

Article

The Black Hole Universe, Part II

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Abstract: In part I of this series, we showed that the observed Universe can be modeled as a local Black Hole of fixed mass $M \simeq 6 \times 10^{22} M_{\odot}$, without Dark Energy: cosmic acceleration is caused by the Black Hole event horizon $r_S = 2GM$. Here, we propose that such Black Hole Universe (together with smaller primordial Black Holes) could form from the hierarchical free-fall collapse of regular matter. We argue that the singularity could be avoided with a Big Bounce explosion, which results from neutron degeneracy pressure (Pauli exclusion principle). This happens at GeV energies, like in core collapse supernova, well before the collapse reaches Planck energies (10^{19} GeV). If our Universe formed this way, there is no need for Cosmic Inflation or a singular start (the Big Bang). Nucleosynthesis and recombination follow a hot expansion, as in the standard model, but cosmological measurements (which are free parameters in the standard model) could in principle be predicted from first principles. Part or all of the Dark Matter could be made up of primordial compact objects (Black Holes and Neutron Stars), remnants of the collapse and bounce. This can provide a faster start for galaxy formation. We present a simple prediction to explain the observed value of $M \simeq 6 \times 10^{22} M_{\odot}$ or equivalently Ω_{Λ} (the fraction of the critical energy density observed today in form of Dark Energy) and the coincidence problem $\Omega_m \sim \Omega_{\Lambda}$.

Keywords: cosmology; dark energy; general relativity; black holes

1. Introduction

A cosmological model predicts the background evolution, composition and structure of the observed Universe given some initial conditions. The standard cosmological model [1,2], also called Λ CDM, assumes that our Universe began in a hot Big Bang (BB) expansion at the very beginning of space-time. Such initial conditions seem to violate the classical concept of energy conservation and are very unlikely [3–6]. Our observed Hubble Horizon has a total mass energy closed to $10^{22} M_{\odot}$ in the form of stars, gas, dust and Dark Matter, which according to the singular start BB model came out of (macroscopic) nothing in the form of some quantum gravity vacuum fluctuations that we can only speculate about and we will never be able to test experimentally because of the enormous energies involved (10^{19} GeV). There is no direct evidence that this ever occurred. This model also requires three more exotic ingredients or patches: Cosmic Inflation, Dark Matter and Dark Energy (DE), for which we have no direct evidence or understanding at any fundamental level. Another problem of the Λ CDM model is why the value of Λ (or DE) is such that $\Omega_{\Lambda} \sim \Omega_M$ today: the cosmological coincidence problem (see [1,2,7,8]). Despite these shortfalls, the Λ CDM model seems very successful in explaining most observations by fitting just a handful of free cosmological parameters.

In reference [9] (paper I, from now on), we propose a new cosmological solution, the Black Hole (BH) Universe (BHU) that can explain the same observations as Λ CDM without the need of Dark Energy (which is just the BHs event horizon). This solution can also be used to model the interior of regular BHs. However, it is not enough to find a new solution to GR. We need to make sure that such a configuration can be achieved in a causal way. A good example of this problem is the infinite Friedman–Lemaître–Robertson–Walker (FLRW) solution. The Hubble rate is the same everywhere, no matter how far, and this is not causally possible [10,11]. Cosmic Inflation alleviates this problem but does not solve it [6]. We propose two possible BHU formation scenarios: (i) one that happens during a rapid expansion (or explosion) and (ii) a version that happens during a free-fall collapse.



Citation: Gaztanaga, E. The Black Hole Universe, Part I. *Preprints* 2022, 1, 0. <https://doi.org/>

Academic Editor: Vesselin G. Gueorguiev, André Maeder

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Both can be applied to a small object, such as a star, or a large object, such as our Universe. The main difference is that for the larger object, the density corresponding to r_S is very low (few atoms per cubic meter), and we can assume a dust ($p = 0$) fluid. This is not the case for small objects, such as stars.

In the appendix, we present the expanding scenario, which requires an existing expansion (such as Cosmic Inflation or a supernova explosion) and a scalar field in a false vacuum potential. This also means that the outside manifold has a different effective Λ term (the true vacuum) and will not be asymptotically flat. So, this shares many of the problems of the standard Λ CDM model that we are trying to avoid, resulting in a more complicated and speculative scenario. Thus, we focus on the hierarchical free-fall collapse to form a BHU, as presented in Section 2, which can start from a uniform low-density cloud of regular matter. Section 3 presents a new anthropic argument to predict the observed value for our BHU mass M and cosmic acceleration. Finally, Section 4 is devoted to our conclusions and discussions.

2. BHU Collapse

BHs are thought to form from gravitation collapse. So, we will study next a simple but generic collapse scenario for pressureless matter.

2.1. FLRW Cloud Collapse

Consider a uniform spherical cloud of dust or CDM (that is, an energy density ρ_m with pressure $p = 0$) with radius R and mass M , surrounded by a region that can be approximated as empty space. The cloud composition is not important to start with, but it could play some role later on. You can imagine an infinite uniform FLRW universe with a very low density, filled with small random amplitude fluctuations of different sizes. The Hubble rate $H^2 = 8\pi G\rho/3$ will be close to zero as in empty Minkowski space. We will focus on one particular positive random fluctuation that slowly collapses, forming a cloud of size R .

Such a cloud will collapse following the FLRW* solution in GR (see paper I). The solution is exactly the same in Newtonian physics [12]. The radius r inside R ($r < R$) is given by $r = a(\tau)\chi$, where χ is a fixed comoving coordinate. The rate of collapse is given by the Hubble–Lemaître law:

$$H^2 \equiv \left[\frac{\dot{a}}{a} \right]^2 \equiv \left[\frac{1}{a} \frac{da}{d\tau} \right]^2 = \frac{8\pi G}{3} \rho \equiv r_H^{-2}. \quad (1)$$

At any time τ (this is proper time for a comoving observer), the collapse rate H is given by the energy density ρ . During collapse, energy–mass conservation requires that $\rho = \rho_m \propto a^{-3}$. Given $a_* = a(\tau_*)$ and $H = H_*$ at time τ_* , this equation can be integrated with solution:

$$a(\tau) = a_* \left[\frac{3}{2} \tau H_* \right]^{\frac{2}{3}} \Rightarrow r_H = \frac{3}{2} \tau. \quad (2)$$

During collapse, H and τ are negative. Note also that during collapse $r_H < 0$, but we will refer to its absolute value to compare to the radial coordinate r . As proper time τ approaches the singularity (at $\tau = 0$), r_H becomes smaller and ρ becomes larger. Structures that are larger than r_H cannot evolve because the time that a perturbation takes to travel that distance is larger than the expansion time. This is illustrated in Figure 1.

The mass M inside R in the FLRW* cloud is (see paper I for details) is:

$$r_S = 2GM = R^3 r_H^{-2}, \quad (3)$$

so that:

$$R = \left[r_H^2 r_S \right]^{1/3} = \frac{9}{4} r_S^{1/3} \tau^{2/3}. \quad (4)$$

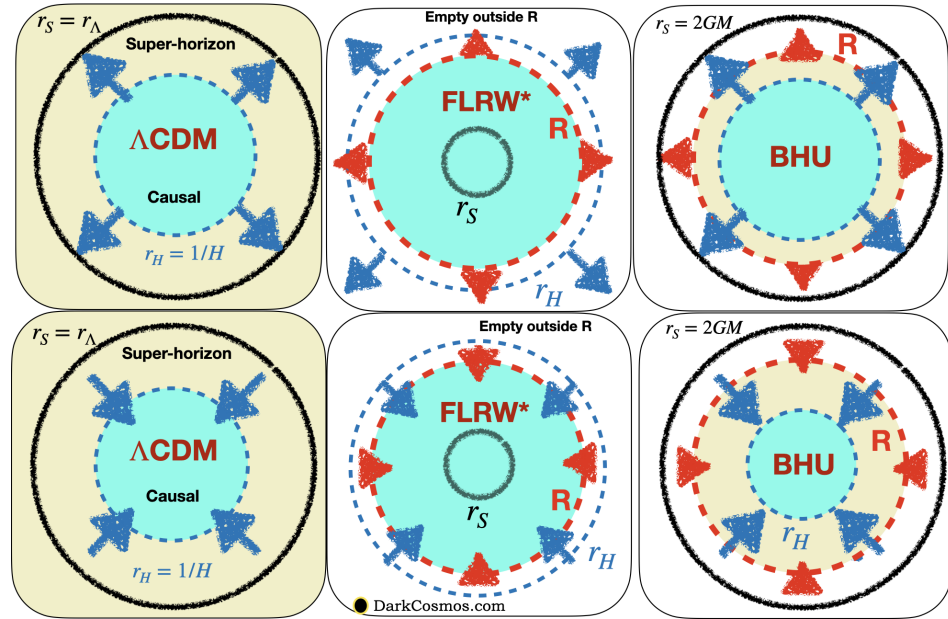


Figure 1. The FLRW metric can be used to model: (LEFT) a Λ CDM infinite universe with $r_\Lambda = r_S = 2GM$, where M is the Λ (or DE) mass inside r_S , (MIDDLE) a stellar cloud with finite mass M inside R (red circle), or (RIGHT) a BHU where R is inside r_S (black circle). In the BHU, $\rho \simeq 0$ outside R , while in Λ CDM, ρ is the same everywhere. The Hubble Horizon $r_H = H^{-1}$ (blue circle) is expanding (top panels) or collapsing (bottom). Matter inside (outside) r_H is colored in cyan (yellow). In our Universe, we have $r_H < R < r_S$. The causal perturbations of size D becomes a super-horizon ($D > r_H$) during collapse and re-enters the Hubble Horizon ($D < r_H$) during expansion, solving the horizon problem without Inflation.

So, we can see that as $\tau \rightarrow 0$, we have $R \rightarrow 0$ and $\rho \rightarrow \infty$. Before collapsing into a BH (i.e., $R > r_S$) we have that $r_H > R$. After BH collapse: $r_H < R < r_S$ (see Figure 1).

Once you form a BH, the collapsed time (i.e., time before singularity) is quite short:

$$\tau_{BH} = \frac{2}{3}r_S \simeq 4.8 \times 10^{-6} \frac{M}{M_\odot} \text{sec} \simeq 1.5 \times 10^{-13} \frac{M}{M_\odot} \text{yr}. \quad (5)$$

The mean energy density of a BH (as measured by an observer outside in flat empty space) at $R = r_S = 2GM$ is:

$$\rho_{BH} = \frac{M}{4\pi r_S^3/3} = \frac{3r_S^{-2}}{8\pi G} \simeq 9.8 \times 10^{-3} \left[\frac{M_\odot}{M} \right]^2 \frac{M_\odot}{\text{km}^3}, \quad (6)$$

regardless of its content. This agrees with Equation (1) for $R = r_H = r_S$, indicating that the collapse eventually produces a BH and ρ keeps growing inside the BH. This of course neglects prepressure or rotation.

This critical ρ_{BH} value should be compared with the atomic nuclear saturation density:

$$\rho_{NS} \simeq 2 \times 10^{-4} \frac{M_\odot}{\text{km}^3}, \quad (7)$$

which corresponds to the density of heavy nuclei and results from the Pauli exclusion principle applied to neutrons and protons. For a Neutron Star (NS) with $M \simeq 7M_\odot$, both densities are the same: $\rho_{BH} = \rho_{NS}$. This relation indicates that it is difficult to form BHs from gravitational collapse with masses smaller than $M \simeq 7M_\odot$, because you somehow need to first overcome the Pauli exclusion principle. It also explains why NS are never larger than $M \simeq 7M_\odot$, as a collapsing cloud with such mass reaches BH density ρ_{BH} before it reaches ρ_{NS} . The maximum observed M for NS is closer to $M \simeq 3M_\odot$ [13], which agrees

with more detailed considerations that include the equation of state estimates. Cold nuclear matter at neutron density is a major unsolved problem in modern physics [13]. Fortunately, we should be able to understand this better in the near future with further modeling (see e.g., [14]) and pulsar and NS observations. This issue could be critical to understand cosmic expansion, as we will discuss here later in Section 2.3.

2.2. Hierarchical or Self Similar Collapse

Imagine a very large (or infinite) and uniform (FLRW) space-time with an initially Gaussian distribution of small random fluctuations δ , so that $\rho = \bar{\rho}(1 + \delta)$. Let us further assume that the amplitude of fluctuations is similar on all scales. By its definition, gravity dominates for masses above the Jeans mass $M > M_J$. For such masses, we can apply the simple free-fall gravitational collapse presented in the subsection above. The positive fluctuations with the smallest masses will collapse first and form a BH for $M \gtrsim 7M_\odot$ or an NS for smaller masses. Larger scales will collapse later and will be made of a uniform mix of already collapsed BH, NS, and background matter. The largest scale perturbations, corresponding to the largest masses, will collapse last.

We do not know the initial particle composition of this low-density fluid, but we can assume that it is similar to the one in our Universe. The density is so low that BHs will behave like collisionless dark matter (CDM). The hierarchical gravitational collapse could also lead to dense cold dark matter (CDM) halos and not necessarily to collapsing BHs. This is the case even if the CDM that we observe today is not made of new exotic DM particles but is made of compact objects with regular matter (such as BHs and NSs). Massive BHs could still form inside CDM halos. So, compact objects could correspond to halos with BHs inside or just naked BHs or NSs.

This leave us with a hierarchical picture, as illustrated in Figure 2, of small BHs inside larger naked BHs or DM halos. Only the smallest BHs will have NSs inside and, as we will see next, only the very large BHs will contain stars and galaxies inside. As we will see later in Section 3, for a Gaussian distribution, we can calculate the probability of structures of different M to form. All this is a crude approximation, as we are neglecting deviations from spherical symmetry and rotation, but it gives us a good idea of how hierarchical formation works [15].

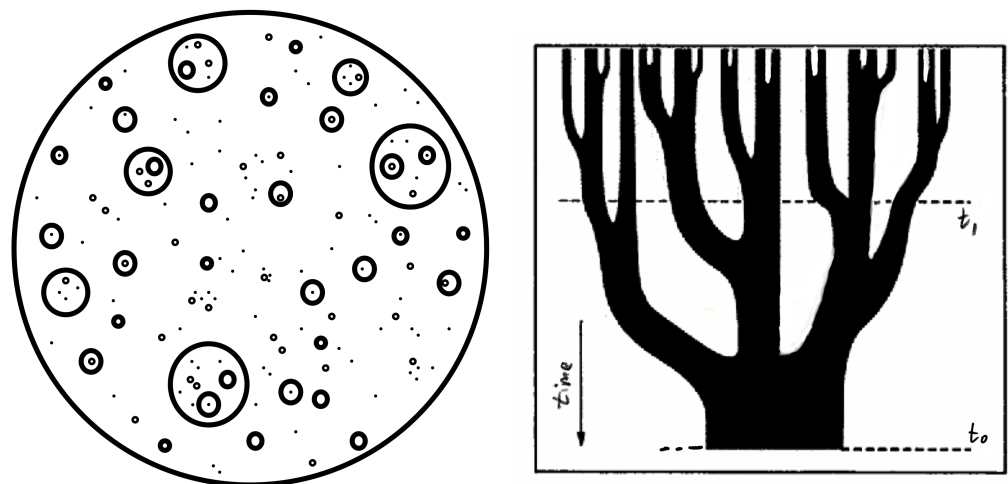


Figure 2. Hierarchical structure of BHs and halos (represented by circles) inside other BHs and halos. The smallest BHs form first. Then, larger-scale fluctuations collapse, including smaller BHs inside. The left panel shows a picture of the structure when the largest BH collapsed (outer circle). The right panel shows a merger tree which corresponds to the time evolution from top (early) to bottom (late) of the collapse of some of the small BHs as they merge into bigger ones. The last instant t_0 (the trunk of the tree) reflects the time when the largest BH has collapsed.

These ideas has been tested in a more realistic way using cosmological N-body simulations (e.g., see [16] and references therein). Note that recent large scale N-body studies are mostly completed in an expanding (and not collapsing) background, which is dominated by Λ (as it happens in our Universe today). In this case, the hierarchy is truncated for the largest objects ($M_* \simeq 10^{15} M_\odot$, see Section 3 below) because there is not enough time for larger compact objects to form (as structure freeze out during the deSitter (dS) phase of the expansion, i.e., close to now). Early N-body simulation without Λ show a much more scale-free hierarchy without truncation (e.g., see [17] and references therein). We expect that this will be even more pronounced when the background is collapsing (instead of expanding) because gravity works in the same direction as collapse.

Typically, once a BH is formed, the internal collapse time in Equation (5) is quite short, and no further structures can form inside. The astronomical time needed to form a star similar to our Sun is measured in units of Gyr or 10^9 yr. This means that according to Equation (5), to form a star similar to our Sun inside a BH, it has to be a very large one: $M \simeq 10^{22} M_\odot$. This is the mass inside our FLRW event horizon $r_S = r_\Lambda$ (for details, see paper I). Such a large BHU has a very low density, $\rho_{BH} = 3r_S^{-2}/8\pi G = \rho_\Lambda$, which is 25% lower than the critical density today $\rho = 3H_0^2/8\pi G$ (a few protons per cubic meter).

According to the equations above, each of the hierarchical BHs collapses into a singularity. However, as mentioned before, quantum mechanics does not allow for matter to become singular. Well before we reach Planck scales ($\simeq 10^{19}$ GeV), we encounter nuclear saturation in Equation (7) ($\simeq 1$ GeV). Although we cannot observe from outside what happens inside, it is reasonable to expect that the collapse will bounce into an explosion, as it happens in core collapse supernovae. The difference with a regular core collapse is that we are inside a BH. During expansion, the action is bounded by the BH event horizon, and the expansion turns into a dS phase, which corresponds to an asymptotically static BHs in proper coordinates (see paper I). So, the collapsing hierarchy is not quite symmetric with the expanding hierarchy. However, there is one case where we can actually observe what happens inside: our own BHU.

2.3. The Big Bounce

From the previous considerations, we conclude that our observed Universe could have formed from the collapse of a very large FLRW cloud. However, when we measured the Hubble–Lemaître law, we observe expansion ($H > 0$) and not collapse ($H < 0$). This means that we must have gone through a Big Bounce sometime in our past. Such a Big Bounce then replaces the role of the hot Big Bang (BB), which requires either Inflation or a singular start. Here are some ideas as to how this Big Bounce could have happened:

(1) Our local FLRW cloud must first collapse to form a BH. Before it collapses, the density of such a large cloud was so small that radiation escaped the cloud, so that $p = 0$. Radial comoving shells of matter are in free-fall collapse and continuously pass $R = r_S$ inside its own BH horizon. We take τ_* in Equation (2) as the time τ_{BH} (a_{BH}) when $r_H = -r_S$, i.e., when the BH forms ($H < 0$). Collapse and expansion are not symmetric because the event horizon is not symmetric: it allows matter to fall in but not to get out. This is key to the BHU model. We then find that the BH forms at time $r_H = -r_S$:

$$\tau_{BH} = \tau_* = -\frac{2}{3}r_S \simeq -11\text{Gyrs}, \quad (8)$$

i.e., before $\tau = 0$ (the BB) or 25 Gyr ago (when we add 14 Gyr from the BB to now).

(2) The collapse continues inside until it reaches nuclear saturation in Equation (7). The Hubble radius corresponding to Equation (7) is only 21 km and contains $7 M_\odot$. So, the collapse mass and scale is similar to that in the interior of a regular collapsing star. Typical supernova explosion energy accounts for a significant fraction of the progenitor rest mass. So, to explain the formation of the typical NS with $1\text{--}3M_\odot$ that we observed in nature, we would need a large progenitor ($3\text{--}9M_\odot$). These values are closed to the $7 M_\odot$ obtained from comparing Equation (7) with Equation (6), indicating that neutron degeneracy pressure

will have some role in the collapse of the Hubble horizon region. Higher densities cannot be reached because of the Pauli exclusion principle. This indicates that the collapse must be halted by neutron degeneracy pressure, causing the implosion to rebound as it happens in stars [18]. Other bouncing mechanisms have been proposed [19–21], but they assume modifications to Classical GR. The CPT-Symmetric Universe idea [22,23] could provide an alternative bouncing mechanism.

(3) The 10^{15} different Hubble size regions within the BHU explode in sync because (ignoring smaller scale fluctuations) the background density is approximately the same everywhere in the collapsing FLRW cloud. The collapse energy ($H < 0$) bounces into approximately uniform expansion ($H > 0$). Radiation, baryons, NSs and primordial Black Holes (PBHs) result from each Hubble size region as compact remnants that can make up all or part of Dark Matter Ω_m . Such compact remnants do not necessarily disrupt Nucleosynthesis or CMB recombination as long as they are not too large [24,25]. The bounce must produce the right amount of diffuse baryons per photon ($\eta = n_B/n_\gamma \simeq 6 \times 10^{-10}$) so that Nucleosynthesis generates the observed primordial element abundance [26]. This will also give the right temperature T_0 for the observed CMB recombination physics.

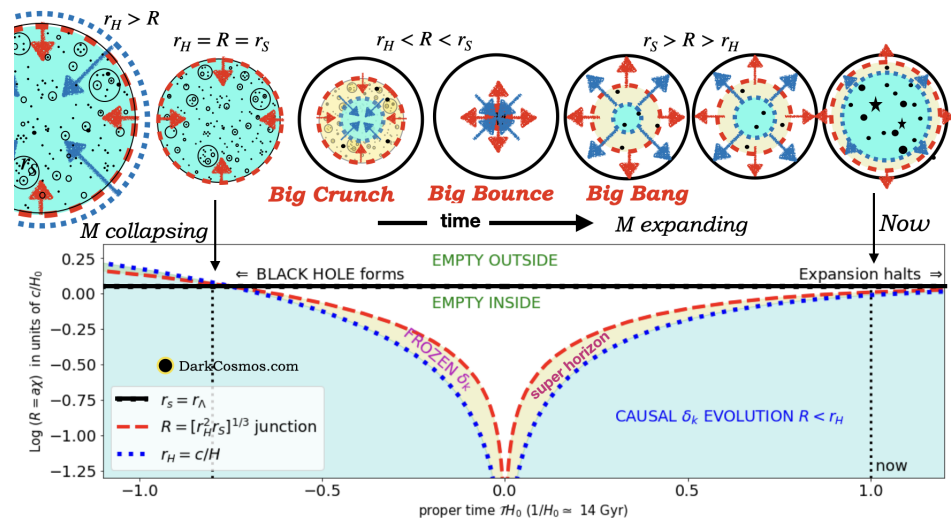


Figure 3. Illustration of the formation of a BHU. Color regions $r < R$ (dashed red line) are filled with a uniform FLRW metric (blue corresponds to $r < r_H$ and yellow corresponds to $r > r_H$) with empty space outside. A cloud of radius R and mass M collapses under gravity. When it reaches $R = r_s = 2GM$, it becomes a BH. The collapse proceeds inside the BH (where a frozen layer $r > r_H$ appears in yellow) until it bounces, producing an expansion (the hot Big Bang). The event horizon r_s behaves like a cosmological constant with $\Lambda = 3/r_s^2$ so that the expansion freezes before it reaches back to $r_s = r_\Lambda \equiv 1/H_\Lambda$. The bottom panels shows the evolution of R and r_H during collapse and expansion in log units of $1/H_0$. The structure in-between R and r_H is frozen and seeds structure formation in our Universe, which could include smaller BHUs, CMB, stars and galaxies, and it replaces the role of Inflation in Λ CDM.

The top panel of Figure 3 illustrates this formation process. The bottom panel of Figure 3 shows an actual numerical calculation for the formation of our Universe. During collapse, the boundary R (falling red dashed line) is fixed in comoving coordinates and follows $R = [r_H^2 r_s]^{1/3}$, where we have fixed $r_s = 2GM$ to its observed value today $r_s = \Omega_\Lambda^{-1/2}/H_0$ with $\Omega_\Lambda \simeq 0.75$ and r_H is just given by Equation (1) with $\Omega_m \simeq 0.25$ and $\Omega_\Lambda = 0$. After the Big Bounce, R follows a null geodesic $ad\chi = d\tau$ (rising red dashed line) with $H(\tau)$ given by effective $\Omega_\Lambda \simeq 0.75$ and $\Omega_m \simeq 0.25$. The Big Bang happened $\tau \simeq H_0^{-1} \simeq 14$ Gyrs ago, and our Universe collapsed into a BH about 25Gyrs ago.

2.4. Comparison with Inflation

Inflation [27–30] is a key ingredient in the standard Λ CDM cosmological model. For a review, see [1,31]. It solves several problems; the most relevant here are: (1) the horizon problem, (2) the source of Large-Scale Structure (LSS), (3) the flatness problem, and (4) the monopole problem. The horizon and LSS problems arise because much of the LSS that we observe today, e.g., BAO in CMB maps, was outside the Hubble Horizon r_H at the time of light emission and therefore could not have had a causal origin. The idea of inflation is that during the very first instances of the Big Bang, the Universe became dominated by some FV or DE, which produced a dS expansion phase (see also our Appendix). Such an exponential expansion solves all the above problems. After expanding by a factor e^{60} , Inflation leaves the universe empty, and we need a mechanism to stop Inflation and to create the matter and radiation that we observe today. This is called re-heating. These require fine-tuning and free parameters that we do not understand at a fundamental level. Inflation is not directly observable or testable, because it occurred when the Universe was opaque and at energies ($> 10^{15}$ GeV) that are out of reach in particle accelerators [1]. Another problem of the Λ CDM model that is not solved by Inflation is why there is DE or Λ , and why the value of Λ is such that $\Omega_\Lambda \simeq \Omega_M$ today: the cosmological coincidence problem (see [1,2,7,8]).

As detailed before, a large fraction of the mass M that collapsed into our BHU is outside r_H , especially close to singular time ($\tau = 0$), as $R = [r_H^2 r_S]^{1/3}$ (see Figures 1 and 3). This clearly solves the horizon problem. Perturbations can be generated during the collapse and exit the horizon before the Big Bounce. Such super-horizon perturbations translate into inhomogeneities in the Big Bounce and can be the source of LSS structures when they re-enter r_H during expansion. Long before Inflation was invented, Harrison [32], Zel'dovich [33] and Peebles [34] proposed that the gravitational instability of regular matter alone can generate a scale-invariant spectrum of fluctuations, which is very similar to the predicted in models of Inflation. This means that both models (Inflation or the BHU) could make similar predictions.

Inflation speculates that reheating could produce the right number of baryons per photon ($\eta = n_B/n_\gamma \simeq 6 \times 10^{-10}$) needed for Nucleosynthesis and CMB recombination. The simplest models of Inflation also predict adiabatic scale-invariant fluctuations (given by an overall amplitude δ_T and slope n_S) in agreement with current observations [1]. However, note that the actual parameters that are fitted to observations ($\delta_T \sim 10^{-5}$, $n_S \sim 1$, $\eta \sim 10^{-9}$, $T_0 \simeq 3K$ or $\Omega_m/\Omega_B \sim 4$) are not fundamental predictions of Inflation but rather free parameters of the Λ CDM model.

In the BHU, the Big Bounce replaces the role of reheating and gives rise to η (diffused baryons to photons) and Ω_m/Ω_B (compact to diffused baryons). The collapse and bounce can also generate the initial spectrum of fluctuations needed to explain the observed cosmic structures ($\delta_T \sim 10^{-5}$ and $n_S \sim 1$). Gravitational instability [32–34] allows perturbations δ to grow causally during the collapse phase, but they quickly exit r_H as the collapse approaches $\tau = 0$. Such causally disconnected regions will therefore have slightly different Ω_m and H_0 at the time close to the Big Bounce (10^{-4} s). These regions correspond to super-horizon perturbations in the CMB that re-enter r_H during the expansion given rise to the structures that we see today in Cosmic Maps.

Because R is always finite, we expect a cut-off in the spectrum of perturbations. This is at odds with the simplest prediction of Inflation. Recent anomalies in measurements of cosmological parameters over very large super-horizon scales agree better with the BHU predictions than with Inflation [35] and indicate that large super horizon fluctuations are not adiabatic, which is in contrast to what is predicted by Inflation. This again could provide evidence for the Big Bounce, which is an out of equilibrium process and cannot be modeled as a perfect fluid or an adiabatic process. Other differences include the abundance of primordial compact remnants, which could make up DM, speed up galaxy formation and produce an intrinsic CMB dipole (see discussion).

The BHU Big Bounce model is speculative in the same way Inflation is speculative. The main difference is that Inflation happens at energies that we will never be able to

test, whereas the Big Bounce collapse within the BHU can be modeled and tested using the same Nuclear Astrophysics (and observations) that are used to understand Neutron Stars, Pulsars or core collapsed Supernovae. Further work is needed to show this in detail. This new model has the potential to explain from first principles some key cosmological observations, such as η , δ_T , T_0 or Ω_m/Ω_B , which are currently fixed by observations as free parameters of the Λ CDM model (see Table 1 for a detailed comparison).

Table 1. Model comparison. Observations that require explanation.

Cosmic Observation	Big Bang (Λ CDM)	BHU Big Bounce
Expansion law	FLRW metric	FLRW metric
Element abundance	Nucleosynthesis	Nucleosynthesis
Baryons per foton $n_B/n_\gamma \simeq 6 \times 10^{-10}$	Free parameter of reheating	Diffuse remnants of collapse
CMB $T_0 = 2.725$ K	Recombination	Recombination
All sky uniformity & LSS seeds	Inflation	Big Bounce
Intrinsic CMB dipole	Local flow	BHU frame
Cosmic acceleration $\omega = -1.03 \pm 0.03$	Dark Energy	BH event horizon size r_S
Age of the Universe: 14 Gyr	Dark Energy	BH event horizon size r_S
Rot. curves, grav. lensing & vel. flows	Dark Matter	Compact remnants from collapse
CMB fluctuations $\delta T = 10^{-5}$	Inflation: free parameter	Perturbations during collapse/bounce
CMB spretral index $n_S = 0.96 \pm 0.01$	Inflation: $1 - \epsilon$	Perturbations during collapse/bounce
$\Omega_m/\Omega_B \simeq 4$	Free parameter	Compact to diffuse rennants
$\Omega_\Lambda/\Omega_m \simeq 3$	Free parameter	Time to de-Sitter phase
Large-scale CMB anomalies	Cosmic Variance (bad luck)	Super-horizon cut-off
Hubble tension	Systematic effects	Super-horizon perturbations
Flat universe $k = 0$	Inflation	Topology of empty space

2.5. Why Is the Universe Flat?

So, if inflation did not happen, why is our Universe flat? The results in paper I can be applied to a FLRW cloud with $k \neq 0$ if we just redefine:

$$r_H^{-2} \equiv H^2 + k/a^2,$$

(9)

so this does not change our BHU model or interpretation. A time-like geodesic of constant comoving radius χ_* contains a constant mass M also for $k \neq 0$. However, what is the motivation to choose a particular topology, other than that of empty space ($k = \Lambda = 0$)? The so-called “flatness problem”, that is solved by Inflation, is only a problem if you assume that the Big Bang Planck singularity (or the postulated initial conditions) will produce different values for the topological curvature k . The same can be argued about a global Λ term. The field equations of GR are local, and they do not change k or Λ by the presence of matter. These are global topological quantities imposed as the starting point to the equations. So, any choice other than $k = \Lambda = 0$ would require some justification that is outside GR. In our analysis, we just assumed the most simple topology, that of empty space with $k = \Lambda = 0$, because we do not start from a past Planck singularity but from almost empty Minkowski space (before the BHU collapse). In the subsequent BHU evolution, k or Λ remain zero, and the singularity is avoided at GeV, well before Quantum Gravity effects (10^{19} GeV), so we do not expect monopoles, a global curvature or Λ to emerge.

3. The Apollonian Universe

Whatever the formation mechanism, one could ask: what was there before our BHU formed? We will assume here that there are other BHUs and regular matter within a larger space-time that we call the Apollonian Universe. Above the Jeans mass $M > M_J$, we can use the Press–Schechter formalism [15] to predict the number of collapsed objects $dn(M)$

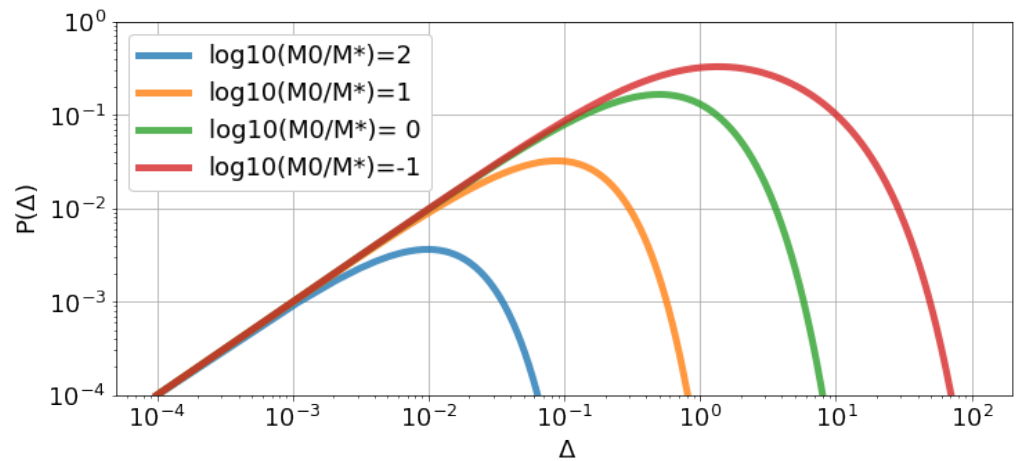


Figure 4. Anthropic probability $P(\Delta)$ in Equation (11) for different values of M_O/M_* . This corresponds to the probability for an observer like us to be in a BH of mass $M = M_O(1 + \Delta)$. M_O corresponds to the minimum time $\tau_O \simeq 10$ Gyr needed for a galaxy and planet similar to ours to form. M_* is the non-linear mass scale. Regardless of M_* , the maximum in $P(\Delta)$ is always within $0 < \Delta < 2$ or $M_O < M < 3M_O$.

of a given mass M . For a scale-free power spectrum $n = 0$ (scalar spectral index $n_s = 1$) close to the one inside our BHU at the largest scales:

$$\frac{dn(M)}{dM} = \sqrt{\frac{1}{\pi}} \left(\frac{M}{M_*} \right)^{1/2} \exp \left(-\frac{M}{M_*} \right) \frac{\bar{\rho}}{M^2}, \quad (10)$$

where M_* corresponds to the gravitational collapse non-linear transition scale. More generally, for different spectral indexes n , we have $(M/M_*)^{(n+3)/6}$. This result can be understood as the abundance of peaks in a Gaussian field [36]. The important point to notice is that large collapsed objects are exponentially suppressed for $M > M_*$. The typical value of M_* increases with time. The value today corresponds to a cluster mass: $M_* \simeq 10^{14} M_\odot$ but was lower in the past. In our observed expanding Universe (inside r_S now), we do not expect objects above $M_* \simeq 10^{15} M_\odot$ to form in the future because structure formation freezes out during the dS phase ($R \Rightarrow r_S$), but this is not a limit for the exterior background where the BHU collapsed because $\Lambda = k = 0$ (see §2.5).

We assume that the probability of having observers like us increases linearly with time for $\tau > \tau_O$ and is zero for $\tau < \tau_O$. So, τ_O is the astronomical time needed for observers like us to exist. Its value must be close to $\tau_O \simeq 13$ Gyrs, corresponding to the age of our galaxy [37], which is only about three times the age of our planet: 4.5 Gyr [38]. The BH collapse time in Equation (5) is proportional to M , so that a large mass $M \simeq 6 \times 10^{22} M_\odot$ in Equation (3) has a typical collapse time of $\tau \simeq 11$ Gyr in Equation (5) (i.e., Equation (8)). The expansion time is longer because of the acceleration caused by the BH event horizon, but during the de-Sitter phase, the Hubble Horizon shrinks and structure formation halts. So, in practice, the relevant timescale is the one given by matter domination in Equation (5): $\tau = 4 GM/3$.

We express M in terms of Δ : $M = M_O(1 + \Delta)$, where $M_O = 3\tau_O/4G$ is the BH mass corresponding to τ_O in Equation (5). The anthropic probability $P(\Delta)$ that an observer lives inside a BH of such mass is then:

$$P(\Delta) \propto \frac{n(M)}{n(M_O)} \Delta = (1 + \Delta)^{-3/2} \Delta \exp \left(-\frac{M_O}{M_*} \Delta \right) \quad \text{for } \Delta > 0. \quad (11)$$

We have divided $n(M)$ in Equation (10) by $n(M_O)$ because we are interested in the relative number of BHs above the ones with the minimal mass M_O . Figure (4) shows

Equation (11) for some values of M_O/M_* . For $M_O \gg M_*$, the probability is dominated by the exponential suppression and $P(\Delta)$ peaks around $\Delta = 0$. This means that most observers will live in a BH with mass M_O . So, an accurate estimation of τ_O provides a prediction for M_O and therefore a prediction for $r_S = 2GM_O$ and $\Lambda = 3/r_S^2$, which is in agreement with the values measured in our BHU. For $M_O \simeq M_*$, the probability $P(\Delta)$ peaks around $\Delta = 1$, which predicts that most observers live in BHs which are two times M_O . For $M_O \ll M_*$, the result is independent of M_* and the peak is at $\Delta = 2$. Thus, regardless of M_* , the maximum probability corresponds to observers in a BH with mass $M_O < M < 3M_O$ or collapse times $\tau_O < \tau < 3\tau_O$, which is very consistent with the measurements in our Universe for τ_O and M in Equation (3). In terms of Λ , this corresponds to $\Lambda_O/9 < \Lambda < \Lambda_O$, where Λ_O is the value corresponding to M_O or τ_O .

4. Conclusions and Discussion

We propose that cosmic expansion originated from the gravitational free-fall collapse of a large and low-density matter cloud. We assume that the background is flat with $k = 0$ and $\Lambda = 0$, as in empty space. The so-called flatness problem, that is solved by Inflation, is only a problem if the BB singularity can somehow create curvature. In the BHU model, the singularity is avoided at GeV (i.e., the energy corresponding to the nuclear saturation density in Equation (7)), well before Quantum Gravity effects (10^{19} GeV), so there is no reason to expect a global curvature or Λ . There is therefore no flatness problem that needs to be solved.

In nature, we never observe cold regular matter with densities larger than that of an atomic nuclei in Equation (7). The reason for this is the Pauli exclusion principle in Quantum Mechanics, which prevents fermions from occupying the same quantum state (which includes position). This neutron degeneracy pressure is responsible for core collapse supernova explosions [18]. We propose here that when the BH collapse reaches nuclear saturation density, it bounces back, as it happens in a supernova core collapse. This Big Bounce occurs at times and energy densities that are many orders of magnitudes away from Inflation or Planck times. Thus, Quantum Gravity or Inflation are not needed to understand cosmic expansion or the monopole problem [28]. Further work is needed to understand the details of such a Big Bounce: to estimate the perturbations, composition, and the fraction of compact and diffuse remnants that resulted from the bounce. This could explain from first principles some of the free parameters in the Λ CDM model, as shown in Table 1.

The Big Bounce could provide a uniform start for the BB, solving the horizon problem (see Figures 1 and 3): super-horizon perturbations during collapse (and bounce) seed structures (BAO and galaxies) as they re-enter r_H during expansion. The main differences with Inflation are the origin of those perturbations and the existence of a cut-off in the spectrum of fluctuations given by R in Equation (4). Such a cut-off has recently been measured in CMB maps [12,39–41]. Galaxy maps are also able to measure this signal [42,43]. The existence of such super-horizon perturbations could be related to the tension in measurements of the cosmological parameters from different cosmic scaletimes [44–48], which have similar variations in cosmological parameters to the measured CMB cut-off anomalies, as reported in [35].

Another difference is the possibility that DM is all made of primordial BHs (PBHs) and Neutron Stars (NSs) remnants from before the BB explosion. According to the PS formalism in Section 3, we expect a significant number of large (super massive) PBHs with $M > 10^9$ as well as a much larger number of smaller ones. This could represent a paradigm shift for models of galaxy formation. There is a tight empirical relation between the BH mass measured in the center of large galaxies and the stellar (bulge) mass or luminosity [49]. This is hard to explain in current models of galaxy formation where galaxies form first and central BHs grow later. In the BHU model, large PBHs could instead be the seeds to form massive galaxies, which together with smaller PBHs and NSs could be the mysterious DM. This could also help explain some recent JWST observations of high redshift massive

galaxies [50,51] and the puzzling observation of very luminous quasars (which trace super-massive BHs) at very high redshift (see [52] and references there in). Smaller BHs could be present as a dormant population, which could be detected with GAIA measurements [53].

The BH collapse time in Equation (5) is proportional to M , so that a large mass $M \simeq 6 \times 10^{22} M_{\odot}$ in Equation (3) is just the right one to allow enough time for galaxies and planets to form before the de-Sitter phase dominates. This provides an anthropic explanation [54,55] as to why we live inside such a large BH. In other words, it is an explanation of why the observed $\Lambda = 3/r_s^2$ is so small and why we live at a time when the expansion is close to the dS phase (the coincidence problem in cosmology [1,2,7,8]). According to Equation (11) (see also Figure 4), the maximum probability corresponds to observers that appear in BHs with $\Lambda_O/9 < \Lambda < \Lambda_O$, where $\Lambda_O = 4/(3\tau_O^2)$ is the value corresponding to $r_s \simeq 3\tau_O/2$ in Equation (8) for the minimum time τ_O needed for observers to exit. If we assume that this time τ agrees with the age of our galaxy, we find good agreement between this prediction and the observed Λ measurements ($\Omega_{\Lambda} \simeq 0.75$). These arguments neglect the global rotation of the FLRW cloud (or the BHU). Such rotation could slow down the expansion rate (see Appendix C in [12]) and play some role in the bounce and collapse time. If our BHU is moving or rotating, within the Apollonian background, this could show as a dipole. A CMB dipole has already been observed [56], but it is usually interpreted as a local flow. This interpretation has recently been challenged by new observations of our local neighborhood (see [57]).

The BHU solution can also be used to understand the interior of regular (stellar or galactic) BHs. Such BHs could just be made of regular (baryonic) collapsing matter, but they will not have time to form regular galaxies or stars inside because within seconds of the bounce (see Equation (5)), the internal dynamics becomes dominated by the de-Sitter phase (caused by their event horizon mass r_s). The bounce proposed here, based in Quantum Mechanics, could avoid both the BH and the BB singularities [58,59], which in the BHUs model both corresponds to the same FLRW cloud collapse (but with different masses). The BHU also eludes the entropy paradox [4] in a similar way as that proposed by Penrose [5]. The difference is that the BHU does not require new laws (infinite conformal re-scaling) or cyclic repetition. Our expansion will end up trapped (and asymptotically static) inside a BH within a larger and older manifold possibly containing other BHUs. We call this the Apollonian Universe (see Section 3). Such an idea provides another layer to the Copernicus Principle, where we are not at the center of our BHU and our BHU is not at the center of everything else.

Funding: This work was partially supported by grants from Spain MCIN/AEI/10.13039/501100011033 grants PGC2018-102021-B-I00, PID2021-128989NB-I00 and Unidad de Excelencia María de Maeztu CEX2020-001058-M and from European Union funding LACEGAL 734374 and EWC 776247. IEEC is funded by Generalitat de Catalunya.

Data Availability Statement: No new data is presented.

Acknowledgments: I want to thank Marco Bruni, Robert Caldwell, Benjamin Camacho, Ramin G. Daghigh, Ravi Sheth and Alberto Diez-Tejedor for their feedback to early drafts of this work and to Angela Olinto and Sergio Assad for their hospitality during the summer of 2022, when the latest version of this paper was completed, extracted from earlier unpublished drafts [60,61], which preceded later review articles [12,62].

Conflicts of Interest: The authors declare no conflict of interest.

Appendix A. Expanding Background

To form a BHU during an explosion or rapid expansion, we need a scalar field and a False Vacuum (FV), ρ_{Λ} , which is a term that is explained next.

Appendix A.1. Scalar Field in Curved Space-Time

Consider a minimally coupled scalar field $\phi = \phi(x_\alpha)$ with:

$$\mathcal{L} \equiv K - V = -\frac{1}{2}\partial_\alpha\phi\partial^\alpha\phi - V(\phi), \quad (\text{A1})$$

The Lagrange equations are: $\bar{\nabla}^2\phi = \partial V/\partial\phi$. We can estimate $T_{\mu\nu}(\phi)$ from its definition:

$$G_{\mu\nu} + \Lambda g_{\mu\nu} = 8\pi G T_{\mu\nu} \equiv -\frac{16\pi G}{\sqrt{-g}} \frac{\delta(\sqrt{-g}\mathcal{L})}{\delta g^{\mu\nu}}, \quad (\text{A2})$$

to find (e.g., see [2]):

$$T_{\mu\nu}(\phi) = \partial_\mu\phi\partial_\nu\phi + g_{\mu\nu}(K - V), \quad (\text{A3})$$

comparing with a perfect fluid in spherical coordinates:

$$T_{\mu\nu} = (\rho + p)u_\mu u_\nu + p g_{\mu\nu}, \quad (\text{A4})$$

we find:

$$\rho = K + V \quad ; \quad p = |K| - V. \quad (\text{A5})$$

In general, we can have $p_\parallel \neq p_\perp$ for non canonical scalar fields (see Equation 5 in [63] for further details). The stable solution corresponds to $p = -\rho \equiv -\rho_{vac}$:

$$\bar{\nabla}^2\phi = \partial V/\partial\phi = 0 \quad ; \quad \rho \equiv \rho_{vac} = -p = V(\phi) = V_i \equiv V_0 + \Delta, \quad (\text{A6})$$

where ϕ is trapped in the true minimum V_0 or some False Vacuum (FV) state $V_i = V_0 + \Delta$. The situation is illustrated in Figure A1. The solution to Equation (A2) for constant $\rho = -p = V_i$ (without matter or radiation) for a general metric with spherical symmetry in physical coordinates:

$$ds^2 = g_{\mu\nu}dx^\mu dx^\nu = -(1 + 2\Psi)dt^2 + \frac{dr^2}{1 + 2\Phi} + r^2 d\Omega^2, \quad (\text{A7})$$

is given by the deSitter (dS) metric:

$$2\Phi = 2\Psi = -r^2/r_\Lambda^2 \equiv -r^2 H_\Lambda^2 = -r^2 8\pi G \rho_\Lambda / 3 = r^2 (8\pi G V_i + \Lambda) / 3. \quad (\text{A8})$$

This metric is static, which indicates that the vacuum solution is in equilibrium. This same dS solution also corresponds to the FLRW metric solution with constant H :

$$ds^2 = f_{\alpha\beta} d\xi^\alpha d\xi^\beta = -d\tau^2 + a(\tau)^2 [d\chi^2 + \chi^2 d\Omega] \quad \text{with} \quad H^2 \equiv \left[\frac{\dot{a}(\tau)}{a(\tau)} \right]^2 = H_\Lambda^2 = 8\pi G \rho_\Lambda / 3. \quad (\text{A9})$$

If we use $\Lambda = V_0 = 0$, so that there is empty space outside (see paper I), this corresponds to a BHU with $r_S = 2GM = \sqrt{3/(8\pi G\Delta)}$, without regular matter or radiation. Note how the smaller the Δ , the larger the M . Note that in general, we would not expect $V_0 = 0$ so that the metric outside is not that of Schwarzschild. This would correspond to a BHU inside a dS or FLRW metric (see bottom right panel of Figure A2 in paper I).

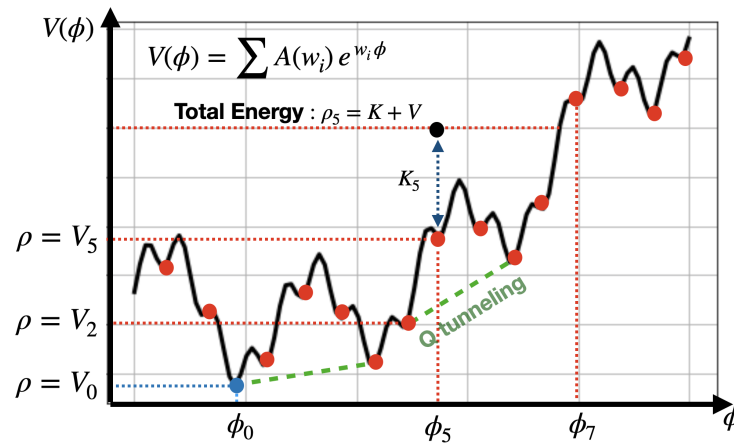


Figure A1. The potential $V(\phi)$, of a classical scalar field $\phi(x)$, made of the superposition of plane waves. A configuration with total energy: $\rho_5 = K_5 + V_5$ (black dot at ϕ_5) can lose its kinetic energy K_5 during expansion (e.g., a supernova explosion or a expanding background) due to Hubble damping and relax into one of the static ($K = 0$) ground states (or FV) $\rho_5 = V_5 \equiv V(\phi_5)$ (red dots). This can

generate a Black Hole (BH.fv) and regular matter from reheating. Each FV has an energy excess $\Delta_i \equiv V_i - V_0$ over the true vacuum at V_0 (blue dot). Quantum tunneling (dashed lines) could allow ϕ to jump between FV, resulting in BH evaporation and new matter/radiation.

Appendix A.2. BHU Formation

Consider a localized field with some fixed total energy $\rho = K + V$ (black dot labeled ρ_5 in Figure A1). In an expanding background (such as a supernovae explosion or Inflation), the field can rapidly lose its kinetic energy (K_5), due to Hubble damping, and end up trapped inside some FV (V_5). If the outside background ($r > r_s$) has at a lower FV, this will generate an expanding BHU until $r_\Lambda = r_s$. Because an additional FV structure can exist within a given FV, the same Hubble damping can form a BHU inside a larger BHU. When K is not fully damped, the classical reheating mechanism around a FV could also be a source of matter/radiation. There is some literature on Bubble Universe formation (e.g., see [64,65] and references therein), but they typically involve quantum gravity ideas or GR extensions. As illustrated in Figure A1, there could be a landscape of nested BHU of different masses and sizes. Such BHUs are similar to primordial BHs [66]. The masses and sizes of such BHU bare no relation with the energy of the expansion (e.g., supernova explosion or inflation) or the host object which originated the expansion. This means that if this mechanism actually occurred, a BH of any mass M can emerge out of violent expansion. For example, a stellar explosion could produce a galactic size BH or one as large as our Universe. The smaller the Δ , the larger the M and r_s . In addition, quantum tunneling (see Figure A1) could generate a cascade of small BHs turning into larger BHs. This could take some time, as the tunneling probability is proportion to $e^{-\Delta t}$. These are puzzling results that strongly depend on the vacuum structure of $V(\phi)$, but they could explain how large mass BHUs formed.

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