m_1, m_2]]]$$

With equation 28 one finds a definition for non-zero mass m_2 , that can be identified with a source mass allowing the gravitation energy interpretation for E_e .

This does not mean the source mass m_2 is equal to the mass of the star-source. Invariance of a quantity does not mean D^* or $D^* | \Delta t$ for this quantity have to be zero. For e.m. radiation and photons, with m_1 equal to the mass 'addition zero', it follows $E_s = A [1/|c(\Delta t)|, m_2] = A [\Delta q_i, m_2]$, and E_s can be interpreted to be the source mass apparent density. With $1/|c(\Delta t)| = M [\Delta q_i, m_1]$ when m_1 the mass 'multiplication unit', there is $c(\Delta t)$ equals the multiplication inverse for the propagation wave particle mass apparent density.

When one defines vacuum with total energy H_0 equal to zero, and with kinetic radiation energy $E = \#n \cdot h \cdot \nu$, for $\#n$ the photon number and wave energy $h \cdot \nu$, one can write Hamiltonian $H = H_0 + \Delta H$, with energy ΔH introduced in (Hollestelle, 2020). Within the time interval description, a time dependent $H = H_0 + \Delta H = E + V + \Delta H$ is the time interval version of the Legendre transform of Lagrangian $L = E - V$, with E kinetic energy and V potential energy and ΔH depending on the t -quantity $\#n$, possibly variable due to interaction within Δt . Applying wave particle duality and with relevant event time interval $\Delta t = [t_b, t_a]$, for radiation 'on the way' while emitted during Δt , energy E_s equals:

$$29 \quad E_s = h \cdot \nu = M [h^+, \Delta t_i]$$

The 'Planck-like' function h^+ is defined with: $h^+ = 1/2 (\Delta^* p \cdot \Delta^* q + \Delta^* q \cdot \Delta^* p)$, and is variable within the one moment time description, with indication Δ^* for variances, different from indication Δ for

time intervals (Hollestelle, 2020). Δt_i is the multiplication inverse for Δt . Due to h^+ one moment time t_b and t_a occur in equation 29. Energy E_s , without interaction or wave function reduction during Δt , is an invariant Δt -quantity during Δt .

Comment 20. From equations 28 it follows $h^+ = m_2$, however within the time interval description, where a traditional dimensional analysis can fail unexpectedly, like for instance with $M [\Delta t, \Delta t] = \Delta t$. This is discussed from the definition for the 'multiplication unit' within the one moment time set, $t, t_0 = t_0, t = t$, in (Hollestelle, 2020).

With $c(\Delta t) = M [\Delta q, \Delta t_i]$ invariant, $c(\Delta t)$ is a Δt - quantity linear in Δt including non-scalar interval multiplication with Δq . The complete propagation surface A from star- source to propagation surface depends on Δt and $c(\Delta t)$ due to time development. Mass m_1 equal to mass 'addition zero' can be allowed due to m_1 and m_2 not being scalars rather Δt -quantities where m_1 , also equal to the mass 'multiplication unit', agrees with $\Delta t_0 = \Delta U$, while E_s does not reduce to zero. Then, for wave particles with 'zero' mass m_1 , ie 'zero mass' photons, this means m_1 is both mass 'addition zero' and mass 'multiplication unit', different from mass 'multiplication zero' agreeing with ΔU_0 , and for any source mass m_2 there is:

$$\begin{aligned} 30 \quad A [m_1, m_2] &= A [m_2, m_1] \\ &= m_2 M [m_1, m_2] = M \\ [m_2, m_1] &= m_2 \end{aligned}$$

Including $\#n$ the number of photons, non-interacting radiation propagation surface energy $E_s = \#n \cdot h \cdot \nu$ increases with $\#n$, however the overall photon mass $m = \sum (i = 1 \text{ to } \#n) m_i = M \#n$, with $M_1 = m_1$ and $M_{i+1} = A [M_i, m_1] = m_1$, remains invariant and equal to m_1 , the mass 'addition zero' for any $\#n$. Within the time interval set and writing all quantities from multiplications with time intervals, wave particle energy $h \cdot \nu = M [h^+, \Delta t_i] = h^+$ in value, where $h^+ = M [\Delta q_i, m_2] = m_2$ in value, the source mass. In the case of 'near' Newtonian situations, with distant star-source, the radiation energy $h \cdot \nu$ relates to source mass m_2 only.

12. Part 2. Star-source radiation and the time interval only set

The description in this paper depends on commutation properties for certain sets. Depending on the considered set dimension, these quantities form reciprocal n -pairs, for the time interval only set, a one-dimensional set, n - pairs are 1-pairs or just pairs and $n = 1$. Where the number of degrees of freedom increases due to these n -pairs, equilibrium means a reduction, the equilibrium requirement, depending on all quantities from the n -pair, and overall invariant Noether charge NC_{set} . The Noether charges turn out to be structure constants for the relevant set, in this paper the time interval only set. It is argued, because of this there exists an n -pair of equivalent energies, kinetic radiation- and gravitation energy, with time development opposed to each other.

Commutation relations, like from equations 1 and 11, assume the complexity of the dimension of the time interval set. The 'working to the right' property introduces an n -pair of quantities, similar to an n -pair of boundaries for time intervals: there is applied only one 'one moment time' parameter and only one relevant event time interval, equations 1 and 11, to derive time interval only derivatives including the derivative 'set rule'. A 'set rule' makes sense with factor Rest non zero, for non trivial commutation relations. This paper's subject is the time interval only set and results are discussed with regard to time development. The following results, part I to V, imply star- source radiation propagation with finite propagation sphere like surfaces different from when radiation extends parallel and stationary.

From the definition for the complete energy for radiation propagation surfaces originating from star sources one finds radiation properties and source properties depend on each other, and on properties of space time. In particular 'close to' symmetry and 'timely' seem reciprocal concepts, discussed in part II. Interestingly one recognizes a space interval related property and a time interval related property where both depend on equilibrium.

The equivalence principle and equivalent reference energies

According to general relativity and the equivalence principle, kinetic energy during acceleration is equivalent to gravitation energy during the decrease of space-like distance of the source masses, disregarding the minus sign, and it is not possible to decide which reference, or interpretation, is due, (Newton, 1686 (c)), (Einstein, 1923 (b)). Similarly a new equivalent reference energy pair is defined for radiation: e.m. kinetic wave energy E_s , and gravitational energy E_e . Similar to arguments for reciprocal quantities, paragraph 4, the reference pair is assumed to be related to each other by a transformation TR, even though they are not reciprocal in value. The energies E_s and E_e have the same value, each from their own reference, not meaning the respective 'reference' spacetime coordinate system, rather like the wave and complementary wave particle description within qm. This equivalence for E_s and E_e differs from the usual relation of kinematics with dynamics, Newton's first law, (Newton, 1686 (a)), since considered is energy only, not action. Radiation propagation implies dispersion free non-interacting radiation complementary with free moving wave particles. Star source radiation propagation relates to invariant e.m. surface field energy including increasing distance from the source and outward time development, (Hollestelle, 2021).

Due to MVT equilibrium assured is $I^* || \Delta t \text{ and } D^* || \Delta t$ having the same result within the time interval only set, paragraph 10. It follows outward radiation is equivalent to inward gravitation in terms of energy. From equation 28, $E_s = A [1/|c(\Delta t)|, A [m_1, m_2]]$, and $E_e = M [1/|c(\Delta t)|, M [m_1, m_2]]$. The radiation kinetic energy E_s reference includes 'addition' or derivatives meaning one way propagation, the gravitation energy E_e reference includes 'multiplication' or integration meaning two way internal 'cross over'. Both results agree with Newton's second law by applying differentiation to equations 28, meaning E_s by derivative to Δt and E_e by derivative to Δq while coordinate Δq being positive and decreasing (Newton, 1686 (b)), (Goldstein, 1950), (Hollestelle, 2021). In particular, the derivatives to resp. Δt or Δq imply opposite signs, introducing the opposite signs for the time development for e.m. radiation and gravitation. The mass quantities related to E_s and E_e regarded to be identical, within equations 28 is similar to the equivalence principle and for the usual interpretation of matter. One can apply parts of the description of reciprocal quantities, paragraph 4, to reference energies E_s and E_e .

From the discussion for the pair of time interval only commutation quantities $cn(\Delta t)$ and $cn'(\Delta t)$, and the specific Lorentz transformation $TL(\Delta t) = \Delta t'$ with invariance for $cn'(\Delta t) = cn(\Delta t')$, correspondence C2, paragraph 4 and 5, introduced is reference transformation TR, with $TR(E_s) = E_e$ and vice versa. Similarly, energies E_s and E_e are Δt - quantities with invariant Δt that allow application of A, M and $D^* || \Delta t$ even while they do not change reference, ie remain or radiation or gravitation.

A trial transformation is: E_s and A [$E_s, \Delta t$] transform to E_e and M [$E_e, \Delta t$], from addition to multiplication or the other way around, where evidently the values for E_s and E_e , for invariant Δt , remain unchanged, similar like for equations 28 with A and with M.

With $D^* || \Delta t [E_s] = E_s$ and $D^* || \Delta t [\Delta t] = \Delta t$ and with $\Delta t = \Delta U$, there is $E_s = M [\Delta t, E_s] = M [E_s, \Delta t]$ and it follows $E_s D^* || \Delta t [M [E_s, \Delta t]] = A [c_1. E_s, c_2. E_s] = A [A [c_1. E_s, c_2. E_s], \Delta t]$, from equations 16 and 18. The trial transformation results in $E_e = TR(E_s) = M [M [c_1. E_e, c_2. E_e], \Delta t] = M [c_1. E_e, c_2. E_e]$ due to $E_e = M [c_1. E_e, c_2. E_e]$ from the multiplication linearity theorem, paragraph 10. This also means: $D^* || \Delta t [E_s] = A [c_1. E_s, c_2. E_s]$ and $D^* || \Delta t [E_e] = M [c_1. E_e, c_2. E_e]$.

Comment 21. Due to $D^* || \Delta t [cn(\Delta t)] = A [c_1. cn(\Delta t), c_2. cn(\Delta t')]$ with derivative constants c_1 and c_2 , and the results of A and M being equal, the derivative $D^* || \Delta t$ of a multiplication of the n-pair of Δt -quantities defined with TL relates to the overall time interval set average for Δt -quantity $cn(\Delta t)$. With $cn(\Delta t') = cn(TL(\Delta t)) = cn'(\Delta t)$ and with $cn(\Delta t)$ different from $cn'(\Delta t)$ only from their time interval domain difference, c_1 and c_2 depend only on these domains. *Comment 22.* Within the e.m. radiation energy E_s reference, the m_1 and m_2 represent the Δt and $\Delta t'$ radiation properties including $c(\Delta t)$ and the star-source, from $E_s = E_e = M [\Delta q, M [m_1, m_2]] = D^* || \Delta q [M [m_1, m_2]]$, equation 28, in terms of complementary gravitational mass. The derivative constants c_1 and c_2 relate to the n-pair of Δt -quantities, with properties changing with Δt together with propagation and time development, and for which their multiplication equals Noether charge $\Delta N = NC_{set}$, paragraph 4, an overall time interval set invariant.

It is inferred that derivatives as an expression for geometry are directly related to source interactions, like gravitation energy, for m_1 and m_2 . Since derivatives are part of the usual wave equation description of q_m , this inference means geometry and gravitation can be united with q_m through the second associative arguments for derivatives. Indeed, one way to derive q_m wave functions is from Noether charges (Arnold, 1974). This inference is a time interval only result due to the existence of reference transformation TR. It is expected that the definition of derivatives can be generalized to space-time intervals and second associative property arguments related to Noether charges, for quantities depending on space-time intervals. There is a difference for time intervals and space intervals, time intervals and time do not allow translational or inversion symmetry where 3-dim. space-like intervals do, and, the propagation sphere surface requirement of invariant measure only emerges from Δt , not from Δq (Hollestelle, 2021). This difference is also discussed introducing structure constants, paragraph 10, and from simultaneity, paragraph 11. Several results are included in the following.

I. Radiation energy and wave particle number

Radiation energy E_s , due to external interaction during Δt , depends on one moment time t where $E_s = E_s(t)$ is a t -quantity, with v remaining invariant like for the photo-electric effect. One finds $E_s(t_a) = \#n$. $E_s(t_b) = (\#n)^2$. $h \cdot v$, quadratic on $\#n$ and linear on frequency v . This differs from internal interaction during Δt with variable frequency v , complementary particle mass m_1 and particle number $\#n$, where energy $E_s = \#n \cdot h \cdot v$ remains invariant and $E_s(t) = E_s(t_b)$ for all t during Δt and the description remains within the time interval only set. From these relations one can verify by measurement the relation $E_s = h \cdot v$ for waves and complementary wave particles respectively. This is one of the few relations from q_m that remain valid within the time interval only set description, like proposed in (Hollestelle, 2021).

II. The 'set rule' and the concepts 'close to' and 'remote'

Scalar multiplication $M[\Delta t_1, \Delta t_2] = \Delta t_3 = a$. Δt_a can be evaluated by applying transformation T_2 , discussed in comment 6, and a 'scale' transformation TC , similar to transformations defined in (Hollestelle, 2020). T_2 includes the specific transforms $T_2[t_b] = t_b$ and $|T_2[t_0]| \ll |t_0|$ and $T_2[t_a] = -1 \cdot t_b$, and TC is the scale transformation transforming $\Delta t = [t_b, t_a]$ to $TC[\Delta t] = [C \cdot t_b, C \cdot t_a]$ with C a scalar and relating boundary parameters t to

$TC[t] = C \cdot t = t_0 \cdot (TC[t_0])^{(-1)} \cdot t$, for a certain given parameter $C[t_0]$, except for $t = t_0$, since t_0 is not a possible boundary parameter. T_2 and TC transform time intervals to time intervals while remaining within the time interval set. Boundary time interval Δt_b like within commutator $[1/t_b, a] \mid \Delta t_b$ is the result of repeated application of T_2 and TC to $\Delta t = [t_b, t_a]$ and is a well defined time interval.

Within equation 19 for $Rest(a) \mid t$, the second and third expression are derived independently and this confirms: derivatives $D^* [a] = D^* \mid \Delta t [a]$ are scalar and both equal to the ordinary commutator $[1/t_b, a]u$. This indicates that situations 'remote' from and 'close to' with respect to situations with H time independent are more similar than the terms suggest. With 'Newtonian' is meant: with H time independent and $\Delta t = [t_b, t_a]$ symmetric. The situations 'remote' from and 'close to' Newtonian can be related, by interpretation from MVT equilibrium of the commutators $[1/t, a] \mid \Delta t$ and $[1/t, a]u$, for any Δt or scalar a .

A change of the relevant time interval Δt from $\Delta t'$ to a different $\Delta t''$ includes a change of the interval boundaries.

The time interval derivative of scalar a to one moment time parameter t_b : write $D^* \mid t_b [a]$, is defined to be equal to time interval derivative $D^* \mid \Delta t_b [a] = [1/t_b, a] \mid \Delta t_b$, all scalars with $\Delta t_b = [t_b', t_b'']$ and similarly for t_a with $\Delta t_a = [t_a', t_a'']$. The Δt_b , and Δt_a , are for any t_b' and t_b'' well defined time intervals, due to T_2 and TC transforming time intervals to time intervals. One finds $[1/t_b, a] \mid \Delta t_b = [1/t, a] \mid \Delta t = D^* \mid \Delta t [a]$. For this to define a time interval set version for Stoke's theorem, it should be $D^* \mid \Delta t [a] = a$. This is valid when invariant scalar a corresponds with overall time interval set invariant Δt -quantity ΔN where $D^* \mid \Delta t [\Delta N] = \Delta N$, discussed in paragraph 10.

Discussed below 'set rule' equations 19 and 20 is $D^* || \Delta t [a] \sim D^* [a]$, a correspondence that for scalar a means the equal sign. One also derives $[1/tb, a] || \Delta tb \sim 1/2$. $[1/tb, a]u$ with both correspondences only valid for 'close' to Newtonian situations. It follows $D^* || \Delta t [a] \sim 1/2$. $[1/tb, a]u$, where both quantities are well defined, is a correspondence valid for 'close to' Newtonian situations, ie Δt is 'close to' symmetrical.

However 'set-rule' equation 19 is valid for all situations, H time dependent or time independent, including both expressions, and confirms both correspondences for all situations. This implies some questions concerning what means 'close to' time independent for H and what means 'close to' symmetrical for Δt . In the description of star source radiation 'close to' or 'remote' from symmetrical for Δt means 'remote' from or 'close to' for the propagation surface relative to the star source, (Hollestelle, 2021). This depends on applying the concept of radiation propagation and group-velocity introducing relevant time interval Δt where $\Delta t = [tb, ta]$ can only be symmetric for $|\Delta t|$ approaching infinity. The meaning of distance and asymmetry in terms of time interval Δt can be an introduction to discuss space time itself.

III. Scalar multiplications, scale transformations, unique linear subsets

Since $M [\Delta t1, \Delta t2] = \Delta t3$ belongs to the time interval set due to the closure theorem, and since any time interval is neutral in Δt , one is assured $\Delta t3$ is linear in Δt : $\Delta t3 = a1 \cdot \Delta t$ for some scalar $a1$. Transformation $T2$, together with scale factor $C = a$, provides meaning for scalar multiplication with a time interval result: defining $a \cdot \Delta ta = \Delta t3 = M [\Delta t1, \Delta t2]$ the $a \cdot \Delta ta = T2 [\Delta ta]$ is a well defined time interval. Now write Δta itself linear in Δt . With $ca = \Delta U$ and $a=1$ multiplication $\Delta t3' = M [\Delta U, \Delta ta] = a1' \cdot \Delta t = \Delta ta$. All $a \cdot \Delta ta$ can be expressed with time intervals $a1 \cdot \Delta t$, linear in Δt . By choosing $\Delta ta = \Delta t$ one finds the specific ca and a where $M [\Delta t1, \Delta t2] = a1 \cdot \Delta t = ca$. These linear subsets are like cosets for 1-dim. sets for Δta .

One finds $a \cdot \Delta ta = M [ca, \Delta ta]$ and $a \cdot \Delta ta = a \cdot M [\Delta U, \Delta ta] = M [a \cdot \Delta U, \Delta ta]$ and this means multiplication with 'multiplication unit' ΔU or with ca both leave the linear subset for Δta invariant. Applied is the symmetrical first associative property for M , paragraph 8. Indeed, it follows $M [\Delta t1, \Delta t2]$ is linear in Δta . The linear subset for Δta is the part of the time interval set defined from, time interval or scalar, multiplication with Δta . It follows $M [\Delta t1, \Delta t2] = a1' \cdot \Delta t = a1 \cdot \Delta t$ with scalar $a1 = a \cdot a1'$ and part of the linear subset for Δt . When $\Delta t2$ equals Δt there is $M [\Delta t1, \Delta t] = \Delta t1$ and this confirms the assumption: any time interval $\Delta t1$ from the time interval set can be written like a multiplication that includes Δt , meaning $\Delta t1$ is linear in Δt . This confirms, in terms of linear subsets, time interval multiplication closure and the closure theorems, paragraph 8.

Comment 23. One moment time parameter t development is defined with addition and Lagrange equilibrium, while time interval development depends on MVT equilibrium. Within the time interval description introduced are two different quantities, t -quantities and Δt -quantities. Δt -quantities like Noether charges depend on multiplication, t -quantities like one moment time t coordinates depend on addition. Transformations 'working to the right' like multiplication with t or $1/t$ and integration or derivatives can be non-linear due to the 'set rule' factor Rest, paragraph 9. This means, non-linear events and linear events can be related by applying these transformations within the time interval description.

IV. Noether charges and structure constants

Usually, structure constants are defined from the multiplication properties for the elements of a group, writing these like exponents of what are termed the generators for the set. From assuming the canonical property for the set-elements, it follows the commutator for two generator elements is linear within the generator set, ie gives another generator set element for result, including certain scalar multiplication. These scalars are termed the structure constants. Taking care of the multiplication Taylor series depends on all higher order ordinary commutators of generators that should reduce to first order ordinary commutators which provides a linear result, depending on the structure constants for the set. In this case, with the group being the time interval only set, multiplication implies a linear result with $M [\Delta t1, \Delta t2] = \Delta t3 = a \cdot \Delta ta$ providing the linearity constants

and this is enough to find the structure constants without introducing the generator set which is prohibited since the time interval only set does not include the canonical property.

For any group, structure constants themselves are independent of the group representation. Indeed, for the time interval only set, the structure constants are independent of the number of necessary and different Δt , ie whether the subset of different Δt is reducible or not. Similarly, the Δt subset is not completely determined by structure constant properties. The time interval only set being 1-dimensional means there is one independent subset, and one independent structure constant. Structure constants relate to the essential properties or quantities of the set, the infinitesimal transformations due to time development. These are the group velocity $c(\Delta t)$ and the differentiation arguments c_1 and c_2 , that together introduce the specific relations for space-time and time development, like radiation propagation.

There is $c(\Delta t) = M[\Delta q, \Delta t] = D^* || \Delta t [\Delta q] = [\Delta t, \Delta q] || \Delta t$ and thus $c(\Delta t)$ is equal to ordinary commutation $[\Delta t, \Delta q]u$ which is independent of t within Δt . Some aspects of the non trivial character of $c(\Delta t)$ are discussed in paragraph 11. For Δq a neutral Δt -quantity, $M[c(\Delta t), \Delta t] = c(\Delta t) = M[M[c(\Delta t), \Delta t], \Delta t]$ provides the linearity constant for Δt , and since ordinary commutation $c(\Delta t) = [\Delta t, \Delta q]u = M[M[c(\Delta t), \Delta t], \Delta t]$ is equal to $c(\Delta t) = M[c(\Delta t), \Delta t]$, this is a structure constant for the time interval set. However, $c(\Delta t)$ is not a scalar.

When measuring $c(\Delta t)$ the time interval Δt is the relevant event time interval, which also is the relevant event time interval for measuring Δq . This can be resolved with variable quantity h^+ , paragraph 11. Following the assumptions for non- interacting star source radiation propagation (Hollestelle, 2020), there is $|\Delta^*q| = |\Delta q|$ and $|\Delta^*p| = |\Delta p|$ and this implies h^+ is a Δt -quantity linear in $|\Delta q|$. $|\Delta p|$ and, $h^+ = M[E, \Delta t]$, linear in Δt .

This is similar to energy $E = M[h^+, \Delta t]$, with reference $E = E_s$ or $E = E_e$, paragraph 11, invariant during time development and without interaction. Both h^+ and Δq are linear in Δt since they are Δt -quantities: $E = M[h^+, \Delta t] = [\Delta t, h^+]u$ and $c(\Delta t) = M[c(\Delta t), \Delta t] = [\Delta t, \Delta q]u$.

Since trivially $E = M[E, \Delta t] = M[M[E, \Delta t], \Delta t]$, both $c(\Delta t)$ and $E = M[E, \Delta t]$ can be considered structure constants and are the same.

Both E and h^+ are only introduced to find that starting from different quantities the structure constants for a set should remain the same, since the number of structure constants depends on the dimension of the set, the time interval only set being 1-dim. meaning there is one independent structure constant.

Interaction can be included with $\Delta q = \Delta q(\Delta t)$ or with $h^+ = h^+(\Delta t)$ and time interval differentiation to Δt without applying one moment time parameter t . Introduce the following invariants $c_- = M[c(\Delta t)_i, |c(\Delta t)|]$ for velocity and $h_- = M[(h. v)_i, |h. v|]$ for energy that render dimensionless multiplication results with $c(\Delta t)$ and E respectively: $s_1 = |M[c(\Delta t), c_-]| = |c(\Delta t)|$, and $s_2 = |M[E, h_-]| = |h. v|$ are structure constants for the time interval set. They are the dimensionless scalar constants for the relevant radiation properties $c(\Delta t)$ and $h. v$. The s_1 and s_2 are written differently due to their different reference origin.

The usual Noether charge NC equals $|h. v| = |M[\Delta q_i, m_2]|$ in value, a result from paragraph 11. Also NC equals $|E|^{-2} \cdot |c(\Delta t)|^{-1} = |m_2|$ in value, a result derived in (Hollestelle, 2021): 'the infinitesimal Noether charges are equal to the infinitesimal derivative difference transformations', and this implies Noether charge NC is the same as the structure constants, it is expected to be related to.

The time interval only set is different from the usual groups considered in relation with structure constants, for instance within elementary particle field theory. One difference is the canonical property usually assumed for these groups to derive the multiplication results for the generators, which however is not valid for the time interval only set, due to the existence of non-zero factors $Rest(a) | t$ and $Rest(a) | \Delta t$, equation 19 and 20. It is inferred that there is only one reciprocal pair $cn(\Delta t)$, $cn'(\Delta t)$, since equation 1 includes only one reciprocal pair cn and cn' , and since the time interval only set is 1-dimensional, only one independent structure constant can be defined from $cn(\Delta t)$ and $cn'(\Delta t)$. This means $s_1 = s_2$ are the same structure constant except for their reference dimensional origin. Assuming the overall time interval and space interval averages for $cn(\Delta t)$ and $cn'(\Delta t)$ are the same

and equal to ΔN , paragraph 4, one finds $D^* || \Delta t [\Delta N] = \Delta N$ and one can define $NCset = A [M [c(\Delta t), \Delta N], c(\Delta t)] = \Delta N \sim N = NC$. Applied are the first associative property for multiplication and similarity for M and A , paragraph 10. It follows the one moment time Noether charge, the usual Noether charge, corresponds with the time interval Noether charge. With $NC \sim A [M [c(\Delta t), \Delta N], c(\Delta t)]$, one finds solution time interval quantity $c(\Delta t) \sim NC$, or differently, $c(\Delta t) = \Delta U$, the time interval 'multiplication unit'.

$NCset$ is equal to $M [s1, c_i] = c(\Delta t)$, with $s1 = |M [c(\Delta t), c_i]| = |c(\Delta t)|$. It follows, the time interval Noether charge $NCset = \Delta N$ and $c(\Delta t)$ are the same, and the group-velocity follows exactly from the commutation quantities, in case of photons $NCset = c(\Delta t)$ is the velocity of light c .

From $s1 = |M [c(\Delta t), c_i]| = |c(\Delta t)|$, $s1$ can be interpreted with radiation group velocity value. From $|h. v| = |M [\Delta q_i, m2]|$, $s2 = |h. v|$ can be interpreted with the source mass apparent density value. Since $s1 = s2$, radiation energy $h. v$ and group velocity $c(\Delta t)$ are directly related to source mass apparent density $m2$. Equation 32 implies the derived constant quantities have similar value as expected.

$$31 \quad NCset = \Delta N = Rest(c(\Delta t)) || \Delta t = c(\Delta t) \sim NC = N$$

$$32 \quad |N| = |c(\Delta t)| = |c|$$

$$s1 = s2 = |c(\Delta t)| = |h. v| = |m2|$$

V. The equation-sign and addition commutation

The equations $A [\Delta U0, \Delta t0] = \Delta t0$ and $\Delta U0 = (1 + (-1))$. $\Delta t0 = A [\Delta t0, -1. \Delta t0]$ seem contradictory, unless one considers $\Delta U0$ has zero domain, like in comment 14, and from inference there is extra freedom when quantities can be subject to 'moving to the other side of the equation-sign' including multiplication with scalar -1. It means introducing a 'commutator for addition'. When quantities are included the reciprocal commutation pair, or n-pair, changes to include these quantities and similarly the Noether charge and structure constants, and the number of degrees of freedom, change. The chosen side with respect to the equation-sign can matter and can have different value, depending on the order of the involved time intervals. To investigate this the time interval only set seems to be a good start, however this is not discussed in this paper. A similar freedom seems to reside within re-writing Newton's laws and equilibrium definitions in a similar way depending on equation-sign side.

The confusion above exists in applying $-1. \Delta t0$ to mean $\Delta t0iv$, the addition inverse, within addition equations like $\Delta U0 = (1 + (-1))$. $\Delta t0 = A [\Delta t0, -1. \Delta t0]$, and interpreting $\Delta U0$ to mean a difference including scalar -1, and can be avoided by 'moving to the other side of the equation-sign' properties similar to those from paragraph 2. The differences due to scalar -1, with $(1 + (-1))$. $\Delta t0 = A [\Delta t0, -1. \Delta t0]$ to mean $A [\Delta t0, \Delta t0iv]$, are part of the one moment time description since this description depends on vectors. However, differently, in the time interval only description the addition inverse is defined from $A [\Delta t2, \Delta t2iv] = \Delta U0$ for any time interval $\Delta t2$.

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