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# Solution of Black Hole Information Paradox in Loop Quantum Gravity

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Abstract:

In this article, Planck star and remnant scenario i.e., solution of black hole information paradox in loop quantum gravity (LQG) are elaborated. Since, Black hole is the final fate of collapse scenario in general relativity; firstly, gravitational collapse with two possible final outcomes i.e., black hole and naked singularity, is outlined. Thereafter, black hole thermodynamics along with four laws of black hole thermodynamics, Hawking radiation, brief introduction to information theory and cause of information paradox are given. Then, solution of black hole information paradox outside the LQG are briefly explained. Thereafter, basic concepts of LQG along with loop quantum cosmology (LQC) are described. LQG can solve information paradox by its natural cut off on the value of quantum volume operator. In LQG, a star namely, Planck star is also proposed that is formed at Planck density and resides at so-called singularity point. Since, LQG removes singularity through big bounce and proposes Planck star which provides enough room for information to escape; hence, it can solve information paradox. LQG predicts remnant of black hole of the order of $10^{-14}m$; which, is under current technological regime. In the remnant scenario, black hole makes quantum transition into white hole and this white hole behaves as a long lived remnants that can solve information paradox. In this quantum transition, a black hole tunnels into a white hole at the end of evaporation.

Keywords: black Hole, black hole thermodynamics, black hole information paradox, loop quantum gravity, Planck star, remnant scenario.

## Contents

# 1 Introduction

General relativity (GR) and quantum physics or quantum field theory (QFT) are fundamental building blocks of modern physics. General relativity involves study of macroscopic part of the universe in curved geometry such as stellar collapse, galactic collapse, cosmological evolution, astrophysics etc. through the study of gravitational interaction; while, quantum field theory involves study of microscopic part of universe through the study of electromagnetic, strong nuclear and weak nuclear interaction. However, there is no sharp boundary between GR and QFT. It means one can use one theory into another theory to understand any phenomenon that involves both frameworks [1].

Study of gravitational collapse involves study of singularity that is introduced at the last phase in case of stellar collapse. Study of coordinate singularity ($r = 2m$) and curvature singularity ( $r = 0$) has raised many questions in physics. Coordinate singularity is actually the boundary of black hole (namely, event horizon) whose actual singularity resides at ( $r = 0$). Due to the strong gravitational pull of black hole even light cannot go beyond event horizon; once, it is trapped inside this boundary. Thus, event of horizon prevents outside observer to communicate with inside observer or vice versa. At the singularity of black hole, general relativity fails, since, it could not remove infinity introduced through singularity. This failure suggests that one needs quantum theory of gravity that can successfully explain singularity that is introduced in stellar collapse, galactic collapse and at the time of big bang [1].

After the discovery that the entropy of black hole is related to its area but not its volume by Bekenstein [2-3]; Hawking suggested that black hole also emits radiation namely Hawking radiation. It means that black hole is not totally black. Hawking successfully derived the formula of black hole entropy i.e., Bekenstein – Hawking entropy formula[4].

Hawking also suggested due to hawking radiation black hole will eventually evaporates and the information of interior of black hole is lost forever [4]. But, information is always conserved according unitarity principle of quantum theory. This principle is the integral part of quantum physics. This raises a paradox; namely, black hole information paradox. Therefore, either considered physical phenomenon is wrong that involves the violation of this principle or one does not know the theory through which information can be recovered [5].

There are many possible solutions of black hole information paradox given by various theories such as AdS/CFT correspondence [6], complementarity [7], AMPS firewall [8], fuzzball [9], soft hair [10], quantum halo [11], and baby universe [12]. Here, all proposed solutions are briefly given; then, solution of black hole information paradox in loop quantum gravity is elaborated.

In this article, general relativity in context with black hole is introduced in the second section. Thereafter, black hole thermodynamics is given in the third section. In the fourth section brief introduction of information is theory, information paradox and proposed solutions

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outside the LQG and inside the LQG are briefly given. In fifth section, loop quantum gravity along with the introduction of loop quantum cosmology and solution of black hole information paradox in LQG i.e., Planck star and remnant scenario are explained.

## 2 Gravitational collapse and black hole thermodynamics

In this section, two possible of outcome of gravitational collapse along with the concept of black hole of thermodynamics are given up to relevant extent in context with black hole information paradox solution in LQG.

### 2.1 Final outcome of gravitational collapse: black hole or naked singularity?

GR proposes that final fate of collapse of a massive star is either black hole or naked singularity. In case of naked singularity, curvature regions formed during the gravitational collapse and ultrahigh density are visible from far away, and there can be observational consequences. Thus, one can have observational quantum gravity effects. In case of gravitational collapse of massive stars, quantum gravity is needed to understand the resulting spacetime singularities [13 - 17].

In case of such collapse, event horizon is created gradually. If any star enters into this horizon before it final stage of collapses i.e., a singularity; then, a black hole is created. If the event horizon does not form or delayed with gravitational collapse; then, a naked singularity is created. Big bang is one of such singularity which was occurred 13.7 billion year ago and visible to far away observer. Another kind of naked singularity is that occurs in case of gravitational collapse of massive star [13 - 17].

It is internal condition (parameters in terms of initial data) within the star such as velocity of collapsing shells, density and pressure (tangential and radial) that determines whether singularity is hidden (black hole) or visible (naked singularity). In case of realistic assumptions, naked singularities also do occur in gravitational collapse. For instance, gradual decrement of density from center to outer layers of collapsing forms naked singularity at the final collapse stage [13 - 17].

The basis of black hole physics put forward Karl Schwarzschild; when he gave a solution of EFEs in which spacetime was assumed spherically symmetric, exhibiting a point mass at the center of symmetry but totally empty and the geometry that is far away from source was assumed flat. Schwarzschild solution is the basis of black hole physics. According to this spherical vacuum symmetric solution, black hole is created. In this solution, the Schwarzschild metric is [13 - 17],

$$ds^2 = \left(1 - \frac{2m}{r}\right) dt^2 - \left(1 - \frac{2m}{r}\right)^{-1} dr^2 - r^2 \left(d\theta^2 + sin^2\theta d\phi^2\right) \tag{2.1}$$

Where, $d\Omega^2 = d\theta^2 + sin^2\theta d\phi^2$ is the volume element and $r = \sqrt{x^2 + y^2 + z^2}$. At $r = 2m$ (event horizon of black hole), coordinate singularity is resided and gravitational potential can have degeneracy in this solution. . At $r = 0$, curvature singularity is occurred [13 - 17].

If $d\theta = d\varphi = 0$ and $ds^2 = 0$; then, the slope of the light cone in case of radial part is $\frac{dt}{dr} = \pm\left(1 - \frac{2m}{r}\right)^{-1}$. If $r \to \infty$ in $r = 2m$ then the slope will be $\frac{dt}{dr} = \pm 1$. For outgoing radial null curves with smaller $r$, $\frac{dt}{dr} = \frac{r}{r-2m}$ $\left(r \to 2m, \frac{dt}{dr} \to \infty\right)$; thus, the light cones will become narrower as one approaches to $r = 2m$. By taking integration in $\frac{dt}{dr} = \pm\left(1 - \frac{2m}{r}\right)^{-1}$, one gets $t = r + 2mln|r - 2m|$ [13 - 17].

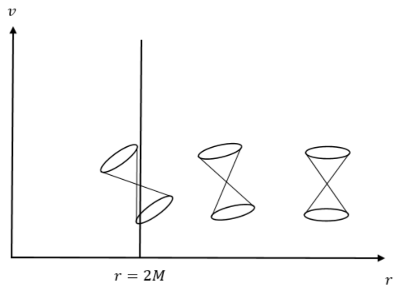


Fig. 2.1: Inward tilting of light cone after r=2M in Eddington Finkelstein coordinates.

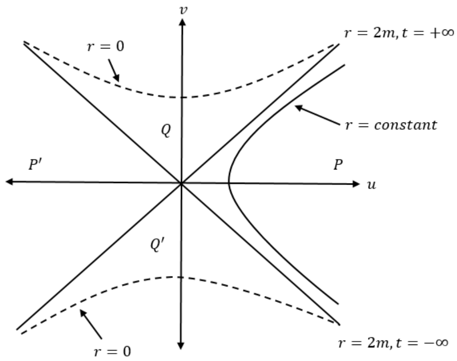


Fig.2.2: A diagram of Kruskal metric with different regions.

In case of radially infalling particles, one gets, $\left(1-\frac{2m}{r}\right)\frac{dt}{d\tau}=1$ and $\left(\frac{dr}{d\tau}\right)^2=\frac{2m}{r}$. Thus, $\frac{dr}{dt}=-\sqrt{\frac{2m}{r}}\left(1-\frac{2m}{r}\right)$ [13 - 17].

The tortoise coordinate i.e., $r^*=r+2mln\left(\frac{r}{2m}-1\right)$ along with two null coordinates i.e., $u=t-r^*$ and $v=t+r^*$ in Eddington –Finkelstein coordinate is crucial to make new Schwarzschild's metric non-singular at $r=2m$ [13 - 17].

The Schwarzschild metric in the form of Eddington –Finkelstein coordinate is written as [13 - 17]

$$ds^2=\left(1-\frac{2m}{r}\right)dv^2-2dvdr-r^2\left(d\theta^2+sin^2\theta d\phi^2\right) \tag{2.2}$$

In Eddington –Finkelstein coordinates, while approaching to $r=2m$ light cones not become very narrow and light cones tilts over that region; since, time and radial coordinate reverse their character inside coordinate singularity i.e., $r=2m$ [13 - 17].

In the 1960s, Kruskal and Szekeres developed coordinate system by considering global approach of GR; which could separate coordinate singularity with actual curvature singularity. Later on Cauchy problem in GR was studied in which global evolution and the role of spacetime topology are clarified. This Kruskal metric can be written as [13 - 17]

$$ds^2=\frac{32m^3}{r}e^{-\frac{r}{2m}}\left(du^2-dv^2\right)+r^2\left(d\theta^2+sin^2\theta d\phi^2\right) \tag{2.3}$$

Where, $u$ and $v$ behaves as global radial marker and global time marker [13 - 17].

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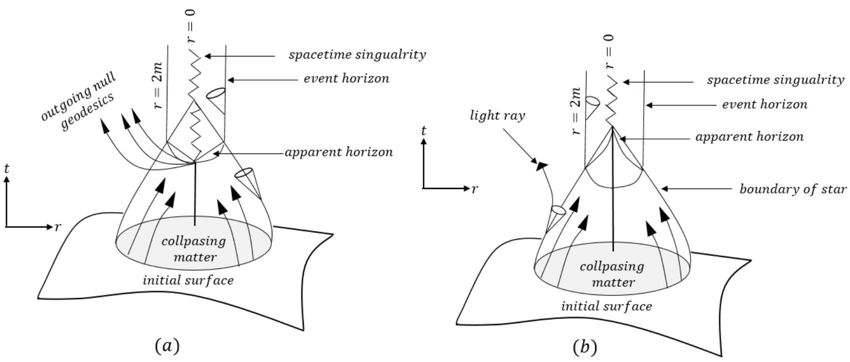


Fig.2.3: (a) Naked singularity as final outcome of gravitational collapse if event horizon is delayed. (b) Spacetime diagram of dynamical evolution in OSD model that create Schwarzschild black hole.

In 1930 Oppenheimer, Snyder and Datt gave OSD model on gravitational collapse of massive object according to GR. Oppenheimer, Snyder and Datt studied the continual gravitational collapse of a pressureless dust, homogeneous density distribution with uniform density matter cloud . The formation event horizon in collapse was proposed in OSD model and is salient feature of this model. As mentioned, event horizon is created before the creation of singularity in case of black hole. Hence, one who resides outside the event horizon cannot communicate with inside observer; since, event horizon behaves at cut off surface. Thus, the Schwarzschild geometry is a integral part of the OSD collapse. This model is proved to be crucial in the understanding of active galactic nuclei, quasars and radio galaxies [13 - 17].

A star that is homogeneous, made of spherical dust with no pressure is converted into black hole at the last stage of collapse. In realistic condition, most of the stars are not fully spherical. These stars are inhomogeneous, and also possess rotation. Thus, during collapse, their shapes are distorted and their rotational speed increases [13 - 17].

In 1969, Roger Penrose conjectured that an event horizon does always form in case of final stage of collapse under reasonable physical conditions. Such a conjecture is known as cosmic censorship conjecture. This conjecture is the base of black hole physics [13 - 17].

One the famous solution of EFEs is the Kerr metric i.e., a rotating axi-symmetric solution. According to one of the conjecture of no hair theorem, end state of collapse is Kerr black hole (mass, spin). Kerr black hole drags their adjacent spacetime in the direction of rotation. So, Penrose suggested that such effects permits a rotating black hole as a source of energy [13 - 17].

In the below table the discrepancies between black hole and naked singularity are briefly summarized; since, it is necessary in understanding remnant scenario and Planck star. Naked singularity is crucial in loop quantum gravity to provide solution of black hole information paradox [14-17].

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| Sr. no. | Black hole | Naked singularity |
|---|---|---|
| 1 | An object in which singularity is censored by event horizon. | An event or object in which singularity is visible or naked. |
| 2 | Information or object that fall in cannot be observed by far away external observer. | Information or object that fall in can be observed by far away external observer one or other way. |
| 3 | It holds cosmic censorship conjecture; hence, it is responsible for information loss and information paradox. | It violates cosmic censorship conjecture; hence, information is not lost and no information paradox. |
| 4 | In case of homogeneity, uniform density distribution, symmetrical and pressureless spherical stellar or galactic collapse, it is created. | In case of inhomogeneity, non-uniform density distribution, non-symmetrical, non-spherical stellar or galactic collapse with non-zero pressure, shear of spacetime, and in case of black hole that rotating or having charge it is created. |
| 5 | Event horizon created before singularity creation and hence black hole. | Event horizon is delayed in case of naked singularity. |

Table no. 2.2: Discrepancies between black hole and naked singularity

There are many other interesting papers on various aspects of naked singularity [18-22].

## 2.2 Laws of black hole thermodynamics

In 1973, Bardeen et al. [23] gave four laws of black hole thermodynamics for axisymmetric solution of EFEs in which stationary state of rotating black hole is axisymmetric. In this equations, Area of event horizon $A$ and surface gravity $\kappa$ is analogous to entropy and temperature of thermodynamics [23-24].

**Zeroth law:** in Black hole thermodynamics, the surface gravity $\kappa$ is constant over the event horizon for a stationary black hole. This law is analogous to zeroth law of thermodynamics [23-24].

**First law:** the relation between two neighbouring stationary axisymmetric solutions exhibiting a perfect fluid with circular flow and a central black hole is given as [23-24]

$$\delta M = \frac{\kappa}{8\pi}\delta A + \Omega_{BH}\delta J_{BH} + \int \Omega \delta dJ + \int \bar{\mu}\delta dN + \bar{\Theta}\delta dS \tag{2.4}$$

Where, $M$ is mass of black hole, $\Omega_{BH}$ is angular velocity of black hole, $J_{BH}$ is angular momentum of black hole, $\delta dJ$ is the change in the angular momentum of the fluid crossing any surface element say $d\Sigma_B$, $\delta dN$ is the change in the number of particles crossing $d\Sigma_B$, $\delta dS$ is the change in the entropy crossing $d\Sigma_B$, $\bar{\mu}$ is the "red-shifted" chemical potential, $\bar{\Theta}$ is the "red-shifted" temperature. Here, $\frac{\kappa}{8\pi}$ is analogous to temperature [23-24]

This law is analogous to first law of thermodynamics i.e., $(\Delta U = Q - W)$ [23-24]

**Second law:** for any black hole, the area of black hole never decrease with time [23-24].

$$\Delta A \geq 0 \tag{2.5}$$

This law is analogous to second law of thermodynamics i.e., $(\Delta S \geq 0)$ [23-24].

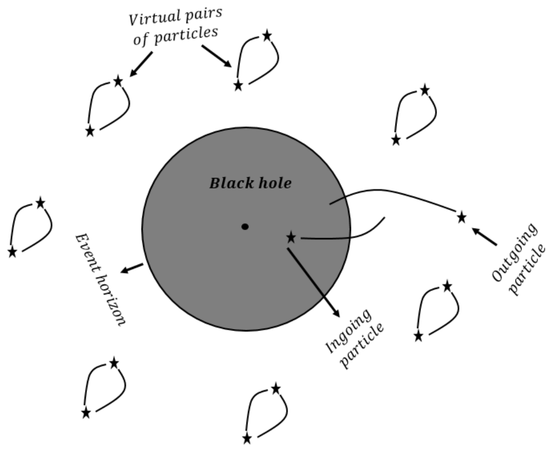


Fig.2.4: Black hole with few virtual pairs of particles near event horizon from infinite pairs in spacetime and one pair in which ingoing particle and out going particle (Hawking particle) is shown.

**Third law:** it is impossible to reduce surface gravity $\kappa = 0$ by any procedure by finite sequence of operations whether it is idealized. This law is analogous to third law of thermodynamics i.e., at absolute zero temperature, the entropy of the system becomes constant.If one throws particles into Kerr black hole to increase its angular momentum and to decrease its surface gravity ; then, the decrement in the value of $\kappa$ occurs per particle thrown in. It gets smaller and smaller as the ratio of angular momentum to mass approaches to i.e., $\frac{J}{M^2} = 1$ and at last, it becomes 0 [23-24].

For $\frac{J}{M^2} \rightarrow 1$, idealistic accretion process also exists; in which, an infinite divisibility of the matter and an infinite time are required with the addition of finite rest mass [23-24].

There is a possibility of naked singularity in case of $\kappa = 0$. in finite number of operations, by reducing $\kappa$ to 0 creates naked singularity through this process and asymptotic predictability is violated that is necessary for black hole physics [23-24].

## 2.3 Bekenstein-Hawking entropy formula and Hawking radiation

In 1971, Bekenstein found that entropy of black hole is proportional to its area but not volume. In the equatorial form, it is written as $S = \alpha A$. Where, $\alpha$ is universal constant [2-3].

Later on, Hawking found that black hole also emits radiation and is not completely black. This radiation is known as Hawking radiation and it also has temperature $T_{BH} = \frac{\hbar c^3}{k 8\pi G M_{BH}}$. Black hole's temperature is proportional to its mass $T_{BH} \propto \frac{1}{M_H}$. But, classical physics says that, black hole can only absorb but cannot emit any radiation. To explain emitting black hole, Hawking suggested that if quantum mechanical effect is considered; then, it causes creation and emission of particles (virtual pair of particles) from black hole. It seems as if black hole is emitting radiation (i.e., Hawking radiation) and it also has temperature. Due to continuous thermal emission; the mass of black hole gradually decreases [4, 24-26].

In case of black hole, it means that one of the particle of virtual pair (it is created due to relativistic quantum mechanical effect near event horizon of black hole) goes in and another particle comes out. So, ingoing particles brought negative energy flux with it and outgoing particles take away positive energy flux with it. Hence, ingoing particles that possess negative energy flux causes event horizon of black hole to shrink towards the singularity; and eventually, this black hole will be disappeared [4, 24-26].

By considering Bekenstein's conjecture, Hawking also gave the equation of entropy of black hole i.e. Bekenstein-Hawking entropy formula [4, 24-26],

$$S_{BH} = \frac{k_\beta A}{4l_p^2} = \frac{k_\beta c^3}{4G\hbar}A = \alpha A \tag{2.6}$$

Where, $l_p^2 = \sqrt{\frac{G\hbar}{c^3}}$ is Planck length, $\alpha = \frac{k_\beta c^3}{4G\hbar}$ is constant, $k_\beta$ is Boltzmann's constant, $\hbar = \frac{h}{2\pi}$ is reduced Planck's constant, $G$ is universal gravitational constant, and $A$ is area of event horizon of black hole [4, 24-26].

# 3 What is information and information paradox ?

In this section, the topics such as the meaning of information in classical and quantum information theory, the unitarity in quantum physics, cause of information paradox in case of black hole and various solution of black hole information paradox outside the field of loop quantum gravity are given up to relevant extent.

## 3.1 Brief overview of information theory

In 1990, Wheeler proposed that information is basics of the physics of the universe. Wheeler used the phrase i.e., "it from bit" [27]

According to Wheeler, everything in physics, such as every particle, every field, and space-time also gets its function, meaning, and existence from yes or no questions i.e., binary choices - bits. Hence, all things and objects are information-theoretic in origin in the universe [27].

In classical information theory, information exhibited in a physical system is the number of $yes/no$ or $high/law$ or $0/1$ questions one needs to get answered to fully specify the system. In case of classical information, the basic unit is the bit [27-30].

In quantum information theory, information of the state of a quantum system is defined by quantum information and qubits are basic units of quantum information. Quantum information is the fundamental physical entity of study in quantum information theory. Quantum information processing techniques is used to manipulate quantum information. Quantum information is defined in terms of Von Neumann entropy [27-30].

In quantum information theory, non-commutative observable cannot be evaluated due to uncertainty principle; since, the eigen state in basis is not matched. Therefore, definitive information about both variables cannot be exhibited by a quantum state [27-30].

Since, information is deeply related to entropy in physics. Hence, for any thermodynamic system, the entropy is written as [30-35]

$$-k_\beta \sum_j p_j ln p_j \tag{3.1}$$

Where, corresponding to a given macrostate of the system, the sum is over all possible microstates and each microstate has a probability $p_j$ of happenning. If it is assumed that each and every microstate has equal probability of occurrence $p_j = \frac{1}{\Omega}$, and $\Omega$ is the number of microstates corresponding to a given macrostate, one can give Boltzmann equation of entropy; which is valid in the micro-canonical ensemble at thermodynamic equilibrium [30-35].

$$S = k_\beta ln\Omega \tag{3.2}$$

The modern information theory was discovered by Ralph Hartley, Claude Shannon and many others scientists as a generalization of entropy. Later on, quantum information theory was also incorporated to this classical information theory for quantum computing, quantum entanglement, quantum teleportation and many other purposes [28-29].

This information theory uses binary system (0 and 1) to study information. For instance, if number of state in terms of this binary in any message i.e., 10101 then $U$ is the number of equiprobable symbols that each character may take for $N = 5$ and $U = 2$. Hence, information must be a function of $u$ as well as $n$ and proportional to length of given message i.e., $N$. It also increases with the number of equiprobable messages $U^N$. Such an information $I$ is written as [30-35]

$$I = Nf(U) = g(U^N) \tag{3.3}$$

Its differentiable solution is given as

$$f(U) = clnU \tag{3.4}$$

Where, $c$ is a any positive constant. As mentioned, information $I$ is a measure of missed or incomplete information. This equation is similar to $S = k_\beta ln\Omega$ [30-35].

Claude Shannon gave generalized equation of information [29] i.e.,

$$I = -c\sum_{j=1}^{U} p_j lnp_j \tag{3.5}$$

which is similar to equation $-k_\beta \sum_j p_j lnp_j$. Shannon also proposed that that the entropy $S$ of a system in terms of information cannot be lower than the entropy of any of its constituent parts [29].

Thereafter, John von Neumann found that quantum state can also by represented by another mathematical object i.e., density matrix instead of wave function. This density matrix is written as [30-35]

$$\rho = \sum_j p_j |\psi_j\rangle\langle\psi_j| \tag{3.6}$$

The density matrix explains a statistical ensemble of different quantum states. It is also used to represent pure or mixed quantum states. The value of probability of any quantum state is unity; then, one can have complete knowledge of that quantum state. Practically it is not possible; since, there is always incompleteness and an ensemble can have different quantum states [30-35].

In case of mixed state, if the wave function $\psi_j$ with probability $p_j$ is there then the expectation value of an observable described by any operator $D$ is [30-35]:

$$\langle D\rangle = \sum_j p_j \langle\psi_j|D|\psi_j\rangle \tag{3.7}$$

By taking expansion of the the states in a basis $\beta_j$ [30-35],

$$|\psi_j\rangle = \sum_k |\beta_k\rangle\langle\beta_k|\psi_j\rangle \tag{3.8}$$

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$$\langle \psi_j| = \sum_l \langle \psi_j|\beta_l\rangle\langle\beta_l| \tag{3.9}$$

By putting these values into above equation, one gets [30-35]

$$\langle D\rangle = \sum_{k,l}\left(\sum_j p_j\langle\beta_k|\psi_j\rangle\langle\psi_j|\beta_l\rangle\right)\langle\beta_l|D|\beta_k\rangle$$

$$\therefore \langle D\rangle = \sum_{k,l}\langle\beta_k|\rho|\beta_l\rangle\langle\beta_l|D|\beta_k\rangle = tr\left(\rho D\right) \tag{3.10}$$

Where, $tr$ is trace [30-35].

In case of one pure state, there will be one non-zero eigen value in density matrix; since, probability is unity. If state is mixed; then, the density matrix has more than one non-zero eigen value. If the eigen values $\rho_j$ implies that the probabilities that the quantum system is in state $j$; then the density matrix is diagonalised. The eigen values of density matrix is equal or greater than zero and it is Hermitian. The trace of density matrix is one [30-35].

In case of quantum system, the von Neumann entropy can be written as [30-35]

$$S = -tr\rho ln\rho = -\sum_j \rho_j ln\rho_j \tag{3.11}$$

This equation gives entropy for mixed quantum states; since, the entropy is zero for pure state. This entropy is used as computing the degree of entanglement between quantum subsystems. If any pure state i.e., $\rho_{total} = |\Psi\rangle\langle\Psi|$ is divided into two part i.e., $M$ and $N$; then, Hilbert space that exhibiting these states is also divided into two separate spaces i.e., $\mathcal{H}_{total} = \mathcal{H}_M \otimes \mathcal{H}_N$. For sub system $M$, the density matrix will be [30-35]

$$\rho_M = tr_N\rho_{total} \tag{3.12}$$

$tr_N$ is the trace over $\mathcal{H}_N$. the entanglement entropy of subsystem $M$ is given by the von Neumann entropy of $\rho_M$ [30-35] i.e.,

$$S_M = -tr_M\rho_M ln\rho_M \tag{3.13}$$

How entropy behaves differently in quantum systems is defined by Entanglement entropy. In case of entangled system, the total entropy is equal to zero; but entropy of subsystems is greater than zero according to Von Neumann entropy equation. One can keep track about the information of whole system; but, one cannot about the part of system [30-35].

## 3.2 Information paradox

The root cause of black hole information paradox is Hawking radiation. In case of black hole that follows classical physics laws, there will be no evaporation; hence, information will not be destroyed. However, one cannot reach to this information due to enormous gravitational pull of black hole. In case of quantum physical treatment, black hole will be evaporated after several years due to Hawking radiation and information is lost [4, 36].

Now, consider a pair of photon near event horizon of black hole in which one photon has negative energy $-E$ and another photon has positive energy $+E$. If $-E$ photon fall into black hole and another travels to infinity (as Hawking radiation) then black hole will eventually be evaporated [37].

To understand information paradox, consider Unruh effect according to which acceleration of observer is proportional to the temperature. It means accelerative observer will feel hot thermal radiations [37].

Consider, two static observers just outside the event horizon of a black hole in Schwarzschild spacetime. One of them is resided at $r_1$, and the other is at $r_2$ ($r_2 > r_1$), which will be pushed to infinity later on. Now consider the inverse of the Schwarzschild radius i.e., the natural acceleration scale $\frac{1}{2M}$ [37].

In case of first observer at $r_1$, its acceleration is far more then $\frac{1}{2M}$, i.e., $a_1 >> \frac{1}{2M}$.If equivalence principle is considered; then, there is no tidal force and observer feels nothing. By comparing static observer (at $r_1$) with a freely falling observer one can say that to maintain $r_1$ distance a static observer has to accelerate itself in outward direction with $a_1$ acceleration and feels radiation of temperature $(\frac{a_1}{2\pi})$ i.e., Unruh radiation. This Unruh radiation is redshifted; since, it is detected by first observer and first observer is propagating towards second observer. Since, second observer far away (at $r_2$) from event horizon; it will not feel Unruh radiation. The magnitude of redshift is given by [37]

$$T_2 = \frac{\mathcal{V}_1}{\mathcal{V}_2} T_1 \tag{3.14}$$

Where, $\mathcal{V}_1$ and $\mathcal{V}_2$ are redshift factor and $T_1$ and $T_2$ are thermal radiation temperatures of first and second observer respectively [37].

If $T_1 = \frac{a_1}{2\pi}$ , the redshift factor at infinity tends to unity; and if first observer is resided close to event horizon whose surface gravity $\kappa = \mathcal{V}_1 a_1$; then, the temperature of the radiation felt by faraway observer is $T = \frac{\kappa}{2\pi}$ [37].

This temperature is possessed by Hawking radiation emitted by the black hole [37].

Since, the area of black hole is proportional to entropy and entropy can be understood in term of information; one can increase the area of black hole by throwing a single or more bits of information into black hole. Consider an example of photon. If knowledge of the point of entry into black hole of photon is unknown; then, such a photon has one bit of information according to Susskind. If wavelength of this photon is of the order of Schwarzschild radius then its energy is $E = \frac{hc}{R_S}$[38].

After the entry of photon into black hole, mass and energy of black hole increases i.e., $\frac{E}{c^2} = \frac{h}{cR_S}$ and $E$. It induces the increment in the radius of event horizon, i.e., $\Delta R_S = \frac{2Gh}{c^3 R_S}$ [38]

Since, the area of event horizon is $A = 4\pi R_S^2$; the area of horizon also increases [38]

$$\Delta A = 4\pi \left(R_S + \delta R_S\right)^2 - 4\pi R_S^2$$

$$\Delta A = 4\pi \left(R_S + \frac{4Gh}{c^3} + \frac{4G^2h^2}{c^6 R_S^2}\right) - 4\pi R_S^2$$

Since, the term $\frac{4G^2h^2}{c^6 R_S^2}$ is negligible it is eliminated and by defining $\frac{Gh}{c^3} = l_P^2$ [38].

$$\therefore \Delta A = \frac{16\pi Gh}{c^3} = 16\pi l_P^2 \tag{3.15}$$

Every time with dropping single bit into black hole, the area of a black hole increases by one Planck area it means that the area of a black hole's horizon is proportional to its entropy or hidden information that is missed information. So, one can say that there is relationship between area of black hole and information that brings towards holographic principle [38-39].

In short, the black hole information paradox can be understood by considering pure state matter that forms a black hole after collapse. After complete evaporation, black hole leaves Hawking radiation which is thermal in nature. Therefore, initial pure quantum state is now converted a mixed state. A transition from pure to mixed quantum state is not unitary. Hence, it violates basic principle of quantum physics [34, 36, 38-39].

This quantum evolution can be understood using an S-matrix i.e., $|\Psi_F\rangle = S|\Psi_I\rangle$; where, $F = final$ and $I = initial$. If the evolution is deterministic; then, it is unitary in principle. It means the initial state can be again obtained from the final state i.e., $|\Psi_F\rangle = S^\dagger|\Psi_I\rangle$ [40].

A final mixed state from an initially pure state is due to the production of particles by black hole (near event horizon) [40].

Consider spacelike slices through the spacetime exhibiting the black hole that penetrate the event horizon and permitting to study the characteristics of the Hawking pairs such as entanglement. Here, semi classical domain is assumed in which quantum gravity effects are not crucial. Therefore, these spacelike slices follows below conditions [5]:

1. The quantum states that is under study; should be exhibited completely on a spacelike slice with intrinsic curvature $\mathbb{R}^{(3)}$. This intrinsic curvature is smaller than the Planck scale everywhere: $\mathbb{R}^{(3)} << \frac{1}{l_P^2}$

2. In $3 + 1$ dimensional spacetime, these spacelike slices should be embedded and it also possess small extrinsic curvature: $\mathbb{K} << \frac{1}{l_P^2}$.

3. The four-curvature of the spacetime should also be small near neighbouring spacelike slice: $\mathbb{R}^{(4)} << \frac{1}{l_P^2}$.

4. Since considered domain is semi classical, any present matter on the slice should not approach the Planck scale where quantum gravity effects becomes dominant and expected to apply.

5. The evolution of states are smooth from one slice to another slice afterwards.

When, above defined spacelike slices are deformed; Hawking pairs are created on the slices. The wavelength of these pairs is of the order of the curvature length scale of the deformation. In case of Schwarzschild black hole that is generic; the wavelength is of the order of Schwarzschild radius. The state of newly created entangled pair is written as [5]

$$|\phi\rangle = \frac{1}{\sqrt{2}}|0\rangle_b|0\rangle_a + \frac{1}{\sqrt{2}}|1\rangle_b|1\rangle_a \tag{3.16}$$

Here, the $a$ particles are going away to infinity and detected as Hawking radiation and the $b$ particles going in towards the singularity [5].

The complete quantum state $|\Phi\rangle$ of the black hole and the pair is written as [5]

$$|\Phi\rangle \approx |\phi\rangle_{matter} \otimes \left(\frac{1}{\sqrt{2}}|0\rangle_b|0\rangle_a + \frac{1}{\sqrt{2}}|1\rangle_b|1\rangle_a\right) \tag{3.17}$$

Where, $|\phi\rangle_{matter}$ is the matter that forms black hole. Since, the impact of matter is negligible on the pair; the equality is only approximate. Now, consider a matter state $|\phi\rangle_{matter}$ as a two-level quantum system i.e. [5],

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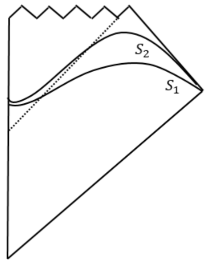


Fig.4.1: A penrose diagram of black hole that created through collapse.

$$|\phi\rangle_{matter} = \frac{1}{\sqrt{2}}|\uparrow\rangle_{matter} + \frac{1}{\sqrt{2}}|\downarrow\rangle_{matter} \tag{3.18}$$

By putting this into equation (4.17) [5]

$$\begin{aligned}|\Phi\rangle \approx &\left(\frac{1}{\sqrt{2}}|\uparrow\rangle_{matter} + \frac{1}{\sqrt{2}}|\downarrow\rangle_{matter}\right)\\ &\otimes \left(\frac{1}{\sqrt{2}}|0\rangle_b|0\rangle_a + \frac{1}{\sqrt{2}}|1\rangle_b|1\rangle_a\right)\end{aligned} \tag{3.19}$$

If the small effect of matter ($\rho << 1$) is considered on the pair; then, it gives formula with full identity instead of approximate identity. This perturbation is permitted; since, it does not modify entanglement properties of system [5]. i.e.,

$$\begin{aligned}|\Phi\rangle \approx &\left(\frac{1}{\sqrt{2}}|\uparrow\rangle_{matter} + \frac{1}{\sqrt{2}}|\downarrow\rangle_{matter}\right)\\ &\otimes \left(\left(\frac{1}{\sqrt{2}} + \rho\right)|0\rangle_b|0\rangle_a + \left(\frac{1}{\sqrt{2}} - \rho\right)|1\rangle_b|1\rangle_a\right)\end{aligned} \tag{3.20}$$

Even though, the below state modification is not permitted [5]

$$\begin{aligned}|\Phi\rangle \approx &\left(\frac{1}{\sqrt{2}}|\uparrow\rangle_{matter}|0\rangle_b + \frac{1}{\sqrt{2}}|\downarrow\rangle_{matter}|1\rangle_b\right.\\ &\otimes \left(\frac{1}{\sqrt{2}}|0\rangle_a + \frac{1}{\sqrt{2}}|1\rangle_a\right)\end{aligned} \tag{3.21}$$

Now, consider the entanglement entropies for the above systems [5].

Tracing over $|\phi\rangle_{matter}$ matter and $b$ for equation (4.19) provides $S_a = ln2$. For equation (4.20) it provides $S_a = ln2 - \rho^2(6 - 2ln2)$. For equation (4.21) it is $S_a = 0$. If $\rho$ is taken zero, then, The entanglement entropy slightly affected by the matter $|\phi\rangle_{matter}$ matter for the Hawking pair will be unperturbed i.e., ln2. The influence of faraway $|\phi\rangle_{matter}$ matter does not affect the final zero-valued entanglement entropy; since it is non-local on the spacelike slice. Therefore, locality infers [5]

$$\left|\frac{S_a}{ln2} - 1\right| << 1 \tag{3.22}$$

According equivalence principle, infalling observer at event horizon does not feel any strangeness (if tidal force of black hole is ignored). Similar to equivalence principle, the regions near event horizon does not contain information about the evaporating black holes that is studied. In case of evaporating black hole, different quantity of Schwarzschild radius wavelength quanta may be detected by different observer only to order 1. But, all observer will admit vacuum state with little error for quanta of wavelength very less than Schwarzschild radius but not below the Planck length [5,41].

The Schwarzschild solution is valid for black hole that is evaporating. Hence, the solution can be applicable during evaporation until the black hole's radius diminishes to the Planck length at any chosen point. By considering the above condition for spacelike slices that exhibits black hole; one should avoid singularity of black hole. By dividing spacelike slices into outside, inside, and connection across the horizon; one gets [5,41],

**Inside horizon** $\Gamma_i : \frac{M}{2} < r < \frac{3M}{2}$ and $r =$ constant. At times before singularity formation, it can be smoothly connected to Schwarzschild origin $r = 0$.

**Outside horizon** $\Gamma_o : r >> 4M$ and also $t =$ constant.

**Connection** $\Gamma_c$: time and space dimension of $\gamma_c$ are of order $M$. it establishes connection between $\Gamma_o$ and $\Gamma_i$ across the event horizon.

The union $\Gamma_i \cup \Gamma_o \cup \Gamma_c$ provides a whole spacelike slice $\Gamma(t, r, c)$. If the evolution of parameters evolves from one slice to other slice smoothly; then [5,41],

$$\begin{aligned} \Gamma_1 = \sigma(t_1, r_1, c_1) \implies \Gamma_2 &= \Gamma(t_2, r_2, c_2) \\ &= \Gamma(t_1 + \delta, r_1 + \delta r, c_1 + \delta c) \end{aligned} \tag{3.23}$$

If $\delta r << M$: then, forward evolution of spacelike slices of the geometry of $\Gamma_c$ remains unchanged. In the segment of $\Gamma_i$, this outcome is getting longer as $\Gamma_o$ shifts in forward direction in time. An entire spacetime exhibiting the black hole is built by gradual succession of these nice spacelike slices. The intrinsic geometries of both $\Gamma_o$ and $\Gamma_i$ remains the same; if, the slices evolves along a timelike normal. Even though, the connecting portion $\Gamma_c$ must be grown as to connect together these segments. To describe for the longer $\Gamma_i$ at successive intervals; this stretching occurs only near $\Gamma_c$ with spatio-temporal dimensions of order $M$ [5,41].

The origin of the Hawking pair generation is such stretching of $\Gamma_c$. At last, the $\Gamma_c$ would eventually become null and then timelike at spacetime; but, the choice of slices in the background of black hole stay spacelike during evolution because of the swapping of time and space coordinates at event horizons. As soon as the black hole reaches Planckian dimensions, Hawking pair production is stopped [5,41].

In case of continuous evaporation of black hole; the entanglement of Hawking pairs modifies. Thermal radiation emitted by normal object conserves information and this information can be retrieved at any stage of process; since, the whole process is unitary. For black holes, a series of spacelike slices with time increment [5,41]:

**Slice 1:** it is a collection of matter $|\phi\rangle_{matter}$ that has not collapsed yet to create black hole.

**Slice 2**: At which, black hole has now created. First Hawking pair is produced; when middle of slice stretches

$$|\Phi\rangle \approx |\phi\rangle_{matter} \otimes \left( \frac{1}{\sqrt{2}} |0\rangle_{b1} |0\rangle_{a1} + \frac{1}{\sqrt{2}} |1\rangle_{b1} |1\rangle_{a1} \right) \tag{3.24}$$

That provides entanglement entropy $ln2$ between the rest of the system and the Hawking radiation [5,41]

**Slice 3:** at this slice, the matter $|\phi\rangle_{matter}$ is unchanged and by the stretching of $\Gamma_c$, the $a_1, b_1$ pair goes outwards. This stretching also induces creation of a new Hawking pair $a_2, b_2$.

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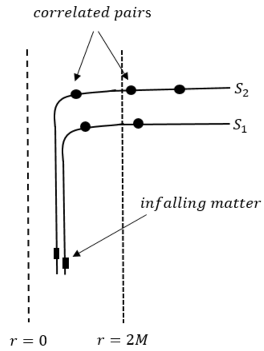


Fig. 4.2: a diagram of Schwarschild' black hole in which the stretching of spacelike slices during evolution of black hole with correlated pairs is shown.

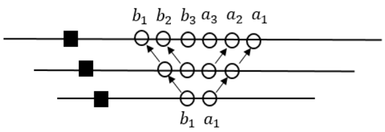


Fig. 4.3: With each new Hawking pair creation earlier one pushing away from event horizon of black hole.

$$|\Phi\rangle \approx |\phi\rangle_{matter} \otimes \left(\frac{1}{\sqrt{2}}|0\rangle_{b_1}|0\rangle_{a_1} + \frac{1}{\sqrt{2}}|1\rangle_{b_1}|1\rangle_{a_1}\right) \otimes \left(\frac{1}{\sqrt{2}}|0\rangle_{b_2}|0\rangle_{a_2} + \frac{1}{\sqrt{2}}|1\rangle_{b_2}|1\rangle_{a_2}\right) \tag{3.25}$$

That provides entanglement entropy $ln2$ between the rest of the system and the Hawking radiation [5,41]

**Slice N+1**: At this slice, $N$ Hawking pairs have been produced:

$$\begin{aligned} |\Phi\rangle \approx |\phi\rangle_{matter} &\otimes \left(\frac{1}{\sqrt{2}}|0\rangle_{b_1}|0\rangle_{a_1} + \frac{1}{\sqrt{2}}|1\rangle_{b_1}|1\rangle_{a_1}\right) \\ &\otimes \left(\frac{1}{\sqrt{2}}|0\rangle_{b_2}|0\rangle_{a_2} + \frac{1}{\sqrt{2}}|1\rangle_{b_2}|1\rangle_{a_2}\right) \\ &\dots \\ &\otimes \left(\frac{1}{\sqrt{2}}|0\rangle_{b_N}|0\rangle_{a_N} + \frac{1}{\sqrt{2}}|1\rangle_{b_N}|1\rangle_{a_N}\right) \end{aligned} \tag{3.26}$$

That provides entanglement entropy $ln2$ between the rest of the system and the Hawking radiation [5,41].

Gradually black hole will emit all of its mass as Hawking radiation and at last, only radiation field will remain i.e., $a_1, a_2, \dots, a_N$. But the final Hawking radiation possess entanglement entropy $Nln2$ which is not entangled with anything. So, density matrix is used to explain the radiation; since, it is in a mixed state. Before complete evaporation of the black hole, the Hawking radiation had entanglement with internal states of the black hole [5,41].

By considering possible corrections, with an magnitude $ln2 - 2\rho$ per pair ($\rho << 1$); the entanglement entropy of the outgoing radiation increases with every new creation of Hawking pair consistently [5,41].

Consider an example, in which unitarity is violated but information is conserved [5,41]. i.e.,

$$|\phi\rangle_{matter} = \mu|1\rangle_{matter} + \nu|0\rangle_{matter} \tag{3.27}$$

Two Hawking pairs i.e., $(a_1, b_1)$ and $(a_2, b_2)$ have come out from the vacuum; after this matter has collapsed to create a black hole. Hence, the complete quantum state of the system is [5,41]

$$\begin{aligned}&\left(\frac{1}{\sqrt{2}}|1\rangle_{matter}|0\rangle_{b_1} + \frac{1}{\sqrt{2}}|0\rangle_{matter}|1\rangle_{b_1}\right)\\ &\otimes (\mu|1\rangle_{a_1} + \nu|0\rangle_{a_1})\\ &\otimes \left(\frac{1}{\sqrt{2}}|1\rangle_{b_2}|0\rangle_{a_2} + \frac{1}{\sqrt{2}}|0\rangle_{b_2}|1\rangle_{a_2}\right)\end{aligned} \tag{3.28}$$

In the beginning, the first Hawking radiation particle i.e., $a_1$ exhibits all of the information related to the matter that collapsed to create the black hole. This information could be retrieved by an observer and so, information is not lost. But creation of Hawking pair is not stopped by the black hole. The second Hawking radiation particle $a_2$ is entangled with its partner $b_2$ with entanglement entropy $ln2$ within the black hole; hence, the final state is mixed after the complete evaporation of black hole. So, unitarity is violated [5,41].

Now consider one Hawking pair $(a, b)$ which is created by a matter state (4.27) that evolves into a black hole [5,41] i.e.,

$$\mu|1\rangle_{matter}|0\rangle_b + \nu|0\rangle_{matter}|1\rangle_b \otimes \left(\frac{1}{\sqrt{2}}|1\rangle_a + \frac{1}{\sqrt{2}}|0\rangle_a\right) \tag{3.29}$$

In which a Hawking radiation particle $a$ is in a pure state but does not have any information about the original matter [5,41].

These are theoretical models in which somehow non-unitarity and information loss is avoided. In reality, normal black hole evaporation cannot avoid non-unitarity as well as information loss; and hence, it creates black hole information paradox [5,41].

## 3.3 Various solution of information paradox outside LQG in brief

The history of physics has witnessed many paradoxes that were occurred in different theories of physics and revolutionize theoretical physics. The black hole information paradox is the top most paradox of modern theoretical physics. Since the discovery of Hawking radiation, there are many solutions proposed by different theoretical communities. In some proposals, the general relativity is modified; while, in some proposals, the quantum physics is modified. Many proposed quantum gravity theories such as string theory, loop quantum gravity and other theories have provided solution for this paradox [1,5].

**AdS/ CFT correspondence:** This theory is one of the duality i.e., gauge/gravity duality, that appeared in string theory that correlates 5D anti De Sitter space (GR) with 4D conformal field theory (QFT). According this theory the Hawking radiation is not completely thermal. It means quantum correction is received by Hawking radiation that stores information at interior of black hole through encoding. Based on this proposal, Hawking suggested that quantum perturbations of the event horizon can permits information to escape from a black hole. This proposal focuses on horizon but not on singularity [6].

**Complementarity:** In this proposal, stretched horizon that is hot as well as physical, is proposed that hovers outside the horizon. According this proposal, an external observer either detect information that comes from horizon or information about interior of black hole by approaching to singularity. The detection of both information is simultaneously not possible. Because, it follows complementarity rule of quantum physics; hence, it does not violate no-cloning theorem of information theory. A proposal, in which monogamy entanglement is combined with complementarity provides existence of firewall [7].

**AMPS Firewall:** According to this proposal, entanglement between outgoing particle (Hawking) radiation and particle that falls inside, is somehow broken instantly after the creation of these particles. This broken entanglement releases enormous amount of energy and creates a wall i.e., firewall. It means that any external observer that approaches to firewall will be incinerated by firewall. This proposal conserves information of interior of black hole; however, it violates GR (namely, equivalence principle) [8].

**Fuzzball:** This proposal belongs to string theory in which black hole is interpreted as fuzzball i.e., ball of string (and branes) of string theory. It removes singularity as well event horizon of black hole. Fuzzball proposal somehow permits information to come out with Hawking radiation [9].

**Soft hair:** This proposal gives more degrees of freedom to black hole. According to this proposal information does not completely enter into horizon of black hole; but, it provides imprint at outside the horizon of black hole [10].

**Quantum halo:** In this proposal of the quantum black hole it has been considered that it interacts in spacetime via small fluctuation and therefore permits information to come out [11].

**Baby universe:** According to this proposal, the information is stored in the universe i.e., baby universe which is somehow isolated and separated from the present universe [12].

**Planck star:** Study of this star also belongs to LQG. In LQG, the Planck star is created when the energy density of collapsing star reaches to the Planck density. In LQG, when the collapsing star reaches Planck density; repulsive force is produced at so-called singularity point due to Heisenberg's uncertainty principle and the black hole is bounced to the white hole. Planck star is resided at this point of quantum transition from the black hole to the white hole. With final explosion, information will come out to external observer [42].

**Remnant scenario:** This proposal belongs to LQG; in which the black hole singularity resolution is more emphasized to solve the black hole information paradox. According to this proposal, information cannot not come out directly; but, it resides in the interior of black hole. It comes out when the black hole evaporation is about to end [43].

In the next section, after necessary introduction of LQG and LQC up to relevant extent; Remnant scenario and Planck star are covered in detail.

# 4 Loop quantum gravity

In this section, introduction and important results of loop quantum gravity are briefly given. Thereafter, loop quantum cosmology (LQC) is briefly explained up to necessary extent. At last, Planck star along with the remnant scenario as possible solution of black hole information paradox are elaborated. LQG is explained in many classic books [1, 44-49] and papers [50-63]. LQC is covered in many books [64-65] and papers [66-68].

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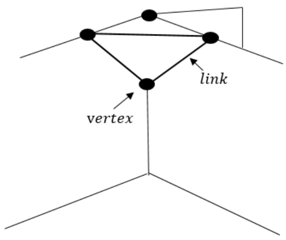


Fig.5.1: A diagram of atom of space.

## 4.1 Brief overview of loop quantum gravity

Study of LQG begins with some conceptual ideas from general relativity such as diffeomorphism invariance and background independent; then, it takes some concepts of quantum field theory to give quantum gravitational picture [1, 44, 50, 55].

LQG, gives quantum picture of space-time. Hence, it is named as quantum geometry. In LQG, geometry is quantized and hence the geometrical observable such as area and volume are promoted as quantum operator. In the beginning, LQG used Dirac's canonical quantization procedure; thus, it is also named as canonical quantum general relativity [1, 44, 50, 55].

LQG proposes discrete structure of spacetime; in which spacetime also possess atomic structure that resembles to matter (i.e., atoms) as well as electromagnetic radiation (i.e., photon). At Planck scale, due to interaction between these quanta of spacetime ; the observed classical spacetime is emerged [1, 44, 50, 55].

To create quantum spacetime, LQG utilizes the combinatorial principle of angular momentum of the spin network that provides a quantum state of the gravitational field. So, a quantum state of space is achieved by spin network; in which, area and volume are not continuous. In LQG, spin network (loops) is not resided in space; but, the spin network is itself creates space. The spin foam is created by the evolution of spin networks [1, 44, 50, 55].

In LQG, the holonomy is very important to generate loop states. In GR, the holonomy is defined as the value difference between initial and final vector transported (in LQG, it is spinor transported) on closed loop during parallel transport operation [1, 44, 50, 55].

The parallel transport operator i.e., $U(dx^{\mu})$ is utilized to differentiate between two different considered field values at different positions. Vector (or spinor) parallel transport relies on path and also exhibits exponential. For finite separations, this parallel transport is described along any arbitrary closed loop or curve $]\omega$. i.e., $U(x,y)$ using the value of connections [1, 44, 50, 55],

$$U(x,y) = \mathcal{P}e^{\left(\int_{\omega} A_{\mu}^{J}(x)t^{J}dx^{\mu}\right)} \tag{4.1}$$

Where, $A_{\mu}^{J}$ is connection and $J \in (0,1,2,3)$, $dx^{\mu}$ is separation in term of displacement between different considered field points, $\mathcal{P}$ implies path ordered integral, $t^{J}$ is gauge group generator, $x$ and $y$ are end point of considered arbitrary curve $\omega$. The path ordered exponential on closed loop behaves as holonomy in LQG; when, the spinor transport in the gauge theory is taken with gauge connection [1, 44, 50, 55].

By considering spinor transport on closed path, one gets the Wilson loop; that is the gauge invariant quantity and obtained from the holonomy of gauge connection. In LQG, this derived holonomy is named as Wilson loop variable and it is invariant under gauge transformation [1, 44, 50, 55]

From parallel transport of spinor around closed loop, one gets the trace of holonomy of this closed path or loop that gives single variable. Thus, this trace is given as [1, 44, 50, 55]

$$W_\omega = Tr\mathcal{P}e^{\left(\oint_\omega igA^J_\mu(x)t^J dx^\mu\right)} \tag{4.2}$$

In year 1986, Ashtekar founded new kind of variable (i.e., Ashtekar's variable) and it is crucial in LQG. This variable simplifies connection formulation of canonical approach of LQG through simplifying the constraints that embraces a complex-valued form for the connection and tetrad variables. In LQG, there are mainly three sort of constraints; namely, Hamiltonian, diffeomorphism or Gauss constraints. The Ashtekar variable makes these constraints free of mathematical anomalies. In LQG, $\mathcal{H}_{EHA}$ is the Einstein-Hilbert-Ashtekar Hamiltonian that is a sum of these (Hamiltonian, diffeomorphism and Gauss constraints) constraints [1, 44, 50, 55].

Since, these all are constraints; these are equal to zero in the canonical approach of LQG. Hence, these constraints behaves as operators and by the action of these operators on a state of quantized spacetime - $|\psi\rangle$; one gets $\mathcal{H}_{EHA}|\psi\rangle = 0$ i.e., the Wheeler – Dewitt equation. In LQG, the Wilson loops intersects and generates the quantum spacetime in which geometrical observable such as area and volume are the quantum operators i.e., the area operator and the volume operator respectively. The states of this quantized spacetime are represented by graphs of spin network. The edge labels of this graphs is obtained by representations of the gauge group ($SU(2)$) and the $SU(2)$ group is a subgroup of $SL(2, C)$ [1, 44, 50, 55].

The complex valued connection $\Gamma^j_c$ replaces the real connection $\omega^{JK}_\mu$ in Ashtekar's formulation. Thus, the resultant variables are [1, 44, 50, 55],

$$\tilde{E}^c_j \rightarrow \frac{1}{i}\tilde{E}^c_j, K^j_c \rightarrow A^j_c = \Gamma^j_c - iK^j_c \tag{4.3}$$

Where, $\tilde{E}^c_j$ is the scalar density or triad electric field, $A^j_c$ is the spatial connection or Ashtekar-Barbero connection, $K^j_c$ the extrinsic curvature. These variables i.e. $A^j_c$ and $\tilde{E}^c_j$ are similar to coordinate and momentum $(q, p)$ [1, 44, 50, 55]

The Ashtekar's formalism behaves as $SU(2)$ gauge theory; since, the Ashtekar's connection formulation variables, i.e. $A^j_c$ and $\tilde{E}^c_j$ acknowledge rotation of $SU(2)$ symmetry with respect to the internal indices. [1, 44, 50, 55].

These constraints are simplified in Ashtekar's variables and written as [1, 44, 50, 55]

$$\mathcal{G}_j = D_c\tilde{E}^c_j \tag{4.4}$$

$$\mathcal{C}_c = \tilde{E}^d_j F^j_{cd} - A^j_c\mathcal{G}_j \tag{4.5}$$

$$\mathcal{H} = \varepsilon^{jk}_l \tilde{E}^c_j \tilde{E}^d_k F^l_{cd} \tag{4.6}$$

$\mathcal{G}_j$, $\mathcal{C}_c$, $\mathcal{H}$ are Gauss, diffeomorphism, and Hamiltonian constraint respectively [1, 44, 50, 55].

The resulting equation of the Einstein-Hilbert-Ashtekar Hamiltonian of GR is [1, 44, 50, 55]

$$\mathcal{H}_{EHA} = N^c\mathcal{C}_c + N\mathcal{H} + T^j\mathcal{G}_j \tag{4.7}$$

Where, $T^j$ is a Lie algebra valued function over spatial surface. $N^c$ and $N$, are shift and lapse, respectively. [1, 44, 50, 55].

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Any state $\psi[A]$ (or functional) is obtained in the canonical approach of LQG through the action of constraints. Thus, the valid solution of equation of motion are [1, 44, 50, 55]

$$\hat{\mathcal{H}}|\psi\rangle = 0 \tag{4.8}$$

$$\hat{\mathcal{C}}_c|\psi\rangle = 0 \tag{4.9}$$

$$\hat{\mathcal{G}}_j|\psi\rangle = 0 \tag{4.10}$$

The Hamiltonian constraint generates time evolution for the state $\psi[A]$. The $\psi[A]$ stays invariant under diffeomorphism constraints. The $\psi[A]$ stays gauge-invariant functions of the connection under the Gauss constraint [1, 44, 50, 55].

Now, the state $\psi[A]$ is written as sum of loops [1, 44, 50, 55] i.e.,

$$\psi[A] = \Sigma_\lambda \psi[\lambda] W_\lambda[A] \tag{4.11}$$

The intersection of these loops creates the quantum spacetime. These loops can give different topology through knotting with each other [1, 44, 50, 55].

The canonical variables behaves as operators in the canonical quantization. Hence, the connection as operator is written as [1, 44, 50, 55],

$$\hat{A}^j_c \Psi(A) = A^j_c \Psi(A) \tag{4.12}$$

Similar to the connection, the triads ( as functional derivative) are written as [1, 44, 50, 55]

$$\hat{\tilde{E}}^c_j \Psi(A) = -i\frac{\delta\Psi(A)}{\delta A^j_c} \tag{4.13}$$

The commutation relations connection and triad is [1, 44, 50, 55]

$$\left[\hat{A}^k_d(y), \hat{\tilde{E}}^c_j(x)\right] = i\delta^c_d \delta^k_j \delta^3(x-y) \tag{4.14}$$

Hence, the triads are the conjugate momenta to the connection. Therefore, the quantum operator for the triad is $E^c_j \rightarrow -i\hbar\frac{\partial}{\partial A^j_c}$ [1, 44, 50, 55].

Using triad and connection operator, the action of $\hat{\mathcal{H}}$ on a Wilson loop is given as [1, 44, 50, 55]

$$\hat{\mathcal{H}} W_\lambda[A] = \varepsilon^{jk}_l E^c_j E^d_k F^l_{cd} W_\lambda[A]$$

$$\hat{\mathcal{H}} W_\lambda[A] = \varepsilon^{jk}_l \frac{\partial}{\partial A^j_c}\frac{\partial}{\partial A^k_d} F^l_{cd} W_\lambda[A] \tag{4.15}$$

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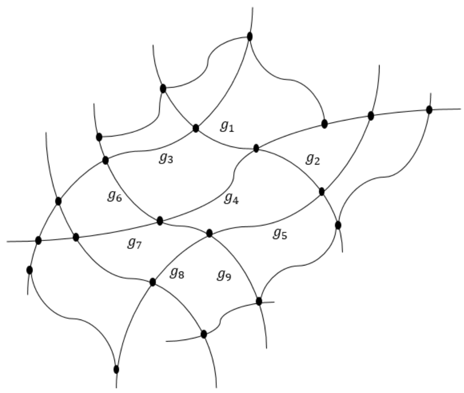


Fig. 5.2: A diagram of spin network.

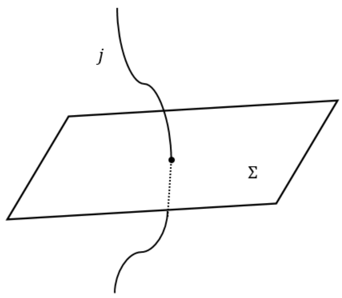


Fig. 5.3: Area generation by puncturing of link in to surface.

Spin network can be obtained by taking a loop state as network $\Theta$ or a graph with edges $e_i$ labelled by elements of some gauge group. In general, the gauge group in LQG is $SU(2)$ or $SL(2, C)$ [1, 44, 50, 55].

$$\psi_\Theta = \psi\left(g_1, g_2, ..., g_n\right) \tag{4.16}$$

Where, $j = 1, 2, 3 \ldots$ and $g_j$ is the holonomy or group element of connection $(A)$ on the $jth$ edge [1, 44, 50, 55].

These loop states are states of LQG. Initially, Penrose gave these representation in spin network. He used combinatorial principle of angular momentum [1, 44, 50, 55].

Each edge are represented by group elements in spin network. In case of LQG these group representation in LQG is angular momentum $j$. Thus, the area operator is [1, 44, 50, 55]

$$\hat{A}_S \Psi_\Theta = l_P^2 \Sigma_k \sqrt{j_k(j_k + 1)} \Psi_\Theta \tag{4.17}$$

It is also written in the another form [1, 44, 50, 55], i.e.,

$$\hat{A}_S \Psi_\Theta = 8\pi\gamma l_P^2 \Sigma_k \sqrt{j_k(j_k + 1)} \Psi_\Theta \tag{4.18}$$

The area is obtained by puncturing or piercing of link in to surface in LQG [1, 44, 50, 55].

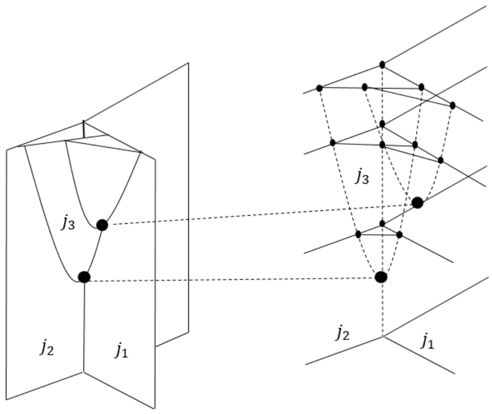


Fig. 5.4: Spin foam creation by evolving spin network.

At the point of intersection, one gets vertices at which the solution of Wheeler-Dewitt equation is obtained in LQG. Hence, the volume operator action is non-zero only at these vertices of the graph $\Theta$. The volume operator is written as [1, 44, 50, 55]

$$\hat{V}\Psi_\Theta = \frac{1}{6}\Sigma_{v\in S\cap\Theta}\sqrt{\epsilon_{bcd}\varepsilon^{jkl}n^b n^c n^d \hat{J}_j \hat{J}_k \hat{J}_l}\Psi_\Theta \tag{4.19}$$

Where, for a sum over a finite number of vertices $v \in \Theta$ that resides $S \cap \Theta$, the integral of the volume operator is reduced [1, 44, 50, 55].

In LQG, vertices must be at least trivalent; since, the volume operator is zero for $v \leq 2$ [1, 44, 50, 55].

In LQG, the spin network provides quantized cross-sectional area and volume through the links and the vertices respectively. At each vertex, an intertwiner is allotted to make the angular momentum conserved at vertices [1, 44, 50, 55].

Spin network provides kinematical picture of LQG; while, spin foam provides dynamical picture of LQG. The evolving spin network generates spin foam. Spin foam theory is covariant approach of LQG in which transition amplitude of quantum geometry is crucial [1, 44, 50, 55].

## 4.2 Introduction to loop quantum cosmology

One of the crucial branch and application of LQG is loop quantum cosmology (LQC); since, singularity is removed by big bounce model in LQC. In LQC, cosmology is studied through quantized spacetime generated by LQG. At cosmological scale, isotropy and homogeneity is very important; therefore, isotropic and homogeneous metric is used in LQC. In case of homogeneity, the value of any physical quantity remains constant at every point; while, in case of isotropy, the value of such physical quantity remains constant in each direction [66-68].

The homogeneous metric in case of any particular spacetime geometry can be written as $\mathcal{M}_\mathcal{G} = (\mathcal{M}, g_{\mu\nu})$ . Where, $g_{\mu\nu}$ is metric and M is a manifold. If any point $x^\mu \in \mathcal{M}$, such a metric is said to be isotropic [66-68].

In classical cosmology, the FLRW metric is written as [66-68]

$$ds^2 = a^2\left(\frac{dr^2}{1-kr^2} + r^2 d\Omega^2\right) \tag{4.20}$$

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Where, $d\Omega^2 = d\theta^2 + sin^2\theta d\phi^2$, $k = -1, 0, +1$ and $a$ is scale factor. The Friedmann equation is (i.e., solution of EFE) written as [66-68]

$$\left(\frac{\dot{a}}{a}\right)^2 + \frac{kc^2}{a^2} = \frac{8\pi}{3}G\rho \tag{4.21}$$

$$2\frac{\ddot{a}}{a} + \left(\frac{\dot{a}}{a}\right)^2 + \frac{kc^2}{a^2} = -\frac{8\pi}{c^2}Gp \tag{4.22}$$

The Friedmann metric in quantum cosmology is written as [66-68]

$$ds^2 = -N(t)^2dt^2 + a(t)^2\left(\frac{dr^2}{1-kr^2} + r^2d\Omega^2\right) \tag{4.23}$$

Where $N(t)$ is lapse function. By putting this metric into the EFE, one can get the vacuum FLRW equation [66-68] i.e.,

$$\left(\frac{\dot{a}}{a}\right)^2 + \frac{k}{a^2} = \frac{8\pi G}{3a^2}H_{matter}(a) \tag{4.24}$$

Where, $H_{matter}$ is the Hamiltonian for any considered matter fields [66-68].
In LQC, two c-number $(\tilde{C}, \tilde{P})$ is defined as [66-68]

$$A_c^j = \tilde{C}\delta_c^j, e_j^c = \tilde{P}\delta_j^c \tag{4.25}$$

Where, $\tilde{C}$ behaves as connection and $\tilde{P}$ behaves as triad for FLRW metric. It is also written as [66-68]

$$|\tilde{P}| = \frac{a^2}{4}, \tilde{C} = \tilde{\Gamma} + \gamma\dot{a} = \frac{1}{2}(k + \gamma\dot{a}) \tag{4.26}$$

Where, $\gamma$ is the Immirzi parameter. The Poisson bracket of $\tilde{C}$ and $\tilde{P}$ is [66-68]

$$\{\tilde{C}, \tilde{P}\} = \frac{8\pi G\gamma}{3}\mathbb{V}_0 \tag{4.27}$$

where, $\mathbb{V}_0 = \int_\Sigma d^3x$ is the volume of a fiducial cell $\mathcal{V}$ in the spatial manifold. In terms of the factors of $\mathbb{V}_0$, $\tilde{C}$ and $\tilde{P}$ can be rewritten as [66-68]

$$C = \mathbb{V}_0^{\frac{1}{3}}\tilde{C}, P = \mathbb{V}_0^{\frac{2}{3}}\tilde{P} \tag{4.28}$$

Thus, the Poisson bracket will be [66-68]

$$\{C, P\} = \frac{8\pi G\gamma}{3} \tag{4.29}$$

In LQG, the graphs states of spin network that resides on different graphs are orthogonal; if, homogeneous or isotropic geometry does not bound these states. Hence [66-68],

$$\langle \Phi_\Theta | \Psi_\Theta \rangle = \delta_{\Theta,\Theta'} \tag{4.30}$$

For two different sets of "momenta" $\{\mu_k\}$ and $\{\mu_{k'}\}$, two states $\psi_1$ and $\psi_2$ are defined that resides on different graphs are orthogonal [66-68].

$$\langle \psi_1 | \psi_2 \rangle = 0 \implies \{\mu_k\} \neq \{\mu_{k'}\} \tag{4.31}$$

Now, the inner product of two states is [66-68]

$$\langle \psi_j | \psi_k \rangle = \delta_{\mu_j,\mu_k} = \begin{cases} 1, if \mu_j = \mu_k \\ 0, if \mu_j \neq \mu_k \end{cases} \tag{4.32}$$

One of c-number i.e., $C$ can be written as operator in the exponential form [66-68] i.e.,

$$C^2 = \frac{sin^2(\mu C)}{\mu^2} + \mathcal{O}(\mu^4) \tag{4.33}$$

Where, $sin(\mu C) = \frac{e^{i\mu C} - e^{-i\mu C}}{2i}$
The commutator of two c-number $(C, P)$ is [ 66-68]

$$[\hat{C}, \hat{P}] = \frac{8\pi\gamma G}{3} \tag{4.34}$$

Another c-number $P$ as the operator $\hat{P}$, is given as [66-68]

$$\hat{P} = (-i\hbar)\frac{8\pi\gamma G}{3}\frac{\partial}{\partial C} = -\frac{8\pi\gamma l_p^2}{3}\frac{\partial}{\partial C}(\because G\hbar = l_p^2) \tag{4.35}$$

On the basis states $\psi_\mu C = exp^{(i\mu C)} \equiv |\mu\rangle$, the action of $\hat{P}$is [66-68]

$$\hat{P}\psi_\mu = \frac{8\pi\gamma l_p^2}{3}\mu\psi_\mu \tag{4.36}$$

In terms of the triad, the physical volume of a unit cell is $V = |P|^{\frac{3}{2}}$. Thus, on the triad eigen state, the volume operator is [66-68]

$$\hat{V}|\mu\rangle = |P|^{\frac{3}{2}}|\mu\rangle = \left(\frac{8\pi\gamma l_p^2}{3}\right)^{\frac{3}{2}} |\mu|^{\frac{3}{2}}|\mu\rangle \tag{4.37}$$

Where, $|\mu|^{\frac{3}{2}}$ is similar to the volume of a fiducial cell $\mathcal{V}$ of the spacetime [66-68]
The action of the volume operator $\hat{V}$ in LQG is again written here [66-68]; i.e.,

$$\hat{V}\Psi_\Theta \sim \Sigma_{v\in S\cap\Theta}\sqrt{\epsilon_{bcd}\epsilon^{jkl}n^b n^c n^d \hat{J}_j \hat{J}_k \hat{J}_l} \tag{4.38}$$

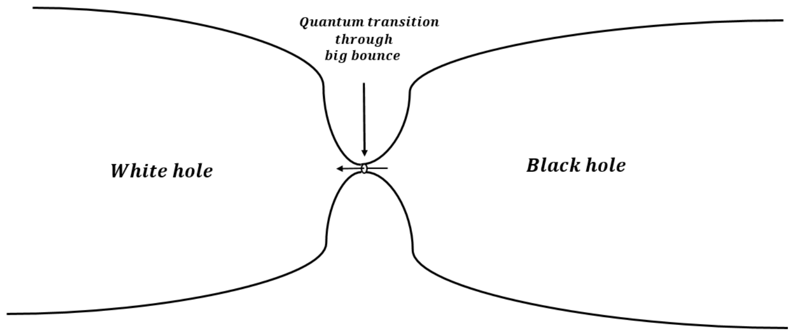


Fig. 5.5: A diagram of big bounce model.

Here, the operator $\hat{J}$ has a minimum eigenvalue of $\frac{1}{2}$ and the smallest possible volume eigenvalue of the volume operator $\hat{V}$ is of the order of $\gamma^{\frac{3}{2}} l_P^3$ [66-68].

LQG, sets cut off value on the volume operator of quantum geometry. In LQC, the factor of $(\widehat{|P|^{-\frac{3}{2}}})$ remains bounded during the evolution of the universe. In case of GR, the scale factor diverges a→0; as, the initial Big Bang singularity is reached. Hence, it yields infinite energy densities for the scalar field ($P \sim a^2$). In case of LQC, $(\widehat{|P|^{-\frac{3}{2}}})$ consists upper limit i.e., $l_P^{-3}$, that guarantees non-occurrence of such divergences. Therefore, LQC replaces big bang by big bounce and provides the mechanism of quantum transition from black hole to white hole [66-68].

## 4.3 Solution of black hole information paradox in LQG

As mentioned, there are mainly two solutions to black hole information paradox in LQG i.e., Planck star and Remnant scenario. There is no sharp boundary between Planck star and remnant scenario as solution of black hole information paradox in LQG. The discrepancies between both solutions can be understood by covering both separately in the next subsections.

### 4.3.1 Planck star

The Planck star was proposed in 2014. In LQC, when energy density is near to Planckian; the quantum gravitational pressure counter balances gravitational pull and removes singularity through reversing the gravitational collapse. Due to this reverse collapse, a black hole is bounced. The Planck star is resided at this point; where, energy density is of order of Planckian. This whole process is shorter in local proper time; but takes several year for the outside observer because of time dilation of huge magnitude. Therefore, bouncing cosmology is the result of pressure produced by quantum gravity effect that prevent gravitational collapse [42].

Since, effect of gravity is controlled by Planckian density, volume of bounce is bigger than Planckian. Therefore, the resulting object is bigger than Planckian by a factor $\left(\frac{m}{m_P}\right)^N$ where, $m_P$ is the Planck mass, $m$ is the initial mass and $N$ positive [42].

In LQC, the Friedmann equation is written as [42]

$$\left(\frac{\dot{a}}{a}\right)^2 = \frac{8\pi G}{3}\rho\left(1 - \frac{\rho}{\rho_P}\right) \tag{4.39}$$

Where, $\rho$ is the energy density of matter, $G$ the universal constant of gravity, $\rho_P = \frac{m_P}{l_P^3} =$

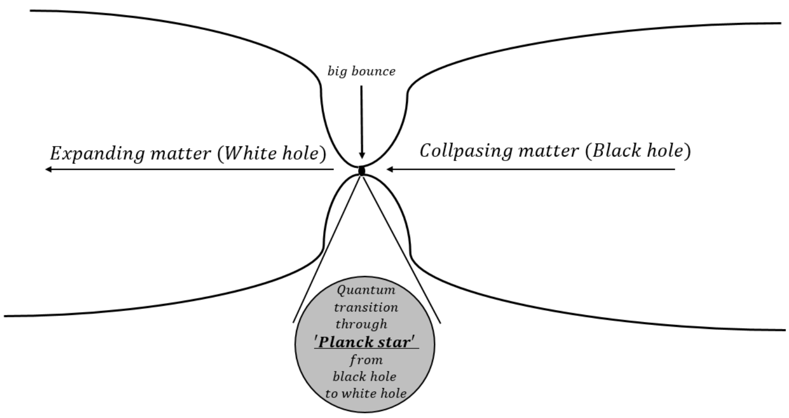


Fig.5.6: Quantum transition through Planck star from black hole to white hole.

$\frac{c^5}{\hbar G^2} = 5.1550 \times 10^{96} kg/m^3$ ($m_P$ is Planck mass, $l_P^3$ is Planck volume and $c$ is speed of light), and dot is time derivative of scale factor [42].

In the equation (5.39), the actual Friedmann equation has been changed by taking quantum gravitational effect into account. Previously, it was believed that quantum gravity manifest itself at Planck length $l_P$; but quantum gravity manifest itself; when, energy density becomes of the order of Planckian [42].

In LQC, collapsing object (i.e., black hole or universe) bounces into white hole or towards expanding region. Since, quantum gravity effect is considered; the Heisenberg's uncertainty principle (for spacetime) plays important role. It produces repulsive quantum gravitational force that stops collapsing body to reach at singularity ($r = 0$) and hence, singularity is disappeared. Thus the volume of bouncing universe at matter dominating phase is $V \sim \frac{m}{m_P} l_P^3$ ($m$ is universe's total mass). Quantum gravity effects starts to appear at this volume to Planck volume [42].

In case of stellar collapse, when extra energy falls into this star; it condenses the core of collapsing star and it becomes heavily compressed core whose density is of the order of Planck density. Thus such collapsing star is converted into Planck star for a while (since, time becomes slower near massive object and for considered volume, mass of Planck star very big) in its local proper time; since, quantum pressure balances the attraction of gravity. The size of Planck star is $r \sim \left(\frac{m}{m_P}\right)^N l_P$ ($m$ is collapsing star's mass, $N$ is positive number). If the value of $N$ is taken $\frac{1}{3}$; then, the resulting Planck star's size is $10^{-10} cm$. Therefore, Planck star is very denser star or a bouncing phase (short lived phase) through which collapsing star avoids singularity and converted into another phase i.e., a white hole [42].

One can study the mathematics of Planck star by considering the Eddington–Finkelstein coordinates that can solve field equations of Einstein. As mentioned in the section (2.2), one can understand the bending of lightcone near inner and outer horizon. In case of Planck star in LQG, light cone tilts inward near outer horizon ($r = 2m$), becomes narrower between inner ($r = r_{in}$) and outer horizon ($r = 2m$) and becomes normal when it enters into Planck region. For Planck star, one can calculate (with natural unit i.e., $G = \hbar = c = 1$) the area of inner trapping horizon by considering the curvature ( here, it is increasing) i.e., $R \sim \frac{m}{r^3}$ (where, $r = \sqrt{\frac{A}{4\pi}}$ is radial Schwarzschild coordinate, the area of constant $r$ sphere is $A$). In this case, the Schwarzschild radius is $r_{SCH} \sim 2m$ (where, $m$ is mass of collapsing star and black hole after collapse) [42]

The curvature $R$ is related to Planck density $\rho_P (R \sim 8\pi\rho_P)$. By taking order $N = \frac{1}{3}$ in

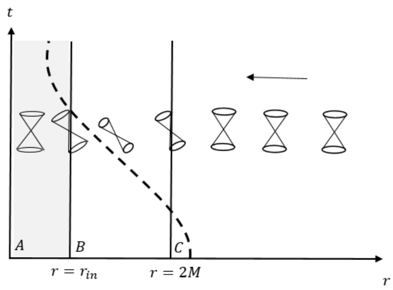


Fig.5.7: Planck star in Eddinton Finkelstein coordinates without evaporation.

the equation $\left(r \sim \left(\frac{m}{m_P}\right)^N l_P\right)$ , one gets $r_{in} = \left(\frac{m}{m_P}\right)^{\frac{1}{3}} l_P$. similarly, the surface area of Planck star will be $A_Q = \left(\frac{m}{m_P}\right)^{\frac{2}{3}} l_P^2$ [42]

By metric of collapsing black hole in Eddington–Finkelstein coordinates is [42]

$$ds^2 = r^2 d\omega^2 + 2dvdr - \mathcal{F}(r)dv^2 \tag{4.40}$$

Where, $\mathcal{F}(r) = 1 - \frac{2m}{r}$ is redshift factor in Schwarzschild' metric and $d\omega^2$ is the metric of a two-sphere [42].

In Eddington-Finkelstein coordinates, the ingoing null geodesic is at constant $u$; while, the outgoing null geodesic follows [42]

$$\frac{dr}{dv} = \frac{1}{2}\left(1 - \frac{2m}{r}\right) \tag{4.41}$$

Therefore, one can say that $r < 2m$ is ingoing; while, $r > 2m$ is outgoing. By taking quantum gravitational effect into account, the redshift factor is modified due to repulsive quantum gravity [42] i.e.,

$$\mathcal{F}(r) = 1 - \frac{2mr^2}{r^3 + 2\varepsilon^2 m} \tag{4.42}$$

By taking expansion in $\frac{1}{r}$, one gets [42]

$$\mathcal{F}(r) = 1 - \frac{2m}{r} + \frac{4\varepsilon^2 m^2}{r^4} \tag{4.43}$$

In the equation (5.43), one obtains the new term means a strong short-scale repulsive force that is because of quantum effects. It stops bending of light cones towards inner side. From equation $r_{in} = \left(\frac{m}{m_P}\right)^{\frac{1}{3}} l_P$, at the lowest zero in the bracket, the ingoing lightlike geodesics tilt back vertical. Thus, going from outer horizon to inner horizon a timelike geodesic move upward that is shown in fig. (5.8). At $r_{in} \sim \varepsilon$, the inner horizon reaches to the new lowest zero ($r_{in}$) and for that, $\varepsilon$ is of the order of $m^N$ ($\varepsilon \sim m^N$). As mentioned in the section (4.2), by considering black hole evaporation through Hawking radiation simultaneously; outer horizon is contracted slowly and inner horizons obtains positive energy. Since, a particle having negative energy falls

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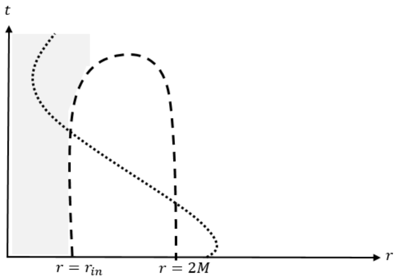


Fig.5.8: Planck star in Eddinton Finkelstein coordinates with evaporation.

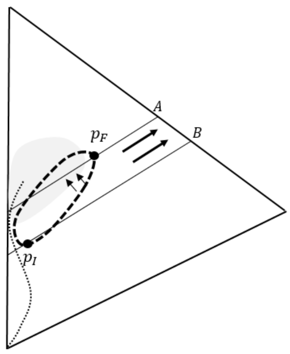


Fig.5.9: Penrose diagram of star life.

in and becomes a particle with positive energy; the expansion of inner horizon takes place. Likewise, $\varepsilon$ becomes time dependent and increases [42].

As mentioned, because of Hawking radiation, outer horizon is contracted; till, it comes to the expanding inner horizon. Hence, there is no horizon (black hole has no longer event horizon and it is now naked singularity with no horiozn) at this place and trapped information can be obtained; since information can easily escapes from this place. This occurs when energy is of the order of Planckian but not at the value of Planck length. As mentioned, such an object (Planck star) is not long lived; but, remnant of this object is long lived and enormously large whose order is bigger than Planck length. In the Fig. 5.9, the quantum gravity effects is shown through shaded area where GR does not work. the dark dashed line shows trapping horizon in which the outer is evaporating (contracting) and the inner is expanding. the dotted line shows outer or external boundary of collapsing star. The $p_I$ and $p_F$ are initial and final point of boundary of external horizon. The arrows are Hawking radiation. There is absence of event horizon at the lowest light line where Hawking radiation is absent [42].

From equation (5.43) and taking time $t = u – r$, one gets red shift factor as

$$\mathcal{F}(r,t) = 1 - \frac{2m(t)}{r} + \frac{4\varepsilon^2(t)m^2}{r^4} \tag{4.44}$$

Where, $m = m(t) = m(collapsetime)$. It means that the Planck star exhibits the memory of the initial collapsed mass even after the evaporation. During black hole evaporation $\varepsilon(t)$ increases; while, $m(t)$ decreases. Hence, inner horizon expands and the outer horizon contracts; due to falling of negative energy particle of Hawking pair. In the theory of Hawking radiation, the time derivative of $m(t)$ and the evaporation time are written as $\frac{dm(t)}{dt} \sim -\frac{1}{m^2 t}$ and $t =$

$\frac{5120\pi G^2 m^3}{\hbar c^4}$ [42].

There are two cases of in the study of Planck star. In the first case, the value of $N$ is $\frac{1}{3}$, the value of $r_{in}$ is $r_{in} = \left(\frac{m}{m_P}\right)^{\frac{1}{3}} l_P$. In the above explanation, this case is elaborated. In the second case the value of $N$ is 1 i.e., $N \sim 1$. It means the value of $r_{in}$ is $r_{in} = \left(\frac{m}{m_P}\right) l_P$ [42].

It is known that near event horizon of black hole, the tidal gravitation force is too strong; hence, equivalence principle of GR is violated. For the second case study, consider that quantum field states is similar to the vacuum near horizon, where the second law of black hole thermodynamics roughly holds; however, the equivalence principle is not violated near horizon and there is a no firewall that incinerate the in falling observer [42].

As mentioned in the section (4.2), each one of the particle of a Hawking pair that goes out, is entangled with the in falling one maximally, that goes on to the Planck star. Thus, information in the Planck star cleanses the Hawking radiation entropy. This internal entropy follows law of Bekenstein bound. With the expansion of inner horizon, its entropy increases; while, the entropy of outer horizon decreases with contraction. When the inner horizon comes to the outer horizon, the information that is resided in a star is ought to at least equal the escaped information through Hawking radiation [42].

Hence, the surface area of a star ought to be $A_F \sim 4S$. Where, $A_F$ is final surface area of the star and $S$ is the total entropy of the Hawking radiation that emitted out and is equal to this point. Here, surfaces are of outer (or external) horizon is also $A$ i.e., four times the remaining entropy in the black hole. Therefore, $A_F \sim A_O - A_F \sim 16\pi m^2 - A_F$ .Where, $A_O$ is the surface area of the horizon initially; prior to the Hawking radiation. Hence, $A_F$ is $A_F = \frac{1}{2}A_O \sim 8\pi m^2$. In the other form, $m_f \sim \frac{1}{\sqrt{2}}m$. This value is approximately equal to Page time (the Page time is the time that a black hole uses to reduce its entropy to the half) [42].

The value of evaporation time is continued to be the same order; since, $t_{evap} \propto m^3$. It is just lessened by a factor $\sim 0.6$. It means that the collapsing star acts as a normal black hole for a long time and normal black-hole astrophysics holds. While, a black hole that considers quantum gravity acts differently; since, the inner core of such a black hole can memorizes the original mass. As per estimation, the $\frac{1}{3}$ mass of total mass is emitted out in the evaporation process. At the end of evaporation, there is absence of horizon in this scenario of Planck star; hence, information that is resided inside the black hole can escape easily due to pressure of quantum gravity. Hence, for an external observer evaporation time is $m^3$ due to huge amount of gravitational shift (almost looks frozen); while for local observer (or in local proper time) this process is quite shorter. It means that black hole is the collapsing star that bounce out via Planck star phase to white hole [42].

As per calculation, the age of the universe is $13.7 \times 10^9 years$. The Planck star may provide experimental signature of quantum gravity i.e., loop quantum gravity. By the study of a spectrum of different masses that are generated in the early era of universe by primordial black holes one can predict life time of such primordial black hole. For instance, the life time of a primordial black hole whose mass is of the order of $10^{12} kg$ is $t_{BH} \sim 14 \times 10^9 years$ [42].

It means that such a black hole is at the end phase of its age. By the age of universe, one can decide the current mass of such a black hole. If energy is suddenly emitted from such an black hole due to effect of quantum gravity; it may be detected. The size of black hole with this mass is achieved through equations $t = \frac{5120\pi G^2 m^3}{\hbar c^4}$, $m_f \sim \frac{1}{\sqrt{2}}m$, and $t_{BH} \sim 14 \times 10^9 years$ [42]; i.e.,

$$r^3 \sim \frac{G\hbar}{348\pi c^2} t = \frac{l_P^2 c}{348\pi} t_{BH}$$

$$\therefore r = \sqrt[3]{\frac{t_{BH}}{348\pi t_P}} l_P \sim 10^{-14} cm \tag{4.45}$$

Where, $t_{BH}$ is life time of black hole, and $t_P = \frac{l_P}{c}$ is the Planck time. $r$ is obtained by a number that is achieved by the ratio of a cosmological scale ($t_{BH}$) to a Planck scale. This ratio is important for observation point of view. Energy range of this wavelength is of the order of $MeV - GeV$ that is detectable by current detectors. However, it is yet to be detected [42].

**How a LQG object *Planck star* solves black hole information paradox?**

If one considers the size of final evaporation of black hole is of the order of Planckian; one cannot recover all amount of information that is trapped inside the black hole. In case of Planck star, this is not big issue; since, the size of black hole at final evaporation is bigger than Planckian size. Therefore, the information that is stored inside can be recovered. Hence, phenomenon of bounce occurs at Planckian density but not at Planck length. Simulteneous occurrence of contraction of outer horizon and expansion of inner horizon establish the situation of naked singularity in which trapped information is released. As per section 4.2, the trapped information cleanses Hawking radiation; thus, there is absence of mixed state and the pure state can be retrieved. Therefore, information is not lost; hence, no paradox is occured in the Planck star study of LQG [42].

For detailed study, there are many papers that cover the conceptual background for Planck star [8, 22, 25, 39, 69-95].

### 4.3.2 Remnant scenario

This scenario was proposed in in 2018 in which the complete life cycle of black hole (i.e. quantum transition of black hole to white hole is crucial ) plays important role in case of information paradox.In the conventional QFTs, a black hole is disappeared after complete evaporation. But in LQG, it tunnels into white hole that is vanished slowly after a long interval by emitting inside mass or trapped information. For macroscopic black holes, the probability of tunnelling is little; however, that increases with mass decrement; when, such a black hole is near to the end of evaporation. The standard tunnelling factor is given as $p \sim e^{-\frac{S_E}{\hbar}}$; where, $\hbar$ is reduced Planck constant, $S_E$ is the Euclidean action [43].

For a stationary black hole, having mass m; if, $S_E \sim \frac{Gm^2}{c}$, then, the standard tunnelling factor will be $p \sim e^{\left(\frac{m}{m_P}\right)^2}$. With $m \to m_P$, this tunnelling factor becomes $\sim 1$ and evaporation is about to end. At the end of evaporation of black hole, it makes transition into a white hole which is long lived and exhibits finite but large interior. Such white hole also exhibits horizon with Planck size [43].

As mentioned, this framework can provide solution of black hole information paradox in which information is not lost through two postulates [43] i.e.,

1. For considered black hole, if curvature is less than the Planck scale, then, the QFT is a nice approximation locally. In other words, no noticeable change happens near horizon; if, black hole is big one.

2. Statistically speaking, inside the horizon of black hole, number of quantum states that are distinguishable from local observables can be greater than $e^{\left(\frac{A}{4\hbar}\right)}$; where, $\hbar$ is reduced Planck constant, $A$ is the surface area of the horizon. In this case, apparent horizon is resided at the boundary of the black hole; since, both apparent and event horizon have different status for dynamical black hole and quantum gravity is considered. After black hole makes transition to white hole, the information that is trapped from long time, is released slowly. Since, Hawking radiation begins to cleanse itself at the Page time according to argument of Page ($e^{\left(\frac{A}{4\hbar}\right)}$ bounds a black hole's total number of states), this postulate is inadequate to solve information paradox. Thus, a firewall consideration

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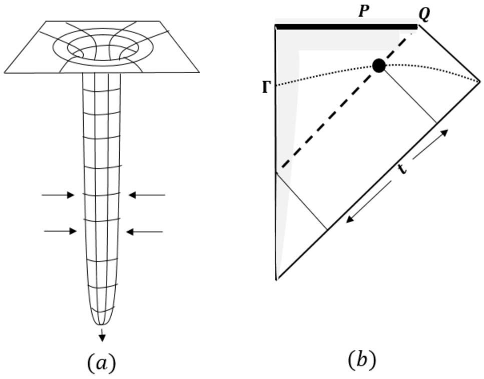


Fig.5.10: (a) Black hole's inner geometry exhibits finite long thin tube. Along with the time, the radius of this tube decreases with length increment. (b) The classical black hole is shown in conformal diagram; in which, effect of quantum gravity is manifested at Region $P$ and $Q$.

is excluded; because, this framework holds the equivalence principle of GR, the unitary evolution, and the field theory applicable for small curvature and local region.

Apart from this, there are also other abstract conditions. To cleanse Hawking radiation, information having entangled entropy $S \sim \frac{m_O^2}{\hbar}$($m_O$ is black hole's initial mass) must be stored on the remnant. Since, remnant possess small mass, it cannot liberate inside information rapidly; hence, it is long lived. Based on unitarity and energy, it exhibits $lifetime \geq \eta_R \sim -\frac{m_O^4}{\hbar^{\frac{3}{2}}}$. The considered metric (black hole background metric) should be steady and fixed under perturbations to liberate information [43].

To understand the complete cycle of black hole, consider a Cauchy surface Γ that travel across the horizon later the gravitational collapse at some time $t$. Here $\Gamma_I$ is inner region of Cauchy surface Γ. The interior of collapsing star is very long tube; whose, length increases with the decrement of the radius of tube (i.e., contraction of tube). For a big interval of time $t$, the volume of inner region $\Gamma_I$ of Γ is proportional to the time from gravitational collapse i.e., $\mathbb{V} \sim 3\sqrt{3}m_O^2 t$ [43].

It means that with the contraction of the area of horizon of the black hole due to Hawking radiation, the interior tube's length increases with time and radius of tube decreases. Hence, a macroscopic black hole having $m_O$ initial mass after gravitational collapse exhibits little mass as well as surface area of horizon [43].

The effect of quantum gravity at two different regions of black hole i.e., $P$ and $Q$ is different. At region $P$, the curvature is big, singularity is enveloped by it and it is space-like separated . The region $Q$ becomes significant at evaporation and it describes horizon [43].

At the time of collapse, when the singularity is about to occur, the Schwarzschild radius $r_s$ decreases with the increment of curvature. Quantum gravity effect prevents curvature not to reach at Planckian length. The metric of this region is given as [43]

$$ds^2 = -\frac{4\left(\eta^2+l\right)^2}{2m-\eta^2}d\eta^2 + \frac{2m-\eta^2}{\eta^2+l}dx^2 + \left(\eta^2+l\right)^2 d\Omega^2 \tag{4.46}$$

Where, $l \ll m$. This metric does not have divergences, singularities and implies a real Riemannian spacetime. Hence, the value of Kretschmann invariant $K \equiv R_{\alpha\beta\gamma\delta}R^{\alpha\beta\gamma\delta}$ will be [43]

$$K(\eta) \approx \frac{9l^2 - 24l\eta^2 + 48\eta^4}{(l+\eta^2)^8} m^2 \tag{4.47}$$

For the big limit of mass, the finite maximum will be $K(0) \approx \frac{9m^2}{l^6}$. By considering $l = 0$, the metric; if, $l \ll \eta^2 < 2m$ (for every values) can be written as [43]

$$ds^2 = -\frac{4\eta^4}{2m - \eta^2} d\eta^2 + \frac{2m - \eta^2}{\eta^2} dx^2 + \eta^4 d\Omega^2 \tag{4.48}$$

Inside the black hole ($\eta < 0$), the Schwarzschild coordinates (inside the black hole) are $t_s = x$ and $r_s = \eta^2$. The metric inside the white hole is achieved at $\eta > 0$. Hence, the equation (5.46) implies transition of black hole geometry to white hole geometry over Planckian region that exhibits bounded curvature [43].

From above metric, one can say that with time lapse, the radius of the tube (i.e., cylindrical in shape) decreases and the length increases. At the region $\eta^2 = 0$, such cylindrical tube gets minimum value and afterwards it bounces back (at a small finite radius $l$). In the bouncing phase, its radius increases and its length decreases. Due to quantum gravity bound, this cylinder not at all gets to zero [43].

In natural units ($G = c = 1$), Planck length will be $l_P = \sqrt{\frac{\hbar G}{c^3}} \sim \sqrt{\hbar}$. This is not appropriate; since, curvature is not singular but bounded. The value of $l$ can be estimated from the requirement that the curvature is bounded at the Planck scale ($K(0) \sim \frac{1}{\hbar^2}$). Hence, the equation $\left(K(0) \approx \frac{9m^2}{l^6}\right)$ becomes $l \sim (m\hbar)^{\frac{1}{3}}$. For a while, reimposing the physical units, one gets $l \sim l_P \left(\frac{m}{m_P}\right)^{\frac{1}{3}}$ ($m \gg m_P$). This value is bigger than the value of Planck length. At the time of transition, from equation (5.46), the three-geometry inside the black hole will be [43]

$$ds_3^2 = \frac{2m}{l} dx^2 + l^2 d\Omega^2 (\because \eta = 0) \tag{4.49}$$

And the Planck star volume is [43]

$$\mathbb{V} = 4\pi l^2 \sqrt{\frac{2m}{l}} (x_{max} - x_{min}) \tag{4.50}$$

The life time of black hole from the gravitational collapse to the beginning of $Q$, one can decide the range of $x$; since, $x = t_s$. At the end of Hawking evaporation the range of $x$ is $((x_{max} - x_{min}) \sim \frac{m^3}{\hbar})$. By taking $l \sim (m\hbar)^{\frac{1}{3}}$, the volume will be $\mathbb{V} \sim \frac{m^4}{\sqrt{\hbar}}$. Due to cut off on volume in quantum gravity, the value of volume never diverges [43].

As mentioned, the region $Q$ describes quantum gravity effect near horizon. The higher bound value is furnished by the Hawking radiation i.e., $\sim \frac{m_O^3}{\hbar}$. Black hole exhibits trapping horizon; while, the white hole exhibits anti-trapping horizon. Both are necessary to comprehend the complete cycle of black hole. The black hole of mass $m_O$ exhibits long tube in its interior. After black hole makes quantum transition (bounce) to white hole, the length of tube gradually with longer time period decreases during the evolution. Therefore, white hole is also known as long lived remnant [43].

From figure (5.11), the life time of the black hole is $\eta_{BH} = b_- - b_0$; where, $b_0$ is the advanced time of the collapse, $b_-$ the beginning time of the quantum transition, and from figure, $b_+$ is the advanced time of end of the quantum transition [43].

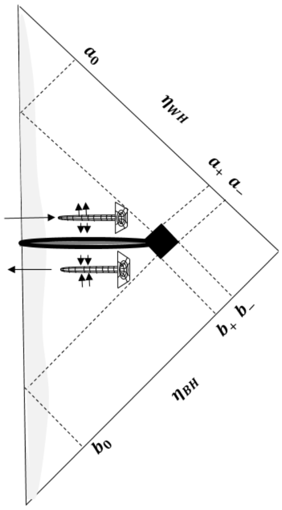


Fig.5.11: Quantum transition from BH to WH with inside geometry depiction.

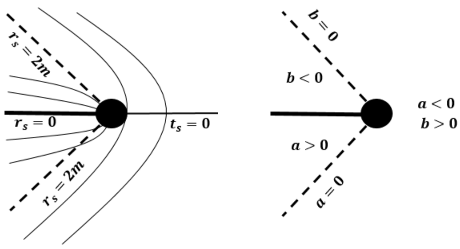


Fig.5.12: a diagram of Q region in which, surface of identical radius of Schwarzschild (left) and the null Kruskal coordinate sign (right) are shown.

Similar to black hole, one can have the white hole lifetime i.e., $\eta_{WH} = a_0 - a_+$. where, $a_o$ is, $a_+$, and $a_-$ are relevant time for white hole [43].

For the region Q, the quantum transition duration is supposed to be $a_+ - a_- = b_+ - b_- \equiv \Delta\eta$ [43].

One can find out the location of $P$ by classical framework; while, the quantum framework is needed for the region $Q$. The process of quantum transition is longer in time [43].

In quantum framework, one can find out the life time of black hole i.e., $\eta_{BH}$ through probability theory. For considered quantum transition, the probability $p$ per unit time is expected to be time independent. Hence for this quantum transition, the normalised probability $\mathbb{P}(t)$ between time interval $t$ and $t + dt$ is controlled by $\frac{d\mathbb{P}(t)}{dt} = -p\mathbb{P}(t)$, i.e., $\mathbb{P}(t) = \frac{1}{\eta_{BH}} e^{-\frac{t}{\eta_{BH}}}$ [43]

One can normalise it i.e., $\int_0^\infty \mathbb{P}(t)dt = 1$. Here, $\eta_{BH}$ is $\eta_{BH} = \frac{1}{p}$ [43]

In the present scenario, unitarity is retrieved if quantum transition will occur at a time $t$. In other words, to retrieve unitarity, one has to consider the full quantum spread of the quantum transition time over different future geometries (since, quantum superposition is present here) [43].

The metric of the whole adjacent area of the $Q$ region can also be written in null Kruskal coordinates; since, it is an extended Schwarzschild metric [43], i.e.,

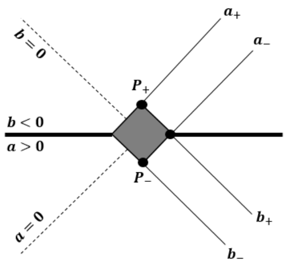


Fig.5.13: The territory of Q transition.

$$ds^2 = -\frac{32m^3}{r} e^{-\frac{r}{2m}} dadb + r^2 d\Omega^2 \quad (4.51)$$

Where, $\left(1 - \frac{r}{2m}\right) e^{\frac{r}{2m}} = ab$

Here, there are two horizons i.e., $b = 0$ and $a = 0$, and two separate regions; where, $a$ and $b$ have different signs. A rapid distinction of the metric component in the equation (5.51) is the result of the rapid change of the value of the radius across the $Q$ region [43].

By defining the boundary of region $Q$ accurately, one can fix the region $Q$. This can be done by recognizing with the diamond that is defined by two points $P_+$ and $P_-$ with coordinates $b_\pm$,$a_\pm$ both outside the horizon, at the same radius $r_P$ and from the bounce time at opposite timelike distances [43],

The corresponding radius $r_P$ indicates that [43]

$$b_+ a_+ = b_- a_- \equiv \left(1 - \frac{r_P}{2m}\right) e^{\frac{r_P}{2m}} \quad (4.52)$$

The corresponding time from the horizon indicates the light lines $a = a_-$ and $a = a_+$ cross on $t_s = 0$, or $a + b = 0$, therefore, $a_- = -b_+$ [43]

The outermost reach of the quantum region is this crossing point, with radius $r_m$ determined by [43]

$$b_+ a_- \equiv \left(1 - \frac{r_m}{2m}\right) e^{\frac{r_m}{2m}} \quad (4.53)$$

There are two parameters i.e., $r_P$ and $\Delta\eta = b_+ - b_- \sim a_+ - a_-$ that completely defines the region. The $r_P$ is the radius at which the quantum transition starts; whereas, $\Delta\eta$ is its duration [43].

A factor i.e., $e^{-\frac{m^2}{\hbar}}$ supress the quantum transition probability per unit time i.e., $p$ exponentially; where, $m$ is mass of the hole at the time of decay. In a time $\frac{m_0^3}{\hbar}$, the mass of black hole contracts to Planck scale due to Hawking evaporation in which the probability density is unit ordered, hence, $\eta_{BH} \sim \frac{m_O^3}{\hbar}$ and $\Delta\eta \sim \sqrt{\hbar}$. Here, $Q$ region exhibits Planckian size [43].

As mentioned, creation of Hawking radiation near event horizon of a hole and this Hawking radiation is reacting back with geometry result into decrement of the surface area of event horizon. Due this decrement, probability of transition of black hole to white hole increases i.e., the surface area of this horizon get to the Planckian order ($A_{BH(final)} \sim \hbar$) after a time interval $\eta_{BH} \sim \frac{m_O^3}{\hbar}$ and the probability of transition become $\sim 1$. At this time, transition surface volume is big [43].

The back reaction of inner part of Hawking radiation plays an important role; since, it going in towards singularity. Due this part, the magnitude of $m$ decreases (mainly, more decrement occurs when process ends) and the magnitude of $x$ coordinate increases [43].

In this case, the order of volume is same as $\mathbb{V} \sim \frac{m^4}{\sqrt{\hbar}}$. Hence, the volume will be $\mathbb{V}_{BH(final)} \sim \sqrt{\hbar} m_O \eta_{BH} \sim \frac{m_O^4}{\hbar}$ [43]

Similar to this, one can derive the volume for the white hole using the Kruskal metric in which Plankian mass Schwarzschild solution is used. The lifetime of white hole that created through quantum transition is written as $\eta_{WH} = a_0 - a_+$. Here, the volume of internal region is conserved during quantum transition. One can write the volume of Planckian curvature region inner side the white hole horizon i.e., $\mathbb{V}_{WH}(a) \sim l^2 \sqrt{\frac{m}{l}} \eta_{WH}$. If, $l \sim m \sim \sqrt{\hbar}$ then, $\mathbb{V}_{WH(initial)} \sim \hbar \eta_{WH}$ [43].

The matching of two volume equation i.e., $\mathbb{V} \sim \frac{m^4}{\sqrt{\hbar}}$ and $\mathbb{V}_{WH(initial)} \sim \hbar \eta_{WH}$ is needed to stick past singularity with the geometry of future singularity. Therefore, one gets $\eta_{WH} \sim \frac{m_O^4}{\hbar^{\frac{3}{2}}}$. It means that white hole having Planck mass is a remnant with long life [43].

#### How *remnant scenario* of LQG solves black hole information paradox?

Before $a_-$, the Hawking radiation get to future infinity that is defined by a mixed state whose entropy is of the order of $\frac{m_O^2}{\hbar}$. Inside the black hole, correlations with field excitations cleanses this Hawking radiation. Due to big inner volume, these excitation can be tackled; even though, mass of the black hole is little. It means that the hole exhibits big capacity for storage of information. Since, remnant is long lived, the trapped information is slowly leaked from horizon (as low energy quanta) and entropy is liberated. Therefore, a particle that went into interior of horizon can escape and cleanse the quantum state of exterior one. This whole process does not violate the law of entropy bound that is urged by unitarity and consideration of energy [43].

Since, white hole having Planck size may be converted into black hole having Planck size; it is instable by nature. But, this Planck size black hole may also make transition into white hole in Planck time. Therefore, one can say that, this remnant is steady and is fixed into quantum super position of black hole and white hole. One cannot distinguish white hole from black hole. Therefore, the remnant scenario can solve the black-hole information paradox [43].

The black hole information paradox can be solved using Such white hole; since, it behaves as a long lived remnant. In LQG, black hole makes quantum transition into a white hole; when, such a black hole is near to the end of evaporation. These white holes having small masses and large finite interiors decays slowly through unitary process and can provide concrete solution to black hole information paradox [43].

For detailed study, there are many papers that cover the conceptual background for remnant scenario [25, 42, 73-75, 80-88, 95-112].

## 5 Concluding remarks

- In this article, intially, GR with gravitational collapse is outlined; thereafter, the discrepencies between black hole and naked singualrity are given.
- To comprehend black hole information paradox, firstly, black hole along with black hole thermodynamics and Hawking radiation are explained.
- Afterwards, the glimpsies of information theory in context with black hole along with the cause of information paradox and solutions of information paradox outside the LQG are briefly elaborated.

- At last, basics of LQG along with the loop quantum cosmology is given. Then, Planck star and remnant scenario are described. This two theories are supposed to be solution of black hole information paradox in LQG.

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