Article

Morphological Simulation of Phyllotactic Spiral-Ring Patterns by the Galactic Spiral Equations from the ROTASE Model

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ABSTRACT

This short paper demonstrates that the galactic spiral equations developed from ROTASE model can be used to morphologically simulate the phyllotactic spiral-ring patterns of plants. It is also noticed during this project that although the numbers in the Fibonacci sequence dominate the phyllotactic numbers, there are still significant phyllotactic numbers are not in the Fibonacci sequence, however, most of those numbers are well fit into a Fibonacci sibling sequence starting with numbers 0 and 2. Many plants show the phyllotactic number pairs of 10-16, 16-26 and 26-42 from the Fibonacci sibling sequence, which is the second dominated sequence in the phyllotaxis.

Keywords: Phyllotactic spiral-ring pattern; Galactic spiral equations; Simulation; Fibonacci sibling sequence

1. Introduction

The spiral as one of the fundamental morphologies of the universe has been well adopted by the nature, specially by the plants, the leaves, florets, and seeds are well organized in repeated patterns in the forms of spirals. Phyllotaxis is the study about arrangement of the leaves, florets, and seeds on the stems and flower buds [1-5]. One of the important research aspects is to simulate the phyllotactic spiral patterns of the plants to understand how such patterns are developed. Shipman and Newell indicate that the principle behind this phenomenon is the maximum efficiency with minimum cost (energy-minimizing bucking pattern) [6], it is the same as for a chemical reaction in which the reaction pass with minimum energy always dominates among all possible passes, and the same as for a commercial business in human society to maximize the profit with minimum cost, therefore, it is an innate property of the every aspect of the nature. Many mathematical spiral equations have been invented in the history of science, like the Archimedean spiral, Euler spiral, Fermat's spiral, hyperbolic spiral, logarithmic spiral, Fibonacci spiral, etc. There are many models have been proposed to simulate phyllotactic patterns of plants in the past. Vogel developed a spiral model in 1979 to simulate the arrangement of sunflower seed head [7], such model is currently the common model to generate sunflower seed head. Fowler, et al. used a cylindrical model to simulate the spiral patterns in pinecones, fir cones and pineapples [8]. Cong et. al. developed repulsion pressure model to simulate spiral phyllotactic patterns of plants [9]. However, a special spiral-ring pattern is adopted by many different plants represented by the spiral aloe (also called Aloe Polyphylla), cacti and flowers as shown in Figure 2. This type of spirals start at the center, gradually extends outwardly, and reaches its limit to form a round boundary edge showing a phyllotactic spring-ring pattern. Simulation of the phyllotactic spring-ring patterns seems a big challenge with those spiral equations mentioned above because none of those spiral equations can produce spiral-ring patterns. However, the author developed a set of galactic spiral equations in recent years from the proposed ROTASE model for the formation of spiral arms of barred galaxies [10,11]. The

ROTASE model is the short name of ROtating Two Arm Sprinkler Emission model. It has been demonstrated that the galaxies with various spiral patterns including spiral-ring patterns and the earth natural spirals can be well simulated with the new galactic spiral equations [10, 12]. This paper will demonstrate that the new galactic spiral equations can be used to simulate the phyllotactic spiral-ring patterns also.

2. The galactic spiral equations from ROTASE model

The following are the primary differential galactic spiral equations derived from the ROTASE model, please refer to the reference [10] for the detail of derivation.

$$\begin{cases} dx = R_b * \frac{y}{\sqrt{x^2+y^2}}\, d\theta \\ dy = R_b * \left(\rho(\theta) - \frac{x}{\sqrt{x^2+y^2}}\right) d\theta \end{cases} \tag{1}$$

The equations (1) can be solved in a polar coordinate system for three different cases, $\rho > 1$, $\rho = 1$ and $\rho < 1$, respectively. The R_b is the half length of the galactic bar; the θ is the galactic bar rotation angle used as time counting in the ROTASE model, and the parameter ρ is the ratio of the X-matter emission velocity over the flat rotation velocity of the galactic disc, the ρ can change with time (θ) in any format. For general application of the equations (1) for the simulation of spirals, the ρ can be treated as a normal parameter which is function of θ. The galactic spiral pattern will be decided by the behavior of the ρ. A mini computer program can be wriiten to calculate the x and y, and the calculated x and y must be rotated counterclockwise by the following equation for final spiral plotting, the counterclockwise rotation is well explained in the reference [11]:

$$\begin{cases} x'(\theta) = x(\theta) * \cos(-\theta) + y(\theta) * \sin(-\theta) \\ y'(\theta) = -x(\theta) * \sin(-\theta) + y(\theta) * \cos(-\theta) \end{cases} \tag{2}$$

Plotting the x'(θ), y'(θ) will produce the calculated spiral, the x' may be changed to -x' depending on the rotation direction of the spirals. If the parameter ρ is less than 1, the spiral will be a spiral-ring pattern with the radius of the ring defined by the equation (3) as:

$$r = \frac{R_b}{1-\rho} \tag{3}$$

The spiral-ring pattern can only be produced by the new galactic spiral equations at the moment compared to all other available spiral equations. The calculation can be carried out by setting the rotation angle θ as the following:

$$\theta_k = k * \delta \tag{4}$$

Where k is the natural number (1,2,3,4,5….) and the δ is the selected angle based on the pattern to be simulated, a good example is the simulation of Sunflower seed head by Vogel with δ equal to golden angle (137.5°) [7].

3. Simulation of phyllotactic spiral-ring patterns of plants

The graphic impression of the output from this calculation is very sensitive to the value of δ as shown in Figure 1, a small change of δ can significantly change the morphological patterns. Figure 1A is the plot calculated with ϱ = 2.5, it shows 5 straight lines from center and separated by 72°, this is because 360°/5 = 72°. However, when the value of δ

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changes only 0.1° less than 72° and keeping the ϱ unchanged, the visual graphics shows a nice 5-spiral pattern as shown in Figure 5B. The 5-spiral pattern becomes much tighter with the δ equal to 71.3° and keeping the ϱ unchanged as shown in Figure 1C.

The value of the parameter ϱ has significantly impact on the visual graphic output also. Figure 1D shows the result with ϱ = 0.985 and δ = 72°, the 5 lines become spiral lines compared to Figure 1A. The graphics becomes a spiral-ring pattern compared to Figure 1B when the δ changes only 0.1° with radius of the ring defined by equation (3). The spiral-ring pattern becomes tighter with δ = 71.3°. Therefore, the unique spiral-ring feature of the galactic spiral equations can be used to simulate the phyllotactic spiral-ring patterns.

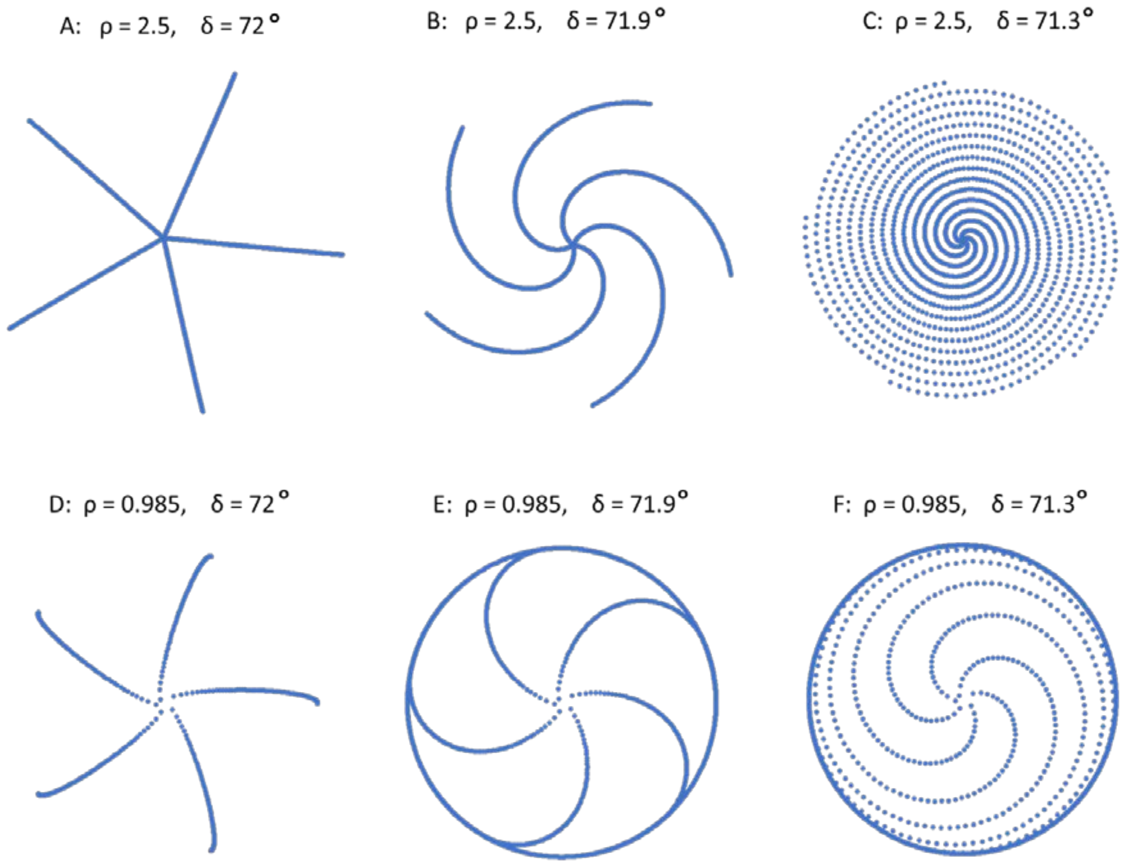

Figure 1, calculated spiral patterns change with ϱ and δ

Figure 2 shows the simulations for 6 different spiral plants and spiral flowers, respectively. The parameters for the simulations are listed in the table 1. One can see that the simulations are excellent, and the galactic spiral equations are well suitable for the morphological simulation of this type phyllotactic spiral patterns.

Table 1, the simulation parameters for Figure 2

	Fig. 2a	Fig. 2b	Fig. 2c	Fig. 2d	Fig. 2e	Fig. 2f

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Name	Aloe polyphylla	Mammillaria Cactus	Pinecone	Aeonium Tabuliforme	Camellia Flower	Camellia Flower
ρ	0.98	0.976	0.99	0.988	0.991	0.9969
θ	71.1°	Red: 35.6° Yellow: 22.4°	Red: 44.85°, Yellow: 27.657°	44.88°	71.82°	71.9°

For this specific application of the galactic spiral equations, the parameter ρ could be treated as a growing parameter of the primordium, the larger the ρ is, the bigger of the ring will be; the angular position of the new-born primordium θ_k of the plants should be decided by the MaxMin-principle with which the new-born primordium has the largest minimum distance with other pre-existing primordia [13-15]. It would be very interesting to see if a theoretical model for the primordium growth can be developed to match or equivalent to the galactic spiral equations (1).

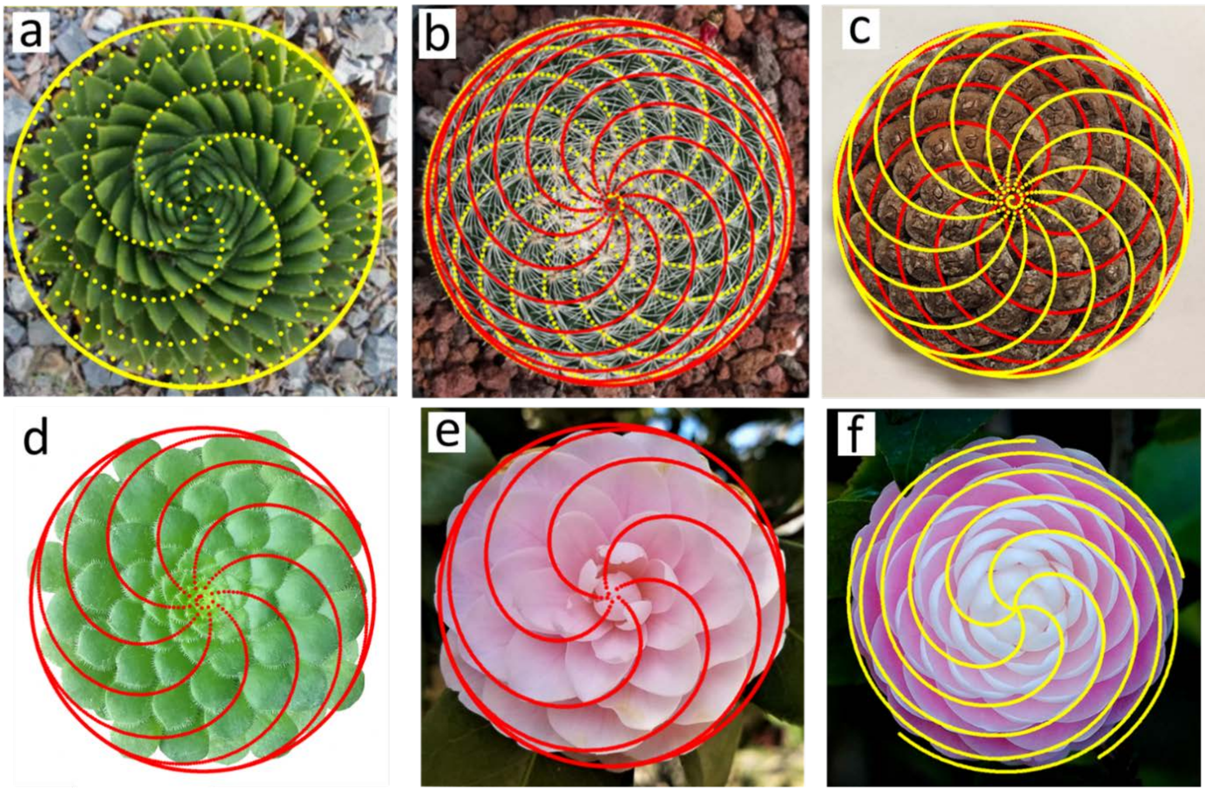

Figure 2, the simulations of 6 plants with spiral-ring patterns by the ROTASE galactic spiral equations.

4. Fibonacci sibling sequence for the phyllotaxis

The number of phyllotactic spirals of plants varies significantly among different species, the phyllotactic spirals form a distinctive class of patterns in nature, the occurrence of such spiral number seems following certain pattern, not random, most of those numbers follow the Fibonacci sequence (0,1,1,2,3,5,8,13,21,34,55,89, 144 ….). However, the author noticed that although the numbers of Fibonacci sequence dominate in the plant phyllotaxis, there are still significant numbers of plants with phyllotactic numbers not belonged to the Fibonacci sequence, plants with phyllotactic numbers 2,4, 6 and 10 are very common, and with numbers 16, 26 and 42 are not rare. The numbers 4,6,10,16,26, and 42 are not in the Fibonacci sequence, but they seem to form a pattern also. Prabowo et. al. proposed Fibonacci-like sequences starting with numbers 2 and 2 as well as starting with numbers 2 and 4 [16]. However, those two Fibonacci-like sequences are included in the Fibonacci sibling sequence starting with numbers 0 and 2:

0, 2, 2, 4, 6, 10, 16, 26, 42, 66….. (5)

Therefore, the phyllotactic numbers 2,4,6,10,16,26 and 42 perfectly match the sequence (5). Figure 2b shows that the mammillaria cactus has the pair of adjacent 10-16 spirals formed by spine clusters. Figure 3 shows 4 additional plants with the numbers of phyllotactic spirals are among the sequence (5) and paired with adjacent numbers of the sequence 10-16, 16-26, 10-16-26 and 26-42; the Figure 3c show 3 sets of different spirals of spine clusters with 3 adjacent numbers in the Fibonacci sibling sequence (5); those paired spiral numbers can be commonly found in succulent/cactus plants. Most likely, the numbers in the sequence (5) are the second dominated phyllotactic numbers in the nature.

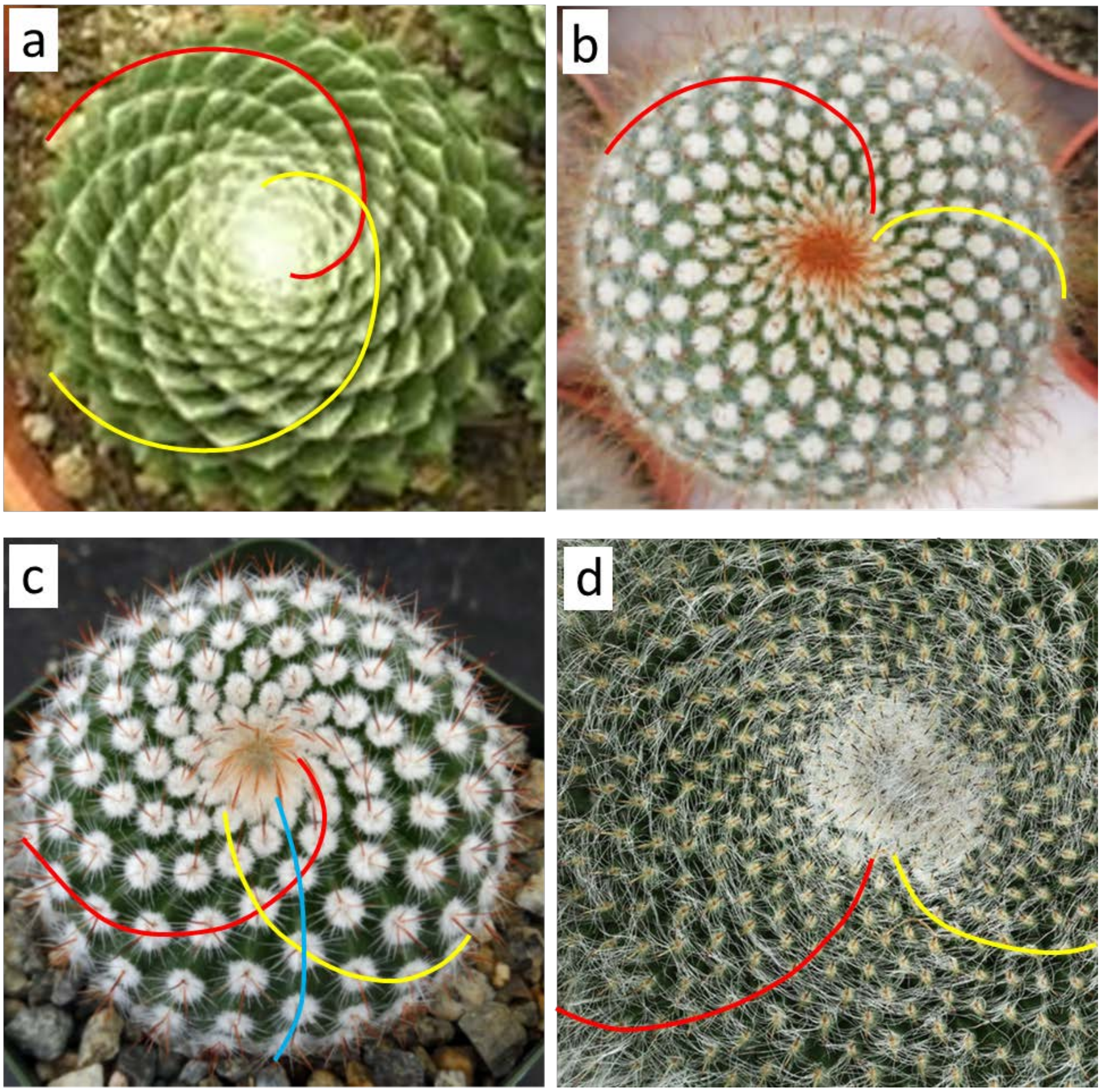

Figure 3, plants with phyllotactic numbers matching the sequence (5)
a: 10-16 spirals, b: 16-26 spirals, c: 10-16-26 spirals, d: 26-42 spirals

4. Discussion

There are still many phyllotactic numbers not belonged to the Fibonacci sequence and the Fibonacci sibling sequence. Another Fibonacci sibling sequence can be proposed as the following with starting numbers 0 and 3:

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$$0, 3, 3, 6, 9, 15, 24, 39\ldots \quad (6)$$

The phyllotactic number 3, 6 and 9 are common in the nature, and the phyllotactic numbers 15 and 24 can be found also. The pairs of adjacent numbers 6-9, 9-15, 15-24 and 24-39 may exist in phyllotaxis of the nature.

5. Conclusion

The results of the simulation demonstrate that the phyllotactic spiral-ring patterns of plants can be nicely simulated by the galactic spiral equations from ROTASE model. Many non-Fibonacci phyllotactic numbers can be grouped into the Fibonacci sibling sequence starting with number 0 and 2.

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