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A New Solution for Measuring Planck's Constant using Compton Scattering

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Abstract

Measured values of the electron mass and Compton wavelength produce a value of Planck's constant with a relative standard uncertainty of 3×10^{-10}. This is only slightly larger than the 1.3×10^{-10} relative standard uncertainty in measurements performed using the Kibble balance. Compton scattering represents an alternative pathway to improving the value of Planck's constant in the future.

Natural units of length, mass, and time offer new pathways to improving the values of physical constants. While extensive values of the Planck units lie beyond the reach of present-day instrumentation, certain product and quotient pairs of Planck units such as the speed of light can be measured with relatively high precision. Better measurements of certain unit pairs will improve the value of the gravitational constant.

Keywords: Planck constant; metrology; Compton scattering; natural units; Kibble balance; Compton wavelength; electron mass

1 Introduction

The System of International Units sets an exact value for Planck's constant based on measurements undertaken by the National Institute of Standards and Technology between the years 2015 and 2017 using the Kibble balance [1]. These measurements reduced uncertainty by more than twofold over previous measurements, achieving a relative standard uncertainty of 1.3×10^{-10}.

Although the Kibble balance was the preferred method for measuring Planck's constant, it is not the only experimental means for obtaining a high precision measurement. Planck's constant can also be determined from measurements of the electron's Compton wavelength and rest mass, each with a relative standard uncertainty of 3.0×10^{-10} [2]. The formula relating these measurements to the reduced

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Planck constant is

$$\hbar = \bar{\lambda}_C m_0 c = 1.054\ 571\ 8176... \times 10^{-34}\ kgm^2/s \tag{1}$$

Although it is not widely understood that the formula gives an accurate value of Planck's constant, the physics behind the formula are mathematically consistent and yield valuable insights [3].

2 Derivation of the results

Equation 1 is derived by representing Planck's constant with natural units in the dimensions L^2MT^{-1} [3–5]

$$\hbar = \frac{l_P^2 m_P}{t_P} = l_P m_P c. \tag{2}$$

In addition, it has been shown that an electron's Compton wavelength and rest mass are inversely proportional [6–8], making the product of wavelength and mass invariant and equal to the product of Planck length and Planck mass [3]

$$\bar{\lambda}_C m_0 = l_P m_P. \tag{3}$$

An important consequence of 3 is that wavelength and mass are equal ratios of the Planck scale where Planck length gives a minimum basis and Planck mass gives a maximum basis

$$\frac{l_P}{\bar{\lambda}_C} = \frac{m_0}{m_P}. \tag{4}$$

To obtain Planck's constant from the electron properties, we substitute 3 into 2 yielding equation 1.

Table 1 summarizes the CODATA values of Compton wavelength and rest mass which produce Planck's constant according to equation 1. Note that measurements of the muon and tau Compton wavelengths and rest masses also produce the constant but with less certainty: 2.2×10^{-8} and 6.8×10^{-5} relative standard uncertainties respectively [2].

Table 1: Lepton properties which determine the value of Planck's constant.

Particle	Compton wavelength $\bar{\lambda}_C$	Rest mass m_0	Reduced Planck constant $\bar{\lambda}_C m_0 c$
Electron	$3.861\ 592\ 6796 \times 10^{-13}$	$9.109\ 383\ 7015 \times 10^{-31}$	$1.054\ 571\ 8176... \times 10^{-34}$
Muon	$1.867\ 594\ 306 \times 10^{-15}$	$1.883\ 531\ 627 \times 10^{-28}$	$1.054\ 571\ 817... \times 10^{-34}$
Tau	$1.110\ 538 \times 10^{-16}$	$3.167\ 540 \times 10^{-27}$	$1.054\ 57... \times 10^{-34}$

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3 Planck scale metrology

It may be reasonably argued that extensive quantities of Planck length, mass, and time lie beyond the reach of experimental measurement [9]. However, certain product and quotient *relationships* between pairs of Planck units are demonstrably within the reach of modern instrumentation; for example, the ratio of Planck length to Planck time. The speed of light has been measured with a relative standard uncertainty of $1.6 \times 10^{-10}\ m/s$ [10], an important consideration in the decision to define c in the System of International Units

$$\frac{l_P}{t_P} = c = 299,792,458\ m/s. \tag{5}$$

An accurate measurement of the speed of light is possible because the intensive ratio of distance and time can be measured on scales much larger than the Planck scale.

Similarly, the product of Planck length and Planck mass has a defined value under the SI as the ratio between two defined constants—Planck's constant and the speed of light

$$l_P m_P = \frac{\hbar}{c} = \frac{l_P m_P \not{c}}{\not{c}} = 3.517\,672\,9417... \times 10^{-43}\ kgm. \tag{6}$$

The inversely proportional relationship between wavelength and mass shown in equations 3 and 4 is responsible for the invariance of Planck's constant. It has been shown that this invariance pertains to integer cycles of elementary particle oscillations and also conserves quantities of wavelength-momentum and time-energy [3].

Planck's constant and the speed of light give a third defined value in the product of Planck mass and Planck time

$$m_P t_P = \frac{\hbar}{c^2} = \frac{\frac{m_P \not{l_P^2}}{\not{t_P}}}{\frac{\not{l_P^2}}{t_P \not{t_P}}} = 1.173\,369\,3920... \times 10^{-51}\ kgs. \tag{7}$$

The significance of these three defined values is *much more* than academic; the Planck units offer an overlooked pathway to obtaining more accurate values of the gravitational constant and other constants that depend on G. Like Planck's constant, the gravitational constant can be represented in natural units of length, mass, and time [3]

$$G = \frac{l_P}{m_P} c^2. \tag{8}$$

The formula indicates that uncertainty in G lies in the ratio of Planck length to Planck mass given an exact value of c. Consequently, the gravitational constant has a relative standard uncertainty of 2.2×10^{-5} which is comparable to the 1.1×10^{-5} relative standard uncertainty in the CODATA values of Planck length and Planck mass.

Improving the value of G requires a more accurate measurement of at least one of the three unde-

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Table 2: Of the six product and quotient relationships between the Planck units, three have exact values based on the exact values of Planck's constant and the speed of light. The other three relationships have uncertainties comparable to the uncertainty in the gravitational constant.

Planck Unit Pair	Equivalent	Value	SI rel. std. uncertainty
$\frac{l_P}{t_P}$	c	$299,792,458\ m/s$	defined
$l_P m_P$	$\frac{\hbar}{c}$	$3.517\,672\,94 \times 10^{-43}\ kgm$	defined
$m_P t_P$	$\frac{\hbar}{c^2}$	$1.173\,369\,39 \times 10^{-51}\ kgs$	defined
$\frac{l_P}{m_P}$	$\frac{G}{c^2}$	$7.426\,160 \times 10^{-28}\ m/kg$	2.2×10^{-5}
$\frac{m_P}{t_P}$	$\frac{c^3}{G}$	$4.036\,978 \times 10^{35}\ kg/s$	2.2×10^{-5}
$l_P t_P$	$\frac{\hbar G}{c^4}$	$8.713\,629 \times 10^{-79}\ ms$	2.2×10^{-5}

fined values in table 2. This is because the three defined values only provide enough information to constrain the proportions between the Planck units and do not reveal the extensive values themselves.

To see why this is the case, consider the three pairs of Planck units with defined values in table 2. Notice that the set contains *either* a product relationship *or* a quotient relationship between a given pair of units, but *not both*. For example, the ratio l_P/t_P is defined but $l_P t_P$ has a large uncertainty by comparison.

If we had precision measurements for both the product *and* the quotient relationship between a pair of units, we could determine a value for the two units with the same level of precision. This is easy to see in the following way. Let a and b represent high precision values of a product and quotient relationship between Planck length and Planck time

$$\frac{l_P}{t_P} = a \tag{9}$$

and

$$l_P t_P = b. \tag{10}$$

From this information we can solve for two equations and two unknowns. Restating 9

$$l_P = t_P a \tag{11}$$

and plugging into 10 gives a solution

$$t_P^2 = \frac{b}{a}. \tag{12}$$

Our misfortune is in having defined values for three Planck unit pairs without a single set of product and quotient relationships for any pair.

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A geometric representation of equations 9 through 12 further illustrates these constraints. We can model the quotient relationship between Planck length and Planck time as two sides of a rectangle. The speed of light gives a ratio between the two sides but not an extensive value of either side. In natural units the ratio is 1:1, but in SI units the t_P side of the rectangle is precisely $299,792,458$ times the length of the l_P side. The ratio between the two sides offers a constraint on the values of l_P and t_P but does not allow us to identify the value of either unit.

Introducing the product relationship $l_P t_P$ gives the area of the rectangle and the two combined constraints give a definite value for each of the units. In this geometric analogy, the defined values of Planck's constant and the speed of light give us defined values for the ratio of length-time and the areas of length-mass and mass-time.

The result is that we have greater precision in the proportions between the Planck units than in the unit values. This is easy to see in the CODATA values of Planck length and Planck time which give a ratio of $299,792,423$ for the speed of light. Although we know this ratio is inaccurate, we don't know how to adjust the two units to correct it. This can only be achieved with more accurate measurements.

Figure 1 illustrates the degree to which CODATA values of Planck length, mass, and time are proportionally inaccurate. In the figure, each node of the triangle represents the current value of a Planck unit and the equilateral triangle formed by these points represents a proportionally accurate relationship between them. Three triangles overlaying the equilateral triangle show the degree to which two of the units are out of proportion given the value of a first unit. For example, the blue triangle with a node on the Planck length indicates that, given the current value of Planck length, the value of Planck mass is too small and the value of Planck time is too large.

Table 3 gives the formulas for calculating proportionally accurate values of the second and third Planck units when given the value of a first unit.

Table 3: Formulas for calculating the values of any two Planck units when given the value of a first unit. Defined values of Planck's constant and the speed of light provide the required constraints.

Given unit value	l_P	m_P	t_P
Planck length	-	$\frac{\hbar}{l_P c}$	$\frac{l_P}{c}$
Planck mass	$\frac{\hbar}{m_P c}$	-	$\frac{l_P}{c}$
Planck time	$t_P c$	$\frac{\hbar}{l_P c}$	-

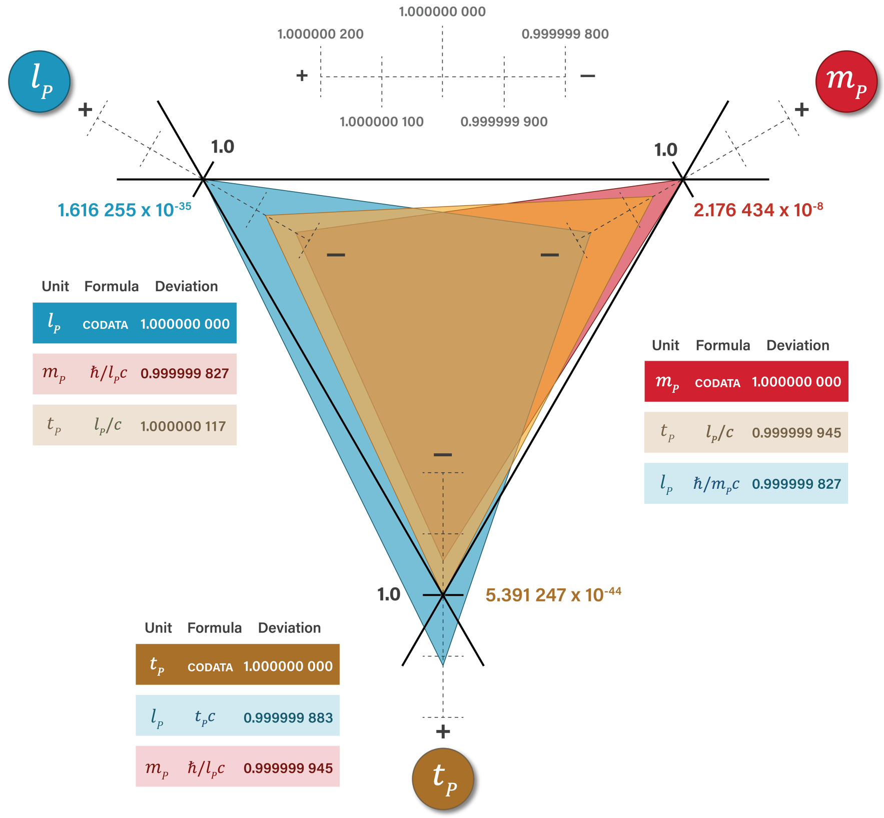

Figure 1: The speed of light and Planck's constant constrain the proportions between Planck length, mass, and time. The equilateral triangle represents a proportionally accurate relationship between the units.

3.1 New measurement approaches

A better understanding and appreciation for the natural units presents an opportunity for devising new measurement solutions that improve the accuracy of physical constants and shed light on the structure of natural phenomena. Measurements of the universal constants are also measurements of the relationships between natural units and an improvement in one elevates the other.

One approach to improving the accuracy of the gravitational constant is to continue refining the instruments and methods for measuring l_P/m_P. However, such measurements depend on accurate measurements of the mass and radius of two bodies and it remains challenging to obtain more precise measurements of the gravitational field between bodies of measurable mass.

An alternate pathway to improving the value of G is to devise new measurement techniques aimed at determining more precise values of m_P/t_P or $l_P t_P$. A more precise measurement of either quantity yields a commensurate gain in the precision of G. This is because a better measurement of any undefined pair in table 2 will improve the values of the Planck units using the formulas in table 4. In particular, we need better values of l_P and m_P given the exact value of c^2. An improvement in these two values will improve the value of G according to equation 8.

Table 4: Formulas for calculating values of the three Planck units with the same precision as a measurement of the Planck unit pairs in the first column. Better values of the Planck units improve the values of universal constants.

Planck unit pair	l_P	m_P	t_P
$\frac{l_P}{m_P}$	$\sqrt{\frac{l_P}{m_P}\frac{\hbar}{c}}$	$\sqrt{\frac{m_P}{l_P}\frac{\hbar}{c}}$	$\sqrt{\frac{l_P}{m_P}\frac{\hbar}{c^3}}$
$\frac{m_P}{t_P}$	$\sqrt{\frac{t_P}{m_P}\hbar}$	$\sqrt{\frac{m_P}{l_P}\frac{\hbar}{c^2}}$	$\sqrt{\frac{t_P}{m_P}\frac{\hbar}{c^2}}$
$l_P t_P$	$\sqrt{l_P t_P c}$	$\sqrt{\frac{\hbar^2}{l_P t_P c^3}}$	$\sqrt{\frac{l_P t_P}{c}}$

The ratio m_P/t_P is found in unit dimensions of force and opens up the possibility of conducting more precise measurements using electromagnetic forces. This is perhaps the most promising pathway for significantly improving the value of G. A greater challenge, however, is measuring $l_P t_P$ which does not appear in the unit dimensions of common natural phenomena.

4 Conclusion

The significance of the natural units is still grievously underappreciated in physics. The historical preference for universal constants in compound unit dimensions is a philosophical preference that overlooks the advantages of more granular information in the natural unit formulas.

A better understanding of Planck's constant in each of its unit dimensions opens up a new pathway for obtaining a more precise value of the constant. The precision with which the electron properties produce Planck's constant was only recently surpassed by measurements using the Kibble balance, demonstrating the viability of the proposed method and theory. The level of agreement between the two methods also confirms the mathematical consistency of the natural units with high levels of precision.

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B Competing interests

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