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Cosmological scales, the universe as a hologram

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Abstract:
Here we deduce the scales of a spherically symmetric, closed system subject to internal repulsive vacuum pressure. Due to periodic boundary conditions of virtual particle loops, it is shown that Ramanujan summation must be physical for the virtual particle case, as to avoid numerical results of either null, very large or infinity. It is thereafter shown that this closed system must be holographic in nature, with mass-energy inversely proportional to the Planck area, up to a yet to be determined geometrical proportionality constant. The validity of this result is strengthened when it is shown that one may obtain the same answer using different methods, either thermodynamic laws or a force balance equation. Implications of this small study is not yet fully understood and is hopefully investigated in subsequent letters.

Keywords: Vacuum theory; Bekenstein-Hawking theory; holography.

1 Introduction

We give a brief outline of the most common mathematical approaches for the concept of vacuum energy and show that these fall in the category of giving as output either a very large number, equation (5) ZERO, equation (7) or divergence, equation (8). Motivated by this we deduce a simpler way of regularizing – that is summing – the quantum vacuum fluctuations by the use of PERIODIC BOUNDARY CONDITIONS of virtual particle loops, and henceforth that Ramanujan summation [1] must be physical for the virtual particle case. Thereafter this result is used when mathematically modeling a closed, spherically symmetric region reasoned to be closely related to or identical to the universe as observed [2]. Finally, the Law of Tully-Fisher [3] is derived up to a geometrical proportionality constant of order unity, and the connection to the singularity theorems is mentioned briefly.

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Using this path, it is reasoned that the so-called cosmological constant Λ must be proportional to the quantity $\hbar^2c^2$, where $\hbar$ is Planck's reduced constant and c is the speed of light in vacuum. This semi-classical approach, equation (12) is coupled to gravity, equation (22) where G is Newton's gravitational constant. In this coupling the ideal stress-energy tensor, equation (21) is used since the vacuum energy can only interact gravitationally and not through other elementary interactions. Finally, an equation of state, or pressure versus energy density relation of $w = -1$ is used for the stress-energy. This is the only self-consistent equation of state for the system considered because the four-velocity u_α of the virtual particles cannot be accurately determined, equation (1). Hence their stress-energy, $T_{\alpha\beta}$ equation (21) must be independent of u_α in order to avoid self-interactions.

In addition, $w = -1$ is the only equation of state of the system which yields independence of the gravitational metric $g_{\alpha\beta}(x)$, equation (22). Therefore, the vacuum energy becomes independent of the metric as is required, and so we avoid any gravitational self-interactions of the system.

This result, equation (23) is used in section 4 to model the observable universe [2] as an idealized black hole. As a consequence, the mass-energy of this system must be inversely proportional to the Planck area, equation (35).

Finally, the Law of Tully-Fisher [3] is derived, section 5. By dimensional analysis the REPULSIVE vacuum acceleration a_c, equation (36) must be proportional to the quantity $\hbar c^3$. When this effect is equated to gravity, equation (37) and solved, the critical centripetal velocity v_c to the fourth power must be proportional to the mass M contained, equation (38). With a proportionality constant of order $\sim \hbar G c^3$. This is similar to the Law of Tully-Fisher. [3]

2 Theoretical review

There have been some previous mathematical models for calculating the vacuum energy density, the usual starting point is the energy-time Heisenberg uncertainty principle

$$\Delta E \Delta t \geq \frac{\hbar}{2} \tag{1}$$

Which implies that the minimum uncertainty that we are interested in is related to the time interval Δt as

$$\Delta E = \frac{\hbar}{2\Delta t} \tag{2}$$

And here the first obvious problem arises, as $\Delta t \to 0$ the vacuum energy ΔE would diverge. This is usually dealt with in a heuristic way by assuming that a theory of quantum gravity would have a shortest possible time scale and length scale given by the Planck units

$$t_p = \sqrt{\frac{\hbar G}{c^5}} \tag{3}$$

$$l_p = ct_p \tag{4}$$

And therefore – using this standard heuristic approach – one would achieve a dominant energy density of the vacuum proportional to

$$\rho_v \propto \frac{1}{l_p^3}\frac{\hbar}{t_p} = \frac{c^7}{\hbar G^2} \tag{5}$$

However, this naive approach cannot be correct, since the value for the energy density of the vacuum, equation (5) is roughly 120 orders of magnitude off from the empirical order of magnitude [4]. Several mathematical methods have been tried to circumvent this, some of which are shown below. However,

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they all yield either a value of ZERO, equation (7) or infinity, hence a cut-off scale is introduced, equation (8). One of the mathematical approaches, also known as ZETA REGULARIZATION⋆, is as follows:

Let the vacuum energy depend on angular frequency $\omega(\mathbf{k})$ and a standard dispersion relation as

$$E = \frac{\hbar}{2}\sum_{r,\mathbf{k}} \omega(\mathbf{k}) \propto \frac{\hbar c}{2}\int_{-\infty}^{\infty} d^3k\|\mathbf{k}\| \tag{6}$$

Where the sum over r, the polarization sum gives a factor of three and is therefore irrelevant in this method. $\|\mathbf{k}\|$ is the norm of the momentum vector $\mathbf{k}$ in three dimensions.

A ZETA REGULARIZATION⋆ of this integral using the properties [5] of the GAMMA FUNCTION $\Gamma(z)$, which is the analytical extension of the factorial [6] and the RIEMANN ZETA FUNCTION $\zeta(s)$ yields the following result for the vacuum energy E.

$$E \propto \int_{-\infty}^{\infty} d^3k\|\mathbf{k}\| = 4\pi\int_0^{\infty} dkk^3 = 4\pi(\int_0^a dkk^3 + \int_a^{\infty} dkk^3) = 4\pi(\frac{a^4}{4} + \frac{1}{\Gamma(\gamma)}\int_0^{\infty}\frac{dt}{t}t^{\gamma}\int_a^{\infty} dke^{-tk})$$
$$= 4\pi(\frac{a^4}{4} + \frac{1}{\Gamma(\gamma)}\int_0^{\infty} dtt^{\gamma-2}e^{-at}) = 4\pi(\frac{a^4}{4} + \frac{a^{1-\gamma}\Gamma(\gamma-1)}{\Gamma(\gamma)}) = 4\pi(\frac{a^4}{4} + \frac{a^{1-\gamma}}{\gamma-1}) = 0 \tag{7}$$

Since $\gamma = -3$ in this case. Hence $E = 0$ using this technique for CONTINUOUS values of k. Here the integral representation [6] of $\zeta(s)$ is used but the geometric series is replaced with an integral.

Other approaches tend to have ugly divergences of either degree four [7] or, sometimes degree two. Meaning that the output of the integration(s) tend to infinity as a polynomial of either degree four or two respectively. In particular the Euclidean[1] Feynman loop propagator [8, 9]

$$i\Delta_F(0) = \int_0^{\Lambda} \frac{dk_E k_E^3}{k_E^2 + m^2} \sim \Lambda^2 \to \infty \tag{8}$$

Tends to infinity with a quadratic degree of divergence, therefore a cut-off scale Λ is introduced at energies where current theories are thought to break down.

In light of the current approaches, it is unclear to the author why one cannot instead embrace the divergences rather than try to AD-HOC get rid of them. There is namely a mathematical technique, known as Ramanujan summation which assigns finite values to divergent series [1]. We will therefore show that Ramanujan summation must be physical for the case of virtual particles. The motivation for this approach is, for example that the technique used for the Casimir effect in quantum field theory⋆[2] [10, 11], consisting of subtracting away infinities yields the same result(s) AS IF one had treated the Ramanujan summation as physical to begin with. Hence without the need of AD-HOC introducing a new term for convergence (what is called a REGULATOR).

[1]Meaning regular spacetime without gravity.
[2]Which is the ZETA REGULARIZATION discussed above.

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3 A fun approach to the Heisenberg vacuum

In this section we derive the vacuum energy associated with the fluctuations of virtual particles, which we will use later.

The scalar quantum harmonic field $\phi(\mathbf{x})$ of a virtual particle closed[3] loop $\mathbf{C}$ must have the following physical property

$$\phi(\mathbf{x}_i) = \phi(\mathbf{x}_f) = \phi(\mathbf{x}_i + \oint_{\mathbf{C}} d\mathbf{C}) = \Delta\nu \cdot \phi(\mathbf{x}_i) \tag{9}$$

Where $\Delta\nu$ is the phase difference between the initial and the final state, $\Delta\nu = e^{i(\nu_f - \nu_i)}$. But since the scalar fields $\phi(\mathbf{x}_i)$, $\phi(\mathbf{x}_f)$ must, by definition be identical it is clear that $\Delta\nu = 1 = e^{2\pi i n}$, for all $n \in \mathbb{N}$ arbitrary number of modes.

Equivalently, we know that for an isotropic system of radius L must have a phase factor $\Delta\nu$ as

$$\Delta\nu = e^{ik_L L} \tag{10}$$

And therefore $k_L L = 2\pi n$ for all $n \in \mathbb{N}$. Thus the quantization in the four-momenta $k_\alpha = \frac{2\pi n_\alpha}{L}$ originates from the PERIODIC BOUNDARY CONDITION of the virtual particle loop. Hence the Ramanujan summation [1], or what is also referred to as the analytical continuation relevant here becomes of the form

$$\zeta(-1) = \sum_{n=0}^{\infty} n = -\frac{1}{12} \tag{11}$$

The Riemann zeta function evaluated at $s = -1$. Consider now the quantum vacuum expectation value E_0 as given by the Heisenberg uncertainty principle

$$E_0 = \langle 0| V(\phi) |0\rangle = \frac{1}{2}k^0 = \frac{1}{2}\sqrt{\eta_{\alpha\beta}k^\alpha k^\beta} \tag{12}$$

Here $V(\phi)$ is the quantum harmonic potential, $(\alpha,\ \beta)$ have values ranging from one to four and where $\eta_{\alpha\beta} = I_4$ is the 4-dimensional identity matrix. We treat the restmass m_R as the fifth reciprocal dimension $m_R = k^4$, because of its apparent validity when writing the dispersion relation as an inner product of the form of equation (12). All units are set as $\hbar = c = 1$. In this way we retrieve the standard dispersion relation through a more compact notation. If we sum over all virtual particle states and polarization states, we obtain.

$$\frac{1}{2}k^0 = \frac{1}{2}\frac{2\pi}{L}\sum_r \sum_{n=0}^{\infty} \sqrt{n^\alpha n_\alpha} \tag{13}$$

Where $k_\alpha = \frac{2\pi n_\alpha}{L}$ due to the PERIODIC BOUNDARY CONDITION of the virtual particle loop. The vacuum energy, equation (12) must transform under Lorentz transformation as

$$E_0 \to \gamma E_0 \tag{14}$$

Where γ is the Lorentz factor, that is for an isotropic spacelike[4] gravitational field g_{RR} we must have

$$\gamma = \sqrt{g_{RR}} \equiv \sqrt{g} \tag{15}$$

[3]Meaning that the start point and end point of the creation/annihilation process coincide, $\mathbf{x}_i = \mathbf{x}_f$.
[4]Since the values of (α, β) range from $(1,\ 4)$ thus does not include the timelike component, equation (12).

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But for consistency with equation (20) the vacuum energy, equations (12, 14) must be a Lagrange multiplier with respect to the metric $g_{\alpha\beta}$. This seeming inconsistency may be resolved by introducing the partition function $\mathbb{Z}[E_0]$ with respect to the radial gravitational field g, hence integrating out the gravitational field dependence as

$$\mathbb{Z}[E_0] = \int_0^\infty \delta g \cdot e^{-\sqrt{g}E_0} = 2(E_0)^{-2} \tag{16}$$

With an implicit unit lengthscale in the exponent. Calculating the vacuum energy E_0, equations (12, 13) for the isotropic case $\sqrt{n_\alpha n^\alpha} = n$ with a loop lengthscale L we arrive at

$$E_0 = \frac{1}{2}k^0 = \frac{1}{2}\frac{2\pi}{L}3\sum_{n=0}^{\infty} n < 0 \tag{17}$$

Where the polarization sum gives a factor of three. Thus the local energy can be negative. Letting the Lagrange multiplier Λ_L be equated to the inverse partition function of the vacuum energy $\mathbb{Z}(E_0)$, equation (16) we achieve an output as

$$\Lambda_L = \mathbb{Z}^{-1}(E_0) = \frac{1}{2}(E_0)^2 = \frac{1}{2}[\frac{3\pi}{L}\sum_{n=0}^{\infty} n]^2 = \frac{\pi^2}{32L^2} \tag{18}$$

Due to equation (11). Setting the radial scale[5] to $L = 1$ m and reinstating $\hbar c$ for each k^0, with dimensions $[\mathrm{m}] = [\mathrm{s}] = [\mathrm{J}]^{-1}$ finally yields the physical quantity[6]

$$\Lambda = \frac{\pi^2}{32}\hbar^2 c^2 \tag{19}$$

Where this proportionality is the key conceptual take-away from this letter, since this proportionality has an order of magnitude of $O(-52)$ m^{-2}, that is having 52 zeros in SI units, which is curiously similar to the value given in the Planck data [4]. The geometrical proportionality constant is however, most possibly erroneous. Not unlike most geometrical proportionality constants considered in this letter, though this is not of the most importance.

Now continuing by the use of general relativity as is required

$$G_{\alpha\beta} + \Lambda g_{\alpha\beta} = \kappa T_{\alpha\beta} \tag{20}$$

Where $\kappa = \frac{8\pi G}{c^4}$, $G_{\alpha\beta}$ is the Einstein tensor, $g_{\alpha\beta}$ the metric and $T_{\alpha\beta}$ the stress-energy. If we use the stress-energy of an ideal fluid

$$T_{\alpha\beta} = (\rho + p)u_\alpha u_\beta + \rho g_{\alpha\beta} \tag{21}$$

With a pressure versus density relation[7] $w = -1$ and together with the field equations in vacuum we achieve the standard relation

$$\Lambda = \kappa \rho_v \tag{22}$$

With Λ as in equation (19) and inverting

$$\rho_v = \frac{\pi}{256}\frac{\hbar^2 c^6}{G} \tag{23}$$

Hence the vacuum energy density within a unit cube region according to this model becomes of order $O(-9)$ J/m^3.

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[5] This defines the unit lengthscale at which we perform our measurements, that is an SI unit of measurement.

[6] It is worth mentioning that the same Ramanujan summation appears in the Feynman loop propagator, (8). Again by the periodic boundary condition $k_\alpha = \frac{2\pi n_\alpha}{L}$ of the virtual particle loop, which is itself described by the Feynman loop propagator.

[7] See section 1.

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4 A spherically symmetric, closed region

Firstly, we model a closed, spherically symmetric region in the simplest case as a charge less, rotation less, though spherically accelerating black hole, therefore subject to Unruh-Hawking radiation, equation (25), and holographic ($d = 2$) Bekenstein entropy [12], equation (26). Hence, the following equations must be valid

$$R = \frac{2GM}{c^2} \tag{24}$$

$$T = \frac{\hbar c^3}{8\pi GkM} \tag{25}$$

$$S = \pi k \frac{R^2}{l_p^2} \tag{26}$$

From equation (23) we have that the vacuum energy density within a unit cube region $L = 1$ m becomes, again possibly with a different geometrical proportionality constant

$$\rho_v = \frac{\pi}{256}\frac{\hbar^2 c^6}{G} \tag{27}$$

The first law of thermodynamics for the closed system being modeled is given by

$$dQ = dU + dW = TdS + pdV = 0 \tag{28}$$

This is essentially the law of energy conservation for the system considered. The total pressure contribution is given by various thermodynamic relations as

$$p = p_m + p_R + p_v \tag{29}$$

$$p_v = -\rho_v c^2 \tag{30}$$

$$p_R = \frac{1}{3}\rho_R c^2 \tag{31}$$

$$p_m = 0 \tag{32}$$

The radiation contribution must average out to zero over large scales. Solving this entire system of equations (24, 25, 26, 27, 28), with V a sphere of radius R, we cancel the minus sign [8] and arrive at

$$M = \frac{2\sqrt{2}}{\pi}\frac{c}{\hbar G} \tag{33}$$

$$R = \frac{4\sqrt{2}}{\pi}\frac{1}{\hbar c} \tag{34}$$

Where we remind ourselves that we have an intrinsic length scale set, equation (18) as $L = 1$ m. These expressions are of order $O(52)$ kg and $O(25)$ m. That is roughly of the empirical orders given [2] though slightly small. Although we would expect that the geometrical proportionality constants would have to be more accurately determined, though this is not of great importance as for the purpose of this letter. Anyways the ordinary mass-energy $E = Mc^2$ of this closed region is expected to be inversely proportional to the Planck area

$$E \propto \frac{L}{l_p^2} \tag{35}$$

Where $L = 1$ m, equation (23). We see here that we have the correct dimensions. The time dependence of these expressions, equations (33, 34, 35) – since the region must [9] be exponentially expanding with

[8] Hence avoiding any imaginary output.
[9] Friedmann equations, [13].

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time in its expansion – is expected to be buried within the temporal dependence of this implicit length scale $L = L(t) \propto a(t)$ the SCALE FACTOR, that is the temporal expression for the spatial expansion of the universe. Set to $L = 1$ m TODAY, similar to the scale factor which is set as $a(t_0) = a_0 = 1$ TODAY. Nevertheless, the fact that the entropy, equation (26), of a closed system always increases [10] **implies** that the closed, trapped region considered must be expanding with time AT ALL TIMES.

5 The Law of Tully-Fisher

Using equation (19) we may find a quantity with dimensions [m/s^2] by taking the root and multiplying by the speed of light squared c^2, again there may be a different geometrical proportionality constant, this is not thoroughly understood. We have divided by $\sqrt{3}$ [13]. Therefore, we obtain a small REPULSIVE[11] acceleration a_c

$$a_c = \frac{\pi \hbar c^3}{4\sqrt{6}} \tag{36}$$

Where we again have an implicit inverse length scale $L = 1$ m. Ignoring the effects of dark matter, it is clear that for a gravitational body bound by an ordinary-matter induced acceleration g is only stable if $g > a_c$ and would no longer be able to sustain itself gravitationally if $g < a_c$, the critical point is when

$$a_c = g \to G\frac{M}{R^2} = \frac{\pi \hbar c^3}{4\sqrt{6}} = \frac{v_c^2}{R} \tag{37}$$

Where R is the radius of a region with corresponding ordinary matter M. Solving our system, equation (37) we arrive at

$$v_c^4 = \frac{\pi \hbar G c^3}{4\sqrt{6}} M \tag{38}$$

This is similar to the ordinary-matter Tully-Fisher relation [3]. Here v_c is the critical centripetal velocity of a gravitational body with ordinary mass M.

We showed above, equations (33, 34), that we may arrive at the scales of a closed, holographic, that is the total degrees of freedom are proportional to the surface area, spherically symmetric region reasoned [2] to be either identical to or closely related to the observable universe TODAY as

$$M \propto \frac{c}{\hbar G} \tag{39}$$

$$R \propto \frac{1}{\hbar c} \tag{40}$$

Up to a yet to be determined accurate geometrical proportionality constant. Now we notice that if we set $v_c = c$ in equation (38) we arrive at the SAME ANSWER up to a proportionality constant, hence avoiding any inconsistencies. We may also use the Nariai limit [14][12] directly with Λ as in equation (19). Hence there are at least three independent or semi-independent ways of arriving at THE SAME answers for these scales up to a yet to be determined geometrical proportionality constant.

[10] By the second law of thermodynamics.

[11] Since $\zeta(-1) < 0$.

[12] Equation five in [14] with $\kappa_0 \approx 0$ since the overall curvature is very close to zero. [15]

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6 A little bit on the singularity theorems

According to the excellent Ph.D thesis of Stephen Hawking [16], if there is a closed, trapped surface and the following assumptions

$$\begin{aligned} &E \geq 0 \text{ for any observer with velocity} \\ &\text{There is a global time orientation} \\ &\text{The universal covering space has a non-compact Cauchy surface } \mathbb{H}^3 \end{aligned} \tag{41}$$

are true, then the space-time considered must contain a physical singularity. Which, for a classical, idealized black hole is located at its center, that is at $r = 0$. Or equivalently, that the equation of state, or pressure versus density relation $w = \frac{p}{\rho}$ must always be greater than minus one third.

$$w > -\frac{1}{3} \tag{42}$$

However, by equations (17, 30) it is clear that the first assumption, or equivalently equation (42) used for proving these singularity theorems do not hold in this case. And therefore that the closed, trapped surface considered in section 4 CAN indeed be free of physical singularities[13].

7 Discussions

It is shown that the model for the holographic case, that is where the total degrees of freedom of the system is proportional to the surface area of the region, is of order as given by the Hubble scale [2]. Though we would expect a different geometrical proportionality constant likely because they need to be more accurately determined. In particular for the dimensional case of $d = 3$, which one would naively expect for a volume dependent entropy, yields $R \propto O(32)$ m which is far off from the Hubble scale [2]. Therefore, any deviation from holography is considered highly unlikely.

The purpose of this letter is for it to work as a steppingstone for further work by people more knowledgeable than its author. It is simply motivated and shown, by periodic boundary conditions of virtual particle loops a connection between virtual particles – where the Feynman diagram contributions tend in general to diverge – and the mathematical technique of Ramanujan summation [1]. The claim is not to care about these divergences, not to try to get rid of them by various techniques, but instead to embrace them by associating their Ramanujan sums, in this case equation (11) to their physical sums.

Others may consider the relative simplicity of the formalism a strength, being as simple as possible but not simpler. And that it has good explanatory power, sections 4 and 5, beyond its original intent, section 3. However, the possibility of SYSTEMATIC ERRORS is clearly present, in addition to numerical delusion or numerology. Two numbers, equation (19) and the value given in the Planck data [4] may APPEAR the same within an order of magnitude but may turn out to be completely unrelated once more accurate measurements of the expansion rate is obtained. In that case sections 4 and 5 would simply be systematic errors of this.

Nevertheless, the derivation of section 3 gives – to date – by far the most accurate model for the vacuum energy as observed [4] and it is considered extremely unlikely that its undeniable empirical matching within an order of magnitude is simply a matter of chance or numerology, therefore the letter may indeed be useful and function as a steppingstone for further work on this topic in the future.

[13]Though this does not IMPLY that the closed, trapped region MUST be singularity free, only that it is theoretically possible. However, section 4 gives STRONG INDICATIONS that the closed, trapped region(s) considered is indeed free of physical singularities, though no PROOF of which is claimed.

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8 Conclusions

Due to all the considerations above, we conclude that it is PLAUSIBLE that up to a yet to be accurately determined proportionality constant that the ordinary mass-energy of the observable bubble universe TODAY is indeed inversely proportional to the Planck area, equation (35) $E \propto \frac{L}{l_p^2}$ $(L = 1 \text{ m})$. In addition, due to comparing with empirical estimations [2], and the possible falsifications of other considerations – in particular the case with a volume dependent entropy[14], equation (26) with $d = 3$ it is concluded that this observable universe is likely holographic in nature, that is mathematically similar to a charge less, rotation less, though spherically accelerating, equation (36) black hole, which by section 6 CAN be free of physical singularities.

Were this novel approach to be true, it is clear that it will be of significant importance. Not only may one derive the properties of the vacuum, equations (19, 23, 36) but simultaneously, section 4 may it infer the inner structure of the most idealized type of black hole by semi-classically, equation (12) including the effects of $\hbar$, which CAN be free of physical singularities, section 6. Consequently, by The Copernican Principle it strongly suggests that black holes indeed contain other universes at various stages, which themselves CAN be singularity free.

It is however unclear whether such universes would have different values for the fundamental constants, further analysis's would have to answer such questions. In general, more investigations into this formalism – if true – is needed in order to find all its implications should they exist.

8.1 Availability of data and materials

All data analysed during this study are included in these published articles:

[3] for the Tully-Fisher relation, equation (38) and [4] for the cosmological constant, equation (19).

No new data was generated during this study. **In addition, there is no conflict of interest.**

[14]Since this yields $R \sim O(32)$ m, far off from the Hubble scale [2].

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