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Some Tightness-Type Properties of the Space of Permutation Degree

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Abstract

In this paper we prove that if the product X^n of a space X has some tightness-type properties, then the space of permutation degree $\mathsf{SP}^{\mathsf{n}}X$ also has these properties. It is proved that the set tightness (resp. T-tightness) of the space of permutation degree $\mathsf{SP}^{\mathsf{n}}X$ is equal to the set tightness (resp. T-tightness) of the product X^n.

Keywords: functor of permutation degree; tightness; set tightness; T-tightness; functional tightness; minitightness
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1 Introduction

At the Prague Topological Symposium in 1981, V.V. Fedorchuk [6] posed the following general problem in the theory of covariant functors which determined a new direction for research in the field of Topology:

- Let $\mathcal{P}$ be some geometric property and $\mathcal{F}$ be a covariant functor. If a topological space X has the property $\mathcal{P}$, then whether $F(X)$ has the same property $\mathcal{P}$? or vice versa, that is, if $F(X)$ has the property $\mathcal{P}$, does it follow that the topological space X has also the property $\mathcal{P}$?

In our case, $\mathcal{P}$ is some tightness-type property, X is a topological T_1-space, and F is the functor of G-permutation degree $\mathsf{SP}^{\mathsf{n}}_{\mathsf{G}}$.

In [6, 7] V.V. Fedorchuk and V.V. Filippov investigated the functor of G-permutation degree and it was proved that this functor is a normal functor in the category of compact spaces and their continuous mappings.

In recent researches a number of authors was interested in the behaviour of certain cardinal invariants under the influence of various covariant functors. For example, in [5, 11, 12, 14, 19, 20] the authors investigated several cardinal invariants under the influence of some weakly normal, seminormal and normal functors.

In [11, 12] some cardinal and geometric properties of the space of permutation degree $\mathsf{SP}^{\mathsf{n}}X$ have been discussed. It is proved that if the product X^n has some Lindelŏf-type properties, then the space $\mathsf{SP}^{\mathsf{n}}X$ also has these properties.

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Moreover, it is shown that the functor SP^n_G preserves the homotopy and the retraction of topological spaces. In addition, it is proved that if the spaces X and Y are homotopically equivalent, then the spaces $\mathsf{SP}^n_G X$ and $\mathsf{SP}^n_G Y$ are also homotopically equivalent. As a result, it has been proved that the functor SP^n_G is a covariant homotopy functor.

The current paper is devoted to the investigation of cardinal invariants such as the T-tightness, set tightness, functional tightness, minitightness (or weak functional tightness), and some other topological properties of the space of permutation degree. Let us mention that tightness-type properties of function spaces have been studied in [10, 18].

The concepts of functional tightness and minitightness (or the weak functional tightness) of a topological space was first introduced and studied by A.V. Arkhangel'skii in [1]. As it turned out, cardinal invariants such as the minitightness and functional tightness are in many ways similar to each other, and for many natural and classical cases they coincide. Moreover, there is an example of a topological space, the minitightness of which is countable, and the functional tightness is uncountable (see [17]).

In [16], the action of closed and R-quotient mappings on functional tightness is investigated. It is proved that the R-quotient mappings do not increase functional tightness. As well as, in [16] it is proved that the functional tightness of the product of two locally compact spaces does not exceed the product of functional tightness of those spaces.

Throughout the paper all spaces are topological spaces, and κ ia an infinite cardinal number.

2 Definitions and Notations

The following definitions and notions will be needed in the sequel.

Definition 2.1 (see [2]). Let A be a subset of a topological space X, the *tightness of A with respect to X* is the cardinal number

$$t(A,X) = \min\{\kappa : \forall C \subset X \text{ such that } A \cap \overline{C} \neq \emptyset \ \exists C_0 \in [C]^{\leq\kappa} \text{ with } A \cap \overline{C}_0 \neq \emptyset\}.$$

If $A = \{x\}$ we briefly write $t(x,X)$ instead of $t(\{x\},X)$. The *tightness* of X is defined as $t(X) = \sup\{t(x,X) : x \in X\}$.

Definition 2.2 ([3]; se also [4, 9]) Let X be a topological space, the *set tightness at a point $x \in X$*, denoted by $t_s(x,X)$, is the smallest cardinal number κ such that whenever $x \in \overline{C} \setminus C$, where $C \subset X$, there exists a family γ of subsets of C such that $|\gamma| \leq \kappa$ and $x \in \overline{\bigcup\gamma} \setminus \bigcup\overline{\gamma}$. The *set tightness* of X is defined as $t_s(X) = \sup\{t_s(x,X) : \ x \in X\}$.

It is clear that for any topological space X we have $t_s(x,X) \leq t(x,X)$ and $t_s(X) \leq t(X)$.

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Definition 2.3 ([8, 9]). For a topological space X the *T-tightness* of X, denoted by $T(X)$, is the smallest cardinal number κ such that whenever $\{F_\alpha\}_{\alpha\in\Lambda}$ is an increasing sequence of closed subsets of X with $cf(\Lambda) > \kappa$, then $\bigcup_{\alpha\in\Lambda} F_\alpha$ is closed.

Let κ be an infinite cardinal, X and Y topological spaces. A mapping $f : X \to Y$ is said to be *κ-continuous* if for every subspace A of X such that $|A| \le \kappa$, the restriction $f \restriction A$ is continuous. A mapping $f : X \to Y$ is said to be *strictly κ-continuous* if for every subspace A of X with $|A| \le \kappa$, there exists a continuous mapping $g : X \to Y$ such that $f \restriction A = g \restriction A$.

Definition 2.4 ([1]; see also [15, 16]). The *functional tightness* $t_o(X)$ of a space X is the smallest infinite cardinal number κ such that every κ-continuous real-valued function on X is continuous.

In [16] the following theorem was proved.

Theorem 2.1 *If X is a locally compact space, then $t_o(X \times Y) \le t_o(X)t_o(Y)$.*

Note that by Theorem 2.1 , $t_o(X^n) = t_o(X)$ for every compact space X and every $n \in N$.

Definition 2.5 ([1]). The *weak functional tightness* (or *minitightness*) $t_m(X)$ of a space X is the smallest infinite cardinal number κ such that every strictly κ-continuous real-valued function on X is continuous.

Clearly, every strictly κ-continuous mapping is κ-continuous. Therefore, for any topological space X we have

$$t_m(X) \le t_o(X) \le t(X).$$

In [15] the following theorems were given.

Theorem 2.2 ([15, Theorem 2.14]) *If X is a locally compact space, then for every space Y,*

$$t_m(X \times Y) \le t_m(X)t_m(Y).$$

Theorem 2.3 ([15, Theorem 2.7; Corollary 2.8]) *For any two spaces X and Y,*

$$t_m(X \times Y) \le t_m(X)\chi(Y).$$

If Y is first countable, then $t_m(X \times Y) = t_m(X)$.

The set of all non-empty closed subsets of a topological space X is denoted by $\mathsf{exp}X$. The family of all sets of the form

$$O\langle U_1, U_2, \ldots, U_n\rangle = \big\{F:\ F \in \mathsf{exp}X,\ F \subset \bigcup_{i=1}^{n} U_i,\ F \cap U_i \ne \emptyset,\ i = 1, \ldots, n\big\},$$

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where U_1, U_2, ..., U_n are open subsets of X, generates a base of the topology on the set $\mathsf{exp}X$. This topology is called the *Vietoris topology*. The set $\mathsf{exp}X$ with the Vietoris topology is called *exponential space* or the *hyperspace* of a space X. We put

$$\mathsf{exp_n}X = \{F \in \mathsf{exp}X : \ |F| \leq n\} \ [7].$$

Let S_n denote the permutation group of the set $\{1, 2, \dots, n\}$, and let G be a subgroup of S_n. The group G acts on the n-th power X^n of a space X as permutation of coordinates. Two points $(x_1, x_2, \dots, x_n), (y_1, y_2, \dots, y_n) \in X^n$ are considered to be *G-equivalent* if there exists a permutation $\sigma \in G$ such that $y_i = x_{\sigma(i)}$. This relations is called *symmetric G-equivalence relation* on X. The G-equivalence class of an element $x = (x_1, x_2, \dots, x_n) \in X^n$ is denoted by $[x]_G = [(x_1, x_2, \dots, x_n)]_G$. The sets of all orbits of actions of the group G is denoted by $\mathsf{SP^n_G}X$. Thus, points of the space $\mathsf{SP^n_G}X$ are finite subsets (equivalence classes) of the product X^n.

Consider the quotient mapping $\pi^s_{n,G} : X^n \to \mathsf{SP^n_G}X$ defined by

$$\pi^s_{n,G}((x_1, x_2, \dots, x_n)) = [(x_1, x_2, \dots, x_n)]_G$$

and endow the sets $\mathsf{SP^n_G}X$ with the quotient topology. This spaces is called the *space of n-G-permutation degree* or simply the *space of G-permutation degree* of the space X.

Let $f : X \to Y$ be a continuous mapping. For an equivalence class $[(x_1, x_2, \dots, x_n)]_G \in \mathsf{SP^n_G}X$ we put

$$\mathsf{SP^n_G}f[(x_1, x_2, \dots, x_n)]_G = [(f(x_1), f(x_2), \dots, f(x_n))]_G.$$

In this way we have the mapping $\mathsf{SP^n_G}f : \mathsf{SP^n_G}X \to \mathsf{SP^n_G}Y$. It is easy to check that the mapping $\mathsf{SP^n_G}$ so constructed is a normal functor in the category of compacta. This functor is called the *functor of G-permutation degree*.

When $G = S_n$ we omit the index or prefix G in all the above definitions. In particular, we speak about the space $\mathsf{SP^n}X$ of permutation degree, the functor $\mathsf{SP^n}$ and the quotient mapping π^s_n.

Equivalence relations by which one obtains spaces $\mathsf{SP^n_G}X$ and $\mathsf{exp_n}X$ are called the *symmetric* and *hypersymmetric equivalence relations*, respectively.

Any symmetrically equivalent points in X^n are hypersymmetrically equivalent. But the converse is not correct, in general. For example, for $x \neq y$ points (x, x, y), (x, y, y) are hypersymmetrically equivalent, but not symmetrically equivalent.

The G-symmetric equivalence class $[(x_1, x_2, \dots, x_n)]_G$ uniquely determines the hypersymmetric equivalence class $[(x_1, x_2, \dots, x_n)]^{hc}_G$ containing it. So, we get the mapping

$$\pi^h_{n,G} : \mathsf{SP^n_G}X \to \mathsf{exp_n}X,$$

representing the functor $\mathsf{exp_n}$ as the factor functor of the functor $\mathsf{SP^n_G}$ [6, 7].

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3 Results

The functor of G-permutation degree SP^n_G preserves κ-continuity of the mappings, i.e. the following holds.

Theorem 3.1 *If $f : X \to Y$ is a κ-continuous mapping, then the mapping $\mathsf{SP}^n_G f : \mathsf{SP}^n_G X \to \mathsf{SP}^n_G Y$ is also κ-continuous.*

Proof. Consider an arbitrary subset $\mathsf{SP}^n_G A$ of $\mathsf{SP}^n_G X$, such that $|\mathsf{SP}^n_G A| \leq \kappa$. Let us prove that the restriction of the mapping $\mathsf{SP}^n_G f$ onto the set $\mathsf{SP}^n_G A$ is continuous.

Put
$$M = pr_i((\pi^s_{n,G})^{\leftarrow}(\mathsf{SP}^n_G A)),$$
where $pr_i : X^n \to X$ is defined as
$$pr_i(z_1, z_2, \ldots, z_n) = z_i,$$
for any $(z_1, z_2, \ldots, z_n) \in X^n$, $1 \leq i \leq n$, and $\pi^s_{n,G} : X^n \to \mathsf{SP}^n_G X$. It is clear that $M \subset X$ and $|M| \leq \kappa$. Take an arbitrary element $[x]_G = [(x_1, x_2, \ldots, x_n)]_G$ from $\mathsf{SP}^n_G A$. Then
$$\mathsf{SP}^n_G f([x]_G) = [(f(x_1), f(x_2), \ldots, f(x_n))]_G \in \mathsf{SP}^n_G Y.$$
Suppose W is a neighborhood of the orbit $\mathsf{SP}^n_G f([x]_G)$ in $\mathsf{SP}^n_G Y$. By definition of the quotient mapping there exist neighborhoods $V_1, V_2, \ldots, V_n$ of the points $f(x_1), f(x_2), \ldots, f(x_n)$ such that $[V_1 \times V_2 \times \ldots \times V_n]_G \subset W$. In this case we have $x_1, x_2, \ldots, x_n \in M$. Since $M \subset X$ and $|M| \leq \kappa$, we have that $f \upharpoonright M : M \to Y$ is continuous. By continuity of f on M, there exist neighborhoods $U_1, U_2, \ldots, U_n$ of the points $x_1, x_2, \ldots, x_n$ satisfying $f(U_j) \subset V_j$ for all $j = 1, 2, \ldots, n$. Then
$$\mathsf{SP}^n_G f[U_1 \times U_2 \times \ldots \times U_n]_G = [f(U_1) \times f(U_2) \times, \ldots, \times f(U_n)]_G \subset W.$$
It means that the restriction $\mathsf{SP}^n_G f \upharpoonright \mathsf{SP}^n_G A$ is continuous at the point $[x]_G$. As $\mathsf{SP}^n_G A$ and $[x]_G$ were arbitrary the theorem is proved.

Theorem 3.2 *For every topological space X we have*
$$t_s\left((\pi^s_{n,G})^{\leftarrow}([x]_G), X^n\right) \leq t_s\left([x]_G, \mathsf{SP}^n_G X\right).$$

Proof. Let $t_s\left([x]_G, \mathsf{SP}^n_G X\right) = \kappa$ and $C \subset X^n$ satisfying $\left(\pi^s_{n,G}\right)^{\leftarrow}([x]_G) \subset \overline{C} \setminus C$. Then we have $[x]_G \in \overline{\mathsf{SP}^n_G C} \setminus \mathsf{SP}^n_G C$. It means that there exists a family $\gamma' \subset \mathsf{SP}^n_G C$ such that $|\gamma'| \leq \kappa$ and $[x]_G \in \overline{\bigcup \gamma'} \setminus \bigcup \overline{\gamma'}$. For every $\mathsf{SP}^n_G S \in \gamma'$ we can choose a set $S \subset C \subset X^n$ such that $\pi^s_{n,G}(S) = \mathsf{SP}^n_G S$. Let $\gamma = \{S : \ \mathsf{SP}^n_G S \in \gamma'\}$ be a family so obtained. It is clear that $|\gamma| \leq \kappa$ and $\left(\pi^s_{n,G}\right)^{\leftarrow}([x]_G) \bigcap (\bigcup \overline{\gamma}) = \emptyset$ and by the closedness of the mapping $\pi^s_{n,G}$ we have $\left(\pi^s_{n,G}\right)^{\leftarrow}([x]_G) \bigcap \left(\overline{\bigcup \gamma}\right) \neq \emptyset$. It means that $t_s\left((\pi^s_{n,G})^{\leftarrow}([x]_G), X^n\right) \leq \kappa$. Theorem 3.2 is proved.

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Theorem 3.3 *If X is a regular space then $t_s(X^n) = t_s(\mathsf{SP}^n_G X)$.*

Proof. Let $\kappa = t_s(X^n)$, $C \subset \mathsf{SP}^n_G X$ and $[y]_G \in \overline{C} \setminus C$. By virtue of the closedness of $\pi^s_{n,G}$, $(\pi^s_{n,G})^{\leftarrow}([y]_G) \cap \overline{(\pi^s_{n,G})^{\leftarrow}(C)} \neq \emptyset$. Let x be an isolated point of $(\pi^s_{n,G})^{\leftarrow}([y]_G) \cap \overline{(\pi^s_{n,G})^{\leftarrow}(C)}$. Clearly, $x \in \overline{(\pi^s_{n,G})^{\leftarrow}(C)} \setminus (\pi^s_{n,G})^{\leftarrow}(C)$. Since $t_s(X^n) = \kappa$, there exists a family $\gamma \subset (\pi^s_{n,G})^{\leftarrow}(C)$ such that $|\gamma| = \kappa$ and $x \in \overline{\bigcup\gamma} \setminus \bigcup\overline{\gamma}$. The set $\left\{(\pi^s_{n,G})^{\leftarrow}([y]_G) \bigcap \left(\overline{\bigcup\gamma}\right)\right\} \setminus \{x\}$ is closed and discrete in $(\pi^s_{n,G})^{\leftarrow}([y]_G)$ and hence X^n. By the regularity of X there exists a closed neighbourhood U of x such that $U \bigcap \left\{\left\{(\pi^s_{n,G})^{\leftarrow}([y]_G) \bigcap \left(\overline{\bigcup\gamma}\right)\right\} \setminus \{x\}\right\} = \emptyset$. Let $\gamma' = \{B \bigcap U : U \in \gamma\}$. It is clear that $x \in \overline{\bigcup\gamma'}$ and $(\pi^s_{n,G})^{\leftarrow}([y]_G) \bigcap (\bigcup\overline{\gamma'}) = \emptyset$. Let $\gamma'' = \left\{\pi^s_{n,G}(B) = \mathsf{SP}^n_G B : B \in \gamma'\right\}$. By the closedness of $\pi^s_{n,G}$ we have $[y]_G \notin \bigcup\overline{\gamma''}$ but clearly $[y]_G \in \overline{\bigcup\gamma''}$ and $|\gamma''| = \kappa$. It means that $t_s(\mathsf{SP}^n_G X) = \kappa$. Theorem 3.3 is proved.

Proposition 3.1 *For any topological space X we have $T(\mathsf{SP}^n_G X) \leq T(X^n)$.*

Proof. Assume that $T(X^n) = \kappa$. It means that for every increasing sequence $\{F_\alpha\}_{\alpha\in\Lambda}$ of closed subsets of X^n with $cf(\Lambda) > \kappa$ we have that $\bigcup_{\alpha\in\Lambda} F_\alpha$ is closed. Since the quotient mapping $\pi^s_{n,G} : X^n \to \mathsf{SP}^n_G X$ is closed onto mapping, it follows immediately that $\{\mathsf{SP}^n_G(F_\alpha)\}_{\alpha\in\Lambda}$ is an increasing sequence of closed subsets of $\mathsf{SP}^n_G X$ and $\bigcup_{\alpha\in\Lambda} \mathsf{SP}^n_G(F_\alpha)$ is closed. It means that $T(\mathsf{SP}^n_G X) \leq \kappa$. Proposition 3.1 is proved.

Theorem 3.4 *If X is a regular space, then $T(\mathsf{SP}^n_G X) = T(X^n)$.*

Proof. According to Proposition 3.1, it suffices to show the following equality: $T(\mathsf{SP}^n_G X) \geq T(X^n)$.

Assume that $T(\mathsf{SP}^n_G X) = \kappa$ and $\{F_\alpha\}_{\alpha\in\Lambda}$ is an increasing sequence of closed subsets of X^n such that $cf(\Lambda) > \kappa$. Put $F = \bigcup_{\alpha\in\Lambda} F_\alpha$ and suppose that there exists a point $x \in \overline{F} \setminus F$.

Let $F'_\alpha = (\pi^s_{n,G})^{\leftarrow}([x]_G) \bigcap F_\alpha$ for every $\alpha \in \Lambda$; the family $\{F'_\alpha\}_{\alpha\in\Lambda}$ is an increasing sequence of closed subsets of $(\pi^s_{n,G})^{\leftarrow}([x]_G)$. Since $(\pi^s_{n,G})^{\leftarrow}([x]_G)$ is finite we have that the set $\bigcup_{\alpha\in\Lambda} F'_\alpha = F \bigcap (\pi^s_{n,G})^{\leftarrow}([x]_G)$ is closed in X^n.

By regularity of X (and hence X^n), there exist two disjoint open sets U and V in X^n such that $x \in U$ and $F \bigcap (\pi^s_{n,G})^{\leftarrow}([x]_G) \subset V$.

Let $F''_\alpha = F_\alpha \setminus V$ for every $\alpha \in \Lambda$. It is clear that $x \in \overline{\bigcup_{\alpha\in\Lambda} F''_\alpha}$ and $(\pi^s_{n,G})^{\leftarrow}([x]_G) \bigcap (\bigcup_{\alpha\in\Lambda} F''_\alpha) = \emptyset$. The family $\{\mathsf{SP}^n_G(F''_\alpha)\}_{\alpha\in\Lambda}$ is an increasing sequence of closed subsets of $\mathsf{SP}^n_G X$. Since $T(\mathsf{SP}^n_G X) = \kappa$ the set $\bigcup_{\alpha\in\Lambda} \mathsf{SP}^n_G(F''_\alpha) = \bigcup_{\alpha\in\Lambda} \pi^s_{n,G}(F''_\alpha) = \pi^s_{n,G}(\bigcup_{\alpha\in\Lambda} F''_\alpha)$ must be closed and by the continuity of $\pi^s_{n,G}$,

$$\pi^s_{n,G}(x) = [x]_G \in \overline{\pi^s_{n,G}(\bigcup_{\alpha\in\Lambda} F''_\alpha)} = \pi^s_{n,G}(\bigcup_{\alpha\in\Lambda} F''_\alpha).$$

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But it is impossible because

$$(\pi^s_{n,G})^{\leftarrow}([x]_G)\bigcap(\bigcup_{\alpha\in\Lambda}F''_\alpha)=\emptyset.$$

This proves that F is closed and so $T(X^n)\le\kappa$. Theorem 3.4 is proved.

If in the above theorem the space X is Hausdorff, then the mapping $\pi^s_{n,G}$ is perfect and the assumption about the regularity of X could be weakened.

Corollary 3.1 *If X is Hausdorff and $T(\mathsf{SP}^{\mathsf{n}}_{\mathsf{G}}X)\le\kappa$, then $T(X^n)\le\kappa$.*

Corollary 3.2 *If X is a locally compact Hausdorff space, then $T(\mathsf{SP}^{\mathsf{n}}_{\mathsf{G}}X)\le T(X^n)\le T(X)$.*

Proposition 3.2 *Let X be any topological space. Then:*

(*a*) $t_o(\mathsf{SP}^{\mathsf{n}}_{\mathsf{G}}X)\le t_o(X^n)$;

(*b*) $t_m(\mathsf{SP}^{\mathsf{n}}_{\mathsf{G}}X)\le t_m(X^n)$.

Proof. Let f be a κ-continuous (resp. strictly κ-continuous) real-valued function on $\mathsf{SP}^{\mathsf{n}}_{\mathsf{G}}X$ and $t_o(X^n)=\kappa$ (resp. $t_m(X^n)=\kappa$). Then the composition $g=f\circ\pi^s_{n,G}$ is κ-continuous (resp. strictly κ-continuous) real-valued function on X^n. We have in both cases that g is continuous. By continuity of $\pi^s_{n,G}$ and $g=f\circ\pi^s_{n,G}$, it follows that f is continuous and hence $t_o(\mathsf{SP}^{\mathsf{n}}_{\mathsf{G}}X)\le\kappa=t_o(X^n)$ (resp. $t_m(\mathsf{SP}^{\mathsf{n}}_{\mathsf{G}}X)\le\kappa=t_m(X^n)$). Proposition 3.2 is proved.

From Proposition 3.2 and Theorems 2.1 and 2.2 we have the following statement.

Corollary 3.3 *Let X be a locally compact space. Then:*

(*a*) $t_o(\mathsf{SP}^{\mathsf{n}}_{\mathsf{G}}X)\le t_o(X^n)\le t_o(X)$;

(*b*) $t_m(\mathsf{SP}^{\mathsf{n}}_{\mathsf{G}}X)\le t_m(X^n)\le t_m(X)$.

It follows immediately from Theorem 2.3 that:

Corollary 3.4 *For every first countable space X, $t_m(\mathsf{SP}^{\mathsf{n}}_{\mathsf{G}}X)\le t_m(X^n)\le t_m(X)$.*

Let us now recall some definitions.

The *weak tightness* $t_c(X)$ of a space X is the smallest (infinite) cardinal κ such that the following condition is fulfilled:

if a set $A\subset X$ is not closed in X, then there are a point $x\in\overline{A}\setminus A$, a set $B\subset A$, and a set $C\subset X$ for which: $x\in\overline{B}$, $B\subset\overline{C}$, and $|C|\le\kappa$.

We say that $A\subset X$ is a *set of type* G_κ in X if there is a family γ of open sets in X such that $A=\cap\gamma$ and $|\gamma|\le\kappa$. A set $A\subset X$ is called κ*-placed* in X if for each point $x\in X\setminus A$ there is a set P of type G_κ in X such that $x\in P\subset X\setminus A$.

Put $q(X)=\min\{\kappa\ge\omega:\text{is }\kappa-\text{placed in }\beta X\}$; $q(X)$ is called the *Hewitt-Nachbin number* of X. We say that X is a Q_κ*−space* if $q(X)\le\kappa$.

Proposition 3.3 *Let X be a compact space. Then $q(\mathsf{SP}^{\mathsf{n}}_{\mathsf{G}}X) \leq d(X)$.*

Proof. It is known (see [2]) that for any Tychonoff space X the following relations are holds:

$$q(X) = t_m(C_p(X)) = t_o(C_p(X)), \quad t_o(X) \leq t_c(X) \leq d(X).$$

So we have

$$q(\mathsf{SP}^{\mathsf{n}}_{\mathsf{G}}X) = t_m(C_p(\mathsf{SP}^{\mathsf{n}}_{\mathsf{G}}X)) = t_o(C_p(\mathsf{SP}^{\mathsf{n}}_{\mathsf{G}}X)) \leq t_c(C_p(\mathsf{SP}^{\mathsf{n}}_{\mathsf{G}}X))$$
$$\leq d(C_p(\mathsf{SP}^{\mathsf{n}}_{\mathsf{G}}X)) \leq d(\mathsf{SP}^{\mathsf{n}}_{\mathsf{G}}X) = d(X).$$

Proposition 3.3 is proved.

Corollary 3.5 *Let X be a compact and separable space. Then $q(\mathsf{SP}^{\mathsf{n}}_{\mathsf{G}}X) \leq \omega$, i.e. the space $\mathsf{SP}^{\mathsf{n}}_{\mathsf{G}}X$ is a Q_ω-space.*

4 Conclusion

An important question in topology is the following. Let F be a functor and $\mathcal{P}$ a topological property. If a space X has the property $\mathcal{P}$, whether $F(X)$ has the same or some other property. This paper is devoted to a study of preservation of tightness-type cardinal invariants (T-tightness, set-tightness, functional tightness, minitightness, weak tightness) of a space X (and its n-th power X^n) under influence of the functor SP^{n} of n-permutation degree. It is shown that for certain classes of spaces some of these cardinal functions are equal for X^n and $\mathsf{SP}^{\mathsf{n}}X$. We hope that these results may be a first step in investigation of similar problems for other known functors.

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