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Stochastic Thermal Load Dispatch Employing Opposition-based Greedy Heuristic Search

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Abstract: A thermal load dispatch problem minimizes the number of objectives viz operating cost and emission of gaseous pollutants together while allocating the power demand among the committed generating units subject to physical and technological system constraints. A stochastic thermal load dispatch problem is undertaken while taking into consideration, the uncertainties, errors in data measurements and nature of load demand which is random. Owing to uncertain load demand, variance due to mismatch of power demand termed as risk, is considered as another conflicting objective to be minimized. Generally multiobjective problems generate a set of non-inferior solutions are generated and supplied to a decision maker to select the best solution from the set of non-inferior solutions. This paper proposes opposition-based greedy heuristic search (OGHS) method to generate a set of non-inferior solutions. Opposition-based learning is applied to generate initial population to select good candidates. Migration to maintain diversity in the set of feasible solutions is also based on opposition-based learning. Mutation strategy is implemented by perturbing the genes heuristically in parallel and better one solution is sought for each member. Feasible solutions are achieved heuristically by modifying the generation-schedules in such a manner that violation of operating generation limits are avoided. The OGHS method is simple to implement and provides global solutions derived from the randomness of the population generated without tuning of parameters. Decision maker exploits fuzzy membership functions to decide the final decision. Validity of the method has been demonstrated by analysing systems in different scenarios consisting of six generators and forty generators.

Keywords: fuzzy theory; heuristic search; stochastic economic load dispatch; risk analysis

1. Introduction

The rising energy demand and diminishing energy reserves have dictated the optimal use of existing resources. The essential intention of economic load dispatch (ELD) of electric power production is to plan the yield of the dedicated generating units so as to meet the load requirement at least operational cost, while fulfilling the system's constraints. ELD problem is a large-scale extremely constrained nonlinear optimization problem.

Economic load dispatch (ELD) assigns the generations as required by the customers keeping in view the several considerations like least transmission losses, minimal discharge of pollutants, multiple fuels, etc. Efforts on resolving ELD problems were using various gradient-based mathematical encoding, such as the Newtonian solution of the optimality conditions, nonlinear programming, linear programming, interior point approaches, quadratic programming, Lambda iterative approach, dynamic programming, Lagrange relaxation, gradient projection method, hybridized integer and linear programming, hybridized linear programming and quadratic programming, [32] etc. has been used to solve ELD. These approaches' strengths include optimality that has been mathematically demonstrated [51], applicability to big problems [51], independence from problem-specific characteristics, and computational speed. These numerical techniques use the unit-incremental cost curves, which increase monotonically, to solve ELD issues.

Unfortunately, real units' input-output properties are inherently very nonlinear. These nonlinear physical properties of generating units are a result of ramp rate restrictions, disconnected prohibited operating zones (POZ), and non-smooth cost functions. Due to this, these techniques may settle for fake / local optimum. In spite of many benefits, gradient methods are incapable of ensuring global optimum solution for discontinuous and non-differentiable objective function [23].

Latest heuristic algorithms emerged as efficient tools for nonlinear optimisation challenges. The algorithms do not need that the objective function must be differentiable and continuous. Such techniques are evolutionary programming (EP) [4,8,11,12,17], genetic algorithm (GA) [2], particle swarm optimization (PSO) and its variants [7,10,16], [20–22,24,25,28,29,32,34,35,38,49,58,62], differential evolution (DE) and its variants [14],[17],[18],[31],[55],[57], ant colony optimisation (ACO) [30,37], bio-geography based optimization (BBO) [48],[51], Taguchi method [19], bacterial foraging optimization (BFO) [23,39,47], cultural self-organizing migrating strategy (CSMS) [33], artificial bee colony (ABC) [50,54,56,61], firefly algorithm (FA) [42], opposition-based harmony search algorithm (OHSA) [43], Self-organising hierarchical PSO (SOH-PSO) [20], PSO with crazy particles (PSO-Crazy) [25], PSO with chaotic and Gaussian approaches (PSO-CG) [21], oppositional real coded chemical reaction optimization [60] and gravitational search algorithm (GSA) [46] etc.. These methods are famous for their capabilities of rapid search of huge solution spaces. Two-phase neural network-based modelling [9], simulated annealing-based goal attainment [3], fuzzy decision trees [6], weight pattern search by fuzzy logic [5], modified shuffled frog leaping algorithm [44], fuzzy logic based bacterial foraging [39], θ -PSO [52], chaotic differential bee colony optimization [56] also attempted to solve the problem. A evolutionary search strategy based on binary successive approximation was suggested by Dhillon et al. [26] as a solution to the economic-emission load dispatch (EELD) problem. The heuristic methods, however, are flawed by the abundance of arbitrary or problem-specific parameters [26].

Nowadays hybrid approaches are in use which blends more than one local and global optimization methods in order to have best features of each algorithm. Recent methods informed in literature are hybrid differential evolution (DE) with biogeography-based optimisation (BBO) (DE-BBO) [31], quantum PSO (QPSO) [29], hybrid genetic algorithm (GA)-pattern search (PS)-sequential quadratic programming (SQP) (GA-PS-SQP) [36], hybridization of EP and SQP (EP-SQP) [4], chaotic differential evolution hybrid with quadratic programming (CDE-QP) [14], hybrid of comprehensive learning PSO and SQP [35], hybrid of distributed Sobol PSO and TABU search algorithm (DPSO-TSA) [38], self-adapted real coded GA [41], fuzzy adaptive chaotic ant swarm optimization hybrid with SQP [45], chaotic PSO hybrid with SQP (CPSO-SQP) [49], differential harmony search algorithm by DE and harmony search (HDE-HS) [53], hybrid PSO with gravitational search algorithm (HPSO-GSA) [58], hybrid PSOGSA based on fuzzy logic [62] and hybrid PSO with SQP (HPSO-SQP) [10]. These heuristic methods provide a quick and decent solution, but they don't always provide the globally optimal (or nearly optimal) solution in a finite amount of time. Heuristic and deterministic methods are used to create hybrid optimization algorithms.

When there are multiple objectives that are incompatible with one another, a decision maker is clearly required. The inescapable multifariousness of complex real-world decision-making (DM) situations is one of their core characteristics. Such problems have a variety of objectives, most of which are incommensurable and frequently at odds with one another. Thus, DM issues in the real world frequently result in the formulation of a multi-objective optimisation problem. Pursuing the most favoured solution from a set of non-inferior solutions is the ultimate goal of multi-objective optimisation. The US Clean Air Act Amendments of 1990 and the increased public awareness of environmental protection have forced utilities to change their design or operational procedures to reduce pollutants and atmospheric emissions from thermal facilities [1]. Numerous methods have been published in the literature for the economic-emission load dispatch (EELD) problem, including the multi-objective optimisation strategy that is being proposed.

Due to the inherent randomness of natural occurrences or the implicit and inaccurate assumptions associated with the method of modelling that is being used, many engineering problems are susceptible to ambiguity. Even though there has been a lot of research on thermal power load scheduling issues, the researchers believe that deterministic prototypes are suited for steady-state situations since they assume deterministic system data. In actuality, the input data contains a great deal of uncertainties and inaccuracies from several sources, such as measurement mistakes and flaws in long- and short-term load predictions. Additionally, power system loads are random variables in real-time processes. The electric power system network has been defined by random variables and researched by several researchers at various levels as a result of the increase in production costs brought on by uncertain factors [1, 13].

In this study, the cost coefficients, emission coefficients, and power demand are treated as random variables while constructing the stochastic model of the multi-objective problem. The output of the generator thus unavoidably becomes random. Random variables are seen as statistically reliant on the other variables defining the system and regularly distributed. The deterministic equivalent of the stochastic model is created from expectations. A function's expected value is obtained by using Taylor's series to expand the function around the mean. As an objective function that must also be minimised is the minimising of deviations resulting from these errors and uncertainties. Thus, the formulation of a multi-objective problem results from taking into account all of these factors during the optimization process. With the help of the proposed search process, this multi-objective problem is solved for a collection of non-inferior solutions, and the best negotiated solution is obtained. Opposition-based learning is used to select the improved solution by comparing the objective functions at a solution's position in the search space to its opposite position during the initialization of the population and also in the algorithm's flow. Heuristics are used by the mutation operator to perturb each gene and search for better genes. Migration introduces a new member from the search space or in the opposite direction from the present point member in order to maintain diversity. Additionally, the method doesn't require any parameter tweaking. This study investigates how OGHS may be used to solve stochastic economic load dispatch and stochastic economic emission load dispatch issues. To demonstrate the viability of the suggested OGHS technique, small and medium power systems are taken into account. The paper is divided in the following sections. Section-2 discusses the formulation of stochastic thermal load dispatch problem. Section-3 and 4 deals with the decision making and constraint handling procedures respectively. Section-5 elaborates the proposed algorithm in detail and Section-6 discusses the test case studies and their obtained results.

2. Stochastic thermal load dispatch problem

The multi-objective load dispatch problem is a multiple non-commensurable objective challenge that minimizes operating cost and gaseous contaminants emission simultaneously. A stochastic EELD problem is devised with the consideration of uncertainties in the system production cost and random nature of load demand [1]. In addition, risk is deemed as an additional conflicting objective to be minimised because of random load and uncertain system production cost.

A. Expected fuel cost

The fuel cost curve is approximated by a quadratic function of the generator power yield P_i :

$$F_1 = \sum_{i=1}^{NG} (a_i P_i^2 + b_i P_i + c_i) + \left| e_i \sin \left(f_i (P_i^{min} - P_i) \right) \right| \quad (1)$$

where a_i, b_i, c_i, e_i and f_i are fuel cost coefficients of i^{th} generator and NG is the number of generators. P_i^{min} is lower limit of power generation of i^{th} generator.

A stochastic version of objective function F_1 is formulated by taking into consideration the cost coefficients and power demand as random. Given that the load demand is unpredictable, the generator output turn out to be random. The expected value of the fuel cost function may be derived by expanding the function, using Taylor's series, about the mean [1]. By taking the expectation of the expanded form, the expected fuel cost obtained and is represented by

$$\bar{F}_1 = \begin{cases} \sum_{i=1}^{NG} \left[\bar{a}_i \bar{P}_i^2 + \bar{b}_i \bar{P}_i + \bar{c}_i + \left| \bar{e}_i \sin \left(\bar{f}_i (P_i^{min} - \bar{P}_i) \right) \right| + \frac{1}{2} \left[2\bar{a}_i + \bar{e}_i \bar{f}_i^2 \left\{ \sin \left(\bar{f}_i (P_i^{min} - \bar{P}_i) \right) \right\} \right] var \bar{P}_i \right] & \forall \bar{P}_i > P_i^{min} \\ \sum_{i=1}^{NG} \left[\bar{a}_i \bar{P}_i^2 + \bar{b}_i \bar{P}_i + \bar{c}_i + \left| \bar{e}_i \sin \left(\bar{f}_i (P_i^{min} - \bar{P}_i) \right) \right| + \bar{a}_i var \bar{P}_i \right] & \forall \bar{P}_i = P_i^{min} \end{cases} \quad (2)$$

where \bar{P}_i is the expected value of the generator output, and \bar{a}_i , \bar{b}_i , \bar{c}_i , \bar{e}_i and, \bar{f}_i are the expected cost coefficients. $var(\bar{P}_i)$ is defined as $C_{P_i} \bar{P}_i^2$, where C_{P_i} is the coefficient of variation of the random variables P_i .

B. Expected emission of gaseous pollutants

The gaseous pollutants emission is modelled and is given below [1]:

$$F_2(P_{gi}) = \sum_{i=1}^{N_g} (\alpha_i P_i^2 + \beta_i P_i + \gamma_i + \delta_i \exp(\xi_i P_i)) \quad (3)$$

where α_i , β_i , γ_i , δ_i and ξ_i are emission coefficients of i^{th} generator.

Taking randomness in load demand in consideration, the expected discharge of gaseous pollutants is represented by

$$\bar{F}_2 = \sum_{i=1}^{N_g} (\bar{\alpha}_i \bar{P}_i^2 + \bar{\beta}_i \bar{P}_i + \bar{\gamma}_i + \bar{\delta}_i \exp(\bar{\xi}_i \bar{P}_i)) + \left(2\bar{\alpha}_i + \bar{\xi}_i^2 \bar{\delta}_i \exp(\bar{\xi}_i \bar{P}_i) \right) var \bar{P}_i \quad (4)$$

where \bar{P}_i is the expected value of the generator output, and $\bar{\alpha}_i$, $\bar{\beta}_i$, $\bar{\gamma}_i$, $\bar{\delta}_i$ and, $\bar{\xi}_i$ are the expected emission coefficients.

C. Expected risk

As the generator outputs P_i are considered as random variables, the expected variations are proportionate to the expectation of the square of the unfulfilled power demand. These anticipated deviations are expected risk and considered as an objective to be minimized [1,13]. The objective is represented as:

$$\bar{F}_3 = E \left[\left(\bar{P}_D + \bar{P}_L - \sum_{i=1}^{NG} \bar{P}_i \right)^2 \right] \quad (5)$$

This on simplification reduces to

$$\bar{F}_3 = \sum_{i=1}^{NG} var(P_i) + \sum_{i=1}^{NG} \sum_{j=1, j \neq i}^{NG} 2cov(P_i, P_j) \quad (6)$$

where $cov(P_i, P_j) = R_{P_i P_j} C_{P_i} C_{P_j} \bar{P}_i \bar{P}_j$ and $R_{P_i P_j}$ is the correlation coefficient of the random variables P_i and P_j and that range from -1 to +1.

D. Expected equality and inequality constraints

When the power network arrangement is fixed and the power demand is arbitrary, then the expected equality constraint is enforced to guarantee real power balance and is expressed as

$$\bar{P}_D + \bar{P}_L - \sum_{i=1}^{NG} \bar{P}_i = 0 \quad (7)$$

and expected generator limits as inequality constraint

$$P_i^{min} \leq \bar{P}_i \leq P_i^{max} \quad (i = 1, 2, \dots, NG) \quad (8)$$

where \bar{P}_i^{min} and \bar{P}_i^{max} are the expected minimum and maximum limits, respectively, of the generator output.

E. Expected transmission loss

According to the well-known Kron's loss formula, the transmission power losses, P_L , are a quadratic function of the power generation. It is expressed as

$$P_L = \sum_{i=1}^{NG} \sum_{j=1}^{NG} P_i B_{ij} P_j + \sum_{i=1}^{NG} B_{i0} P_i + B_{00} \quad (9)$$

The power generations P_i are random variables reliant on each other. B_{ij} , B_{i0} and B_{00} are deemed as B coefficients with uncertainties. The expected transmission losses \bar{P}_L using Taylor's series are expressed as [1]

$$\bar{P}_L = \sum_{i=1}^{NG} \sum_{j=1}^{NG} \bar{P}_i \bar{B}_{ij} \bar{P}_j + \sum_{i=1}^{NG} \bar{B}_{i0} var(P_i) + \sum_{i=1}^{NG-1} \sum_{j=i+1}^{NG} 2\bar{B}_{ij} cov(P_i, P_j) + \sum_{i=1}^{NG} \bar{B}_{i0} \bar{P}_i + \bar{B}_{00} \quad (10)$$

where \bar{B}_{ij} , \bar{B}_{i0} and \bar{B}_{00} are the expected B-coefficients.

From the above equations, the stochastic economic emission problem is characterized as a multi-objective optimization problem specified as

$$\text{Minimize } [\bar{F}_1, \bar{F}_2, \bar{F}_3]^T \quad (11a)$$

Subject to

$$\sum_{i=1}^{NG} \bar{P}_i = \bar{P}_D + \bar{P}_L \quad (11b)$$

$$\bar{P}_i^{min} \leq \bar{P}_i \leq \bar{P}_i^{max} \quad (i = 1, 2, \dots, NG) \quad (11c)$$

The objective is to get the expected generation schedule, \bar{P}_i , ($i = 1, 2, \dots, NG$) by employing proposed OGHS algorithm.

3. Decision making

Due to the decision maker's ambiguous decisions, his aims could be vague. The membership functions used to define fuzzy sets express the degree of membership in particular fuzzy sets and have values between 0 and 1. 1 indicates complete set satisfaction, whereas 0 indicates complete set unsatisfaction. The DM determines the membership function by taking into account the minimum and maximum values of each objective function concurrently with the rate of rise of membership function. μ_{F_i} . Presuming that μ_{F_i} is a precisely monotonic diminishing and continuous function [1] defined as

$$\mu_{F_i} = \begin{cases} 1 & \bar{F}_i \leq \bar{F}_i^{min} \\ \frac{\bar{F}_i^{max} - \bar{F}_i}{\bar{F}_i^{max} - \bar{F}_i^{min}} & \bar{F}_i^{min} < \bar{F}_i < \bar{F}_i^{max} ; (i = 1, 2, 3) \\ 0 & \bar{F}_i \geq \bar{F}_i^{max} \end{cases} \quad (12)$$

The membership function's value represents how well (on a scale from 0 to 1) a solution has met the \bar{F}_i objective.

In multi-objective optimization problem, where more than one contradictory objective is considered, min-max fuzzy operation is employed for decision making to choose the best compromised result. Mathematically, it can be expressed as

$$\mu^k = \min(\mu_{\bar{F}_j}^k (j = 1, 2, \dots, N_{ob})) (k = 1, 2, \dots, N_p) \quad (13)$$

4. Constraint handling

Both direct and indirect approaches can be used to tackle the non-linear, constrained optimization issue. Contrary to indirect methods, which turn the constrained optimization issue into an unconstrained problem and then solve it as an unconstrained minimization problem, direct methods explicitly integrate constraints. Heuristics that are detailed in the subsections are used to explicitly control equality and POZ constraints. When fixing the generation within the generation limits, ramp-rate limits are taken into account.

In the procedure of obtaining solution, it may fall out of the feasible range in search space with the breach of some constraints linked with the power dispatch problem. A constraint handling algorithm is developed for this purpose, which addresses the problem of violation of constraints, based on direct constraint handling methodology. The disparity in power demand constraint of k^{th} member is calculated as

$$\bar{D}_{PD}^k = \bar{P}_D + \bar{P}_L^k - \sum_{i=1}^{N_g} \bar{P}_i^k \quad (14)$$

This disparity in power demand is dispensed among all the generator units by randomly choosing s^{th} generating unit, \bar{P}_{gs}^k of k^{th} member from whole population that is perturbed as

$$\bar{P}_s^k = \begin{cases} \bar{P}_s^k + \Delta \bar{P}_s^{max} & ; D_{PD}^k > 0 \\ \bar{P}_s^k - \Delta \bar{P}_s^{min} & ; D_{PD}^k < 0 \end{cases} \quad (15)$$

Algorithm-I: Constraint handling procedure for k^{th} member

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• DO
  ○ FOR  $i = 1, N_g$ 
    ▪ Compute  $D_{PD}^k$  using Eq. (14)
    ▪ IF  $(|D_{PD}^k| \leq \epsilon)$  EXIT
    ▪ Select a generator  $P_s^k$  randomly from NG units of  $k^{th}$  member of population
    ▪ IF  $(D_{PD}^k < 0)$  THEN
      * Compute  $\Delta P_s^{min}$  using Eq. (16)
      * Update  $P_s^k$  using Eq. (15)
    ▪ ENDIF
    ▪ IF  $(D_{PD}^k > 0)$  THEN
      * Compute  $\Delta P_s^{max}$  using Eq. (17)
      * Update  $P_s^k$  using Eq. (15)
    ▪ ENDIF
  ○ ENDFOR
• WHILE  $(|D_{PD}^k| > \epsilon)$ 
RETURN
```


Calculated perturbation is contained within the prescribed range and ramp-rate limits as described below.

$$\Delta P_s^{min} = \begin{cases} |D_{PD}^k| & ; \text{if } |D_{PD}^k| < \Delta P_s^{min} \\ \Delta P_s^{min} & ; \text{otherwise} \end{cases} \quad (16)$$

where $\Delta P_s^{min} = r[\bar{P}_s^k - P_s^{min}](|D_{PD}^k|/\bar{P}_D)$ and r is a uniform random number in the range of (0-1).

Perturbation is calculated and violation of limits of generation is contained within recommended range and ramp-rate limits and is expressed below

$$\Delta P_s^{max} = \begin{cases} |D_{PD}^k| & ; \text{if } |D_{PD}^k| < \Delta P_s^{max} \\ \Delta P_s^{max} & ; \text{otherwise} \end{cases} \quad (17)$$

where $\Delta P_s^{max} = r[\min(P_s^{max}, P_s^0 + UR_i) - P_s^k](D_{PD}^k/P_D)$ and r is a uniform random variable.

Any generating unit is chosen only once in a cycle. This procedure is repeated until $|D_{PD}^k|$ reaches some infinitesimally small value. The stepwise procedure is detailed in Algorithm-I.

5. Proposed Opposition based greedy heuristic search method

The multi-objective power dispatch problem is solved using a heuristic search strategy that is suggested in the study. Expected risk is seen as a second objective that should be minimized in addition to expected operational cost, which is taken as the objective function, \bar{F}_1 . The overall member function $\mu^k = \min(\mu_{F_j}^k (j = 1, 2, \dots, N_{ob}))$, is maximized. If the objective function improves, the decision to choose a member and obtain a superior member is deemed a "success"; if not, it is deemed a "failure". The members are randomly initialized. Utilizing opposition-based learning to the choice variables, a good, varied set of population is obtained after random initialization of the members. The diversity is preserved through migration that is also based on opposition-based learning.

5.1. Random initialization

In random initialization, the initial NP members (solutions) are produced randomly within the search space making use of uniformly distributed random numbers as

$$\bar{P}_i^k = \bar{P}_i^{min} + r_i^k(\bar{P}_i^{max} - \bar{P}_i^{min}); (i = 1, 2, \dots, N_g; k = 1, 2, \dots, N_p) \quad (18)$$

where r_i^k is uniform random number for i^{th} generator and k^{th} member, NP is population size.

5.2. Opposition-Based Learning

Heuristic optimization strategies start with a randomly selected member and then increase its quality to get the best solution. The difference between these initial estimations and the ideal solution affects calculation time. However, it can be enhanced by taking advantage of the chance to start with a better solution while also checking its own opposing solution [17]. The superior initial answer is chosen, either randomly or according to its opposite guess. Therefore, the convergence can be sped up by starting with the estimate that is closer than the other, as determined by its objective function. The same method can be constantly applied to every solution in the current population as well as to original solutions. The population in opposition is obtained as

$$\bar{P}_i^{k+NP} = \bar{P}_i^{min} + \bar{P}_i^{max} - \bar{P}_i^k; (i = 1, 2, \dots, N_g; k = 1, 2, \dots, N_p) \quad (19)$$

5.3. Objective function evaluation

Proposed strategy can be employed to single objective and multi-objective situation. The goal is to minimize operating cost, F_1 , in scalar objective problem using Eq. (1). In multi-objective setting, fuzzy max-min operator is utilized to manage the conflicting

objective functions after the individual objective functions' membership values are found using Eq. (14), as

$$\mu^k = \min(\mu_{\bar{F}_j}^k \ (j = 1, 2, \dots, N_{ob})) \ (k = 1, 2, \dots, N_p) \quad (20)$$

For each generated solution, the fuzzy operator "min-max" evaluates the amount of satisfaction that is simultaneously obtained by all of the objective functions. The decision-making process selects the best-negotiated option with the highest level of satisfaction, or cardinal priority rating. Eq. (13) is treated as objective function to be maximized for multi-objective problem solved by OGHS algorithm.

5.4. Mutation Policy

The task of mutation policy is to deliver ability for good exploration with the crucial obligation of the solution improvement. Mutation strategy is centred on random perturbation. One dimension in the search is thought as a gene. Every gene of current member is modified by doing perturbation with random size in both directions in such a way so that the solution remains within feasible range. This considered current member is renewed to its best mutated inheritor at the end of every mutation procedure. The mutation is termed as a success if the mutated member is superior one from the previous one, or else the mutation is ignored. Mathematically, the approach is detailed as below:

For the mutation, k^{th} expected member, \bar{P}_i^k from population is chosen randomly and is regarded as a candidate member as P_{il}^{old} . Evaluate objective function either F_{1l}^{old} using Eq. (1) for single objective optimization problem or $\mu_i^{old} (= \min\{\mu_{jl}^{old}; j = 1, 2, \dots, N_{ob}\})$ and membership functions, μ_{jl}^{old} using Eq. (13) for multi-objective optimization problem. This candidate member of i^{th} unit is perturbed as indicated below:

$$\bar{P}_{ilm}^{new} = \bar{P}_{il}^{old} + (-1)^m r_i \alpha \Delta_i u_{il} \ ; \ (l = 1, 2, \dots, N_g; m = 1, 2; i = 1, 2, \dots, N_g) \quad (21)$$

where $u_{il} = \begin{cases} 1 & ; i = l \\ 0 & ; i \neq l \end{cases}$ and r_i is uniformly distributed random number. α is a adaptive factor that reduces the step length in every iteration.

The calculation of perturbation is within the normalized permitted range of generation as defined below:

$$\alpha \Delta_i = (\alpha(\bar{P}_i^{max} - \bar{P}_i^{min}) P_D) / \sum_{i=1}^{N_g} (\bar{P}_i^{max} - \bar{P}_i^{min}) \ ; \ (i = 1, 2, \dots, N_g) \quad (22)$$

The genes (generator outputs) are confined within their prescribed during the mutation process as

$$\bar{P}_{il1}^{new} = \begin{cases} \bar{P}_{il}^{old} - r_i \alpha \Delta_i u_{il} & ; \text{if } r_i \Delta_i < \bar{P}_{il}^{old} \\ \bar{P}_i^{min} & ; \text{otherwise} \end{cases} \quad (23)$$

$$\bar{P}_{il2}^{new} = \begin{cases} \bar{P}_{il}^{old} + r_i \alpha \Delta_i u_{il} & ; \text{if } r_i \Delta_i < P_i^{max} \\ \bar{P}_i^{max} & ; \text{otherwise} \end{cases} \quad (24)$$

Evaluate objective function \bar{F}_{1lm}^{new} using Eq.(1) or $\mu_{im}^{new} (= \min\{\mu_{jim}^{new} \ (j = 1, 2, \dots, N_{ob}; m = 1, 2)\})$ and membership function μ_{jim}^{new} is evaluated using Eq. (13).

Selection operator is utilised to select the improved perturbed member (solution). The selection is as stated below ($m = 1$):

For economic thermal power dispatch problem:

$$\bar{P}_{il}^{new} = \begin{cases} \bar{P}_{ilm} & ; \bar{F}_{1lm}^{new} < \bar{F}_{1l,m+1}^{new} \\ \bar{P}_{il,m+1} & ; \text{otherwise} \end{cases} \quad (25)$$

For multi-objective thermal power dispatch problem:

$$\bar{P}_{il}^{new} = \begin{cases} \bar{P}_{ilm} & ; \mu_{lm}^{new} > \mu_{l,m+1}^{new} \\ \bar{P}_{il,m+1} & ; \text{otherwise} \end{cases} \quad (26)$$

Update the objective either \bar{F}_{1l}^k or $\mu_i^k (= \min\{\mu_{1i}^k, \mu_{2i}^k\})$ corresponding to \bar{P}_{il}^{new} . Updated gene is taken for $(i + 1)^{th}$ generation as

For economic thermal power dispatch problem:

$$\bar{P}_{il}^{old} = \begin{cases} \bar{P}_{il}^{new} & ; \bar{F}_{1lm}^{new} < \bar{F}_{1l}^{old} \\ \bar{P}_{il}^{old} & ; otherwise \end{cases} \quad (27)$$

For multi-objective thermal power dispatch problem:

$$\bar{P}_{il}^{old} = \begin{cases} \bar{P}_{il}^{new} & ; \mu_{lm}^{new} > \mu_l^{old} \\ \bar{P}_{il}^{old} & ; otherwise \end{cases} \quad (28)$$

and best value is chosen for k^{th} member of population as $\bar{P}_i^k = \bar{P}_{Ngl}^{old}$ and its corresponding objective function either \bar{F}_1^k or μ^k

5.5. Random Migration Operator

With the advancement of the algorithm, the population's diversity and capacity for exploring the search space rapidly decrease, and the grouped entities are unable to mutate into new, superior race. Randomly selected individuals begin migrating in order to overcome this restriction, improve the exploration of the search space, and lower the selection pressure for a small population. The i^{th} generator of k^{th} member is randomly migrated as follows

$$\bar{P}_i^{k+Np} = \begin{cases} \bar{P}_i^{min} + \bar{P}_i^{max} - \bar{P}_i^k & ; p \leq p_{mig} \\ \bar{P}_i^{min} + r_i^k(\bar{P}_i^{max} - \bar{P}_i^{min}) & ; otherwise \end{cases} \quad (i = 1, 2, \dots, N_g; k = 1, 2, \dots, N_p) \quad (29)$$

where p is uniform random number and p_{mig} is the probability of migration.

Algorithm-II gives detail to implement the proposed OGHS.

Algorithm-II: OGHS

- Initialize the $2N_p$ population using Eqs. (18) and (19)
- Fix violated constraints, if any using constraint handling Algorithm – I
- Evaluate objective function μ^k using Eq. (20)
- Select best population N_p out of $2N_p$
- Select global best solution
- Compute step length Δ_i as per Eq. (22)
- **FOR** iteration = 1, max_iteration
 - $\Delta_i = \alpha \Delta_i ; (i = 1, 2, \dots, N_g)$
 - **FOR** $k = 1, N_p$
 - **FOR** $i = 1, N_g$
 - * Select l^{th} member randomly and consider objective function $\mu_i^{old} (= \min\{\mu_{jl}^{old} ; (j = 1, 2, \dots, N_{ob})\})$ using Eq. (20)
 - * Calculate \bar{P}_{il1}^{new} and \bar{P}_{il2}^{new} as per Eq. (23) and (24)
 - * Fix violated constraints, if any, using constraint handling Algorithm – I
 - * Evaluate objective function $\mu_{im}^{new} (= \min\{\mu_{jim}^{new} (j = 1, 2, \dots, N_{ob}; m = 1, 2)\})$ and μ_{jim}^{new} is evaluated using Eq. (20).
 - * $\bar{P}_{il}^{new} = \bar{P}_{il1}^{new} ; \mu_i^{new} = \mu_{i1}^{new}$
 - * IF $(\mu_{i2}^{new} > \mu_{i1}^{new}) \{ \mu_i^{new} = \mu_{i2}^{new} \text{ and } \bar{P}_{il}^{new} = \bar{P}_{il2}^{new} \}$
 - * IF $(\mu_i^{new} > \mu_i^{old}) \{ \mu_i^{old} = \mu_i^{new} \text{ and } \bar{P}_{il}^{old} = \bar{P}_{il}^{new} \}$
 - **ENDFOR**
 - $\bar{P}_i^k = \bar{P}_{il}^{old}, \mu^k = \mu_i^{old},$
 - **ENDFOR**
 - **FOR** $k = N_p + 1, 2N_p$
 - Generate population using random migration as per Eq. (29)
 - Call Algorithm – 1 to satisfy energy balance constraint equation
 - Evaluate objective function $\mu^k (= \min\{\mu_j^k (j = 1, 2, \dots, N_{ob})\})$ and μ_j^k is evaluated using Eq. (20).
 - **ENDFOR**
 - Select best population N_p out of $2N_p$
 - Select global best solution
- **ENDFOR**

6. Test Systems and results

By investigating stochastic multi-objective thermal load dispatch issues for small and medium power systems, the validity of the suggested strategy has been demonstrated. Analysis is considered for stochastic economic load dispatch (SELD) and stochastic economic-emission load dispatch (SEELD) situations. By assigning one of the objectives full weight and ignoring the others, the minimum values of the objectives are met. When the assumed weight value is 1.0, the objective is given full weight, and when it is zero, the objective is disregarded. The summary of considered power systems to implement SELD and SEELD is given in Table-1. In addition, the values of the coefficients of variation and correlation coefficients assumed in the study in all the cases are $C_{p_i} = 0.01$ to 0.1 ; $\forall (i = 1, 2, \dots, N_g)$ and $R_{p_i p_j} = -0.03$ to 0.03 ; $\forall (i = 1, 2, \dots, N_g; j = 1, 2, \dots, N_g; i \neq j)$ with the step interval of 0.01. The population size, NP is taken as 100 for every experiment. Maximum

iterations are set to 1000 for all the experiments. Robustness of the suggested algorithm is confirmed by 100 mutually exclusive trial runs.

Table 1. Outline of undertaken power system for analysis.

Example	N_g	VPL	Transmission losses	Remarks
1	6 [1]	×	√	SELD
2	40 [8]	×	×	SELD
3	40 [8]	√	×	SELD
4	40 [8,40]	√	√	SELD
5	40 [8,40]	√	×	SEELD

6.1 Example-1: 6-thermal generator electric power system problem

Six generator systems are used to show the effectiveness of the suggested strategy. In Table 2, the projected fuel price, gas emissions, and generator operating limitations are shown. In Table 3, the expected average transmission loss coefficients are shown. Three power demands of 700 MW, 900 MW, and 1100 MW are examined in this scenario.

Changes in the predicted fuel cost's % deviation from deterministic fuel cost in relation to the coefficient of variation, C_{p_i} and correlation coefficient, $R_{p_i p_j}$ is shown in Fig.1. The variation in percentage deviation in expected pollutant emission from deterministic pollutant emission with respect to the coefficient of variation, C_{p_i} and correlation coefficient, $R_{p_i p_j}$ is shown in Fig.2 respectively. Figure 3 depicts the expected risk as a function of the coefficient of variation, C_{p_i} and correlation coefficient, $R_{p_i p_j}$. Table-4 displays the percentage deviation of expected fuel cost from its deterministic value, the percentage deviation of expected pollutant emissions from its deterministic value, the expected risk, and the expected generation schedule at different coefficients of variation, ($C_{p_i} = 1\%$ and $C_{p_i} = 10\%$) and correlation coefficient ($R_{p_i p_j} = \pm 0.03$) for the load demands of 700MW, 900MW and 1100MW respectively.

Table 2. The fuel cost, economic emission constants and generator operating limits [1].

Gen	Fuel cost coefficients			Emission coefficients			p_i^{min} (MW)	p_i^{max} (MW)
	a_i (Rs/MW ² h)	b_i (Rs/MWh)	c_i (Rs/h)	α (kg/h)	β (kg/h)	γ (kg/h)		
1.	0.15247	38.53973	756.7989	0.00419	0.32767	13.85932	10	125
2.	0.10587	46.15916	451.3251	0.00419	0.32767	13.85932	10	150
3.	0.02803	40.39655	1049.998	0.00683	0.54551	40.2669	35	225
4.	0.03546	38.30553	1243.531	0.00683	0.54551	40.2669	35	210
5.	0.02111	36.32782	1658.57	0.00461	0.51116	42.89553	130	325
6.	0.01799	38.27041	1356.659	0.00461	0.51116	42.89553	125	315

Table 3. Average expected transmission loss coefficients (in MW⁻¹).

0.002022	-0.000286	-0.000534	-0.000565	-0.000454	0.000103
-0.000286	0.003243	0.000016	-0.000307	-0.000422	-0.000147
-0.000533	0.000016	0.002085	0.000831	0.000023	-0.000270
-0.000565	-0.000307	0.000831	0.001129	0.000113	-0.000295
-0.000454	-0.000422	0.000023	0.000113	0.000460	-0.000153
0.000103	-0.000147	-0.000270	-0.000295	-0.000153	0.000898

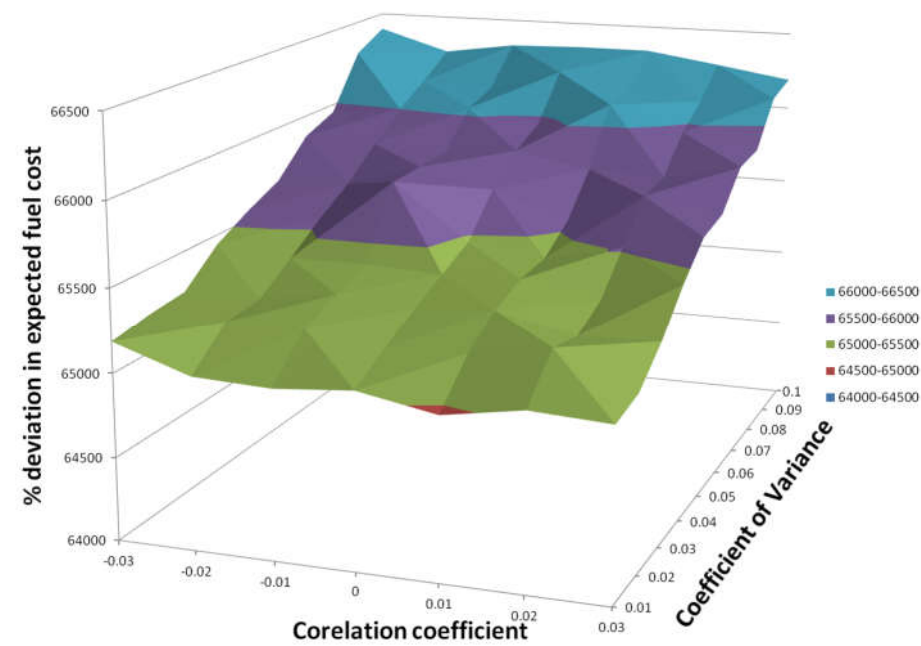


Figure 1. Percentage deviation from the expected fuel cost objective versus the correlation coefficient $R_{P_iP_j}$ and coefficient of variance C_{P_i} .

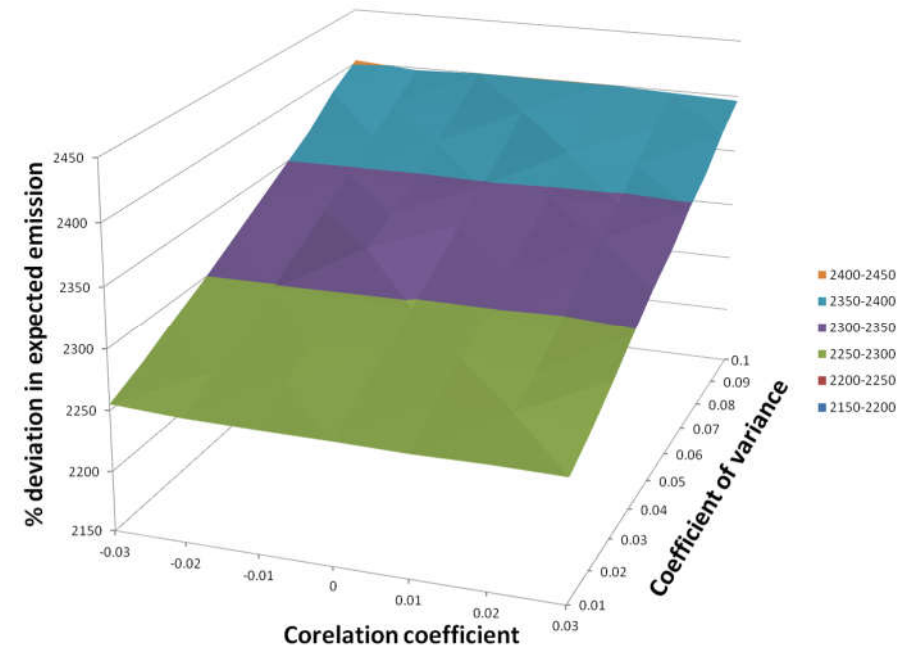


Figure 2. Percentage deviation in expected emission of gaseous pollutants along versus correlation coefficient $R_{P_iP_j}$ and coefficient of variance C_{P_i} .

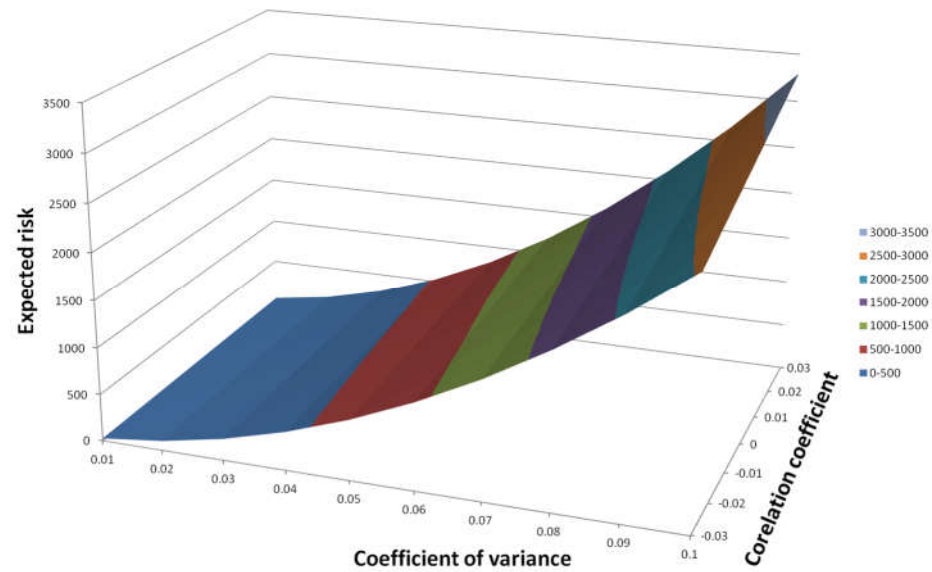


Figure 3. Expected risk versus correlation coefficient $R_{P_i P_j}$ and coefficient of variance C_{P_i} .

Table 4. Values of objective functions and generation plans for load demands of 700MW, 900MW, and 1100MW at various coefficients of variation and correlation coefficients.

P_D	$R_{P_i P_j}$	C_{P_i}	μ	F_1	F_2	F_3	P_1	P_2	P_3	P_4	P_5	P_6
700	0.00	0.0	0.65170	39037.44	1078.698	0.00	97.67036	72.7025	61.97384	105.0953	227.6541	169.2725
	-0.03	0.01	0.54891	40155.08	1044.428	8.71476	115.9856	87.75609	77.8606	113.9939	190.7215	155.0199
	-0.03	0.10	0.50028	42125.18	1096.881	842.4584	125	105.1449	97.5015	126.8699	160.908	142.6784
	0.03	0.01	0.49165	41003.92	1042.469	11.21391	125	96.20328	87.371	124.6204	171.1524	144.6385
	0.03	0.10	0.42348	41711.22	1090.179	1125.297	125	100.0111	89.14144	125.4332	168.8068	143.8901
900	0.00	0.00	0.35549	49933.38	1687.299	0.00	99.51418	74.99191	63.43766	148.6791	325	252.7803
	-0.03	0.01	0.34756	50564.48	1589.907	18.09034	120.3983	90.53859	82.77207	152.7629	292.9168	222.7011
	-0.03	0.10	0.32113	51865.32	1651.258	1591.262	125	100.4988	94.59234	159.0681	272.8531	217.8652
	0.03	0.01	0.32937	51123.79	1560.996	20.35286	125	102.9383	90.36965	158.878	273.7344	215.371
	0.03	0.10	0.28547	52424.08	1638.733	2017.158	125	114.0865	98.22901	163.6979	261.6143	212.2133
1100	0.00	0.00	0.20497	64189.68	2259.394	0.00	125	131.5968	116.3736	210	325	315
	-0.03	0.01	0.11956	65187.36	2255.252	25.31987	125	150	129.2949	210	325	292.294
	-0.03	0.10	0.1129	66393.91	2402.271	2552.278	125	150	131.4576	210	325	295.3218
	0.03	0.01	0.12209	65020.35	2250.679	32.6899	125	150	121.8579	210	325	296.099
	0.03	0.10	0.11747	66191.11	2396.016	3294.397	125	150	122.9375	210	325	299.4826

When the coefficient of variation, C_{P_i} , remains constant, the percentage deviation in expected fuel cost decreases at a very small rate in relation to the variation in correlation coefficient, $R_{P_i P_j}$. However, when the coefficient of variation, C_{P_i} , changes and the correlation coefficient, $R_{P_i P_j}$, is fixed at one value, the percentage deviation in expected fuel cost rises more quickly. When the correlation coefficient, $R_{P_i P_j}$, is changed while the coefficient of variation, C_{P_i} , remains constant, the percentage deviation in expected emission exhibits a nearly constant trend. Furthermore, when the coefficient of variation, C_{P_i} , varies while the correlation coefficient, $R_{P_i P_j}$, remains constant, the percentage deviation in expected emission increases more rapidly. Similarly, expected risk is nearly constant with correlation coefficient, $R_{P_i P_j}$, at constant coefficient of variation, C_{P_i} , and increases at a faster rate with coefficient of variation, C_{P_i} , at constant correlation coefficient, $R_{P_i P_j}$.

6.2. Example-2: 40 generator system neglecting transmission loss (with convex characteristics of generator system)

In this example, a 40-thermal generator power system is studied. The operating cost coefficients are derived from [8] for a load demand of 10500MW. In this case, the system is modelled with convex generator characteristics without taking into account valve point loading effects. Transmission losses are ignored. Figures 4 and 5 illustrate, respectively, the percent deviation of the expected fuel cost from its deterministic value and the expected risk together with the correlation coefficient $R_{P_i P_j}$ and coefficient of variation C_{P_i} . As the correlation coefficient $R_{P_i P_j}$ is raised, it can be seen in Fig. 4 that the percent variation in projected fuel cost increases. At greater coefficients of variation, C_{P_i} , this rate of increase is more rapid. The similar rise pattern is shown in Figure 5. The rate of rise is more for higher coefficients of variations, C_{P_i} , as compared to lower coefficients of variations, C_{P_i} . Table 6 displays the values of the objectives, including the predicted risk and the percentage difference between the expected fuel cost and its deterministic value. Table 7 displays the anticipated generating schedule and anticipated fuel costs for the scenario with independent variables (deterministic case)

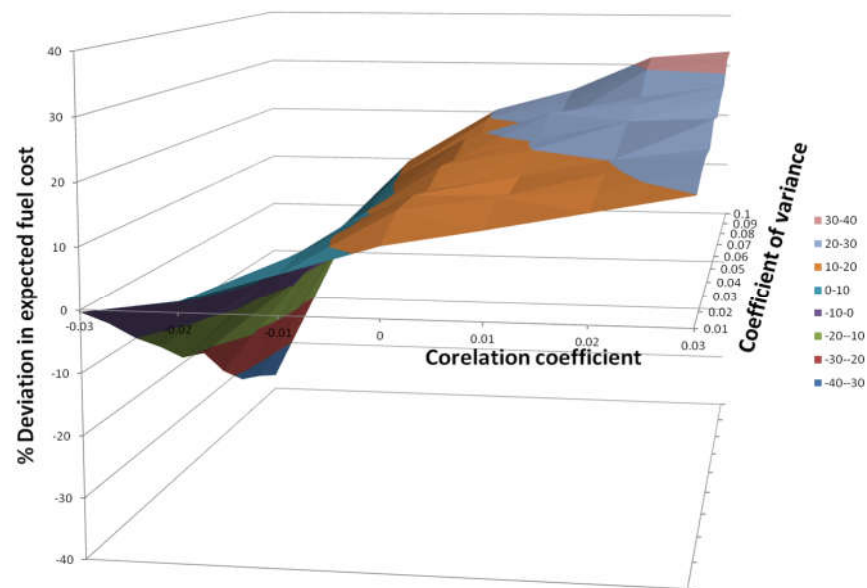


Figure 4. Percentage deviation in expected fuel cost objective with correlation coefficient $R_{P_i P_j}$ and coefficient of variance C_{P_i} .

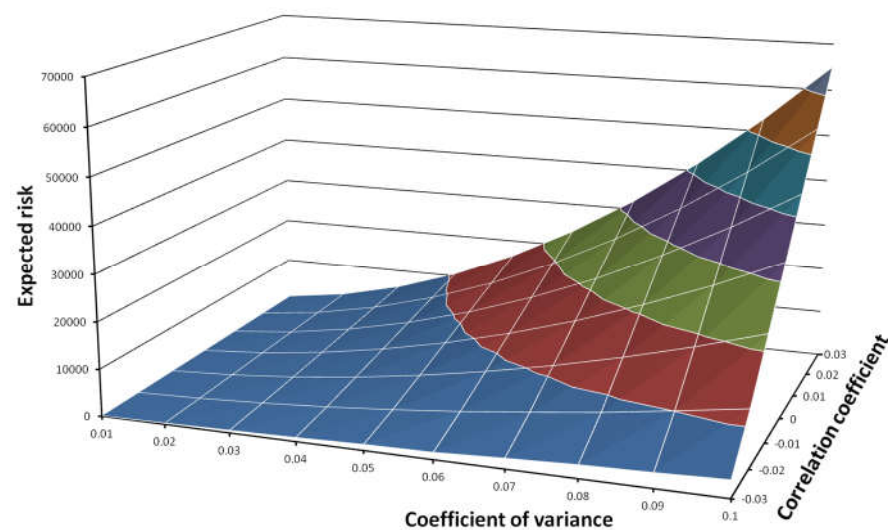


Figure 5. Expected risk with correlation coefficient $R_{P_i P_j}$ and coefficient of variance C_{P_i} .

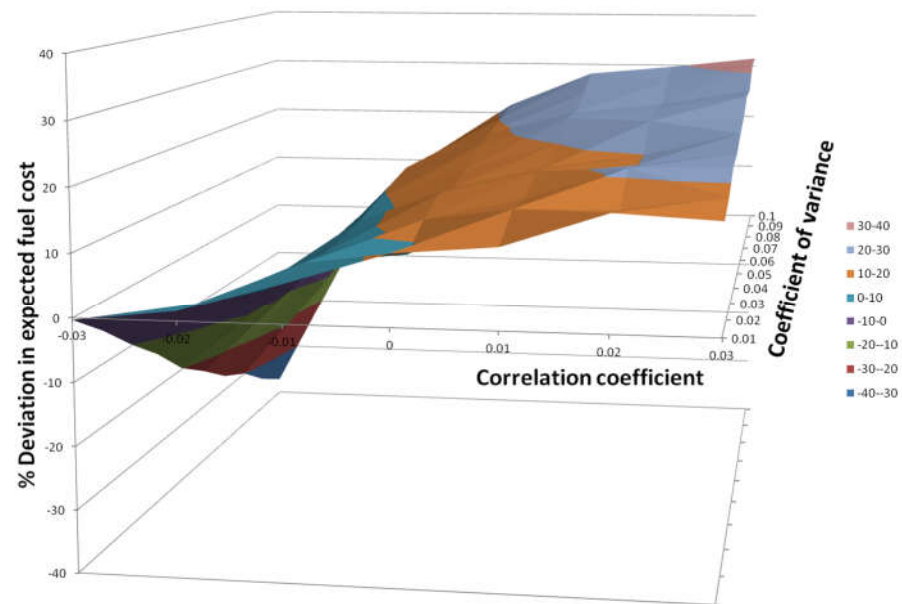
6.3. Example-3: 40 generator system without transmission loss (with non-convex characteristics of generator system)

This example system's case study includes 40 non-convex fuel cost characteristics with effects of valve point loading [8]. Losses in the transmission are disregarded. The load demand is taken to be 10500MW. Fig-6 and Fig-7 show the percentage variation in expected fuel cost from its deterministic value as well as expected risk versus correlation coefficients $R_{p_i p_j}$ and coefficient of variation C_{p_i} . Fig-6 and Fig-7 show a nearly identical rate of increase in the percentage deviation of predicted fuel and variance of power demand mismatch as were shown in Fig-4 and Fig-5, respectively. Even though the total rise is lower than in Example-2, there is no discernible difference between the deviation patterns in the convex and non-convex system case analyses that are being taken into consideration. The sinusoidal term in Eq. 2 for the valve point loading effect is what causes the estimated operational cost of generators to increase or decrease. Table 6 lists the values of the objectives, including the estimated risk and the percentage difference between the predicted fuel cost and its deterministic value. Table 7 shows the anticipated generating schedule and fuel cost, taking into account the situation of independent variables (deterministic case).

The results for the deterministic scenario are compared with those by other techniques provided in literature, as shown in Table 5, to demonstrate the competitiveness of the recommended algorithm. 30 mutually independent test runs of the suggested approach were performed to evaluate the algorithm's resilience. Table 5 also includes the least, highest, average, and standard deviation values from these 30 algorithm test runs that are mutually incompatible. The same results from other methods that are documented in literature are also presented for comparison.

Table 5. Statistical values for 40 generator system ELD problem.

Method	Fuel cost			Standard deviation
	Minimum	Average	Maximum	
CTPSO[32]	121694.6056	121944.3959	–	–
CSPSO [32]	121435.9581	121945.0564	–	–
COPSO [32]	121411.8975	121499.9769	–	–
CCPSO [32]	121403.5362	121445.3269	–	–
CSOMA[33]	121422.10	–	–	–
MBFA [39]	121415.65	–	–	–
FCASO-SQP [45]	121456.98	122026.21	–	–
CPSO-SQP [49]	121458.54	122028.16	–	–
IABC [50]	121412.75	–	121503.58	–
IABC-LS [50]	121412.73	–	121471.61	–
MsEBBO/mig [51]	121415.520	121521.68990	121476.25170	36.40770
MsEBBO/mut[51]	121416.288	121585.01860	121500.92790	32.74280
MsEBBO/sin[51]	121415.3090	121479.36570	121421.65560	11.56960
MsEBBO[51]	121412.53440	121450.00260	121417.18770	5.79960
θ -PSO [52]	121420.90	121509.84	121852.42	–
HPSO-GSA [58]	121412.5682	–	–	–
MABC/P/Log [61]	121412.590	121431.580	121493.190	18.160
MABC/D/Cat [61]	121412.540	121431.780	121503.760	19.161
OGHS	121403.6	121431.3	121494.5	25.718

**Figure 6.** Percentage deviation in expected fuel cost objective along with correlation coefficient $R_{P_iP_j}$ and coefficient of variance C_{P_i} .

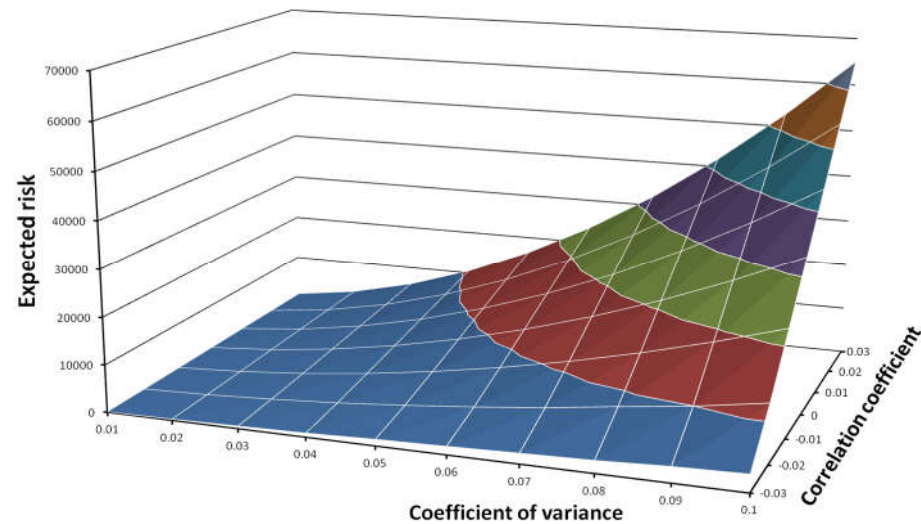


Figure 7. Expected risk with correlation coefficient $R_{P_i P_j}$ and coefficient of variance C_{P_i} .

6.4. Example-4: 40 generator system considering transmission loss (with non-convex characteristics of generator system)

This case study includes implications of valve point loading and non-convex fuel cost features for 40 generators [8]. In this instance, the transmission losses are also included. Expected load demand is considered as 10500MW. Fig-8 and Fig-9 show, respectively, the predicted fuel cost % deviation from its deterministic value and the estimated risk with respect to the correlation coefficient, $R_{P_i P_j}$, and coefficient of variation, C_{P_i} . At greater coefficients of variation, C_{P_i} , the rate of increase in percentage deviation in projected fuel cost and expected risk is steeper. Although the total rise is less than the prior case, Example-3, the growth pattern is identical to an earlier case taken under investigation. The sinusoidal term in Eq. 2 for the valve point loading effect is what causes the estimated operational cost of generators to increase or decrease. Table 6 lists the values of the objectives, including the estimated risk and the percentage difference between the predicted fuel cost and its deterministic value. Table 7 shows the anticipated power generation schedule and fuel cost for the scenario with independent variables (deterministic case).

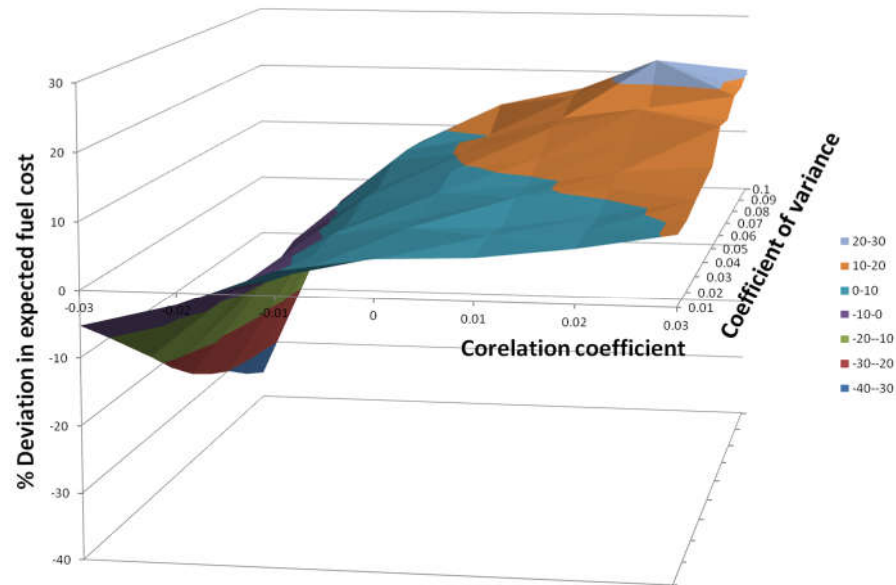


Figure 8. Percentage deviation in expected fuel cost objective versus correlation coefficient $R_{P_i P_j}$ and coefficient of variance C_{P_i} .

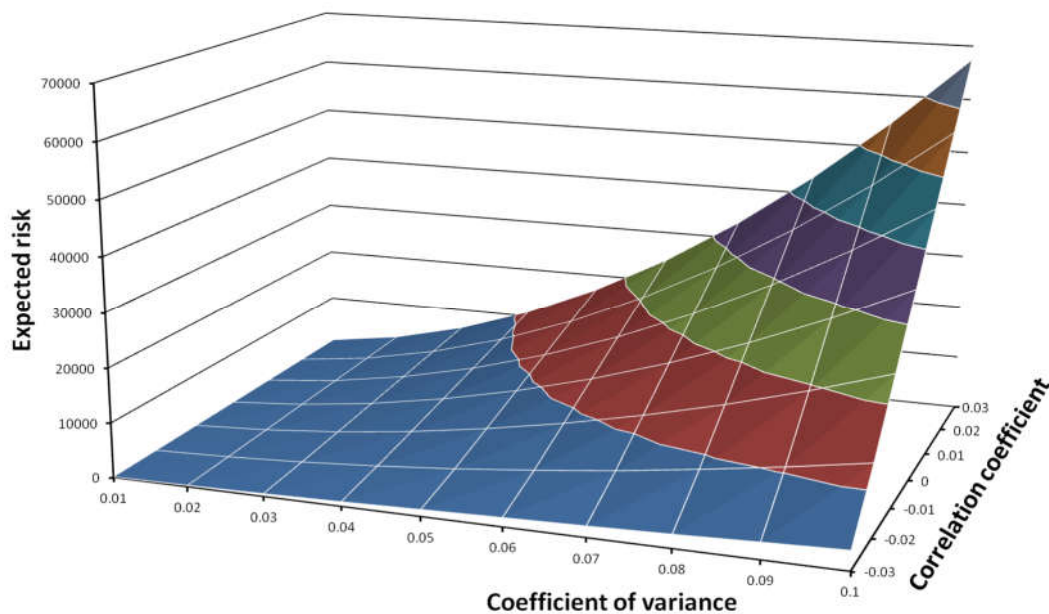


Figure 9. Expected risk versus correlation coefficient $R_{P_i P_j}$ and coefficient of variance C_{P_i} .

6.5. Example-5: 40 generator system economic-emission load dispatch neglecting transmission loss (with non-convex characteristics of generator system)

In this example, a 40-generator system with a non-convex fuel cost objective function considering valve point loading is used to illustrate stochastic economic emission load dispatch. The transmission losses are also taken into account. Information on emission coefficients and operating cost coefficients is taken, respectively, from [8] and [40]. Expected load demand is considered as 10500MW. Figures 10 through 12 illustrate the expected risk as well as the expected fuel cost as compared to its deterministic value and the percentage deviation in pollution emission from its deterministic value in relation to the correlation coefficient $R_{P_i P_j}$ and the coefficient of variance, C_{P_i} . The percentage variation in predicted fuel cost and expected risk from Figs. 10 and 12 showed the same rate of increase as in previous case studies, however the overall increase was less substantial than in case 2. Even though transmission losses are ignored in this example study, the

environmental target was taken into account, which resulted in a lower overall rise. On the other hand, as the correlation coefficient, $R_{P_iP_j}$, increases at various coefficients of variation, C_{P_i} , the deviation in the emission of gaseous pollutants does not exhibit any fixed monotonic pattern. The values of the objectives, including the percentage deviation in projected fuel cost from its deterministic value, the percentage deviation in pollutant emission from its deterministic value, and the predicted risk, are shown in Table 6. For the case of independent variables, Table 7 displays the anticipated generation schedule, fuel cost, and gaseous pollutant emissions (deterministic case)

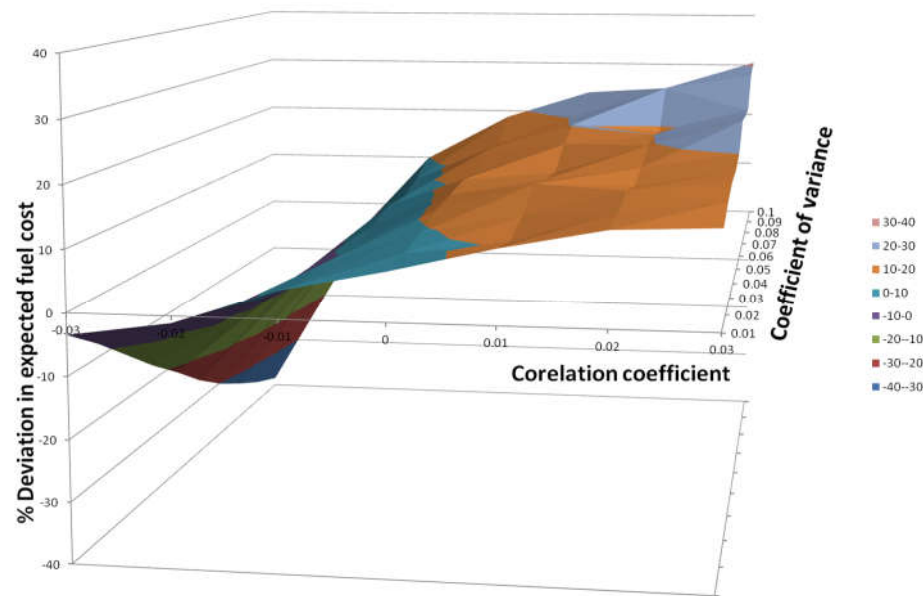


Figure 10. Percentage deviation in expected fuel cost objective versus correlation coefficient $R_{P_iP_j}$ and coefficient of variance C_{P_i} .

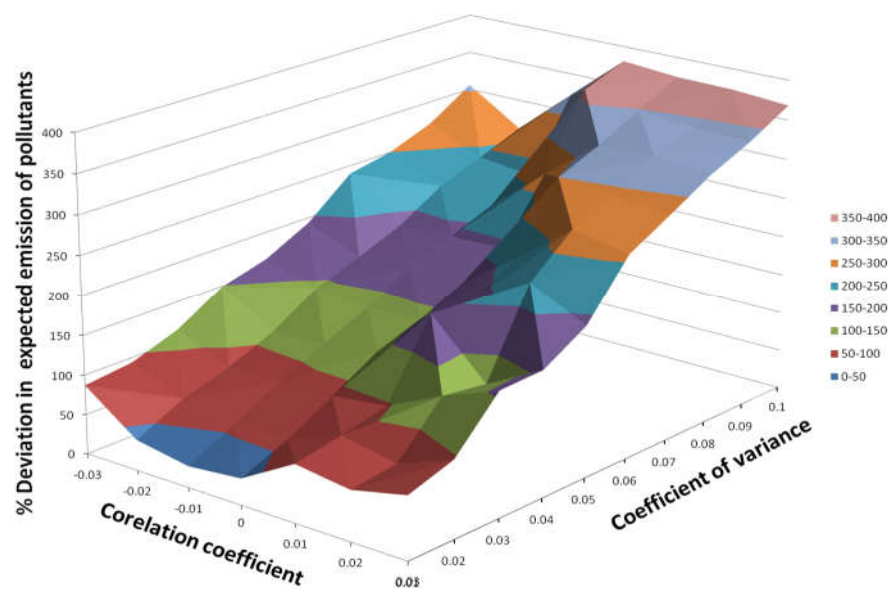


Figure 11. Percentage deviation in expected emission objective versus correlation coefficient $R_{P_iP_j}$ and coefficient of variance C_{P_i} .

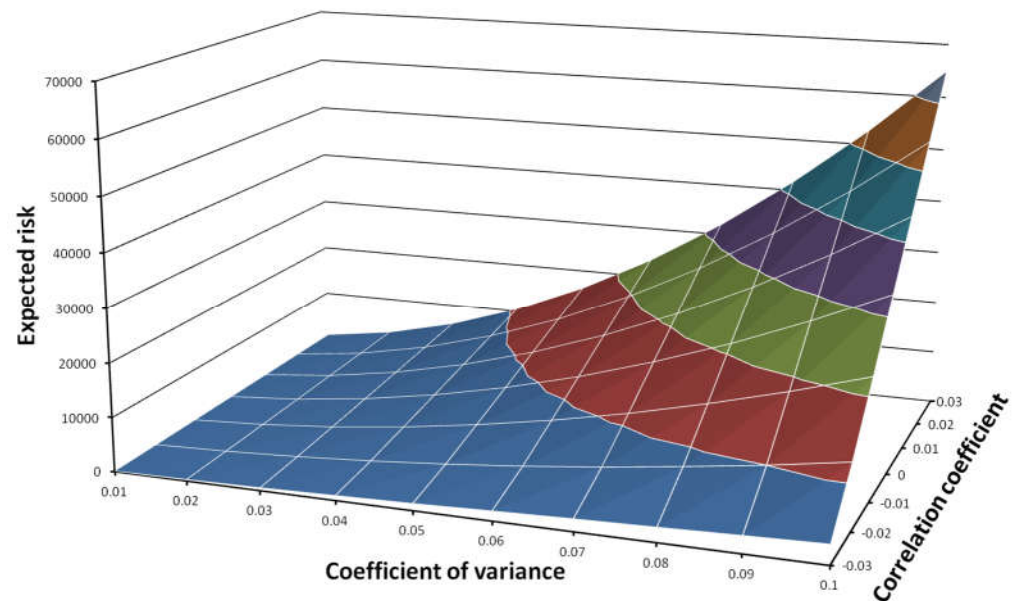


Figure 12. Expected risk versus correlation coefficient $R_{P_i P_j}$ and coefficient of variance C_{P_i} .

6.6. Sensitivity

There are no factors that can be changed or tuned for the suggested algorithm because the sole component that impacts algorithm acceleration, namely, α , is made adaptable in nature. This demonstrates the algorithm's independence.

6.7. Conclusion

The investigation on the fluctuation in predicted costs and pollutant gas emissions as a result of random measurements in generator characteristics and erratic load demand is presented in this paper. Stochastic economic load dispatch and stochastic economic-emission load demand example case studies are analysed and presented. In order to find the most comprehensive answers to the optimization problem, the opposition-based greedy heuristic search (OGHS) method is suggested in this study. The primary advantage of this search method is that it can be applied easily to solve any optimization problem, regardless of its complexity. The process begins with good population and struggles for the better member during mutation and has ability of migrating a new member or it's opposite to preserve diversity. This demonstrates the greedy conduct of the algorithm. Heuristics constrains the movement of solution procedure within feasible region owing to the physical limitation of the system components involved. Direct constraint handling procedure guarantees to satisfy the equality constraints. This method is efficiently applied to different examples of economic load dispatch which intrinsically are having discontinuities and non-differentiability in their objective functions. The suggested algorithm doesn't have the necessity of tweaking the algorithm parameters as these are made adaptive. The present analysis shows the risk involved due to the randomness of parameters. Furthermore, in the study of the randomness of parameters for the cases in which valve point loading effect is also considered. The expected operating cost of generators have percentage rise with respect to the correlation coefficient and coefficient of variance due to dependency on the sinusoidal term of expected cost function.

Table 5. Percentage deviation in expected fuel cost, expected emission of pollutants and the expected risk objectives for various correlation coefficient, $R_{P_i P_j}$ and coefficient of variance, C_{P_i} for 40 generators power system.

	Example-2				Example-3				Example-4				Example-5			
$R_{P_i P_j}$	-0.03	-0.03	0.03	0.03	-0.03	-0.03	0.03	0.03	-0.03	-0.03	0.03	0.03	-0.03	-0.03	0.03	0.03

C_{P_i}	0.01	0.10	0.01	0.10	0.01	0.10	0.01	0.10	0.01	0.10	0.01	0.10	0.01	0.10	0.01	0.10
μ	0.242410.59236	^{0.0964} ₇	0.09673	0.24844	0.5744	0.09773	0.09527	0.2819	0.56758	0.16486	0.17406	0.24303	0.57783	0.08949	0.10196	
F_1	120932.76341.4	14561	161278.	121004	76224.3	142196.	159679.	123506.	83924.2	143967.2	157466	120961.	77070.3	144933.	163569.	
	9	1	8.4	9	8	4	5	8	1			4		8	4	
F_2	--	--	--	--	--	--	--	--	--	--	--	377780.	819709.	361426.	946159.	
												5	9	4	4	
F_3	63.978	3632.68	655.51	64959.4	61.974	3608.77	657.310	64970.1	59.222	3469.93	681.510	67400.5	63.678	3548.46	656.007	64945.4
		5	7	5		1		7		3		1		6		8
P_1	114	114	^{113.99} ₉	114	114	114	114	114	114	114	114	114	114	114	114	114
P_2	114	114	114	114	113.999	114	114	114	114	114	114	114	114	114	114	114
P_3	97.399	120	120	119.999	97.399	120	120	120	120	120	120	120	97.400	120	120	120
P_4	190	190	190	190	190	190	190	190	190	190	189.999	190	190	190	190	190
P_5	92.661	97	97	96.999	88.584	97	97	97	89.940	97	97	96.999	97	97	97	97
P_6	140	140	140	140	140	140	140	139.999	140	140	140	140	140	140	139.999	140
P_7	300	298.062	300	300	300	300	300	300	300	300	300	300	300	300	300	300
P_8	300	300	300	300	300	300	300	300	300	300	300	300	300	300	300	300
P_9	300	300	300	300	300	300	300	300	300	300	300	300	300	300	300	300
P_{10}	204.799	300	300	300	204.800	300	300	300	279.601	300	300	300	204.799	300	300	300
P_{11}	243.599281.829	^{374.99} ₆	375	243.602	287.731	374.996	375	312.123	289.766	374.997	375	168.799	285.442	375	375	
P_{12}	243.600281.892	^{374.99} ₆	375	243.601	287.638	374.995	375	243.605	289.542	375	375	243.601	286.03	375	375	
P_{13}	214.761343.792	^{379.98} ₃	371.383	304.532	353.289	385.665	369.543	125	336.612	125.002	125	304.521	350.262	375.107	371.126	
P_{14}	304.525344.066	^{381.16} ₂	371.038	304.54	353.575	387.457	371.442	304.522	352.204	356.596	351.46	393.567	350.526	381.79	371.058	
P_{15}	394.208	343.95	378.97	370.768	304.523	353.736	386.232	370.082	304.52	352.471	387.546	377.36	304.528	351.045	382.536	369.48
P_{16}	304.524344.483	^{381.97} ₈	370.439	304.523	353.668	387.787	369.466	389.573	352.848	394.196	391.019	304.52	350.793	379.304	370.989	
P_{17}	472.753	436.88	^{399.53} ₆	415.982	472.782	445.617	399.626	417.998	468.46	442.615	436.579	428.915	471.495	442.779	399.566	414.418
P_{18}	472.88	436.896	^{399.53} ₆	416.426	472.063	445.704	399.526	418.598	469.621	442.602	448.043	438.111	471.123	277.607	399.524	414.202
P_{19}	488.028458.146	^{421.52} ₃	423.99	487.875	467.159	421.501	423.465	484.425	463.939	439.668	437.526	486.078	465.179	421.728	419.974	
P_{20}	487.89	458.191	^{421.51} ₄	421.970	487.407	466.994	421.523	426.397	484.332	463.699	433.587	431.973	486.230	465.169	421.455	418.848
P_{21}	503.319470.264	^{433.55} ₂	357.759	503.383	310.155	350.389	355.826	499.745	475.681	441.538	435.176	501.114	476.737	433.063	358.789	
P_{22}	503.059301.225	348.48	358.45	502.325	479.091	433.577	355.946	499.414	475.696	434.679	431.346	501.579	477.067	349.466	359.066	
P_{23}	503.886470.540	^{433.50} ₀	356.590	503.221	479.002	349.842	355.878	499.454	475.666	433.893	433.148	502.504	307.258	433.502	360.434	
P_{24}	503.745470.411	^{347.19} ₉	356.803	503.133	310.063	348.049	358.104	498.056	475.438	433.527	402.755	501.96	476.798	347.208	357.760	
P_{25}	502.528	470.64	^{345.97} ₁	357.627	501.941	479.043	349.95	355.950	495.081	475.241	343.872	362.629	500.240	477.001	344.724	358.718
P_{26}	502.266470.468	^{347.39} ₁	356.257	502.599	479.047	433.589	355.616	495.791	475.478	433.507	369.820	500.204	477.364	433.469	361.324	
P_{27}	10.004	50.855	^{111.04} ₆	150	10.001	47.719	100.334	150	10.01	91.064	112.542	150	10	48.745	107.514	150
P_{28}	10	50.822	^{111.61} ₉	150	10.004	47.680	100.691	150	10	91.331	105.843	150	10	48.754	110.498	150
P_{29}	10	50.806	^{113.55} ₅	150	10	47.807	100.705	150	10	57.938	107.775	150	10	48.688	111.128	150

P ₃₀	88.808	97	96.965	96.999	97	97	96.994	96.999	96.998	97	97	96.999	89.521	97	97	97
P ₃₁	190	190	190	190	190	190	190	190	190	190	190	189.999	190	190	190	190
P ₃₂	190	190	190	190	190	190	190	190	190	190	190	189.999	190	190	190	190
P ₃₃	190	190	190	190	190	190	190	190	190	190	189.999	190	190	190	190	190
P ₃₄	164.802	199.999	200	200	164.808	200	200	200	200	200	200	200	164.816	200	200	200
P ₃₅	164.808	168.663	^{199.99} ₈	200	164.827	200	199.999	200	200	200	199.999	200	164.799	200	200	200
P ₃₆	164.802	166.814	200	200	164.853	166.176	200	200	200	200	200	200	164.804	200	200	200
P ₃₇	110	110	110	110	110	110	110	110	110	110	109.999	110	110	110	110	110
P ₃₈	110	110	^{109.99} ₉	109.999	110	110	110	110	110	110	110	110	109.999	110	109.999	110
P ₃₉	110	110	110	110	110	110	109.999	110	110	110	110	110	110	110	110	110
P ₄₀	488.362	458.305	^{421.49} ₈	422.517	487.672	467.102	421.57	423.688	484.481	463.631	435.942	434.244	486.796	464.756	346.419	421.814

Table 6. Generation schedule along with objective function values for the case of independent variables (deterministic case).

Example-2				Example-3				Example-4				Example-5			
F ₁	121411.9			F ₁	121411.9			F ₁	123468.3			F ₁	121685.8		
F ₂	--			F ₂	--			F ₂	--			F ₂	353785.9		
P ₁	110.8136	P ₂₁	523.2806	P ₁	110.8136	P ₂₁	523.2806	P ₁	113.0465	P ₂₁	523.2803	P ₁	110.8006	P ₂₁	522.4756
P ₂	110.8384	P ₂₂	523.2811	P ₂	110.8384	P ₂₂	523.2811	P ₂	114	P ₂₂	523.2794	P ₂	110.7995	P ₂₂	522.6317
P ₃	97.40414	P ₂₃	523.2807	P ₃	97.40414	P ₂₃	523.2807	P ₃	120	P ₂₃	523.2797	P ₃	97.39978	P ₂₃	523.2175
P ₄	179.737	P ₂₄	523.2797	P ₄	179.737	P ₂₄	523.2797	P ₄	179.7334	P ₂₄	523.2796	P ₄	179.7325	P ₂₄	523.0742
P ₅	87.8691	P ₂₅	523.2832	P ₅	87.8691	P ₂₅	523.2832	P ₅	97	P ₂₅	523.2802	P ₅	87.79984	P ₂₅	521.5883
P ₆	140	P ₂₆	523.2806	P ₆	140	P ₂₆	523.2806	P ₆	140	P ₂₆	523.2794	P ₆	140	P ₂₆	520.914
P ₇	259.5999	P ₂₇	10	P ₇	259.5999	P ₂₇	10	P ₇	259.602	P ₂₇	10	P ₇	300	P ₂₇	10
P ₈	284.6023	P ₂₈	10	P ₈	284.6023	P ₂₈	10	P ₈	284.6002	P ₂₈	10	P ₈	284.598	P ₂₈	10
P ₉	284.6008	P ₂₉	10	P ₉	284.6008	P ₂₉	10	P ₉	284.6064	P ₂₉	10	P ₉	284.5977	P ₂₉	10
P ₁₀	130	P ₃₀	96.99962	P ₁₀	130	P ₃₀	96.99962	P ₁₀	130	P ₃₀	96.99976	P ₁₀	204.6555	P ₃₀	87.80145
P ₁₁	168.8007	P ₃₁	190	P ₁₁	168.8007	P ₃₁	190	P ₁₁	168.8	P ₃₁	190	P ₁₁	168.7987	P ₃₁	159.7331
P ₁₂	94	P ₃₂	190	P ₁₂	94	P ₃₂	190	P ₁₂	243.5995	P ₃₂	190	P ₁₂	168.7997	P ₃₂	190
P ₁₃	214.7596	P ₃₃	190	P ₁₃	214.7596	P ₃₃	190	P ₁₃	125	P ₃₃	190	P ₁₃	214.7599	P ₃₃	189.9993
P ₁₄	394.2798	P ₃₄	164.8072	P ₁₄	394.2798	P ₃₄	164.8072	P ₁₄	304.5193	P ₃₄	200	P ₁₄	304.5194	P ₃₄	200
P ₁₅	394.279	P ₃₅	199.9995	P ₁₅	394.279	P ₃₅	199.9995	P ₁₅	394.2793	P ₃₅	199.9995	P ₁₅	304.5186	P ₃₅	199.9986
P ₁₆	304.5201	P ₃₆	199.9995	P ₁₆	304.5201	P ₃₆	199.9995	P ₁₆	394.2794	P ₃₆	199.9995	P ₁₆	304.5188	P ₃₆	199.9995
P ₁₇	489.2791	P ₃₇	110	P ₁₇	489.2791	P ₃₇	110	P ₁₇	489.2794	P ₃₇	110	P ₁₇	489.2798	P ₃₇	109.9988
P ₁₈	489.2805	P ₃₈	110	P ₁₈	489.2805	P ₃₈	110	P ₁₈	489.2805	P ₃₈	110	P ₁₈	489.2786	P ₃₈	110
P ₁₉	511.281	P ₃₉	110	P ₁₉	511.281	P ₃₉	110	P ₁₉	511.2792	P ₃₉	110	P ₁₉	511.2686	P ₃₉	110
P ₂₀	511.2793	P ₄₀	511.2798	P ₂₀	511.2793	P ₄₀	511.2798	P ₂₀	511.2791	P ₄₀	511.281	P ₂₀	511.2517	P ₄₀	511.187

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