Preprints (www.preprints.org) | NOT PEER-REVIEWED | Posted: 21 March 2023 doi:10.20944/preprints202209.0151.v3

Some Comments on a Time Interval Only Description

Harmen H. Hollestelle
Independent researcher, Common Room, Bandung 40191, West-Java, Indonesia

Abstract
The time interval set approach depends on results for radiation propagation from star sources, ie spherically symmetric, where properties relate to propagation sphere surfaces, finite for any realistic event and measurement. In contrast the usual vector and one moment time set approach from Newton's laws, applies to stationary parallel radiation. A time interval only description necessarily has to start from it's own definitions, and properties for time intervals have to be defined with time intervals. This paper is devoted to deriving time interval only set properties commutation, addition and multiplication and derivative, and to define how these can correspond consistently to one moment time set properties. Time development within the time interval description equilibrium depends on the well-known 'mean velocity theorem' (MVT) and the time interval version of the Legendre transform, defined in earlier papers.
Radiation propagation 'away from' and gravitation 'towards' the star source can be part of the description. In the discussion parts these properties are applied within astrophysics. Included is a integral derivation of star source radiation energy for both zero and non zero wave particle mass. The time interval set is not one of the usual sets encountered. However it's non-linearity or non-commutativity property can be related to Noether charges and structure constants, that equal the radiation wave group-velocity, for photons the light velocity *c*, and, different from the usual derivative to one moment time, to specifics of the time interval only set derivative to time intervals.

Key words: time interval, measurement event, derivative, group-velocity, Noether charge, structure constants, star cloud, radiation propagation, complementarity, gravitation, equivalence principle

1. Introduction

Within the time interval description of radiation, values for quantities occur to be finite, from finite radiation propagation group-velocities and from interaction with finite changes. For radiation, star source wave radiation is assumed, with spherical symmetric propagation and wave particle complementarity, to derive properties for the, overall, time interval set. For infinite time intervals the results for the time interval description are the same as for Newtonian vector kinematics, the usual one moment time description, e.g. [Arnold, 1], [Goldstein, 2]. With time intervals being infinite, in this paper termed Newtonian situations, the Hamiltonian H remains time independent, and similarly time intervals being finite implies a measurement event or H is time dependent. Measurement events relate to finite time intervals, where however without interaction the situation remains dispersion free and repeated measurements do not indicate change.

Defined is the relevant event time interval, time interval Δt, the reference interval for any measurement event. The photo-electric effect is an example of an interaction where H is time dependent. The time interval description including time interval dependent derivatives, commutation and equilibrium (MVT) relations, where all time intervals, when H time dependent, are finite and asymmetrical, is defined in earlier papers, e.g. [Hollestelle, 3], [Hollestelle, 4]. Measurements are interrelated with finite time intervals, infinite time intervals however are not excluded by the time interval description. Within a 'dialogue' approach, the Heisenberg view of qm is described among others in [Beller, 5]: particle properties like positions or paths can only be accessed from sequences of interactions or measurements. Within the Heisenberg view the uncertainty relations are the basis of qm [Roos, 6]. According to Beller this is an 'operational' view, which rejects causal space-time description compared to several of more 'conventialist' or 'realist' views which could emphasize the adequacy/consistency of unified concepts besides the reference to 'genuine' aspects of reality (terms by Beller). This philosophical discussion is not part of this paper however. Uncertainty relations continue to be part of qm if only because they relate qm subjects, like elementary particles, photons and electrons, to 'Newtonian' experimental subjects or quantities, accessible from measurements, [Roos, 6], [Sakurai, 7]. Within the time interval description, and assuming de Broglie complementarity, one can derive uncertainty relations, different from the usual uncertainty relations when the Hamiltonian is time dependent, however recovering these when H time independent. In this paper time interval set commutation relations are part of the approach to describe time development. Part I includes the definition of the time interval only description from correspondence by applying the operator concept 'working to the right'. Part II includes applications towards star source radiation in terms of Noether charges and structure constants.

Part I. The time interval only description and t-quantities and Δt-quantities.
Within the time interval description, a quantity can be a t-quantity, variable with one moment time coordinate t during time interval Δt, or a Δt-quantity, invariant during Δt while Δt invariant, for instance like a Δt average, and possibly variable when Δt changes within the overall time interval set. Due to the specific definition of time interval set equilibrium this means linearity in t resp. linearity in Δt. Time interval set equilibrium depends on the 'mean velocity theorem' (MVT), and results are the same as for Lagrangian equilibrium when Δt is infinite and symmetric, however can differ when Δt finite and asymmetric. A correspondence for t-quantities and Δt-quantities

Preprints (www.preprints.org) | NOT PEER-REVIEWED | Posted: 21 March 2023 doi:10.20944/preprints202209.0151.v3

is introduced in paragraph 2 to 5, this allows the time interval only description that not anymore depends on one moment time quantities.
Properties for the time interval only set, like addition, multiplication and derivatives are discussed in the subsequent paragraph 6 to 10. The value of correspondence and the possibility of a time interval only set as a set is also argument for these separated sets in physics. Other results in this paper include Noether charges and structure constants in terms of time interval commutation properties and depend on (a-)symmetry evaluations. These results are valuable by themself, however the value of the time interval only description is that it is an approach to time development, different from the Newtonian kinetic and one moment time approach, due to the application of only time intervals and MVT equilibrium.

The time interval only description, correspondence, and the 'infinite regress' problem.
The time interval and MVT equilibrium description includes time interval boundaries, ie one moment time quantities, due to time intervals and derivatives to time intervals within its results. To find a time interval only description, one can let one moment time quantities correspond with time interval quantities while not plainly equated with time intervals. In this way correspondence can resolve the 'infinite regress' problem in [Hollestelle, 3], when defining equilibrium with the Hamiltonian being the time interval version of the Legendre transform of the Lagrangian L: with time intervals plainly inserted for time interval boundaries, ie inserted for one moment time quantities, this would mean, due to the derivative in the time interval version of the Legendre transform, time interval boundaries return within the definition for time interval equilibrium causing the regress. The term correspondence sometimes is applied otherwise for a relation of qm concepts that correspond with experimental phenomena [Roos, 6], ie interpretation of concepts, an application partly resembling the one in this paper.

Part II. Star source radiation.
The time interval only description assumes star-source radiation which is spherical symmetric rather than parallel. The originating process of star-source radiation, while related to source mass, is not investigated. Assumed is radiation is in 'dispersion free' motion without interaction and it's propagation surface energy remains invariant during time development, while emission energy per time interval depends on the invariant source. Even so, this implies finite time intervals and a finite wave group-velocity, the average for the positive group-velocities for the propagation surface. A propagation surface assumes a space-like limit for simultaneous propagation waves, related to a finite time interval. For mass zero related waves, ie e.m. waves, the group-velocity is assumed equal to c, the finite velocity of light, following GR. The group-velocity for non zero mass waves acquires free particle momentum in terms of $p = h.\ k$, de Broglie complementarity, [Beller, 5].

Radiation and gravitation.
Newton's second law relates kinetics with source related forces, applying vectors and space time coordinates, within a one moment time description, [Goldstein, 2], [Newton (I), 8]. In this paper equilibrium is due to energy rather than action and this allows to discard the question of inertial forces. The interpretation of radiation from time interval perspective includes field energy, change and both finite or infinite intervals and assumes star-source radiation. The time interval only set properties depend on this.
Where the equivalence principle in GR implies equivalence for gravitation energy and kinetic energy, the time interval description implies equivalence for gravitation energy and radiation wave energy, ie the kinetic propagation surface energy due to star source radiation. The subject of 'action at a distance' is unresolved yet within physics, however in GR simultaneity is introduced applying a finite velocity of light. Simultaneity for radiation in this paper implies radiation waves emitted and 'on the way' during Δt, however only when Δt is measurable, this is discussed in paragraph 11.
For both zero mass e.m. radiation waves and massive particle waves, an integral description of these phenomena is derived, without other assumptions, within the time interval only description. For time interval Δt finite and asymmetric, discussed are the concepts 'remote' and 'close to', in terms of space intervals and radiation star sources being localizable or not, and in terms of simultaneity, paragraph 11 and 12.

The 'sign' of radiation and the 'sign' of gravitation from opposite time development.
A finite radiation propagation surface obviously should be asymmetrical due to its time development and propagation away from the source. Interestingly the same can be mentioned for sources including electric charge. In terms of surface energy fields this means the surface tends to increase or decrease distance from the source due to its geometry and asymmetry, and such fields have been studied before, including with 'spike' dependent energy with the usual curvature constants, or with geometry dependent 'surface-depth' interaction energy, [Hollestelle, 9]. Curie's principle, [Curie, 10], turns out to agree with Newton's second law, for asymmetric propagation surfaces with wave group-velocity related time development. Within a one moment time description radiation propagation is away from the source where gravitation as a source for movement is directed towards the mass sources. Not considered in this paper is a Higgs mechanism, the result for zero temperature seems related to this, paragraph 11. When considering actions and related derivatives within the time interval set, it is possible to describe asymmetrical time development and gravitation with, opposite to

radiation propagation, time development towards the sources.

2. Commutation relations and derivatives in terms of 'working to the right'

For operators in field theory the concept 'working to the right' usually is introduced from commutation relations and derivatives to one moment time coordinates: operators are 'working to the right' on state vectors, [Roos, 6]. In this paragraph a correspondence for the one moment time set and the time interval set, applying commutation relations and derivatives to time intervals, includes the 'working to the right' property for time intervals instead of operators. Correspondence, then, prepares a way to approach the 'infinite regress' problem and to derive time interval only set properties. For this purpose one moment time and time interval commutation relations are necessary however for a complete discussion other coordinates and intervals, for instance applying a covariant approach, should be included. From the 1-dimensional time interval set one derives two commutation quantities cn and cn' with one independent commutation relation. With higher dimensional sets, different numbers of quantities and relations are involved.

For t-quantities, the usual description is one where these depend on one moment time t, and Lagrangian equilibrium. The Newtonian kinematic description is a one moment time description. The usual commutation is defined with [A1, A2]u = A1(t). A2(t) - A2(t). A1(t), indicated with subscript u, for two t-quantities A1 and A2 at the same one moment time t. With the multiplication . and the usual + and – indications, the following equations are assumed that together define commutation relations for one moment time t. It is assumed that multiplication with t is order dependent.

1 t. cn' = cn. t

1/t. cn = cn'. 1/t

2 [t, cn]u = t. (cn – cn')

[1/t, cn]u = - (cn – cn'). 1/t

The cn and cn' are assumed to be different from each other, coordinate t not commutation free, and the only commutation quantities related to one moment time coordinate t.

The commutation equations relate one moment time t to the one moment time derivative to t: d/dt, including derivatives for cn and cn' and derivatives for multiplication A1. A2, meaning A1 'multiplication' A2, writing d/dt [A1. A2] for t-quantities A1 and A2. Derivative d/dt is 'working to the right', unto the square brackets, on every part of a multiplication within the usual first order differential calculus, [Newton (II), 11], with the following distributive, ie symmetrical second associative, property for derivatives:

3a d/dt [A1. A2] = d/dt [A1]. A2 + A1. d/dt [A2] = 2. A1. d/dt [A2] + Rest

3b Rest = (d/dt [A1]. A2 - A1. d/dt [A2])

Assumed is the order of the parts on the right side can be changed for A1 and A2 for the one moment t description derivative d/dt. Trivially however essentially in the second part of this equation d/dt is 'working to the right' on A1 or A2 only. The third part is decisive for the interpretation of equations 1 and 2. While equation 3 includes only one step, with d/dt 'moving to the right' this leaves to the left a quantity, ie at the first step: the quantity A1 'moving to the left' of d/dt, including multiplication with a scalar, this is scalar 2, and a factor Rest between ordinary brackets. This is similar to one moment time t or 1/t 'moving to the right' leaving to the left cn' or cn respectively in equations 1. Equations 1 and 2 and equation 3 together define the properties of cn and cn' and d/dt. The derivative defined with equation 3 is discussed more generally in paragraph 9, from the introduction of the possibly nonlinear derivative 'set rule' to interprete the factor Rest, which can be different for each set.

Within the time interval description, multiplication, addition and derivatives can be applied to t-quantities and to Δt-quantities. From equations 1 and 2 similar equations can be derived for the same cn and cn' now regarded to be Δt-quantities instead of t-quantities. Assumed is that the described situation is 'remote' from H time independent, with time interval Δt = [tb, ta] asymmetric and finite such that |tb| << |ta|. In this case cn and cn' are invariant during Δt and can be regarded Δt-quantities, within the time interval description including MVT equilibrium this means they can be described being linear with Δt, and are not described as t dependent, linear or otherwise. The indications A and M mean addition and multiplication within the time interval set, A is evaluated in paragraph 7, M is evaluated in paragraph 8. Equations 10 in paragraph 5 are commutation relations for Δt-quantities cn and cn' without reference to one moment time t and part of a complete and closed time interval only description.

Time interval derivative. The one moment time description for the usual derivative to one moment time t of quantity A1, is d/dt [A1], and within the time interval description with D*|t [A1] = D* [A1] = A [1/t. A1(t), –1. A1(t). 1/t] = [1/t, A1]u, the usual commutator, disregarding a factor 1/2, [Hollestelle, 4]. MVT equilibrium

Preprints (www.preprints.org) | NOT PEER-REVIEWED | Posted: 21 March 2023 doi:10.20944/preprints202209.0151.v3

implies linearity in t for any t-quantity A1 = A1(t) including D* [A1] = +/- A1. 1/t, the + or - depending on A1 increasing or decreasing with t and with positive or negative sign. It follows D* [A1] remains invariant during Δt. This is part of the t-quantity perspective. The indications A and M are not applied for one moment time parameters like t or 1/t. Similarly the time interval description for the derivative to time interval Δt of quantity A1, D*||Δt [A1], is defined with the time interval commutator, D*||Δt [A1] = A [1/tb. A1(tb), –1. A1(ta). 1/ta] = [1/t, A1]||Δt, for time interval Δt = [tb, ta], and depends on equations 1 and 2.

3. Correspondence C1 for t-quantities with Δt-quantities

The time interval derivative to one moment time t, for A1 positive and increasing, is: D* [A1] = A1. 1/t, paragraph 2. Due to MVT equilibrium D* [A1] remains invariant during Δt. D* [A1] can be regarded a t-quantity as well as a Δt-quantity and this allows for the definition of correspondence for t-quantities and Δt-quantities. Applying time interval Δt averages for A1 and one moment time t, and assuming < A1. 1/t >||Δt = < A1 >||Δt. 1/< t >||Δt, it follows in terms of t-quantities only:

4 A1. 1/t = < A1 >||Δt. 1/< t >||Δt

This relation is valid for any t-quantity A1 and follows from MVT equilibrium and D*[A1]. The average for time interval Δt of parameter t: < t > = < t >||Δt depends on the (a-)symmetry of time interval Δt. The following properties for time interval Δt = [tb, ta] are applied [Hollestelle, 3]: when H time dependent Δt has finite measure and is asymmetrical with |tb| << |ta|, while for H time independent Δt has infinite measure with |tb| = |ta| each infinite. For asymmetric Δt: parameter t average < t > = 1/2(ta + tb) = t0, with t0 the "multiplication unit" for one moment time parameters during Δt, meaning t0. t = t. t0 = t. Left out are factors 2. Similarly, there is time interval Δt equals the "multiplication unit" (ΔU) for the time interval set.

The average for Δt of any t-quantity A1 can be considered from Δt-quantity perspective just as well as from t-quantity perspective since < A1 >Δt is invariant for Δt. With the MVT equilibrium property for Δt-quantities, linearity with Δt, and introducing scalar a and time interval multiplication M, it follows < A1 > = M [a, Δt] = Δt3. From one moment time t, a t-quantity however not a Δt-quantity, one defines correspondence by choosing < t > to correspond with time interval unit ΔU: < t > ~ ΔU. Above it was argued < t > = t0 for Δt asymmetric. In this way, indicated with ~, an average of any t-quantity A1 can correspond with a time interval quantity. It follows in equation 4 the right side can be from Δt-quantity perspective while above it is from t-quantity perspective. With subscript i is indicated the multiplication inverse. The left side of equation 5 is in terms of t-quantities, the right side in terms of time intervals and Δt-quantities:

5 A1.1/t ~ M [Δt3, ΔUi] = M [M [a, Δt], ΔUi]

When changing from t-quantity to Δt-quantity perspective, Δt-quantities are regarded part of the time interval set, and since they are linear in Δt, every Δt-quantity is equal to some Δt3 = M [a, Δt]. For any Δt-quantity being a time interval, a is scalar number. This is discussed in paragraph 8, and paragraph 12, part III, the closure theorem for the time interval set.

Definition. Correspondence C1. For the H time dependent asymmetrical situation, averaging one moment time t for Δt, there is < t >||Δt = t0, the one moment time set 'multiplication unit', different from the 'multiplication zero', with t0. t = t. t0 = t. When t0 corresponds with Δt0 per definition as a term always, and in the asymmetric case corresponds with time interval set 'multiplication unit' ΔU, different from time interval set 'multiplication zero' ΔU0, one finds: < t >||Δt = t0 ~ Δt0 = ΔU for asymmetric Δt.
For the H time independent and symmetrical Δt situation with |tb| = |ta| there is < t > = t0 equals t + -1.t = t0, with t0 in this case both the 'multiplication zero' and the 'addition zero', corresponding with Δt0 in this case being equal to ΔU0 both the 'multiplication zero' and 'addition zero' for the time interval set. In this symmetric case < t >||Δt = t0 ~ Δt0 = ΔU0, not equal to ΔU. The one moment time t average < t >||Δt = t0 corresponds with the specific time interval termed Δt0, meaning correspondence of one moment time t0 with the time interval set 'multiplication unit' or 'multiplication zero' due to H being time dependent or not.

Comment 1. The difference in one moment time t average < t >||Δt ~ Δt0 equal to 'multiplication unit' or 'multiplication zero' for quantities and properties while remaining within physics is interesting in its own.
Comment 2. The defining property for ΔU is: M [ΔU, Δt1] = M [Δt1, ΔU] = Δt1, and for the multiplication inverse Δt1i: M [Δt1, Δt1i] = M [Δt1i, Δt1] = ΔU, for any time interval Δt1. One can argue, not regarding uniqueness however, ΔU can be identified with Δt, with Δt the relevant event time interval, from M [ΔU, Δt] = Δt while also M [Δt, Δt] = Δt, a relation derived independently for addition A for all Δt1 from interpretation in paragraph 7 and for M in paragraph 8. It also follows Δt0 = Δt is a solution for Δt0. The discussion in paragraph 7 implies some uniqueness properties for Δt.
The multiplication inverse Δti for Δt, is not well defined yet to belong to the time interval set. When M [ΔU, Δt] = Δt and M [Δti, Δt] = ΔU = Δt one finds Δt is a solution for Δti: M [Δt, Δt] = ΔU. This particular solution Δti = Δt is well defined and belongs to the time interval set.

<u>Comment 3.</u> A indicates addition and one defines A [Δt1, Δt0] = A [Δt0, Δt1] = Δt1 and A [Δt1, Δt1iv] = A [Δt1iv, Δt1] = Δt0, for Δt0 'addition zero' for the time interval set and Δt1iv addition inverse with subscript iv for any time interval Δt1. Multiplication with scalar -1 of any time interval Δt1, M [-1, Δt1], is not necessarily equal to the addition inverse Δt1iv for Δt1, and is not a well-defined time interval when Δt1 is a time interval and Δt asymmetric.

4. Pairs of reciprocal commutation quantities and pairs of Noether charges

The commutation quantities cn(t) and cn'(t) are neutral t-quantities within the time interval description: they are invariant with t during Δt, ie with constant values within their domain (cn-domain or cn'-domain) within Δt, and one can define cn(t) = cn and cn'(t) = cn' during Δt, and dependence on Δt, the Δt-quantity perspective, is described by introducing the positive average scalar density D(cn, Δt). There is < cn(t) >||Δt = ∫||Δt dt [cn]. 1/|Δt| = N. D(cn, Δt) with D(cn, Δt) = |cn- domain|. 1/|Δt-domain| a Δt invariant scalar quantity. Applied is the time interval integral to one moment time t: for instance I* [2] = ∫||Δt dt [2] = 2. |Δt|, the domain for a t-quantity equal to scalar 2 being the Δt-domain Δt. In the Δt-quantity perspective < cn(t) >||Δt is equal to invariant parameter N only when the cn-domain measure for Δt is equal to the Δt-domain measure: in this situation there is D(cn, Δt) = |Δt|. 1/|Δt| = 1. It is assumed that this is not always the case due to a specific equilibrium requirement for cn and cn'.

Similar to D(cn, Δt) defined is D(cn', Δt) for cn' and both densities depend on Δt and on the quantities cn and cn' resp. The equilibrium requirement is assumed to imply D(cn, Δt) + D(cn', Δt) = 1, both for the same Δt. For certain related time intervals Δt and Δt', when the respective domains equal upper limit 1, there is D(cn', Δt') = D(cn, Δt) = 1. When regarding invariant parameter N from the t-quantity perspective, and with cn and cn' the same sign, assuming the usual addition properties, one finds in terms of neutral t-quantities:

6 $\quad$ cn + cn' = N. D(cn, Δt) + N. D(cn', Δt) = N

Rewriting with M and A for time intervals one finds the corresponding Δt-quantity perspective. Equation 6 describes a requirement for neutral t-quantities cn = cn(t) and cn' = cn'(t) where parameter t is the variable with invariant Δt. Arguments for this requirement are discussed in comment 5, and paragraph 12, part III. It exists due to cn, cn' and N, being not ordinary constants, rather physical quantities defined within the time interval MVT equilibrium description.

The t-quantities cn(t) and cn'(t) are termed "reciprocal" quantities due to equation 6. A change of perspective to corresponding Δt-quantities cn(Δt) and cn'(Δt) and time intervals implies the equilibrium requirement means A [(cn(Δt), cn'(Δt)] = ΔN, including < N > = N ~ ΔN, invariant with Δt for the complete time interval set. Followed is correspondence C1, paragraph 3. N and ΔN being variable with Δt is not considered in this paper. In the symmetric case all t-quantities in equation 6 remain invariant and the addition remains valid. For the asymmetric situation with < t > ~ Δt0 = ΔU one finds: when < D(cn, Δt) >||Δt ~ ΔU and cn(Δt) = M [ΔN, ΔU] = ΔN, it follows < D(cn', Δt) >||Δt ~ ΔU0 and cn'(Δt) = M [ΔN, ΔU0] = ΔU0, and the other way around, while invariant |ΔN| remains the upper limit value for |cn(Δt)| or |cn'(Δt)|. For invariants related to the usual time development with one moment time t and the usual Noether charges referred is to [Noether, 12], [De Wit, Smith, 13]. The equilibrium requirement in Δt-quantity perspective while Δt remains the variable is:

7 $\quad$ A [cn(Δt), cn'(Δt)] = A [M [ΔN, ΔU], M [ΔN, ΔU0)]] = A [ΔN, ΔU0] = ΔN

In support for requirement equations 6 and 7, an argument includes a well known theorem for time and space averages. Addition A [A1, A2], where A1 and A2 are Δt-quantities and correspond to t-quantities, in this paragraph preliminary applied for addition with ΔU and ΔU0, is defined in paragraph 6 and 7. Domain zero measure is not considered for cn(Δt) or cn'(Δt) since this means these quantities are not time interval related rather one moment time t related. It can be argued ΔN in equation 7 is an approximation for the complete time interval set average < cn(Δt) >||set, equal to the cn(Δt) space average [Arnold, 1], considering cn'(Δt) = cn(Δt') where Δt' is the above specified time interval different from Δt while there is the same upper limit value for | cn(Δt)| and |cn'(Δt')|. In this case the addition A [cn(Δt), cn'(Δt)] in equation 7 remains valid and equal to the invariant time interval set average and equals ΔN being invariant with Δt, ie invariant with Δt change for the overall Δt set.

The equilibrium requirement is supported by the symmetric second associative property, equation 3, due to the property for the usual Noether charge, NC with D*||Δt [ΔN] = NC, when NC regarded a neutral t-quantity. Due to similarity for A and M, discussed in paragraph 10, it follows ΔN = A [cn(Δt), cn'(Δt)] = M [cn(Δt), cn'(Δt)]. The invariant usual Noether charges, NC, should refer to the relevant event time interval Δt that is decisive for measurements.

<u>Comment 4.</u> *It is implied the NC depend on the one moment time description. The usual NC are the same as the neutral t-quantities cn and cn', reciprocal t-Noether charges, within the time interval description when limited to the finite time interval Δt, the relevant event time interval. Δt is relevant for deciding what are constants, ie*

Preprints (www.preprints.org) | NOT PEER-REVIEWED | Posted: 21 March 2023 doi:10.20944/preprints202209.0151.v3

invariant quantities, through measurement within Δt.
From time interval perspective, the usual NC correspond with the Δt-Noether charges, similarly limited to finite time interval Δt, however for a variable Δt. The pair of reciprocal Δt-Noether charges are the same as the Δt-quantities cn(Δt) and cn'(Δt), while ΔN = A [cn(Δt), cn'(Δt)] is an invariant for the overall time interval set. In fact cn(Δt) and cn'(Δt) both have the same limit value, and a difference exists only because of domain differences depending on the variable, ie Δt, equations 6 and 7. ΔN similarly can be termed a ΔN-Noether charge corresponding with the usual NC when the complete time interval set with variable Δt is considered.

Comment 5. For any relevant event time interval Δt, cn(Δt) and cn'(Δt), being averages and time interval Δt-quantities, resemble for instance < T >||Δt and < V >||Δt, ie the kinetic energy and potential energy quantities being invariant during Δt because of averaging however variable with Δt. For kinetic energy T and potential energy V, both t-quantities and not necessarily invariant during Δt and within the usual one moment time description, Lagrangian equilibrium requires total energy T + V remains invariant during Δt just like < T >||Δt + < V >||Δt. The quantities T and V are in principle independent. However, for T and V and Lagrangian equilibrium, this means some relation applies, ie a requirement to maintain equilibrium.

5. The 'infinite regress' problem and the definition of time interval boundaries, and correspondence C2

The usual qm description of radiation, including de Broglie complementarity, applies the one moment time description, and assumes parallel plane waves and invariant energy, ie parallel ray propagation [Sakurai, 7]. In contrast, star source radiation assumes sphere surface propagation, and the term 'ray' is not applied in this paper, rather propagation.

The description in terms of particular coordinates for star source radiation follows [Hollestelle, 3], [Hollestelle, 4]. The space coordinate q, within both the one moment time and time interval description, indicates a coordinate place at the radiation propagation sphere surface such that coordinate origin qc is 'close' to q. It is meant q(ta) = q and q(tb) = qb, where qb indicates the star source coordinate place, for Δt = [tb, ta]. The space coordinate q implies a possible measurement place on the propagation surface. The line of sight from the star source at place qb to place q at the propagation surface relates to space interval Δq. Interval measure |Δq| = |q – qb| increases with propagation development. Place q remains at the propagation surface and qc remains near q during propagation. From these definitions the one moment time description seems still integrated within the time interval description. Any usual Lorentz transformation TU should leave metric distances unchanged. It is argued to introduce new Lorentz transformations TL that leave metric propagation surface measures unchanged since star source radiation energy remains unchanged during propagation while being proportional to the metric measure of the propagation surface.

There is, for time interval Δt = [tb, ta] boundary tb relates by definition, not regarding causality, to boundary ta, with tb = tb[ta]:

8 $\qquad$ 1/tb = - 1/ta (1 - (c. q(ta))^2)

The constant c with dimension similar to the multiplication inverse of q, assures (c. q) remains dimensionless. Defined are the transformations A and B and B(-1) for which the Hamiltonian remains invariant: A(t) with A(ta) = tb, and B(t) with B(ta) = -1. tb and its reverse B(-1). Transformation A defines any time interval Δt with [tb, ta] = [A(ta), ta]. A series of transformations B(-1) transforms Δt to another Δt', and includes a re- scaling of one moment time t, which takes care (c. q(ta))^2 << 1 for equation 8 and transformation A to remain valid for defining Δt with ta and tb = A(ta) with positive and negative sign for these resp. boundaries.

Continuing from the above definitions one finds: two different solutions exist: q(t) = +/- q0. 1/t0. t and q(t) = +/- q0. t0. 1/t. The factor (c. q0) for q(t) = q0 at t = t0 is an invariant, and equal to multiplication M [c, q0] within the time interval set, where both c and q0 remain invariant during Δt.

Definition. Correspondence C2. A correspondence can be found by applying a specific Lorentz transformation TL, where space- and time coordinates q and t transform to resp. q + qL and t + tL, with qL and tL constant space- and time coordinate values. Transformation B(-1) is equal to a Lorentz transformation TL, with t' = TL(t), where one moment time t and t' correspond with time interval Δt = [tb, ta] and Δt' = [tb', ta'], and where q' = q(ta') remains well-defined and 'close to' qc' = q(tc') like q = q(ta) 'close to' qc = q(tc). When |t'. 1/t| = |(t + tL). 1/t| > 1 and with equation 8 in terms of transformation tb = A(ta), this means that for TL related to a series of B(-1): t' = TL(t) is quadratic in t, [Hollestelle, 4]. By applying this TL one defines the time interval Δt-quantities cn(Δt) and cn'(Δt) with cn'(Δt) = cn(Δt'). This defines correspondence C2: the one moment time commutation pair cn and cn', equation 1, corresponds with the time interval set commutation pair cn(Δt) and cn'(Δt). Correspondence C2 is the same as correspondence C1, paragraph 3.

From equation 8 and from correspondence C2, it follows equations 9. *The one moment time parameter t-quantities on the left side correspond with time interval Δt-quantities on the right side:*

9a |ta|. 1/|tb| = |cn'(Δt)|. 1/|cn(Δt)|

9b ta. tbi ~ M [cn'(Δt), cni(Δt)]

Comment 6. From MVT equilibrium it follows D*[q(t)] = +/- q(t). 1/t, applying one moment time parameter t within the time interval description like equation 8 and the definition, paragraph 2, with two solutions, q(t) linear in t and q(t) linear in 1/t. Due to MVT equilibrium and D* [q(t)] being independent of t during Δt, q(t) necessarily is linear in t and the linear in 1/t solution can exist only when it is the *same solution*, ie also linear in t.

For multiplication inverse ti = 1/t for t, where ti not necessarily belongs to the one moment time set, one finds: ti. t = t. ti = t0, with 'multiplication unit' t0, paragraph 3, which leaves possibly many solutions ti. To give meaning to ti, defined is: solution ti is to belong to the one moment time set, and when t1 and t2 belong to the one moment time set, multiplication t1. t2 = t3 does similarly. One moment time set properties like these are studied in [Hollestelle, 3]. For Δt boundaries ta and tb the relation with t0 is: ta + tb = 2. t0 and ta. Tb = 1/2. (ta^2 + tb^2) approaching to ta. Tb = 1/2. ta^2 for |ta| >> |tb|, valid for situations with H time dependent, with Δt highly asymmetric. These one moment time tb and ta properties correspond with the relevant event time interval Δt properties M [Δti, Δt] = M [Δt, Δt] = Δt = ΔU, comment 2. From this correspondence one derives equation 9.

The multiplication inverse Δti for Δt exists due to the relation M [Δt, Δt] = ΔU, and at least a solution is Δti = Δt which is linear in Δt. Within the time interval description I* and D* indicate integration and differentiation to one moment time t and similarly I*||Δt and D*||Δt indicate integration and differentiation to time interval Δt. All Δt-quantities are regarded linear in Δt and can be reduced to time intervals, this is discussed in paragraph 8, the closure theorem. One derives for the Δt-quantity cn(Δt) in the time interval description:

10 I*||Δt [cn(Δt)] = M [Δt, A [cn(Δt), –1. cn'(Δt)]] = A [cn(Δt), – 1. cn'(Δt)]

D*||Δt [cn(Δt)] = M [A [cn'(Δt), –1. cn(Δt)], Δti] = A [cn'(Δt), – 1. cn(Δt)]

With equations 9 and 10 the 'infinite regress' problem for the definition of time intervals is resolved. These equations are time interval only relations, and do not require one moment time parameter t. They include multiplication with Δt resp. Δti and are the time interval equivalent of equations 1 and 2, ie they define commutation relations for time intervals and define time intervals 'working to the right'. These equations are generalized to apply to the complete time interval set in paragraph 10.

Within the time interval description results for I*||Δt and D*||Δt start from MVT equilibrium and the linear in Δt property just like for I* and D*, from MVT equilibrium follows the linear in t property. Derivation of equation 10 includes I*||Δt [A1] = [t, A1]||Δt and D*||Δ [A1] = [1/t, A1]||Δt, from paragraph 2 and [Hollestelle, 3], and application of equation 9, correspondence C1: t0 ~ Δt0 = ΔU = Δt for I*||Δt [A1], and linearity in Δti for D*||Δt [A1], explaining the occurrence of Δt and Δti within equation 10. Equation 10 is a time interval only set result. However in terms of one moment time quantities cn and cn', the results seem opposed to each other.

When it is assumed the situation is 'remote' from H time independent, Δt is asymmetric with |ta| >> |tb| and |cn'(Δt)| >> |cn(Δt)|, equation 9. For cn'(Δt) similar equations can be derived. The indication * is reserved for operators, however applied for integration and differentiation by exception. From equation 10 one finds I*||Δt [cn(Δt)] and D*||Δt [cn(Δt)] to be similar, in agreement with Δti = Δt and comment 6, except for the order of cn(Δt) and cn'(Δt). The order difference can be disregarded since from the correspondence t0 ~ ΔU = ΔUi it follows I*||Δt [cn(Δt)] = D*||Δt [cn(Δt)], ie the results in terms of time intervals for integration and differentiation are the same, at least for Δt-quantities cn(Δt) and cn'(Δt). Since cn(Δt) is linear in Δt, and any Δt-quantity can be represented linear in cn(Δt), exactly similar results for equation 10 are valid for all time intervals and Δt-quantities, not only for cn(Δt) and cn'(Δt). This general result implies validity of the time interval set 'multiplication closure' theorem, and of the time interval set 'multiplication linearity' theorem, derived in resp. paragraph 8 and 10.

6. The time interval only set

Commutation properties are usually defined within a one moment time description and depend on the vector like space time coordinates and quantities that can form combinations like addition or multiplication. Time development and change can be included in the one moment time description with the derivative to parameter t. The time interval description includes both t-quantities and Δt-quantities. However, for a description with time intervals only, specific properties for the time interval set like addition and multiplication are necessary, combinations for time intervals that before did not have meaning yet. In [Hollestelle, 3] two elementary parameters define time interval Δt = [tb, ta] and its boundaries tb an ta within a one-dimensional time concept. It is argued these two elements are such that they agree with the way time can be measured or counted and they are independent of, preliminary to, one moment time t. Any time interval is based on these parameters where a

 doi:10.20944/preprints202209.0151.v3

measurement includes counting towards ta, forward to ta in the future, while counting from tb in the past. Any time interval includes a part of the past tb belongs to and a part of the future ta belongs to. These boundaries tb and ta, one moment time t and the vector concept still remained within the time interval description. From the results of paragraph 5 one finds a time interval only set and its description without one moment time parameters.

7. Time development and the time interval only set

Time development in this paper means that there is the same 'time' for the cosmological universe together and all its parts agree on this 'time' and no part is late or early in reference to this 'time'. Not is meant time is a zero-dimensional concept without development and not is meant agreement is variable or is chosen. Rather, time development proceeds together and 'simultaneous' time measurements should give 'the same' results. A definition for 'simultaneity', in paragraph 11 and 12, starts from the discussion of star source radiation. A second property is, time intervals do not exist outside themselves: they don't add time from outside to themselves and remain only with themselves. One finds:

11 Δt1 'addition' Δt1, to the same time interval, leaves Δt1 invariant: A [Δt1, Δt1] = Δt1

Δt1 can be any time interval within the time interval only set. This confirms the interpretation of time interval only set addition with domain addition where addition of two identical domains results in the same domain.

Within the time interval only description addition is defined starting from the introduction of 'addition zero' time interval Δt0 such that addition of Δt0 with any other time interval Δt1 leaves Δt1 invariant, A [Δt0, Δt1] = A [Δt1, Δt0] = Δt1, similar to comment 4. Within the time interval only description events and properties are regarded for a real situation with the specific relevant event time interval Δt, with A [Δt1, Δt] = Δt1, similar to comment 3. The 'addition inverse' Δtiv for a finite relevant event time interval Δt equals Δt, i.e. A [Δtiv, Δt] equals A [Δt, Δt] = Δt. One finds Δt 'addition' -1. Δt does not equal Δt, where -1. Δt means with opposite signs for the boundaries. The -1. Δt is not the 'addition inverse' for Δt and is not well defined for finite time intervals: it does not belong to the time interval set. One might argue Δt 'addition' -1. Δt equals -1. Δt from the above definition where Δt0 equals Δt. Notice that Δt0 is not a zero-measure time interval in the sense of domain measure equal to zero. The order within addition seems to matter. A [Δt1, Δt2] means addition for any two time intervals Δt1 and Δt2.

8. Time interval only set multiplication: the multiplication closure theorem

Similar to addition A one can define multiplication M [Δt1, Δt2] for any two time intervals Δt1 and Δt2, from the introduction of 'multiplication unit' ΔU, while the relevant event time interval remains Δt. Like with addition it is not clear immediately what multiplication means for the time interval only set. Several properties for M are discussed in the following.

I. Multiplication closure. The result of multiplication is assumed to belong to the time interval only set: M [Δt1, Δt2] = Δt3 and this is termed multiplication closure of the time interval only set. The resulting Δt3 is a time interval, supported by the time interval set 'multiplication unit' ΔU relation M [ΔU, Δt1] = Δt1. The following properties are valid. The multiplication inverse Δt1i for any time interval Δt1 is a time interval itself, including the multiplication property: M [ΔU, Δt1i] = Δt1i. Multiplication inverse Δti for Δt equals Δt itself, Δt being the relevant event time interval, comment 3. M [Δt1, Δt1i] = ΔU for any Δt1, assuming there exists at least one multiplication inverse time interval Δt1i for each Δt1. The multiplication closure theorem is derived below.

II. Time development. From similar arguments for the validity of equation 11, multiplication M [Δt1, Δt1] = Δt1 is to be valid for any time interval Δt1.

III. Associativity. The symmetrical first associative, 'series', property for M: M [M [g1, g2], g3] = M [g1, M [g2, g3]] is assumed valid, the symmetrical second associative, 'parallel' or distributive, property for M: M [g1, M [g2, g3]] = M [M [g1, g2], M [g1, g3]] is not necessarily valid, for gi any three time intervals. The first and second associative property both to be valid can be contradictory. The usual derivative of a multiplication includes the second associative property, equation 3. The derivative to a t- or Δt-quantity includes one moment time t or time interval Δt commutations, from paragraph 2. In this paper derived are the associative properties for the time interval only set, associative properties for ordinary variables or vector sets are well known within group theory, [Jacobson, 14].

For time interval only set multiplication the following definition is feasible, equation 12. The integral I*||Δt2 to time interval Δt2 is introduced, just like the usual integral I* to one moment time t, with the integral domain equal to the Δt2-domain:

12 M [Δt1, Δt2] = M [M [< Δt1 >||Δt2, Δt2], Δt2] = M [< Δt1 >||Δt, Δt2] = I*||Δt2 [Δt1]

Applied is equation 11 in the multiplication M version. This definition depends on the order of Δt1 and Δt2. I*|| Δt2 symmetrical for Δt1 and Δt2 is possible however not discussed in this paper. With time interval Δt1 invariant during Δt and Δt1 = < Δt1 >||Δt, and Δt = Δt0, there is < Δt1 >||Δt = M [< Δt1 >||Δt, Δt] = M [< Δt1 >||Δt2, Δt2] and < Δt1 >||Δt2 = M [M [< Δt1 >||Δt2, Δt2], Δt2i] = M [M [< Δt1 >||Δt, Δt], M [Δt2i, Δt]] = M [Δt1, Δt2i]. This provides the second and third part of equation 12, including the average < Δt1 >||Δt2, and the integral to Δt2, I*|| Δt2 [Δt1] = M [< Δt1 >||Δt, M [Δt, Δt2i]] = < Δt1 >||Δt2. Applied is comment 3 and the symmetrical first associative property for M, property III above.

With unchanged multiplication order for Δt1 and Δt2, and with Δt2 = Δt, no changes occur for equation 12 in principle and it immediately follows:

13 M [Δt1, Δt] = < Δt1 >||Δt = I*||Δt [Δt1] = Δt1

The difference for averaging and integration to time interval Δt2 or to Δt, follows from the above equations. When time interval Δt1 and Δt2 do not extend to outside Δt, ie the common domain for Δt1 or Δt2 with Δt equals Δt1 or Δt2, and with one moment time t being continuous within Δt these averages are evident. For Δt1 = ΔU, the ‘multiplication unit’ for the time interval set, one finds evidently:

14 M [ΔU, Δt] = I*||Δt [ΔU] = Δt

15 M [ΔU, Δti] = I*||Δti [ΔU] = Δti

Comment 7. Closed multiplication implies Δt3 = M [Δt1, Δt2] is a time interval and validity of Δt3 = a. Δta, scalar a depending on time interval Δta, implies the existence of solution Δta being a time interval. Not all scalar arguments a can be allowed when M [Δt1, Δt2] = a. Δta = Δt3 is a proper time interval, where for all time intervals the future domain part measure exceeds the past domain part measure. When Δta is a time interval, -1. Δta is not when H is time dependent and Δt asymmetric.

For any time interval, transformation A, from equation 8, is the defining and necessary relation for its one moment time set boundaries. Δt-quantities being linear with Δt is due to MVT equilibrium and the derivative definition, paragraph 2. The closure theorem implies the same: Δt3 being not only a Δt-quantity, also a time interval linear with Δt when Δt1 and Δt2 are time intervals.

Closure theorem I. The multiplication result Δt3 can be rewritten with a time interval multiplication Δt3 = a. Δta = M [ca, Δta]. Applying the symmetrical first associative property for any quantity ca, ca = M [ca, Δt] = M [ca, M [Δtai, Δta]] = M [M [ca, Δtai], Δta], ie there exists a time interval average multiplication with ca = M [< ca >||Δta, Δta], implying ca is a time interval. Time interval averages, for any invariant quantity, are time intervals, from < ca >||Δta = M [ca, Δtai] discussed below equation 12. When applying the first associative property with several steps, one finds: M [ca, Δta] = M [M [< ca >||Δta, ai. M [ca, Δta]], Δta] is equal to M [< ca >||Δta, ai. M [ca, M [Δta, Δta]]] = M [< ca >||Δta, ai. M [ca, Δta]] = M [< ca >||Δta, Δta] = ca. There is ca = Δt3 = a. Δta and Δta = Δt is an existing solution for M [ca, Δta] = ca, comment 3, for any allowed scalar a, comment 7. This solution implies ca = a. Δt and Δt3 = a. Δt and both ca and Δt3 are time interval and linear in Δt.

Solutions for Δt3 can be found in another way directly from a. Δta = M [ca, Δta] and property M [Δta, Δta] = Δta. These solutions for Δt3, for any Δta, not only Δta = Δt, should exist since the time interval set is linear in Δt, ie 1-dimensional, due to MVT equilibrium. These solutions are found by starting from ca = e1. Δt and Δta = e2. Δt, with result Δt3 = e1. e2. Δt linear in Δt and a = e1.
Solution Δta = e2. Δt is linear in Δt and includes any scalar e2, thus in fact any time interval Δta can be a solution. The consideration that any time interval is invariant during Δt and linear in Δt, is discussed in paragraph 12, part III. For the specific scalar a = 1 the solution Δta exists and is a time interval and Δt3 = M [Δt1, Δt2] is a time interval. Time interval only multiplication exists and is equal to scalar multiplication at least for a = 1: a. Δta = M [ca, Δta] with ca = ΔU = Δt. The scalar a is the resultant for ca = Δt and Δta = Δt3, however with reference to comment 7.

Closure theorem II. The relation Δt3 = a. Δta = M [ca, Δta] depends on the argument scalar a having ‘positive’ sign. For argument a with a ‘negative’ sign the scalar multiplication does not follow the requirements for comment 7. From the first associative property for M with g1 equal to any scalar a one finds linearity with scalar a for multiplication M [Δt, Δta], with Δt3 = a. Δta = a. M [Δt, Δta] = M [a. Δt, Δta], equal to Δt3 = M [ca, Δta] with solution ca = a. Δt, linear in Δt. From the above derivation there is Δt3 = ca = a. Δt linear in time interval Δt, ie itself a time interval, and it follows validity for the closure theorem for any scalar a, with reference to comment 7: *multiplication M is closed within the time interval set.*

Comment 8. The first associative property for a derivative of a multiplication seems valid if only due to the preserved order of the gi in series, property II. More precisely, this property is related to the following commutation properties for t-quantities NC(t) that are linear in parameter t due to MVT equilibrium. NC(t) is defined from the neutral t-quantities cn and cn’ with D* [NC(t)] = cn and it follows from equation 1:

16 t|1. NC(t)|2. cn’ = NC(t)|1. t|2

17 $1/t|1.\ NC(t)|2 = NC(t)|1.\ 1/t|2.\ cn$

Indication |1 or |2 means the t-quantity at this place depends on parameter t with the specific value t = t1 or t = t2. Equations 16 and 17 are not definitions, rather they are derived from the relation D* [NC(t)] = cn and equation 1 for t-quantities cn and cn' from MVT equilibrium within the time interval description. One finds the order of |1 and |2 is preserved when reversing the order of parameter t and quantity NC(t) within the equations, due to the specific commutation properties. For the overall time interval set and within time interval Δt perspective, cn and cn' correspond with the Δt invariant Δt-quantities cn(Δt) and cn'(Δt), linear dependent on Δt, and confirmed is the first associative property for a derivative at least for cn(Δt) and cn'(Δt).

9. Derivatives to time intervals and the time interval only set derivative 'set rule'

Similar like for multiplication M, paragraph 8, one moment time t can be left out from addition A within the time interval only set. Proposed is a definition for A [Δt1, Δt2] that includes multiplication M [Δt1, Δt2] and the derivative to time interval Δt2, D*||Δt2. When applied is Δt2 = Δt, it follows equation 21 and 22, where the asymmetrical second associative property is assumed for the derivative to Δt, D*||Δt, evidently similar with one moment time derivative d/dt, equation 3.

Comment 9. The definitions for M and A depend on the existence of a real event where the relevant event time interval is Δt. Only with this Δt the properties for Δt0 and ΔU can be considered sensible. Multiplication does not make sense with Δt2 not equal to Δt because it includes a time interval average during Δt2, where however the measurement event time interval, the time interval that is also the reference for averaging, is equal to Δt. Multiplication is included in A and therefore also addition A depends on relevant event time interval Δt, due to the following definition, equation 18.

18 $A\ [\Delta t1, \Delta t2] = D^*\|\Delta t2\ [M\ [\Delta t1, \Delta t2]]$

19 $A\ [\Delta t1, \Delta t] = D^*\|\Delta t\ [M\ [\Delta t1, \Delta t]] = \Delta t1$

20 $A\ [\Delta t0, \Delta t] = D^*\|\Delta t\ [M\ [\Delta t0, \Delta t]] = \Delta t0$

For Δt equal to Δt0, the Δt0 'addition zero' by definition, comment 4, equation 19 is confirmed since addition with Δt leaves any Δt1 invariant. Equations 19 and 20 result from the identification Δt2 equals Δt like was considered for multiplication in equation 12 and 13, due to the argument from comment 9. Equation 18 can be derived to be valid for Δt1 and Δt2 identified with Δt-quantities cn(Δt) and cn'(Δt). Since cn(Δt) is linear in Δt due to MVT equilibrium this supports generalization of equation 18 to the time interval set. The multiplication linearity theorem, paragraph 10, is necessary for this generalization. A discussion follows in paragraph 12, part III, on linear subsets. The first associative property is applied and cancelling terms are left out. Terms cancel due to A [Δt1, Δt] = Δt1.

The one moment time derivative 'set-rule' and the asymmetric second associative property
The one moment time derivative of a multiplication g1. g2, with g1 and g2 arbitrary scalar t-quantities, can apply the asymmetric arguments c1 and c2: d/dt [g1. g2] = c1. d/dt [g1]. g2 + c2. g1. d/dt [g2], similar to equation 3 in the one moment time description for the derivative to one moment time t, when scalars c1 = c2 = 1. When, equation 3, both two parts are valued equal, the symmetric second associative, ie distributive, property is valid.

Assume g1. g2 = a. g3 with g1 = a, an invariant scalar, and g2 any t-quantity. The derivative 'set-rule' should define the result, when d/dt is 'moving to the right' of scalar a: d/dt [a. g3] = c1. d/dt [a]. g3 + c2. a. d/dt [g3] = a. d/dt [g3], in the symmetric one moment time description, equation 3. The scalar a is 'moving to the left' or released from d/dt, and in this case the factor Rest equals zero.

The time interval only set and the time interval derivative 'set-rule'
Now the perspective is changed to Δt-quantities and the time interval set. There are properties: A [a1. g1, a2. g1] = (a1 + a2). g1, for instance with A [1. g1, 1. g1] = A [g1, g1] = 2. g1, where a1 and a2 are scalar numbers and g1 belongs to the quantity set. These properties are usually assumed valid for the one moment time set. They do not exist within the time interval set where property A [Δt1, Δt1] = Δt1 is essentially different and depends on domain addition instead of vector addition, like for equations 19 and 20.

Comment 10. The asymmetric second associative property for the derivative to time interval Δt, D*||Δt [M [Δt1, Δt2]], with Δt1 and Δt2 any two time intervals, is:

21 $D^*\|\Delta t\ [M\ [\Delta t1, \Delta t2]] = A\ [c1.\ M\ [D^*\|\Delta t\ [\Delta t1], \Delta t2], c2.\ M\ [\Delta t1, D^*\|\Delta t\ [\Delta t2]]]$

with possibly un-equal scalars c1 and c2 for an asymmetric derivative and equal scalars for a symmetric derivative for M [Δt1, Δt2]. In the particular case Δt1 = a. Δta including any scalar a and time interval Δta, and Δt2 identified with Δt, one finds:

22 $A\ [a.\ \Delta ta, \Delta t] = D^*\|\Delta t\ [a.\ \Delta ta] = A\ [c1.\ M\ [D^*\|\Delta t\ [a], \Delta ta], c2.\ M\ [a, D^*\|\Delta t\ [\Delta ta]]]$

This is not equal to the one moment time derivative ‘set rule’: d/dt [a. g1] = a. d/dt [g1].

The time interval derivative ‘set rule’, equation 23, is derived from equation 18. Applied is, ‘addition zero’ Δt0 equals Δt and ‘multiplication unit’ ΔU similarly equals Δt, paragraph 3. For multiplication with scalar a left out are brackets, writing for instance M [a. Δt1, Δt] = a. Δt1. The time interval derivative ‘set-rule’ concept is such, that scalar a is ‘moving to the left’ and outside of the derivative D*||Δt brackets with similarity to equation 22 and does not remain to the right and inside brackets. The ‘set-rule’ itself originates from equation 18. This ‘set rule’ is non-trivial since D*||Δt [a], for invariant scalar a, not necessarily equals ΔU0 and does not correspond with d/dt [a], equal to the one moment time set ‘addition zero’, which itself equals the ‘multiplication zero’. Differentiation provides correspondence for t-quantities with Δt-quantities only when the time interval derivative ‘set rule’ is introduced.

Comment 11. The existence of scalar multiplication M [a, Δt2] = a. Δt2, for any Δt2 is assured, in the sense of being equal to a certain time interval Δt3, for any invariant scalar a, not being time interval. For scalars the usual one moment time indications and definitions for M and A like . and + apply. Due to the multiplication closure theorem, paragraph 8, for any scalar a, with reference to comment 7, the result for M [a. Δt, Δt2] = a. Δt2 = Δt3, is a time interval. With solution Δt2 = Δt and Δt3 belonging to the time interval set, this allows the application of M and A and I*||Δt2 and D*||Δt2 to these Δt3, and this is a multiplication linearity or subset property, similar to the general property discussed in paragraph 10, and 12 part III. Time interval derivative D*||Δt2 [a] exists independent of this for any scalar a.

It follows from direct evaluation of A and M, applying derivatives D*||Δt [Δt1] = [1/t, Δt1]||Δt, and D* [A1] = [1/t, A1]u, for time interval Δt1 including one moment time boundaries tb and ta and one moment time quantity A1, paragraph 1, D*||Δt [a. Δt1] does not have to be linear in scalar a. In equation 23 and 24, factors Rest, Rest(a)|t and Rest(a)||Δt, are introduced for the non linear part within the derivative. The result, although with similarity to the second associative property constants c1 or c2 for equation 22, includes the derivative ‘moving to the right’ specific for any set and ‘set rule’. This is a time interval only set result.

The time interval derivative ‘set rule’.

23 D* [a. Δt1] = A [a. D* [Δt1], Rest(a)|t]

Rest(a)|t = [1/tb, a]u. Δt = M [D* [a], Δt]

24 D*||Δt [a. Δt1] = A [a. D*||Δt [Δt1], Rest(a)||Δt]

Rest(a)||Δt = M [D*||Δt [a], Δt]

Like for any ‘set rule’, scalar a is ‘moving to the left’. The derivative D*||Δt [a. Δt1] does not include scalar a within D*||Δt [Δt1], while Rest(a)||Δt is added to the right. This agrees with the interpretation of D*||Δt ‘moving to the right’, like for one moment time parameter t or time interval Δt, equation 1 and 10, leaving a commutation quantity to the left. The factor Rest is invariant during Δt, and therefore can be considered from t-quantity or a Δt-quantity perspective. Rest(a)||Δt is derived by applying equations 4 and 5 and the correspondence relations from paragraph 3.

The properties for t0, including t. t0 = t0. t = t for any t, are defined in [Hollestelle, 3]. The usual commutation [1/tb, a]u has to be evaluated carefully. For a = 1 there is [1/tb, 1]u equals: tbi. 1 – 1. tbi, with tbi = 1/tb, indeed part of the one moment time set, paragraph 5. However, this is not necessarily equal to (1 – 1). tbi, the ‘multiplication zero’ equal to the ‘addition zero’ for the one moment time set, due to nonzero commutation relations, and since the specific scalar 1 is ‘multiplication unit’ for the scalar set one finds tbi. 1 – 1. tbi = tbi + (tbi)iv only, ie equal to the one moment time ‘addition zero’ only, when for all t, one moment time addition inverse tiv = -1. t. The usual commutation can be resolved by rewriting it corresponding to a time interval commutation that equals a time interval derivative, and regains due to MVT equilibrium commutation value zero, paragraph 1. Applied is time interval operator t* leaves scalar a invariant and the derivation includes a transformation of scalar a by multiplication with parameter t. Operator t* [A1] = M [D*||Δt [M[t, A1]], Δt] for any one moment time quantity A1, [Hollestelle, 4]. This confirms, for any scalar a: [1/tb, a]u ~ D* [a]||Δt.

From equation 4, with one moment time average < t > = t0, D* [a] equals < a >. 1/< t > = a. t0i, for scalar a positive and invariant with t. Assuming H time dependent one moment time ‘multiplication unit’ t0 corresponds with time interval ‘multiplication unit’ ΔU. Recall ΔUi = ΔU = Δt0 and Δti equals Δt, comment 3. It follows D*||Δt [a] and Rest(a)||Δt both equal a. ΔU. Rest(a)||Δt is a time interval related Δt-quantity, implying equation 24 is meaningful as a time interval only relation.

For any scalar a, Rest(a)|t = M [a. t0i, Δt] corresponds with a. M [ΔU, Δt] = a. ΔU. This equals Rest(a)||Δt = M [a. ΔU, Δt] = a. ΔU, equal to ΔU for a = 1. In this case, for a = 1, one finds D* [a. Δt1] = a. D* [Δt1] and D*||Δt [a. Δt1] = a. D*||Δt [Δt1] as it should be.

Preprints (www.preprints.org) | NOT PEER-REVIEWED | Posted: 21 March 2023 doi:10.20944/preprints202209.0151.v3

Comment 12. Rest(a)|t, the right side part for derivative D* [a. Δt1] and equation 23, can be derived independently without applying the second part, the usual commutator [a, 1/tb]u. The one moment time derivative D* and the usual commutator being equal, paragraph 2, is confirmed by the validity of both Rest(a)|t expressions in equation 23. This is also discussed in paragraph 12, part II.

Comment 13. M [ΔU0, Δt1] = ΔU0, with ΔU0 the time interval set 'multiplication zero', corresponding with the scalar set 'multiplication zero'. The domain measure for ΔU0 is zero and ΔU0 = 0. Δt1, for any finite Δt1 including ΔU0 = 0. ΔU. With zero domain measure, ΔU0 is not a proper time interval, and resembles a one moment time parameter and does not belong to the time interval set. Time intervals with zero domain measure are not included in the time interval set in this paper.

The derivative asymmetric second associative property and arguments c1 and c2.
With c1 and c2 scalars defining the asymmetrical second associative property for derivative D*||Δt [M [Δt1, Δt2]] with equation 21, one finds, from D*||Δt2 [Δt1] = Δt1 for Δt2 equal to Δt, equation 19, and with a = 1:

25 D*||Δt [M [Δt1, Δt]] = A [c1. M [Δt1, Δt], c2. M [Δt1, Δt]]

Within the last part, M [Δt1, Δt] = Δt1, due to Δt = ΔU and Δt = Δt0 for the time interval only set.

26 A [Δt1, Δt] = D*||Δt [M [Δt1, Δt]] = A [c1. Δt1, c2. Δt1] = Δt1

This is the time interval only set relation equivalent to the one moment time set relation where usually c1 = c2 = 1. When both c1 and c2 equal 1 it follows: A [Δt1, Δt1] = Δt1. This is the same result derived from time development interpretation, paragraph 7. Recall c1 and c2 are not any scalars, they have the meaning of second associative property arguments for D*||Δt [M [Δt1, Δt2], equation 21, and are possibly asymmetric and unequal. Where equation 22 depends on the second associative property for derivative D*||Δt, instead equations 23 and 24 introduce Rest(a)|t and Rest(a)||Δt, without applying associative property arguments. These are two interpretations for the derivative, one with associative property and one with 'moving to the right' property, introduced in paragraph 2. When both c1 and c2 equal 1 and with Δt1 = Δt, the time interval only set relation seems to correspond and to be similar to the usual one moment time set relation: d/dt [t. t] = c1. (1. t) + c2. (t. 1) = 2. t, however it is not. The time interval set differs from the usual one moment time set also in this way.

10. Time interval only set multiplication linearity theorem, wave equations

With the introduction of averages and time interval set commutation relations, one moment time parameters remain included within the time interval description. For the time interval only description, the integration and derivative definition follow from equations 12 and 18 in terms of time interval only set multiplication M and addition A. Equations 27 and 28 are valid for Δt1 equal to resp. cn(Δt) or cn'(Δt) and Δt2 = Δt due to the results of paragraph 5. They are valid for the time interval only set for any two time intervals Δt1 and Δt2 due to the multiplication linearity theorem.

27 I*||Δt2 [Δt1] = M [Δt1, Δt2]

28 D*||Δt2 [Δt1] = A [Δt1, Δt2]

Comment 14. In the time interval description operators are indicated with *, depending on a 'operator quantity' for their interpretation. Operator quantities are indicated with |operator*|, for example the time operator t* relates to quantity |t*| = t. Such operator quantities are not defined for I* and D* and I*||Δt2 and D*||Δt2, and, even while 'working to the right' like operators, these are not operators.

It follows from I*||Δt2 and D*||Δt2 having equal results for Δt2 = Δt, equation 10 and discussion, similarly M and A have equal results, at least for cn(Δt) and cn'(Δt). This does not depend on the perspective: to one moment time t or to time interval Δt, since M and A are time interval multiplication and addition for both perspectives and, from equation 12 and 18 and the discussion following 'set rule' equations 23 and 24, with Δt2 = Δt the relevant event time interval, I*||Δt and D*||Δt for cn(Δt) and cn'(Δt) correspond exactly with I* and D* for cn and cn'.

M and A depend on time interval properties resembling common domain and combined domain respectively. Their difference, due to equation 10 including I*||Δt [cn(Δt)] and D*||Δt [cn(Δt)], depends on the order for cn(Δt) and cn'(Δt) and does not return in the result itself in terms of time intervals. I*||Δt and D*||Δt, and M and A can have the same results, since this follows from D* [M] = A and D* [M] = M, applying properties A = A [Δt1, Δt] = Δt1 and M = M [Δt1, Δt] = Δt1 which are valid for any Δt1, due to the time interval set properties Δt = Δt0 and Δt = ΔU.

Multiplication linearity theorem. Equations I||Δt2 [Δt1] = M [Δt1, Δt2] and D*||Δt2 [Δt1] = A [Δt1, Δt2] are valid for any two Δt-quantities, ie any two time intervals Δt1 and Δt2 from the time interval only set.*

The theorem can be derived directly from the definitions for M and A however these definitions have general validity only by inference. Instead applied are MVT equilibrium and the time interval only set derivative. The derivation for the closure theorem, paragraph 8, differs from the following.

Due to specific Lorentz transformations TL there is cn'(Δt) = cn(Δt') defining Δt' from Δt, correspondence C2 depends on this time development, paragraph 5. The specific TL and change Δt to Δt' such that Hamiltonian H remains unchanged is derived in [Hollestelle, 4]. For other changes Δt to Δt1 ≠ Δt', H can be variable. For transformation TL with Δt to Δt', the theorem is valid due to the results of paragraph 5. For all other transformations the linearity of Δt1 with Δt is assured, even when the linearity constants do not agree with the specific transformation TL. From linearity of Δt1 with Δt it follows, the arguments for Δt' transformed from Δt are valid for any Δt1 transformed from Δt.
From equation 1 defined is cn'(Δt) = cn(Δt'). Since cn(Δt) and cn(Δt') are linear in the relevant event time interval, ie Δt and Δt' respectively, this implies a linear transformation Δt to Δt'. This relation can be reversed unless for instance D*||Δt for cn(Δt) or cn'(Δt) equals the time interval set 'addition zero' ΔU0. In the reverse case Δt' is linear in cn(Δt) and thus Δt' is linear in Δt. In fact, for any time interval Δt1, the implied transformation from Δt then equals a linear multiplication of Δt.
For any Δt2 the transformation Δt1 to Δt2 equals a time interval addition, applying a nonspecific Lorentz transformation, where H can be variable. A linear transformation exists due to this addition, and the theorem does apply for these Δt2 since a linear transformation means a linear multiplication result, due to MVT equilibrium. Similarly, when Δt2 linear in Δt1 a linear transformation does exist, this is discussed in paragraph 12, part III. *The theorem is valid for any time interval Δt1 and Δt2, and for any Δt-quantities being linear in Δt, except for the no-reverse case.*

An interpretation of change for Δt is necessary. For the changed situation the validity of the following is to be ensured: M' [Δt1, Δt'] = M'' [Δt1, Δt''] = Δt1, ie with the situation change the relevant event time interval Δt' changes to Δt'' while specific properties for M and A do not change and are defined with the relevant event time interval indication Δt, one of these properties Δt = ΔU. It depends on the situation present: before change Δt' = Δt, after change Δt'' = Δt, where situations for Δt' or Δt'' share properties, however not all properties, to assure change.

The no-reverse case.
In this case a change for Δt does not change cn or cn': cn(Δt') = cn(Δt'') and cannot be reversed. For a change Δt' to Δt'' with Δt' ≠ Δt'', and where Δt' = Δt'' is not considered, with Δt' = Δt before change, and the factor +/-1 not yet specified while +1 for positive increasing cn(Δt'') etc, the time development equation for invariant cn(Δt') is A [cn(Δt''), cn(Δt')iv] = M [+/-1. D*||Δt [cn(Δt')], A [Δt'', Δt'iv]] = Δt0. For D*||Δt [cn(Δt')] = ΔU0, one finds A [cn(Δt''), cn(Δt')iv] = ΔU0. The time interval ΔU0 = Δt0 is both 'multiplication zero' and 'addition zero' for the time interval only set and has time interval domain measure zero and is not a proper time interval, and cn(Δt) and cn'(Δt) are not well defined, comment 13.

For D*||Δt [cn(Δt'')] ≠ ΔU0 one finds:

I. A change from Δt' = Δt to Δt'' = Δt means like above: A [cn(Δt''), cn(Δt')iv] = M [+/-1. D*||Δt [cn(Δt')], A [Δt'', Δt'iv]] = Δt0 equal to the time interval "addition zero" since cn(Δt'') = cn(Δt') is invariant.

II. From I: cn(Δt'') = A [M [+/-1. D*||Δt [cn(Δt')], Δt''], cn(Δt')] = A [M [cn(Δt'), Δt''], cn(Δt'')] = cn(Δt'). This follows from equation 10, with Δt' = Δt and Δt'iv = Δt' before change and Δt'' = Δt, and Δt''iv = Δt'' after change. Applied is +/-1. D*||Δt [cn(Δt'')] = M [cn(Δt''), Δt''] = cn(Δt'') for all Δt'', equation 10. At least one of Δt' and Δt'' equals Δt and [Δt'', Δt'] = [Δt', Δt''].

III. From II: A [cn(Δt''), cn(Δt')iv] = M [cn(Δt'), A [Δt'', Δt'iv]] = Δt0. There is cn(Δt')iv = cn(Δt'')iv = A [Δt'', Δt'iv] and the solution cn(Δt'') is linear in Δt'' for all Δt''. Evenso from II, the derivative +/-1. D*||Δt [cn(Δt'')] = cn(Δt''), and cn(Δt'') is linear in Δt''. These apply the first associative property.

The no-reverse case, with D*||Δt [cn(Δt'')] ≠ ΔU0 for any Δt'', means cn(Δt'') is linear in Δt'', and this does not agree with cn(Δt'') invariant. *This completes the derivation for the multiplication linearity theorem.*

From equations 27 and 28 for cn(Δt) or cn'(Δt) one finds equations 29 to 34.

29 I*||Δt [cn(Δt)] = A [cn(Δt), A [ΔU, cn(Δt)]] = A [cn(Δt), cn(Δt)]

30 I*||Δt [cn'(Δt)] = M [cn'(Δt), Δt] = A [cn'(Δt), A [ΔU, cn'(Δt)]] = A [cn'(Δt), cn'(Δt)]

31 A [a. Δt1, Δt] = D*||Δt [M [a. Δt1, Δt]] = D*||Δt [a. I*||Δt [Δt1]] = A [a. D*||Δt [I*||Δt [Δt1]], Rest(a)||Δt]
Equation 31 is valid including with cn(Δt) or cn'(Δt) instead of Δt1. Not necessarily cn(Δt) follows the requirement of time interval asymmetry like Δt itself for situations when H time dependent, and – 1. cn(Δt) is valid however -1. Δt is not. From equation 10 for I*||Δt it follows:

32 a. cn(Δt) = D*||Δt [a. I*||Δt [cn(Δt)]] = A [a. D*||Δt [I*||Δt [cn(Δt)]], Rest(a)||Δt] = A [a. D*||Δt [A

Preprints (www.preprints.org) | NOT PEER-REVIEWED | Posted: 21 March 2023 doi:10.20944/preprints202209.0151.v3

[cn(Δt), -1. cn'(Δt)]], Rest(a)||Δt]

a. cn'(Δt) = D*||Δt [a. I*||Δt [cn'(Δt)]] = A [a. D*||Δt [I*||Δt [cn'(Δt)]], Rest(a)||Δt] = A [a. D*||Δt [A [cn(Δt), -1. cn'(Δt)]], Rest(a)||Δt]

For scalar a = 1, Δt1 = cn(Δt) and Δt2 = Δt there is Rest(a)||Δt is nonzero and equal to cn(Δt). Scalar a can be 'moving to the left' from I*||Δt without any nonzero factor Rest, linearly, however from D*||Δt only non-linearly with 'set rule' equation 24 including Rest(a)||Δt. Due to property A [Δt1, Δt1] = Δt1 for any time interval Δt1 including Δt1 = cn(Δt), the Rest(a)||Δt can be left out and one finds D*||Δt, and I*||Δt exactly the same.

33 a. cn(Δt) = D*||Δt [a. I*||Δ [cn(Δt)]] = A [a. D*||Δt [I*||Δt [cn(Δt)]], Rest(a)||Δt]

a. cn'(Δt) = D*||Δt [a. I*||Δt [cn'(Δt)]] = A [a. D*||Δt [I*||Δt [cn'(Δt)]], Rest(a)||Δt]

For a = 1 and with Δt-quantity cn(Δt) linear with Δt, one finds:

34 cn(Δt) = D*||Δt [I*||Δt [cn(Δt)]] = A [D*||Δt [I*||Δt [cn(Δt)]], cn(Δt)]

cn'(Δt) = D*||Δt [I*||Δt [cn'(Δt)]] = A [D*||Δt [I*||Δt [cn'(Δt)]], cn'(Δt)]

Wave equations and structure constants.
These equations are similar to second order wave equations. From the discussion below equation 7 it follows ΔN equals both addition or multiplication of the reciprocal pair of commutation quantities. The derivative D*||Δt [ΔN] = A [cn(Δt), cn'(Δt)] = ΔN and similarly the second derivative D*||Δt [D*||Δt [ΔN]] = ΔN and equations 34 apply to ΔN. These equations are due to the derivatives being time interval set, MVT equilibrium, derivatives. Recall ΔN is an invariant for the overall time interval set. ΔN, like any Δt-quantity, is part of the linearity 'subset' for cn(Δt) or cn'(Δt). This is discussed in paragraph 12, part III, for linear subsets for the time interval only set.

From combinations of commutations, being multiplications within the generator set, one finds the set structure constants from the second derivatives, [De Wit, Smith, 13], [Veltman, 15]. Structure constants for the time interval set depend on D*||Δt [D*||Δt [C]] for some combination C, and from multiplication time development property, paragraph 8 property II, multiplication within the generator set equals multiplication within the time interval set, and one finds C = ΔN. The second derivative invariants are Lorentz transformation TL invariants, due to TL being a surface measure preserving transformation, unlike the usual Lorentz transformations TU. Due to this the overall TL invariant ΔN can be interpreted directly to be a structure constant.

From equations 1 and 10 one finds the reciprocal pair-like commutation properties of the one moment time set and the time interval only set respectively. The commutation constants form a n-pair and the existence of an overall equilibrium invariant, in this case ΔN = M [cn(Δt), cn'(Δt)], implies for structure constants:

Comment 15. The dimension of the set decides on the number of structure constants within one set n-pair.

Comment 16. For the time interval only set the n-pair is a pair, due to both time and the time interval only set being one dimensional, and any time interval is linear in Δt due to the closure theorem.

11. Discussion. Star-source radiation propagation and time development

Star source radiation, interpreted with propagation sphere surfaces, is different from radiation with stationary parallel propagation. The metric surface measure, not meaning the usual line element dependent metric, for propagation surfaces is invariant with change of relevant event time interval Δt that indicates time development within the time interval description for radiation fields [Hollestelle, 4]. This description assumes de Broglie complementary and radiation is interpreted with waves or complementary wave particles of zero or non-zero mass with a group-velocity c(Δt), *c* for photons. An effort to describe the resulting folded, wrinkled, radiation propagation surface for changing Δt is the following variation on a theorem from topology. This has not been derived completely.

Propagation surface A related to radiation time interval Δt can be idealized with a regular sphere Q(Δt), that does not fulfill the surface measure requirement. However A can be approximated with a surface P, the union of a constant number n of disjoint, open en dense, subsets Pi, applying a construction P-HY from topology, [Hocking, Young, 16]. Similar to this is construction P-PS, with number n variable for the limit subsets Pi, to cover the surface Q in the infinite limit of a step counting parameter, with P the indecomposable continuum limit and, due to a variable width parameter v, varying metric surface measure for the limit subsets Pi. By arranging n and v, it is inferred always the metric surface measures can be mA(Δt) > mQ(Δt) for the surface A to be a 'folded/wrinkled' cover, for Q.
Assumed is c(Δt) remains invariant. It is argued the metric requirement for A does not apply the linear metric, the line element, even when the metric surface measure of a regular sphere usually relates to the metric radius measure squared. The limit parts Pi being 'linear'-like, due to decreasing v, does not imply the ordinary line element metric.

 doi:10.20944/preprints202209.0151.v3

Comment 17. The construction of subsets to describe the propagation wave with surface P does not mean it is implied the propagation surface is divided into photon path destination parts. The discussion of propagation starting from the concept of photon paths in qm is not the subject of this paper.

For Δt invariant situations, number parameter n ≥ 1 is un-decided. When n is independent and un-decided it supports a degree of freedom. For any n, each of the n limit parts Pi(v, n) is dense in Q and the surface measure mA remains equal to mP = n. mQ ≥ mQ.
One interpretation is, energy does not change with n, and a change of n implies a symmetry transformation. Any n ≥ 1 is realistic for propagation surface A, to remain a covering for the regular sphere Q. A second interpretation is, implied is a situation with zero temperature. Several already existing results for star source radiation allow for a zero temperature. Two applications are summarized below.
A trial description for star-cloud radiation, including temperature dependence for the overall star-cloud and for the star sources themselves is given in [Hollestelle, 4]. A zero temperature means, with space interval ΔQi for star i at space interval Δqi towards the average < Δqi >||cloud, the average < ΔQi >||cloud is approximately zero while the cloud source is localizable which has implications for the symmetry of the star-cloud itself in space. Another zero temperature result is, star source i at Δqi, for all i, is ‘remote from’ the propagation surface and q0. The equations of state for high or low temperature equilibrium are derived in [Hollestelle, 9], for a sphere surface cloud, however within the one moment time description. A ‘remote’ localisable cloud source of non-zero mass matter wave radiation at a measurement place for Δt’ will appear like a ‘remote’ cloud source with near to zero mass matter wave radiation at the measurement place for Δt’’ with |Δt’| << |Δt’’|.

Comment 18. Simultaneous events. Simultaneous events defined from the propagation surface A need an interpretation, depending on ‘timely’ time intervals for radiation measurements, which is assured by Δt, the overall relevant event time interval, being ‘timely’. The term ‘timely’ implies: a time interval Δt’ is timely when it is itself measurable with time interval measure result |Δt’|, ie from one measurement within Δt, [Hollestelle, 3]. This resembles Einstein’s discussion of simultaneity and ‘locality’ for space-time, [Einstein, 17], however the differences with simultaneity for time intervals are severe due to time intervals not being translational invariant in the present description. Simultaneous measurements for any Δt’ and Δt’’ are possible when |Δt’| < |Δt| and |Δt’’| < |Δt|, with Δt’ and Δt’’ being both timely and the relevant event time intervals for these resp. measurements. With propagation surface energy is meant the integral energy, from the star source to the propagation surface, with simultaneous propagation, that is all radiation emitted and ’on the way’ during Δt, [Hollestelle, 4].

Simultaneous distribution theorem. Any distribution is assumed to be a simultaneous distribution. For the 2-dim radiation propagation surface A a distribution Pi, i = 1 to n > 1, is possible because it does not imply similarly a distribution Δti ≠ Δt. Any distribution for A remains a cover for the same Q and thus remains with the same Δt, time developement being the same for A and Δt by definition. A distribution for the radius for A implies a distribution for time interval Δt which is not allowed due to Δt remaining the relevant event time interval. This theorem is related to the introduction of Lorentz transformation TL, with surface measure preserving property, paragraph 5.

Star-source wave propagation with zero or non-zero complementary particle mass.
Assume non-zero mass m1 for radiation wave particles, complementary to radiation from a star-source, and introduce the mass m2 where mA proportional with M [m1, m2]. Field energy Ee is linear with M [m1, m2] and inverse linear with |Δq|, where |c(Δt)| = |M [Δq, Δti]| is the wave group-velocity. The group-velocity in fact depends on the space time metric, and equals the ‘apparent velocity’ of the associated complementary wave particle, [Goldstein, 2], [Hollestelle, 3]. In this case the space time metric is assumed diagonal and symmetric in the space coordinates, to allow the above expression for c(Δt). This can be interpreted with gravitational field energy Ee for situations without external interaction. The e.m. wave propagation surface energy, ie kinetic energy Es, equals Ee due to mass m2, with m2 in this way related to, dependent on, propagation surface properties. From the definition for c(Δt), there is Δqi = M [Δti, c(Δt)i] = c(Δt)i:

35 Es = A [1/|c(Δt)|, M [m1, m2]], equal to A [1/|c(Δt)|, m2] for m1 mass ‘multiplication unit’

36 Ee = M [Δqi, M [m1, m2]] = D*||Δq [M [m1, m2]] = D*||Δt [M [1/|c(Δt)|, M [m1, m2]]]

With equation 35 and 36 one finds a definition for non-zero mass m2, that can be identified with the source mass to decide on the gravitation energy interpretation for Ee. D* or D*||Δt of an invariant does not have to be zero. With 1/|c(Δt)| = D*||Δq [m1] = M [Δqi, m1] for m1 the mass ‘multiplication unit’, there is *c(Δt) equals the multiplication inverse for the propagation wave particle mass apparent density.* There is for photons, when m1 equal to the mass ‘multiplication unit’, Es = A [Δqi, m2], and *Es can be interpreted to be the source mass apparent density.*
When one defines vacuum with total energy H0 equal to zero, and with kinetic radiation energy E = #n. *h. v*, for #n the photon number and wave energy *h. v*, one can write Hamiltonian H = H0 + ΔH, with the energy ΔH introduced in [Hollestelle, 3]. Within the time interval description, a time dependent H = H0 + ΔH = E + V + ΔH

Preprints (www.preprints.org) | NOT PEER-REVIEWED | Posted: 21 March 2023 doi:10.20944/preprints202209.0151.v3

is the time interval version of the Legendre transform of Lagrangian L = E – V, with E kinetic energy and V potential energy and ΔH depending on the t-quantity #n, possibly variable due to interaction within Δt. Applying de Broglie complementarity and with relevant event time interval Δt = [tb, ta], from the above ref., radiation propagation energy, for radiation emitted and 'on the way' during Δt, Es equals:

37 Es(< Δt) = h. ν = M [h+, Δti]

The 'Planck-like' function h+ is defined with: h+ = 1/2 (Δ*p. Δ*q + Δ*q. Δ*p), within the one moment time description and with . and + for t-quantities and Δ* for variances, different from Δ for intervals [Hollestelle, 3]. Δti is the multiplication inverse for Δt. Due to h+ one moment time tb and ta occur in equation 37. Energy Es, without interaction or wave function collapse during Δt, is invariant and a neutral Δt-quantity during Δt.

However, from equations 35 and 36 it follows h+ = m2. With c(Δt) = M [Δq, Δti] invariant, c(Δt) is a Δt-quantity linear in Δt including non-scalar multiplication with Δq. The complete propagation surface A from star-source to propagation surface depends on Δt and c(Δt) due to time development. Mass m1 equal to mass 'addition zero' can be allowed due to m1 and m2 not being scalars rather Δt-quantities where m1, the mass 'multiplication unit', corresponds with Δt0 = ΔU, while Es does not reduce to zero. *Then, for wave particles with 'zero' mass m1, ie photons, this means m1 is both mass 'addition zero' and mass 'multiplication unit'*, different from mass 'multiplication zero' corresponding with ΔU0, and for any source mass m2 there is:

38 A [m1, m2] = A [m2, m1] = m2

M [m1, m2] = M [m2, m1] = m2

Including #n the number of photons, non-interacting radiation propagation surface energy Es = Es(< Δt) = #n. *h. ν* increases with #n, however the overall photon mass m = ∑ (i = 1 to #n) mi = ∑ (i = 1 to #n) Mi = m1, where Mi+1 = M [Mi, m1] = m1 and M1 = m1, remains invariant and equal to m1 the mass'addition zero' for any #n. *Within the time interval set and writing all quantities from multiplications with time intervals, wave particle energy h. ν = M [h+, Δti] = h+, and in turn h+ = M [Δqi, m2] = m2 the source mass. Thus in the case of 'near' Newtonian situations, with distant source, the radiation energy h. ν relates only to source mass m2.*

12. Discussion. Star-source radiation in terms of the time interval only set

The description in this paper depends on commutation properties for certain sets. Depending on the considered set dimension, these quantities form reciprocal n-pairs, for the time interval only set, a one-dimensional set, n-pairs are 1-pairs or just pairs and n = 1. Where the number of degrees of freedom increases due to these n-pairs, equilibrium brings constraints and reduction, the equilibrium requirement, depending on all quantities from the n-pair, introducing overall invariant ΔN-Noether charges. The Noether charges turn out to be structure constants for the relevant set, ie in this paper the time interval only set. It is assumed due to equilibrium there exists a n-pair energies, kinetic radiation and gravitation, with opposed to eachother time development.
Commutation relations, like from equation 1 and 10, assume the dimension of one moment time and the time interval set, ie 1-dimensionality. Both the 'working to the right' property and 'moving to the left' property form a n-pair, ie similar to a n-pair of boundaries for time intervals: there is applied one 'one moment time' parameter and one time interval, equations 1 and 10, to define time interval derivatives including the 'set rule'. A 'set rule' only makes sense for non zero commutation relations. This discussion can assume all other coordinates and intervals, like mentioned in paragraph 2, however time is the subject for this paper and only time interval related results are included. The following results imply star-source radiation propagation with finite propagation sphere surfaces where in usual qm field theory radiation fields extend parallel and stationary.

The equivalence principle and equivalent reference energies.
According to general relativity and the equivalence principle, kinetic energy during acceleration is equivalent to gravitation energy during the decrease of space-like distance of the source masses, and it is not possible to decide which reference is due, [Newton (III), 18], [Einstein, 19]. Similarly a new equivalent reference energie pair is defined: Es: e.m. kinetic wave energy, and Ee: gravitational energy. To follow the arguments for reciprocal quantities, paragraph 4, the reference pair is assumed to be related to each other by a transformation TR. The energies Es and Ee have the same value however each from their own reference, not meaning the respective spacetime coordinate system, rather like the wave and complementary wave particle description within qm. A reference equivalence for Es and Ee differs from the usual relation for kinematics and dynamics, Newton's first law, [Newton (I), 8], since considered is energy, and not included is action. Radiation propagation implies dispersion free noninteracting radiation complementary with free moving wave particles. Dispersion free star source radiation propagation relates to invariant e.m. surface field energy including outward time development, ie towards infinity, [Hollestelle, 4]. Due to MVT equilibrium assured is I*||Δt equals D*||Δt, within the time interval only set results are the same, paragraph 10. It follows outward radiation is equivalent to inward gravitation in terms of energy.

Preprints (www.preprints.org) | NOT PEER-REVIEWED | Posted: 21 March 2023 doi:10.20944/preprints202209.0151.v3

From equation 35 and 36, Es = A [1/|c(Δt)|, A [m1, m2]], and Ee = M [1/|c(Δt)|, M [m1, m2]]. The Es kinetic reference includes 'addition' or derivatives, the Ee gravitation reference includes 'multiplication' or integration. Both results agree with Newton's second law from applying differentiation to equations 35 and 36, Es due to the derivative to Δt and coordinate Δq being positive and decreasing and Ee due to the derivative to Δq, [Hollestelle, 4], [Goldstein, 2] and [Newton (I), 8]. In particular, the derivative to resp. Δt or Δq implies opposite sign, giving an argument for the opposite of the actions related to radiation and gravitation.

The mass quantities for both equations 35 and 36 being regarded identical is the basis of the equivalence principle and for the usual interpretation of matter. Continued is with applying the description of reciprocal quantities to the reference energies.

Following the discussion for the pair of commutation quantities cn(Δt) and cn'(Δt), ie Δt-Noether charges, and the specific Lorentz transformation TL(Δt) = Δt' with equal values for cn'(Δt) = cn(Δt'), correspondence C2, paragraph 5, introduced is reference transformation TR, with TR(Es) = Ee and vice versa. Still, energies Es and Ee are Δt-quantities that allow application of A, M and D*||Δt while remaining within the same reference. Transformation TR relates the equivalent pair Es and Ee, just like the specific Lorentz transformation TL relates a n-pair, with n = 1, of reciprocal quantities while the energy remains invariant, ie the equilibrium requirement. The equilibrium requirement implies surface energy remains invariant during transformation TL for time interval Δt, where the quantities cn and cn' include the 'moving to the right' property of Δt.

A trial transformation is TR: Es and A [Es, Δt] to Ee and M [Ee, Δt], from addition to multiplication or the other way around, where evidently the values for Es and Ee, for Δt, remain unchanged, similar like for equations 35 with A and 36 with M.

With D*||Δt [Es] = Es and D*||Δt [Δt] = Δt, since Δt = ΔU there is Es = M [Δt, Es] = M [Es, Δt] and it follows Es = D*||Δt [M [Es, Δt]] = A [c1. Es, c2. Es] = A [A[c1. Es, c2. Es], Δt], equation 18. The reference transformation results in Ee = TR(Es) = M [M [c1. Ee, c2. Ee], Δt] = M [c1. Ee, c2. Ee] due to Ee = M [c1. Ee, c2. Ee] from the multiplication linearity theorem, paragraph 10. This also means: D*||Δt [Es] = A [c1. Es, c2. Es] and D*||Δt [Ee] = M [c1. Ee, c2. Ee].

<u>Comment 19.</u> *The derivative D*||Δt of a multiplication of TL related quantities, due to D*||Δt [cn(Δt)] = A [c1. cn(Δt), c2. cn(Δt')], relates, with derivative constants c1 and c2, to the overall time interval set average for Δt-quantity cn(Δt). With cn(Δt') = cn(TL(Δt)) = cn'(Δt) and cn(Δt) different from cn'(Δt) only from the time interval densities, it is these densities for cn(Δt) and cn'(Δt) that decide on c1 and c2.*

<u>Comment 20.</u> *Within the e.m. wave energy Es reference, the m1 and m2 represent the Δt and Δt' radiation properties including c(Δt) and star source properties, from Es = Ee = M [Δqi, M [m1, m2]] = D*||Δq [M [m1, m2]], equation 36, in terms of complementary gravitational mass. The derivative constants c1 and c2 relate to the n-pair of Δt-quantities, Δt-Noether charges, with properties changing with Δt, ie together with propagation and time development, and for which their multiplication Noether charge ΔN, paragraph 4, is overall time interval set invariant.*

It is inferred that derivatives as an expression for curvature/non-Cartesian geometry are directly related to source interactions, like gravitational energy for m1 and m2. Since derivatives are part of the usual wave equation description of qm, this inference means geometry and gravitation can be united with qm through the second associative arguments for derivatives. Indeed, one way to derive qm wave functions is from Noether charges [Arnold, 1]. This inference is a time interval only result due to the existence of reference transformation TR. It is expected that similarly derivatives can be generalized to space-time intervals and second associativity property arguments related to Noether charges for properties depending on space-time intervals. There is a difference for time intervals and space intervals, [Hollestelle, 4], time intervals and time do not allow to reverse/turn backwards where 3-dim. space-like intervals do, and, the propagation sphere surface requirement of invariant measure only emerges from Δt, not from Δq. This difference is also discussed introducing structure constants, paragraph 10, or from simultaneity, paragraph 11.

I. Radiation energy and wave particle number.
Wave energy Es, due to external interaction during Δt depends on one moment time t where Es = Es(t) is a t-quantity, with invariant ν like for the photo-electric effect. Es(ta) = #n. Es(tb) = (#n)^2. h. ν and depends quadratic on #n and linear on frequency ν. This differs from internal interaction with variable frequency ν, complementary wave particle mass m1 and number #n, where Es = #n. h. ν remains invariant and Es(t) = Es(tb) for all t during Δt and the description remains within the time interval only set. From these relations one can verify by measurement the relation Es = h. ν for wave particle and complementary wave particles and waves respectively, this is one of the few relations from qm that remain within the time interval only set description.

II. The 'set rule' and the concepts 'close to' and 'remote'.
Scalar multiplication M [Δt1, Δt2] = Δt3 = a. Δta can be evaluated by applying transformation T1 and 'scale'

transformation T2, termed B and C in [Hollestelle, 3] and discussed below equation 8. T1 includes the specific transforms T1 [tb] = tb and |T1 [t0]| << |t0| and T1 [ta] = -1. tb, and T2 is a scale transformation transforming Δt = [tb, ta] to T2 [Δt] = [C. tb, C. ta] with C a scalar and boundary parameters t to T2 [t] = C. t = t0. 1/T2 [t0]. t, for a certain given T2 [t0], except for t = t0, since t0 is not a possible boundary parameter. T1 and T2 transform time intervals to time intervals while remaining within the time interval set. Δtb within commutator [1/tb, A]|| Δtb, is the result of repeated application of T1 and T2 to Δt = [tb, ta].

Within equation 23 for Rest(a)|t, the second and third expression are derived independently and confirm: derivative D* [a] = D*||Δt [a] equals the ordinary commutator [1/tb, a]u. This indicates that situations 'remote' from and 'close to' with respect to situations with H time independent are more similar than the terms suggest. The situations 'remote' from and 'close to' Newtonian, with Newtonian meaning: with H time independent, can be related, by interpretation from MVT equilibrium of the commutators [1/t, a]||Δt and [1/t, a]u for any scalar a and any Δt. With the below arguments a correspondence for 'remote' and 'close to' is derived.

A change of the relevant time interval Δt from Δt' to a different Δt'' includes a change of the interval boundaries. The time interval derivative to one moment time parameter tb, is defined to be equal to D*||Δtb [a] = [1/tb, a]||Δtb with Δtb = [tb', tb''] and similarly for ta with Δta = [ta', ta'']. Recall that these Δtb, and Δta, are for all tb' and tb'' well defined time intervals, due to T1 and T2 transforming time intervals to time intervals. One derives [1/tb, a]||Δtb = [1/t, a]||Δt, and thus [1/tb, a]||Δtb ~ D*||Δt [a]. For this to define a time interval set version for Stoke's theorem, it should be D*||Δt [a] = a. This is valid for scalar a equal to a multiplication like ΔN, paragraph 10.

Discussed below 'set rule' equations 23 and 24 is correspondence D*||Δt [a] ~ D* [a]. One also derives [1/tb, a]||Δtb ~ 1/2. [1/tb, a]u with both correspondences only valid for 'close' to Newtonian, H time independent, situations. It follows D*||Δt [a] ~ 1/2. [1/tb, a]u, where both quantities are well defined, is a correspondence valid for 'close to' Newtonian situations, ie Δt is 'close to' symmetrical. However 'set-rule' equation 23 is valid for all situations, H time dependent or time independent, including both expressions, and confirms both correspondences for all situations. This implies some questions concerning what means 'close to' time independent for H and what means 'close to' symmetrical for Δt. In the description of star source radiation 'close to' or 'remote' from symmetrical for Δt means 'remote' from or 'close to' for the propagation surface from the star source, [Hollestelle, 4]. The meaning of distance and asymmetry in terms of time interval Δt can be an introduction to space time itself and action at a distance.

III. Scalar multiplications and scale transformations. Linear subsets within the time interval only set.
From M [Δt1, Δt2] = Δt3, a time interval due to the closure theorem, and since any time interval is neutral in Δt, there is assumed Δt3 is linear in Δt: Δt3 = e. Δt for some scalar e. Transformation T2, together with scale factor C = a, provides meaning for scalar multiplication with a time interval result: a. Δta = Δt3 = M [Δt1, Δt2] with a. Δta = T2 [Δta] remains a well defined time interval. Now write Δta itself linear in Δt. With ca = ΔU and a = 1 multiplication Δt3' = M [ΔU, Δta] = e'. Δt = Δta. All a. Δta can be expressed with time intervals e. Δt linear in Δt. By choosing Δta = Δt one finds the specific ca and a where M [Δt1, Δt2] = e. Δt = ca. Linear subsets resemble cosets for 1-dim. sets with certain properties.

One finds a. Δta = M [ca, Δta] and a. Δta = a. M [ΔU, Δta] = M [a. ΔU, Δta] and this means multiplication with 'multiplication unit' ΔU or with ca both leave the linear subset for Δta invariant. Indeed, it follows M [Δt1, Δt2] is linear in Δta. The linear subset for Δta is the part of the time interval set defined from, time interval or scalar, multiplication with Δta. It follows M [Δt1, Δt2] = a. e'. Δt = e. Δt with scalar e = a. e' and part of the linear subset for Δt. When Δt2 equals Δt there is M [Δt1, Δt] = Δt1 and this confirms the assumption: any time interval Δt1 from the time interval set can be written like a multiplication including Δt, and is linear in Δt. This confirms, in terms of linear subsets, multiplication property A and the time interval set closure theorem, paragraph 8.

Comment 21. Non-linear and linear events. Within the time interval description introduced are two different quantities, t-quantities and Δt-quantities. It is possible to relate non-linearity, related to Δt-quantities like Noether charges that depend on multiplication, with linearity, related to t-quantities like one moment time t coordinates that depend on addition, by applying transformations 'working to the right' like multiplication with t or 1/t and integration or derivatives. This means, non-linear events are related to linear events by applying one of these transformations within the time interval description.
Two reciprocal t-quantities, possibly the same, after multiplication quadratic in t, correspond with a Δt invariant quantity, ie with a t^(0) Δt-quantity, paragraph 4. In contrast, a t-quantity, linear in t, due to addition remains t-quantity, with linearity from time development by addition, and remains possibly variable during Δt. Similarly a t^(-1) t-quantity equals a t^(+1) t-quantity.

IV. Structure constants.

Usually, structure constants are defined from the multiplication properties for the set or group elements writing these like exponentials of generator set elements. From assuming the canonical property for the set elements, it follows the commutator for two generator elements can be assumed to be linear within the generator set, ie gives another generator set element for result, including a scalar multiplication. These scalars are termed structure constants. Taking care of the multiplication Taylor series depends on writing out all higher order ordinary commutators of generators that should reduce to first order ordinary commutators which provides a linear result, depending on the structure constants for the set. For the time interval set, multiplication implies a linear result with M [Δt1, Δt2] = Δt3 = a. Δta providing the linearity constants and in this case this is enough to find the structure constants, without applying the generator set.

For any set, structure constants themselves are independent of the set representation. Indeed, for the time interval only set the structure constants are independent of the number of necessary and different Δta, ie whether the subset of different Δta is reducible or not. Similarly, the Δta subset is not completely determined by structure constant properties. There being only one independent subset for the time interval only set, confirms the time interval only set being 1-dimensional, and the existence of only one independent structure constant. A structure constant is to relate to the essential properties or quantities of the set, in this case these are the group velocity c(Δt) and the differentiation arguments c1 and c2, that together introduce the specific relations for space-time and time development, like radiation propagation.

There is c(Δt) = M [Δq, Δti] = D*||Δt [Δq] = [Δt, Δq]||Δt is equal to ordinary commutator [Δt, Δq]u which is independent of t within Δt. Some aspects of the non trivial character of c(Δt) are discussed in paragraph 11. For Δq a neutral Δt-quantity, M [c(Δt), Δt] = c(Δt) = M [M [c(Δt), Δti], Δt] provides the linearity constant for Δt, and ordinary commutator [Δt, Δq]u = M [M [c(Δt), Δti], Δt] with M [c(Δt), Δti] = c(Δt) a structure constant for the time interval set. However, c(Δt) does not have dimension of scalar.

When measuring c(Δt) the time interval Δt is the relevant event time interval, which also is the relevant event time interval for measuring Δq. This can be resolved with the quantity h+. Following assumptions for non-interacting star source radiation propagation [Hollestelle, 3] there is |Δ*q| = |Δq| and |Δ*p| = |Δp| and this implies, h+ is a Δt-quantity linear in |Δq|. |Δp|, and due to being equal to M [E, Δt], h+ is linear in Δt.
E = M [h+, Δti] for E invariant during propagation and without interaction. Both these relations are equal to ordinary commutator relations and both h+ and Δq are linear in Δt since they are Δt-quantities. It follows E = M [h+, Δti] = [Δt, h+]u and this ordinary commutator equals E = [E, Δt] = M [M[E, Δti], Δt] with M[E, Δti] = E, a structure constant for the time interval set like c(Δt). Thus, c(Δt) = M [Δq, Δti] and E = M [h+, Δti] can be considered as structure constants. Both E and h+ are only introduced to find that starting from different quantities the structure constants for a set remain the same, the number of structure constants depending just on the dimensionality of the set, where the time interval set is 1-dimensional.
Interaction can be included with Δq = Δq(Δt) or with h+ = h+(Δt) and time interval differentiation to Δt without applying parameter t. The wave group-velocity c(Δt) and radiation energy E(Δt*)*, Δt-quantities, can be interpreted to be structure constants disregarding quantity dimensionality. Now introduce the following invariants c_ = M [c(Δt)i, |c(Δt)|] for velocity and h. ν_ = M [(h. ν)i, |h. ν|] for energy that render dimensionless multiplication results with c(Δt) and E respectively*: sc1 = M [c(Δt), c_] = |c(Δt)|, and sc2 = M [E, h. ν_] = |h. ν| are structure constants for the time interval set. These are the dimensionless scalar values for the relevant properties c(Δt) and h. ν.*

The time interval only set is different from the usual groups considered in relation with structure constants, ie within elementary particle field theory. One difference is the canonical property usually assumed for these groups to derive the multiplication results for the generators, which however is not valid for the time interval only set, due to the existence of non-zero factors Rest(a)|t and Rest(a)||Δt, equation 23 and 24. It is inferred that there is only one pair of cn(Δt), cn'(Δt), since equation 1 includes only one reciprocal pair cn and cn', and there is only one independent structure constant for the time interval only set itself, from the cn(Δt) and cn'(Δt) Δt-quantities, the set being 1-dimensional. This means sc1 = sc2, they are the same structure constant. Similar arguments are valid for interval quantities with a dimension different from the time interval set, for instance Δq-quantities and the space interval only set. Due to equation 23 the factor Rest(a)|t is included in Noether charge NC, that can be derived by assuming both the overall time interval and space interval averages for cn(Δt) are the same, following a theorem from [Arnold, 1], and equal to ΔN. One finds the time interval derivative to Δt of the complete space integral ΔN to be NC = A [M [c(Δt), ΔN], c(Δt)], depending on the first associative property for multiplication and applying similarity for M and A, paragraph 10. Thus NC is equal to the time interval derivative of overall time interval average ΔN = M [cn(Δt), cn(Δt')], ie NC is equal to D*||Δt [M [cn(Δt), cn(Δt')]] = A [a. D*||Δt [Δta], Rest(a)||Δt], for a = c(Δt) and Δta = ΔN and Rest(a)||Δt = c(Δt). From paragraph 10, D*||Δt [ΔN] = ΔN implying ΔN = NC and NC = M [c(Δt), NC] with solutions c(Δt) = NC, or, c(Δt) equals 'multiplication unit' ΔU.

Δt-Noether charge Δt-NC equals the energy |h. ν|. This Δt-NC is equal to cn(Δt) or cn'(Δt), since there is only

Preprints (www.preprints.org) | NOT PEER-REVIEWED | Posted: 21 March 2023 doi:10.20944/preprints202209.0151.v3

one Δt-quantity Noether charge due to dimensionality for the time interval only set.
A second result is ΔN-NC = ΔN = M [cn(Δt), cn(Δt')], the sum of the reciprocal commutation n-pair. The ΔN-Noether charge can be derived to be ΔN-NC = |mA|^2 = |E|^2, and ΔN-NC = M [cn(Δt), cn(Δt')] = A [cn(Δt), cn(Δt')]. This equality is valid due to E = h. v and due to the results of paragraph 10. |h. v| = m2 is a result from paragraph 11, ΔN-NC = |E|^2 is derived in [Hollestelle, 4].
The usual Noether charge is termed NC, paragraph 4. The non-linear property Rest(c(Δt))||Δt = c(Δt) of the time interval only set is included in Noether charge NC: NC = M [sc1, c_i] = c(Δt), ie NC equals the radiation wave group-velocity, with sc1 = M [c(Δt), c_] the structure constant for the time interval only set depending on c(Δt). From NC = M [Rest(c(Δt))||Δt, ΔN] = M [c(Δt), ΔN] = M [NC, ΔN] follow the solutions for the time interval Δt-Noether charges: Δt-NC = cn(Δt) and ΔN-NC = ΔN = c(Δt) are the same, ie *the commutation relations derive exactly the field group-velocity, in case of photons, cn(Δt) equals the velocity of light c*. Consider the Δt-NC are the same as the usual NC when the complete overall time interval set is regarded, paragraph 4. Time intervals with domain measure non zero only are included, comment 13.

V. The equation-sign and the commutator for addition.
The equations A [ΔU0, Δt0] = Δt0 and ΔU0 = (1 - 1). Δt0 = A [Δt0, -1. Δt0] seem contradictory, unless one considers zero domain for ΔU0, comment 13, and infers there is extra freedom when quantities can be subject to 'moving to the other side of the equation-sign' by multiplication with scalar -1. It means introducing a 'commutator for addition'. When quantities are added the related reciprocal pair, or n-pair, changes to include these quantities and similarly for the Noether charge and structure constants, and the number of degrees of freedom. The chosen side with respect to the equation-sign can matter and can have different value, depending on the order of the involved time intervals. This is a subject not for this paper, however to investigate this the time interval only set seems to be a good start. A similar freedom seems to reside when re-writing Newton's laws and equilibrium definitions in a similar way depending on equation-sign side.

The confusion exists in applying -1. Δt0 to mean Δt0iv, the addition inverse, within addition equations like above like ΔU0 = (1 - 1). Δt0 = A [Δt0, -1. Δt0], and interpreting ΔU0 to mean a difference including scalar -1, and can be avoided by 'moving to the other side of the equation-sign'. These differences, due to scalar -1, with (1 - 1). Δt0 = A [Δt0, -1. Δt0] equal to A [Δt0, Δt0iv], are part of the one moment time description since this description depends on vectors. However, in the time interval only description the addition inverse is defined from A [Δt2, Δt2iv] = Δt0 for any time interval Δt2.

References.
[1] Arnold, V. I. (1974): Mathematical Methods of Classical Mechanics, Russian/English translation 1989, K. Vogtmann and A. Weinstein, Springer
[2] Goldstein, H. (1950): Classical Mechanics, Addison-Wesley
[3] Hollestelle, H. (2020): Time, Equilibrium, and General Relativity, Int. Jrnl. of Science & Tech., vol. 8, 5
[4] Hollestelle, H. (2021): Some Results for a Time Interval Approach to Field Theory and Gravitation, preprints202106.0684.v3
[5] Beller, M. (1999): Quantum Dialogue, Univ. Chicago Press
[6] Roos, W. J. (1978): Grondslagen van de quantummechanica (Fundaments of quantum mechanics), postgraduate course, Utrecht Univ.
[7] Sakurai, J. J. (1978): Advanced Quantum Mechanics, Addison-Wesley
[8] Newton, I. (1686) (I): The Mathematical Principles of Natural Philosophy, Latin/English translation 1729, A. Motte, Benjamin Motte Editors, Book I, includes the axioms or laws of motion
[9] Hollestelle, H. (2017): Thermodynamics of a Set of Closed Curved Surfaces and Noether's Theorem, Int. Jrnl. of Science & Tech., vol. 5, 1
[10] Curie, P. (1894): Sur la symétrie dans les phénomènes physiques, symétrie d'un champ électrique et d'un champ magnétique. J. Phys. Theor. Appl., 3 (1), Hal archives-ouvertes
[11] Newton, I. (1686) (II): Benjamin Motte Edition 1729, Book I, includes the principles of derivatives by infinitesimal limit
[12] Noether, E. (1918): Invariante Variationsprobleme (Invariant Variation Problems), German/English translation 1971, Tavel, M. A., Transport Theory and Statistical Physics, 1(3), arXiv:physics0503066v3
[13] De Wit, B. and J. Smith (1986): Field Theory in Particle Physics, Vol. I, North Holland Physics/Elsevier
[14] Jacobson, N. (1974): Basic Algebra I, Freeman
[15] Veltman, M. (1974): Representations of Groups and Applications to Elementary Particle Physics, postgraduate course, Utrecht Univ.
[16] Hocking, G., S. Young (1961): Topology, Addison-Wesley
[17] Einstein, A. (1921): Geometry and Experience, Lecture at the Prussian Academy of Sciences, 1921, The Collected Papers of Albert Einstein, 2002, Princeton Univ. Press
[18] Newton, I. (1686) (III): Benjamin Motte Edition 1729, Book I, Definition I, includes the definition of the quantity of matter, related to acceleration, proportional to mass known by its weight
[19] Einstein, A. (1923): The Foundation of the General Theory of Relativity, in Lorentz, H., Einstein, A., Minkowski, H. and H. Weyl: The Principle of Relativity, reprint of the original publication 1952, Methuen and Company, translated by Perrett, W. and G. Jeffery, Dover Publishers

Preprints (www.preprints.org) | NOT PEER-REVIEWED | Posted: 21 March 2023 doi:10.20944/preprints202209.0151.v3