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*Article*

# Introducing Non-hierarchical RSM and MIGA for Performance Prediction and Optimization of A Centrifugal Pump under the Nominal Condition

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**Abstract:** In order to improve the operation performance of the multi-stage double-suction centrifugal pump and reduce the internal energy loss of the pump, this paper proposes a single-objective optimization design method based on non-hierarchical response surface model (RSM) and the multi-island genetic algorithm (MIGA). Nine parameters, such as the blade outlet width and blade wrap angle, were used as design variables, and the optimization objective was the efficiency under design conditions. In total, 149 sets of valid data were obtained under the latin hypercube sampling method (LHS), the corresponding thresholds were set for efficiency and head, and 99 sets of valid data were obtained. A cross-validation analysis of the sieved data was carried out based on non-hierarchical RSM, global optimization of the efficiency was carried out using MIGA, and numerical verification was carried out via CFD. The research results show that compared with hierarchical RSM, non-hierarchical RSM can approximate the nonlinear relationship between the objective function and the design variables with higher accuracy, and the model fitting $R^2$ value was 0.919. The efficiency was improved by 3.717% after optimization. The overall prewhirl of the impeller inlet after optimization decreased, the internal speed of the volute significantly improved, the large-area vortex at the volute and the outlet pipe was eliminated, the impact loss at the volute separating tongue disappeared, and the overall hydraulic performance of the pump was improved. The total entropy output value of the optimized pump was reduced by 4.79 (W/K), mainly concentrated in the reduction in the entropy output value of the double volute, and the overall energy dissipation of the pump was reduced.

**Keywords:** multi-stage double-suction centrifugal pump; non-hierarchical RSM; MIGA; optimization

## 1. Introduction

As general mechanical equipment in the field of fluid machinery, pumps are widely used in production and life for the purpose of conveying fluid media. For the multi-stage double-suction centrifugal pumps used in the fields of sewage treatment, water diversion irrigation, and industrial water supply, during the operation of large flow and high head, due to the complexity of the structure, it is easy to cause internal flow disorder, resulting in low overall efficiency of the pump [1-2].

However, current pump manufacturers and users have higher and higher requirements for pump performance, and obtaining a high-efficiency pump type has become key. In the field of hydraulic machinery, the use of numerical simulation methods to optimize the mechanical properties of pumps has been widely used [3]. Traditional pump design occurs via a combination of numerical calculations and experiments; the design process is very complicated, and the calculation process takes a long time. At present, with intelligent optimization algorithms being applied more and more widely, optimization design that combines numerical calculations and an intelligent optimization algorithm is also very common. The operational speed and accuracy of this combination method are greatly improved compared with those of the original model. This can reduce the labor and experimental costs, and a better pump model is ensured. Yunguang et al. [4] proposed to use a radial basis function (RBF) neural network to optimize the impeller of a turbo centrifugal pump, with sampling based on the latin hypercube sampling (LHS) method, the results show that the optimized model efficiency and head are improved compared with the original model 5.74% and 4.85%. Jie et al. [5] combined the Kriging model with numerical analysis to find the optimal design parameters for a torque converter impeller, thereby improving the performance of the torque converter. Jamshid et al. [6] proposed a hybrid analysis framework based on an artificial neural network (ANN) to evaluate the probability of failure of sewage pumping stations, which accurately predicted the safety margin of the pump and reduced the computational burden. Nataraj et al. [7] used response surface model (RSM) and computational fluid dynamics (CFD) to design an impeller to improve the performance of a centrifugal pump, resulting in a 2.06m increase in total head and a 65.22W reduction in power dissipation. Peng et al. [8] used RSM to study and optimize the jet pump, taking the pressure amplitude and time-averaged power dissipation of a jet pump as a response to achieve maximum pressure amplitude and minimum power consumption. The final results showed that RSM is feasible as an evaluation method for optimizing jet pumps. Khaled et al. [9] optimized the efficiency of a pump based on RSM and the multi-objective genetic algorithm, and they used geometric parameters including the number of blades, impeller speed, etc. As design variables to predict the performance of the pump under stable and transient conditions, and also predict corrosion. The Kriging model, radial basis neural network, and artificial neural network are generally applicable to occasions with a large sample size, while RSM is suitable for occasions with a small sample size, which can obtain better fitting accuracy and randomness in the case of more design variables [10].

As a commonly used statistical analysis technique, RSM has the characteristics of strong applicability and wide application range, among which it can effectively locate the individual effects and interactions between parameters [11]. In the optimization process with many design variables, there is generally a high-intensity nonlinear programming between the objective function and the design variables. At present, most scholars use low-order polynomial functions to fit the objective function. Tijana et al. [12] studied the usefulness of combinations based on RSM and ANN in characterization, modeling, and optimization, and they found better results with the prediction of second-order polynomial functions by comparing the fitted $R^2$values of linear and second-order response surface polynomial functions. Xuhe et al. [13] proposed an optimization strategy for developing a turbine runner model based on CFD technology, a second-order response surface model and a multi-objective genetic algorithm. Taking six geometric parameters such as the blade load as design variables, some design problems of the turbine runner are effectively solved, and the calculation cost is reduced. Han et al. [14] proposed an integrated method based on second-order RSM and the genetic algorithm to analyze the influence of various parameters of the standpipe inlet and outlet and obtain an optimal design; finally, the total head loss coefficient and the inflow and outflow velocity distribution coefficients were reduced by 4.687%, 11.765%, and 38.596%, respectively. However, compared with low-order polynomial functions, using high-order polynomial functions to fit functions can obtain higher prediction accuracy. In order to obtain more reliable test data for an air source heat pump, Eleonora et al. [15] used fourth-order RSM to expand the data sample; they examined the performance of the air heat source pump by changing the water supply temperature of the indoor terminal under different environmental conditions, and they obtained the performance of the air source heat pump. An optimal configuration of the system was found, minimizing power consumption while maintaining interior comfort.

We can improve the quality of the higher-order RSM, by eliminating unnecessary terms, also reducing the uncertainty of model prediction, and improving the fitting accuracy. This kind of polynomial that randomly ignores some lower-level terms is called a non-hierarchical polynomial [16]. Nuo et al. [17] proposed an efficient stochastic model update method based on statistical theory and developed incomplete fourth-order polynomial RSM. Combining RSM with Monte Carlo Simulation (MCS) reduces computation and enables fast random sampling. Teppei et al. [18] applied interactive layered RSM to the parameter optimization of photonic crystal nanocavities, and they demonstrated the effectiveness of this method for parameter optimization.

In this paper, the improved response surface method is used to optimize the design of the impeller and volute of a multi-stage double-suction centrifugal pump. On the basis of CFD numerical simulation, the LHS method is used to generate data and filter them. Based on the improved fourth-order incomplete polynomial RSM, the nonlinear relationship between the efficiency and geometric parameters is established, using the multi-island genetic algorithm (MIGA) to solve an optimal point of efficiency and carry out numerical verification, the pump performance before and after optimization is compared and analyzed to obtain the impeller and volute design parameters with higher fitting accuracy and better running stability. The state and characteristics of the internal and external flow fields before and after pump optimization, and an analysis of the entropy production performance are presented.

## 2. Pump model parameters and computational method

### *2.1. Hydraulic Model*

The first-stage single-suction impeller and the secondary double-suction impeller of the multi-stage double-suction centrifugal pump use the same impeller hydraulic model. In order to better eliminate the radial force of the impeller when the pump is running, the volute of the flow passage adopts a double volute design. At the design operating point, the design performance parameters of the pump are: Q=540m3/h, design head H=132m, speed n=1490r/min, specific speed ns=64. The formula for calculating the specific speed is as follows:

$$n_s = \frac{3.65nQ^{1/2}}{H^{3/4}} \tag{1}$$

The overall 3D pump fluid domain was modeled in the model design software UG NX, as shown in Figure 1. After the water enters the suction chambers on both sides, it flows into the middle symmetrical flow channel perpendicular to the axis through the single-suction impellers on both sides, then flows into the double-suction impeller from the flow channel, and finally discharges through the middle pressure water chamber. The specific details of the flow through the impeller are shown in Figure 2. The main design parameters of the multi-stage double-suction centrifugal pump are shown in Table 1.

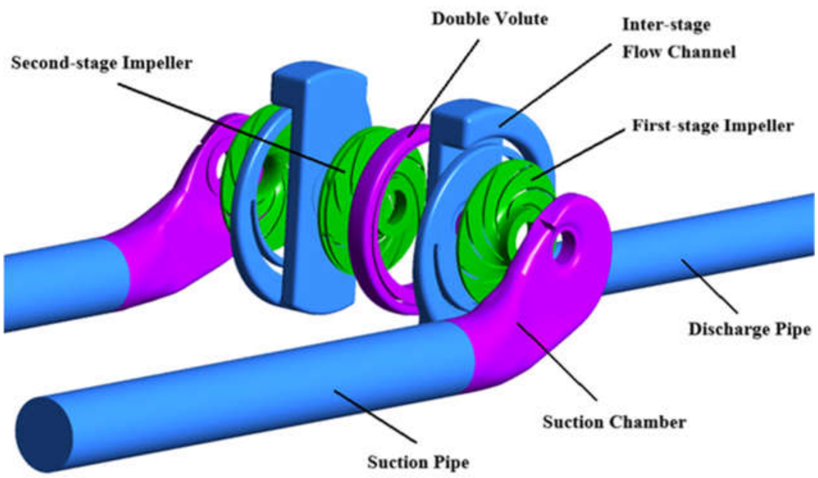


**Figure 1.** Computational domain of the multi-stage double-suction centrifugal pump.

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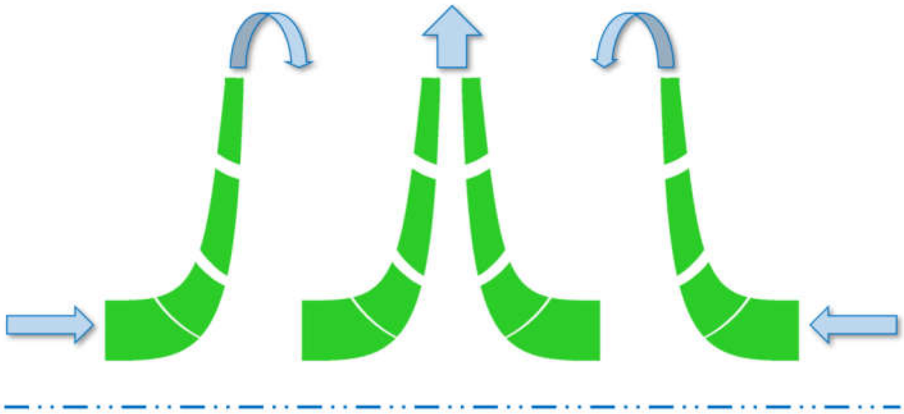

**Figure 2.** Schematic diagram of water flow through the impeller.

**Table 1.** Main design parameters of the multi-stage double-suction centrifugal pump.

| Parameter | Abbreviation | Value |
|---|---|---|
| Flow rate | $Q$ | 540m³/h |
| Head | $H$ | 132m |
| Rotating speed | $n$ | 1490r/min |
| Specific speed | $n_s$ | 54.09 |
| Impeller inlet diameter | $D_1$ | 196mm |
| Impeller outlet diameter | $D_2$ | 485mm |
| Impeller outlet width | $b_2$ | 15mm |
| Volute inlet width | $b_3$ | 70mm |
| Volute inlet diameter | $D_3$ | 495mm |
| Volute outlet diameter | $D_4$ | 200mm |
| Number of blades | $Z$ | 8 |

*2.2. Mesh Generation and Numerical Calculation*

For the multi-stage double-suction centrifugal pump, due to the complexity of its double volute internal structure, ANSYS ICEM was used to generate unstructured meshes. The impeller, suction chamber, interstage runner and other components were based on the commercial software TurboGrid with high precision and good convergence performance of high-quality structural grid. In order to better satisfy the subsequent high-precision flow field analysis and more accurately characterize the complex flow phenomena around the solid wall, the mesh of the solid surface was refined. Part of the computational domain grid is shown in Figure 3.

The CFD in the commercial software ANSYS CFX was used to study and analyze the hydraulic characteristics of the pump. The turbulence model adopted was the shear stress transfer model (SST k-ω) widely used in multi-stage double suction centrifugal pumps [19-20]. In order to meet the requirements of the above turbulence model, the maximum y+ of the walls of the volute tongue and the impeller blade was less than 10, and the maximum y+ of the other walls was less than 50.

In the independence analysis of the effect of the number of grid cells on the numerical calculation results, a total of five groups of independent grid numbers were generated, and the calculation results of the corresponding lift and efficiency are shown in Figure 4. After grid-independence analysis, the grid size was finally determined. The final number of cells was 130.858×105. The boundary conditions of the pump were set to total pressure inlet and mass flow outlet. The computational domain generated a total of 11 networks

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including the suction chamber, suction pipe, first-stage impeller, inter-stage flow channel, second-stage impeller, double volute and outlet pipe grid. The number of grid cells for each computational domain is shown in Table 3.

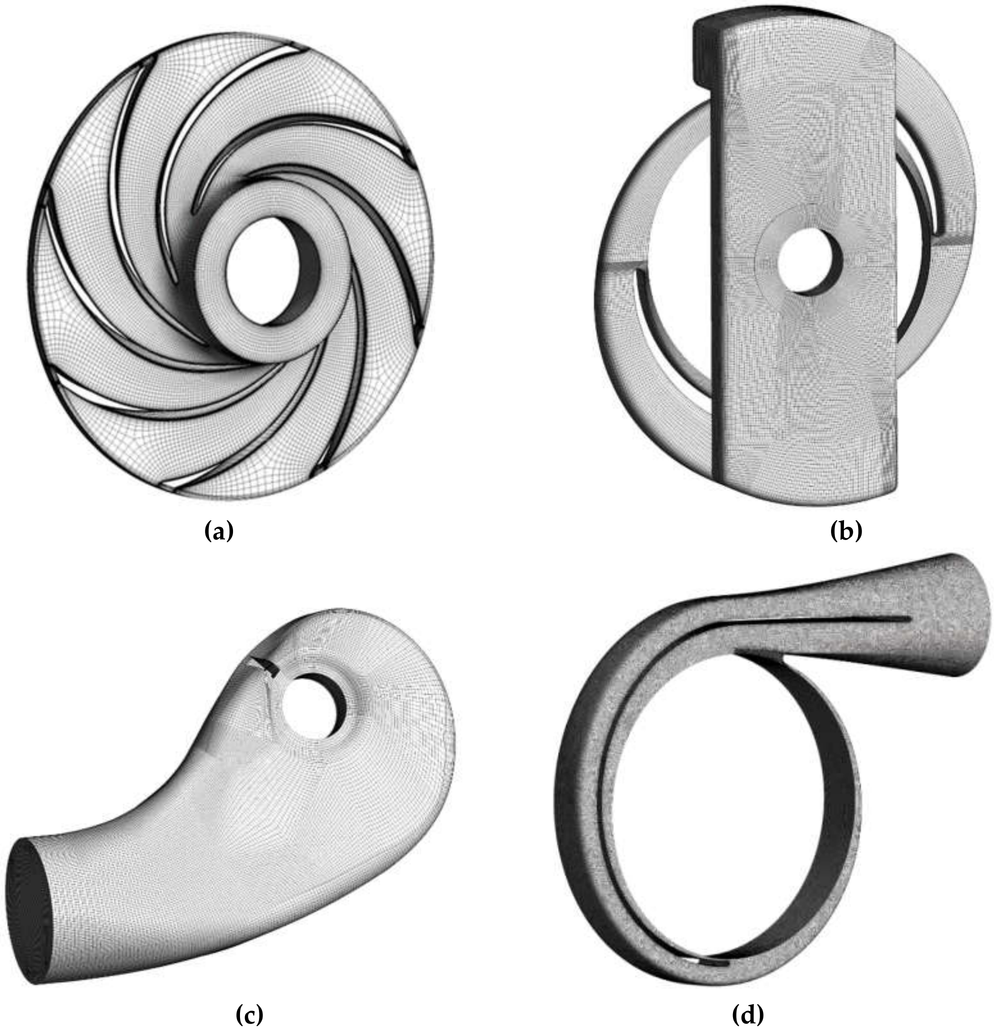


**Figure 3.** Meshes of the calculation domain: the impeller **(a)**, the inter-stage flow channel **(b)**, the suction chamber **(c)**,and the volute **(d)**.

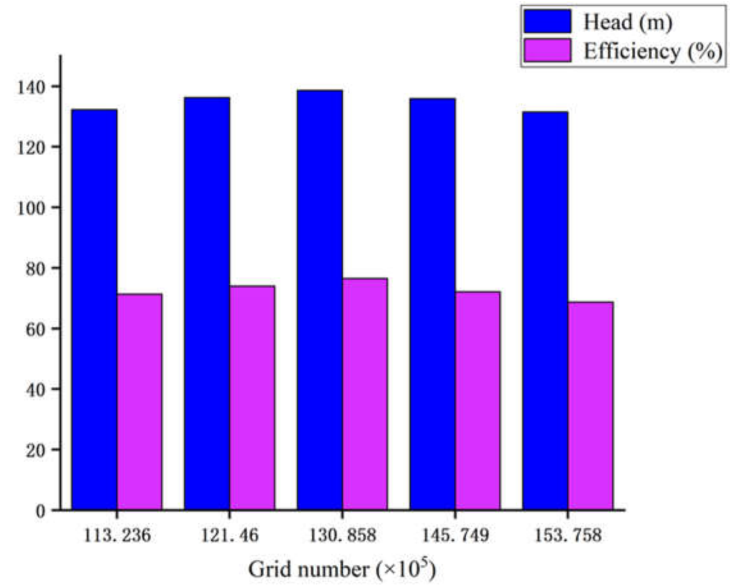


**Figure 4.** Result of grid independence analysis.

**Table 2.** The number of grid cells for each computational domain.

| Domain | Number of grid cells(×10⁵) | Number |
|---|---|---|
| Suction pipe | 1.67 | 2 |
| Suction chamber | 7.78 | 2 |
| Impeller | 12.45 | 4 |
| Inter-stage flow channel | 12.78 | 2 |
| Double volute | 27.10 | 1 |
| Discharge pipe | 9.48 | 1 |

*2.3. Experimental verification*

A comparison between the test results and the numerical calculation results is shown in Figure 5. It can be seen from the figure that the trends of the test curve and the numerical calculation curve are almost the same. Since the energy loss generated by the pump itself was not fully considered during the test, the test results of head and efficiency were generally lower than the numerical calculation results. At the design operating point, the numerical calculation result of the pump was 76.512%, the test result was 73.705%, and the absolute error of the two was 2.807%. Under non-design conditions, the error between the numerical calculation results and the experimental results did not increase greatly, so the numerical simulation method in this paper is reliable and can be used for subsequent optimization studies.

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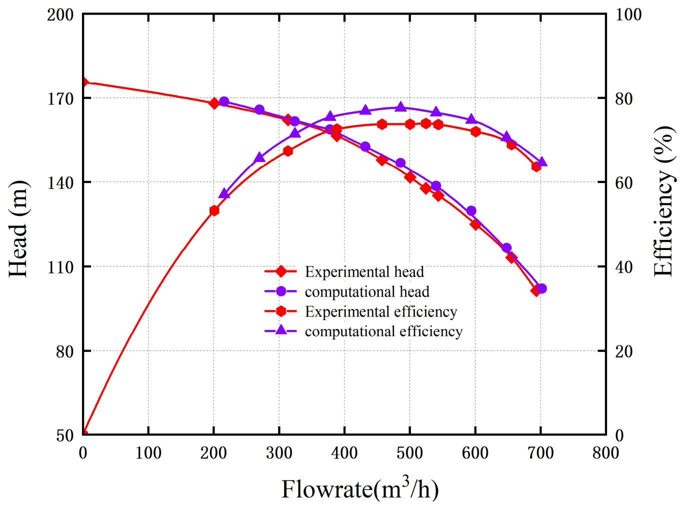


**Figure 5.** Comparison of experimental results and numerical calculation results.

## 3. Optimization process

Figure 6 presents the optimization flow chart for this paper. The efficiency under the design condition of the multi-stage double-suction centrifugal pump was selected as the optimization objective, the nine design parameters of the pump were used as the optimization variables, and respective boundary conditions were set for the nine variables. The latin hypercube sampling (LHS) method was used to generate 149 groups of valid sample data, the performance of the original scheme was compared, the data were screened, the functional relationship between the objective function and the design variables was established, and the objective function was fitted based on the improved response surface model (RSM) using multi-island genetic algorithm (MIGA). This algorithm finds the optimal efficiency point for CFD verification and finally obtains the optimal geometric parameter design of the volute and the impeller.

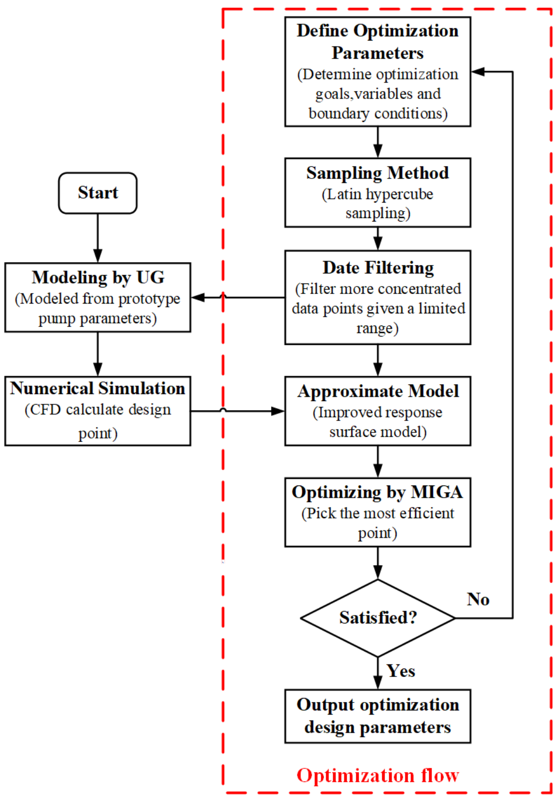


**Figure 6.** Optimization flow chart.

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### 3.1. Optimization objective

Due to the long-term and continuous operation of the pump and the frequent operation under low load or variable load, the operating point of the pump easily deviates from the high-efficiency area; the operating efficiency of the pump is then greatly reduced, and a large amount of energy is wasted. In order to save energy and reduce the internal energy loss, so as to improve the operating efficiency of the two-stage split centrifugal pump, this paper takes the efficiency at the design operating point as the optimization goal. The efficiency equation is as follows:

$$\eta = \frac{Q_d}{3600} \times \frac{P_{2t} - P_{1t}}{T \times \omega} \quad (2)$$

where $Q_d$ is the flow rate at the design operating point, m$^3$/h; $P_{2t}$ and $P_{1t}$ are the total pressure at the inlet and outlet respectively, Pa; $T$ is the torque of the impeller, $N\cdot m$; and $\omega$ is the rotational speed of the impeller, rad/s.

### 3.2. Design variables and parameter ranges

Since this paper only addresses the design and optimization of the blade profile, in order to reduce the complexity and error of the overall calculation, the diameter of the impeller inlet and outlet and the thickness of the blade were kept unchanged. There were nine design variables to be optimized and controlled, and the range of each design variable is shown in Table 3. In the table, $x_1$ represents the outlet width of the blade, which is used to control the variation range of the size of the impeller on the axial projection diagram. $x_2$ and $x_3$ represent the inlet placement angles of the rear and front cover plates of the blade, respectively, and $x_4$ and $x_5$ represent the outlet placement angles of the rear and front cover plates of the blade, respectively. The blade wrap angle was set as the design variable $x_6$; variable $x_7$ is the Stepanoff number that controls the change of the cross-sectional area in the volute; and $x_8$ and $x_9$ represent the volute inlet width and the starting position of the volute baffle, respectively.

**Table 3.** Boundary range of design parameters.

| Design parameter | Lower limit | Upper limit |
|---|---|---|
| $x_1$ | 10 | 20 |
| $x_2$ | 25 | 35 |
| $x_3$ | 20 | 30 |
| $x_4$ | 20 | 30 |
| $x_5$ | 20 | 30 |
| $x_6$ | 115 | 135 |
| $x_7$ | 0.15 | 0.3 |
| $x_8$ | 70 | 90 |
| $x_9$ | 150 | 180 |

### 3.3 Latin hypercube sampling method

As an important step in the optimization process of an experimental design, it is necessary to choose an appropriate sampling technique. Since there are many variables in this optimization design, in order to obtain better space filling randomness, accuracy, and robustness for the sample parameters. The LHS method was used to generate 149 sets of valid data for the defined nine variables and the range of the design variables. In order to further reduce the error of the sample and obtain more concentrated data sample points, thereby improving the convergence and fitting accuracy of the data, corresponding thresholds were set for the head and efficiency in the sample data. Set the threshold of head to 135m, and the threshold of efficiency to 78%. Finally, three partial data sets as shown in Figure 7 were screened out; the 99 groups of valid data screened in the second part were selected for subsequent model training and prediction.

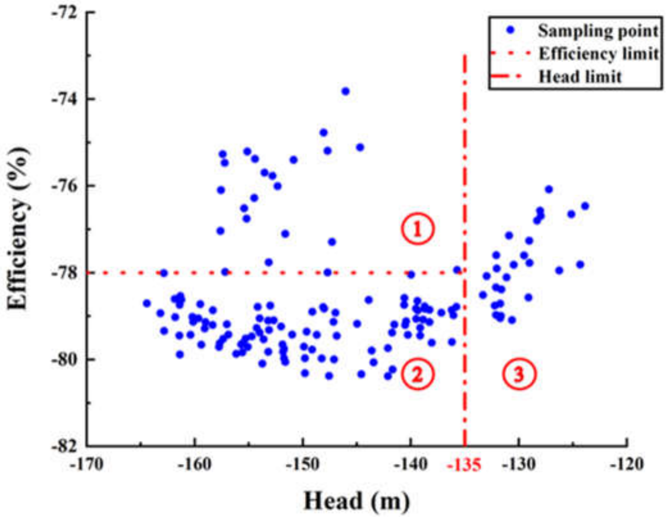


**Figure 7.** Sample data screening diagram.

*3.4. Non-hierarchical response surface methodology(RSM)*

As a common approximation model established between the objective function and design variables, RSM has multiple selectable polynomial orders, such as first- (linear), second-, third-, and fourth-order polynomial functions. Based on the multi-parameter optimization design in this paper, in order to improve the accuracy of the model prediction results, the fourth-order response surface model was selected for fitting calculations. The fourth-order response surface polynomial function expression is as follows:

$$f(x) = a_0 + \sum_{i=1}^{n} b_i x_i + \sum_{ij(i<j)} c_{ij} x_i x_j + \sum_{i=1}^{n} d_i x_i^2 + \sum_{i=1}^{n} e_i x_i^3 + \sum_{i=1}^{n} g_i x_i^4 \quad (3)$$

where $x$=($x_1$,$x_2$,…,$x_n$), $x_i$(i=1,2,…,n) are design variables, $a_0, b_i, c_{ij}, d_i, e_i, g_i$ are the regression coefficients of each polynomial, and the number of hierarchical polynomials is 1+9+(81-9)/2+9+9+9=73. The non-hierarchical RSM was selected in this paper to use non-hierarchical polynomials to analyze and verify the accuracy of the model.

*3.5 Optimization algorithm*

As an improved genetic algorithm based on the traditional genetic algorithm, MIGA is a pseudo-parallel genetic algorithm based on population grouping. The function of diversity and preventing premature maturity solves the problem that traditional genetic algorithms have in which are prone to falling into local optima [21-22].

Based on the 99 groups of sample data obtained by screening the original data, the above-mentioned fourth-order response surface polynomial function was used to establish the relationship between the optimization objective and the design variables, the MIGA was used for optimization, and the performance of the impeller was finally verified. The parameter settings of the optimization algorithm are shown in Table 4.

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**Table 4.** The parameter settings of the optimization algorithm.

| Option | Value |
|---|---|
| Sub-population size | 10 |
| Number of islands | 10 |
| Number of generations | 50 |
| Rate of crossover | 1.0 |
| Rate of mutation | 0.01 |
| Elite size | 1 |
| Rel tournament size | 0.5 |
| Penalty multiplier | 1000 |
| Penalty exponent | 2 |
| Default variable bound | 1000 |
| Max failed runs | 5 |

## 4. Results

### *4.1. Approximate model fit accuracy*

In order to verify the accuracy of the approximate model, this paper compares the accuracy of the model prediction of the third-order and fourth-order response surface polynomials in the hierarchical and non-hierarchical models. The $R^2$ value is used to represent the degree of agreement between the approximate model and the sample points. The closer the value is to 1, the higher the prediction accuracy of the approximate model. For the fourth-order response surface polynomial, the number of polynomials when layered is 73, and for the third-order response surface polynomial, the number of polynomials when layered is 64. Figure 8 a, b presents the corresponding $R^2$ values of the third-order and fourth-order polynomials under the hierarchical polynomial. It can be seen that the $R^2$ value is higher under the third-order hierarchical fitting, and the fitting effect is better. Figure 8 c, d presents the $R^2$ values corresponding to 40 third-order and fourth-order polynomials without layering. In this optimization process, the cross-validation method was used for error analysis, and 50 groups of random data were selected for cross-validation error analysis. At the same time, automatic three-dimensional modeling and numerical simulation were performed on these 50 groups of data, and the corresponding calculation results were finally obtained. After many instances of repeated training, it can be seen that the fitting effect of the fourth-order non-hierarchical model is better than that of the third-order model. The combined accuracy is higher than that of layering.

In this model verification, when the fourth-order model is non-hierarchical, and when the number of polynomials is selected as 40, a higher fitting accuracy can be obtained. Table 5 shows the design variable values before and after optimization. The efficiency of the optimal scheme is 80.939%, which is 4.427% higher than the 76.512% before optimization. The efficiency value verified by CFD is 80.229%, and the relative error is 0.88%. Therefore, the optimization model has good reliability and can be accurately used for pump performance prediction.

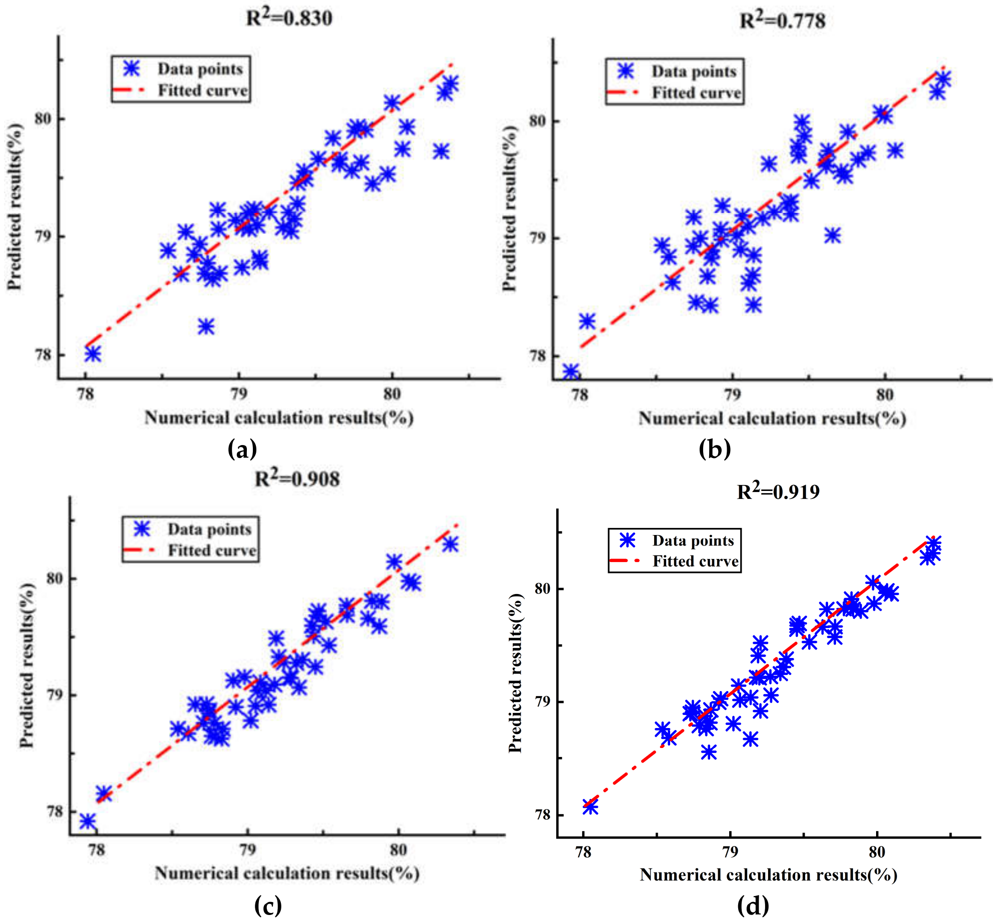


**Figure 8.** Corresponding values of $R^2$ for polynomials: cubic hierarchy **(a)**, quartic hierarchy **(b)**, cubic non-hierarchy **(c)**, quartic non-hierarchy **(d)**.

**Table 5.** Design variable values before and after optimization.

| Variables | $b_2$/mm | $\beta_{1h}$/° | $\beta_{2h}$/° | $\beta_{1s}$/° | $\beta_{2s}$/° | $\varphi$/° | $K_s$ | $b_3$/mm | $\theta$/° |
|---|---|---|---|---|---|---|---|---|---|
| Original | 15 | 33.82 | 25.38 | 25.53 | 25.15 | 124 | 0.2982 | 70 | 190 |
| Optimal | 14.74 | 32.58 | 26.15 | 20.03 | 26.58 | 127.74 | 0.1992 | 70.01 | 165.32 |

*4.2 Sensitivity analysis*

In order to verify the influence of the design variables on the performance and efficiency of the pump, sensitivity analysis was carried out for the nine variables in the optimal design. Table 6 shows the corresponding coefficient values of each polynomial using the fourth-order non-hierarchical 40th-degree polynomial. It can be seen from the table that the coefficients of $x_1$, $x_4$, $x_6$, $x_8$, and $x_9$ are negative numbers; that is, the blade outlet width $b_2$, the blade front cover inlet placement angle $\beta_{1s}$, the blade wrap angle $\varphi$, the volute outlet width $b_3$, and the double-volute starting position $\theta$ of the diaphragm have a negative effect on the overall efficiency of the pump. The blade wrap angle $\varphi$ and the starting position $\theta$ of the diaphragm of the double volute have a significant impact on the hydraulic power of the pump. The blade outlet width, the blade front cover inlet placement angle, and the volute outlet width have little influence on the overall performance of the pump and can be almost ignored. Because the coefficients of $x_3$ and $x_7$ are positive values, the outlet placement angle $\beta_{2h}$ of the rear cover plate of the blade and the Stepanoff number $K_s$ have a positive impact on the overall efficiency of the pump, of which the Stepanoff number $K_s$ has a greater influence. The influence of the placement angle $\beta_{2h}$ at the outlet of the rear cover plate of the blade is small.

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**Table 6.** Corresponding coefficients for each polynomial.

| Term | Coefficient | Term | Coefficient | Term | Coefficient | Term | Coefficient |
|---|---|---|---|---|---|---|---|
| $x_1$ | -9.82 | $x_4^2$ | $1.98\times10^{-3}$ | $x_2x_8$ | $-3.43\times10^{-3}$ | $x_6^3$ | -0.04 |
| $x_3$ | 0.38 | $x_6^2$ | 7.26 | $x_3x_6$ | $-1.20\times10^{-3}$ | $x_7^3$ | $2.54\times10^{4}$ |
| $x_4$ | -0.49 | $x_7^2$ | $-9.13\times10^{3}$ | $x_3x_7$ | 0.19 | $x_8^3$ | $-1.56\times10^{-4}$ |
| $x_6$ | -612.25 | $x_8^2$ | $3.61\times10^{-2}$ | $x_3x_8$ | $-1.82\times10^{-3}$ | $x_9^3$ | $-4.11\times10^{-3}$ |
| $x_7$ | 1418.98 | $x_9^2$ | 1.01 | $x_4x_6$ | $1.60\times10^{-3}$ | $x_1^4$ | $1.07\times10^{-3}$ |
| $x_8$ | -2.79 | $x_1x_7$ | 2.28 | $x_4x_7$ | 0.22 | $x_2^4$ | -5.70 |
| $x_9$ | -111.13 | $x_1x_8$ | $-4.20\times10^{-3}$ | $x_7x_9$ | -0.09 | $x_6^4$ | 7.52 |
| $x_1^2$ | 1.19 | $x_1x_9$ | $3.00\times10^{-3}$ | $x_8x_9$ | $1.06\times10^{-3}$ | $x_7^4$ | $-2.67\times10^{4}$ |
| $x_2^2$ | -0.10 | $x_2x_4$ | $3.71\times10^{-3}$ | $x_1^3$ | -0.06 | $x_9^4$ | 6.23 |
| $x_3^2$ | $-2.41\times10^{-3}$ | $x_2x_7$ | 0.25 | $x_2^3$ | $4.49\times10^{-3}$ | *const* | $2.40\times10^{4}$ |

*4.3 Inner flow analysis*

Figure 9 presents a comparison of the impeller inlet peripheral speed before and after optimization. The inlet peripheral speed of the impeller has an important influence on the pump head. If the peripheral speed is too large, it easily forms a prewhirl at the inlet and affects the impeller head. In the steady calculation, due to the uneven distribution of the dual-inlet flow channels and the water suction chamber, for the second-stage impeller, the inter-stage flow channels have a great influence on the flow distribution. Under the centrifugal force of the first-stage impeller, the overall increase in the velocity distribution of the second-stage impeller is higher than that of the first- stage impeller. Compared with that before optimization, the peripheral velocity distribution of the first-stage impeller inlet shows almost no great change. After optimization, the average circumferential speed at the inlet of the second-stage impeller is 4.45[m s^-1], and the average circumferential speed of the second-stage impeller inlet of the original scheme is 6.14[m s^-1]. Thus, compared with the original model, the overall prewhirl of the impeller inlet has decreased, so the hydraulic performance of the impeller is been significantly improved, and the optimized second-stage impeller has improved significantly.

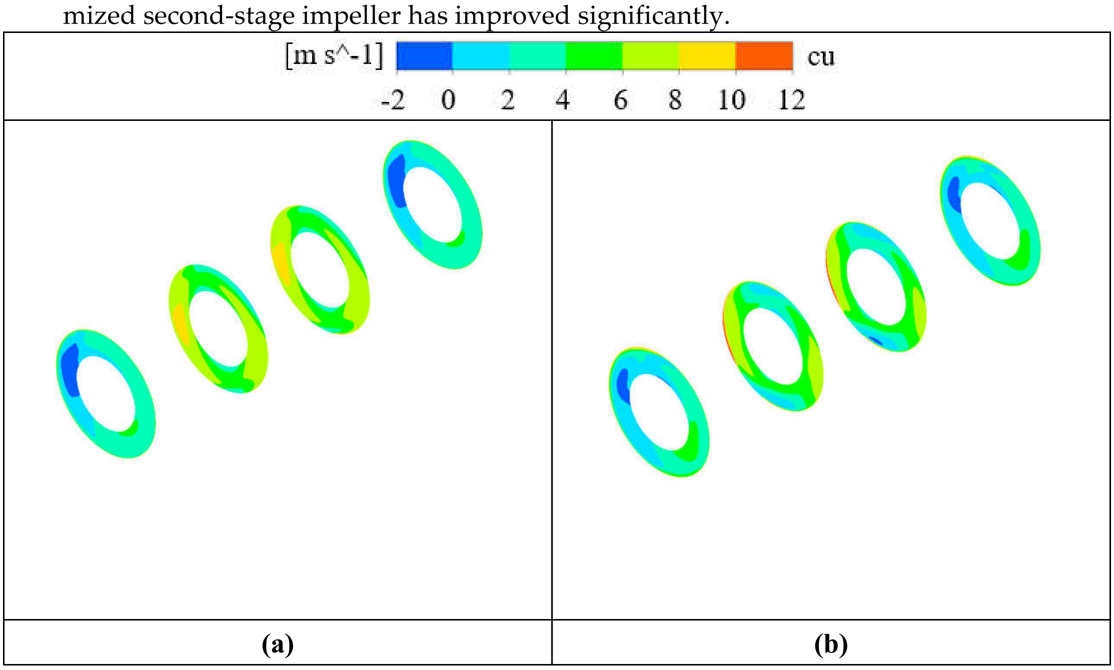


**Figure 9.** Comparison of the peripheral speed distribution of impeller inlet before **(a)** and after **(b)** optimization.

Figures 10 and 11 presents a comparison of the speed distribution before and after the optimization of the first-stage impeller and the second-stage impeller, respectively. It can be seen from the figures that the internal flow of the first-stage impeller of the original scheme fits better with the blade profile, while the hydraulic performance of the optimized first-stage impeller is not improved to a certain extent, but flow separation occurs in two of the flow channels. For the optimized second-stage impeller, the internal flow of the second-stage impeller of the original scheme is smooth overall, and the streamlines in the other optimized flow channels show no obvious change, but there is a backflow phenomenon in the upper flow channel, resulting in a vortex.

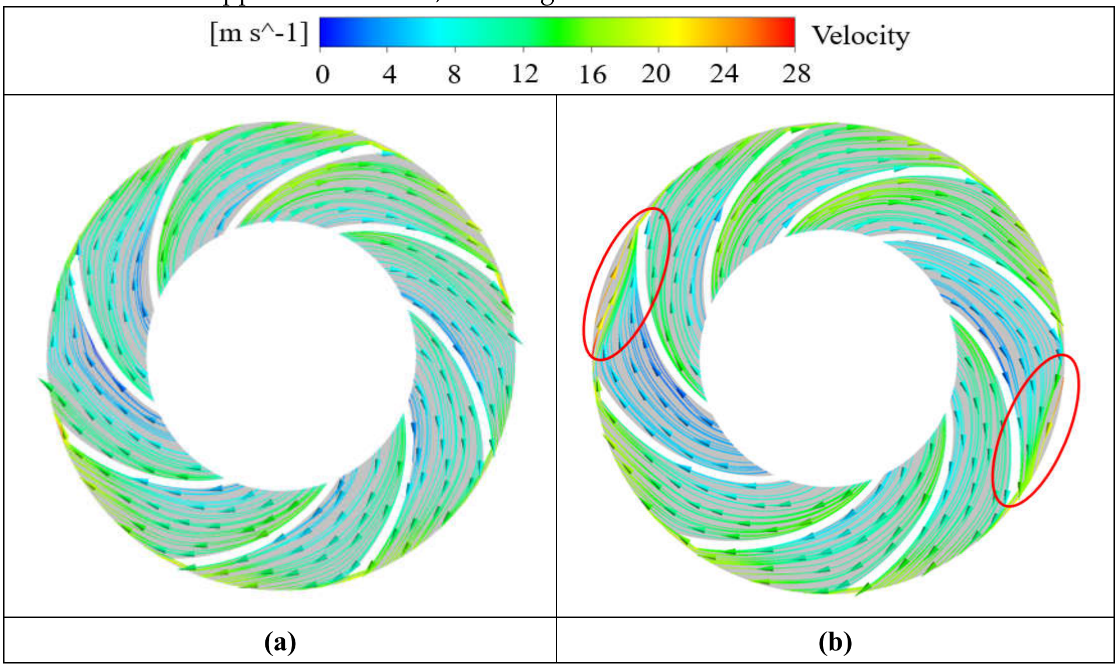


**Figure 10.** Comparison of the first-stage impeller velocity distribution before **(a)** and after **(b)** optimization.

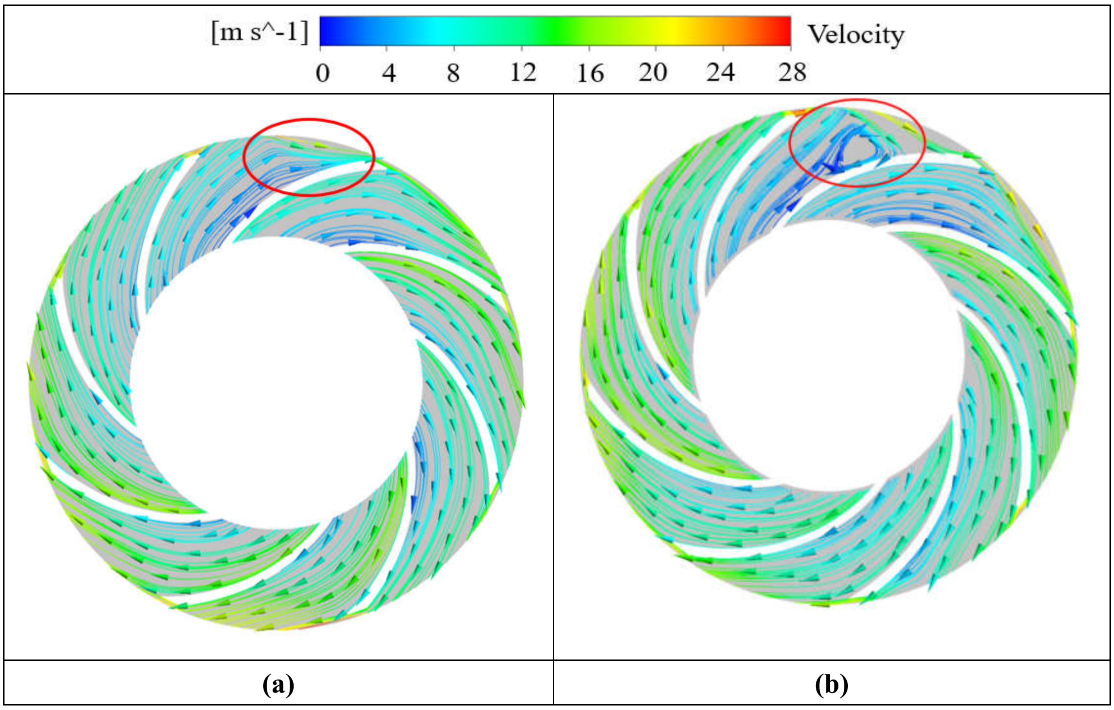


**Figure 11.** Comparison of the second-stage impeller velocity distribution before **(a)** and after **(b)** optimization.

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Figure 12 presents a comparison of the distribution of the velocity streamlines of the double volute before and after optimization. It can be seen from the figure below that the velocity of the volute after optimization is significantly improved, the vortex at the volute and the outlet pipe is eliminated, and the impact loss at the volute tongue is eliminated. The overall velocity inside the volute is reduced and the velocity at the outlet tube is increased, so the overall hydraulic performance of the volute is improved. Since the volute is a static water-passing component, we generally think that it has little effect on the pump efficiency. However, as an energy recovery component that converts kinetic energy into pressure energy, the volute has a considerable impact on the efficiency of the pump. Therefore, improvement of the internal flow performance of the volute can effectively improve the overall operating efficiency of the pump [23].

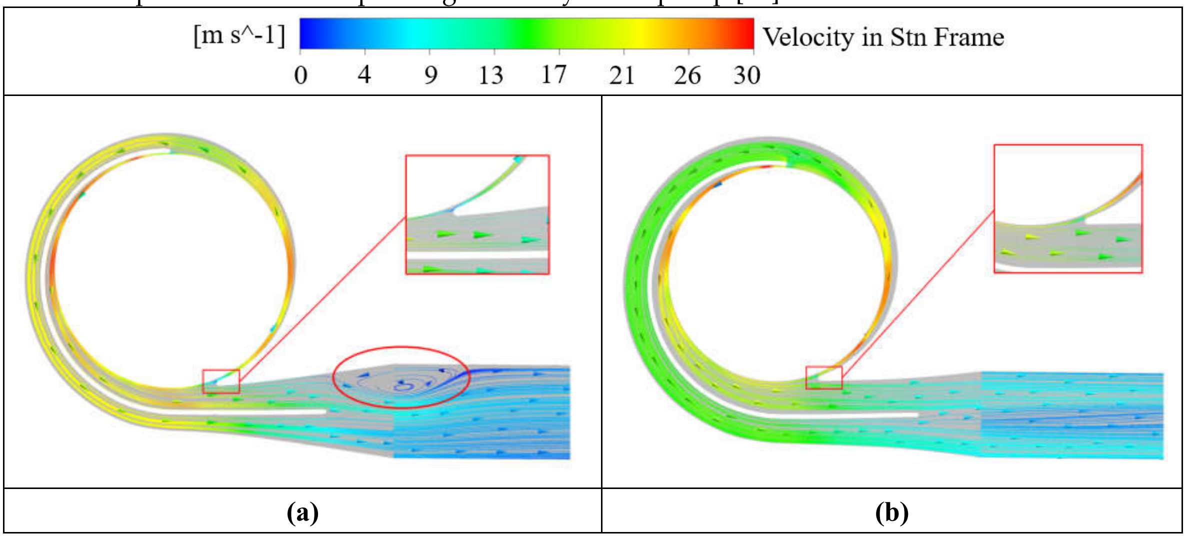


**Figure 12.** Comparison of the velocity streamline distribution of the double volute before **(a)** and after **(b)** optimization.

*4.4 Characteristic analysis of the entropy field*

Due to the phenomena of secondary flow, backflow, pressure pulsation, and flow separation that aggravate energy dissipation during pump operation, energy dissipation can be effectively evaluated by comparing the entropy production results before and after optimization. For this entropy production analysis, the entropy production including wall dissipation, turbulent dissipation, and direct dissipation is considered based on the Reynolds time-averaged turbulent motion.

Table 7 shows the entropy production values corresponding to the wall, turbulence and direct dissipation of different components before and after optimization, where Imp 1$^{st}$1 represents the first-stage impeller on the left in Figure 1, Imp 1$^{st}$2 is the first-stage impeller on the right, Vol is the double volute, and Imp 2$^{nd}$ is the second-stage impeller. It can be seen from the table that the entropy production of various dissipations of the optimized first-stage impeller is increased, and the wall dissipation of the second-stage impeller is reduced to a certain extent. The dissipation of the volute in all three parts is reduced, mainly concentrated in the dissipation of the wall, which is reduced by 9.07(W/K) compared with that before optimization.

Figure 13 presents a comparison chart of the entropy production results of the first-stage impeller, the second-stage impeller, and the double volute before and after optimization. As can be seen from the figure, due to the optimization of the structure of the double volute, the entropy production value after optimization is reduced by 9.64 (W/K), and the energy dissipation of the volute is significantly reduced. The other parts may have a small increase in entropy production due to the deterioration of the optimized flow state, but the overall entropy production of the pump is decreased by 4.79 (W/K), so the overall energy loss of the pump is reduced, the performance is improved, and the optimization effect of the volute is better than that of the impeller.

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**Table 7.** Entropy production for the wall, turbulence, and direct dissipation of different components before and after optimization.

| | Original | | | Optimization | | |
|---|---|---|---|---|---|---|
| Dissipation type | Wall | Turbulence | Direct | Wall | Turbulence | Direct |
| Imp $1^{st}1$ | 12.12 | 2.32 | 2.04 | 12.44 | 3.01 | 2.77 |
| Imp $1^{st}2$ | 12.13 | 2.32 | 2.04 | 12.29 | 3.05 | 2.82 |
| Vol | 35.25 | 7.82 | 0.60 | 26.18 | 7.18 | 0.67 |
| Imp $2^{nd}$ | 23.95 | 5.53 | 4.57 | 23.84 | 6.34 | 5.32 |

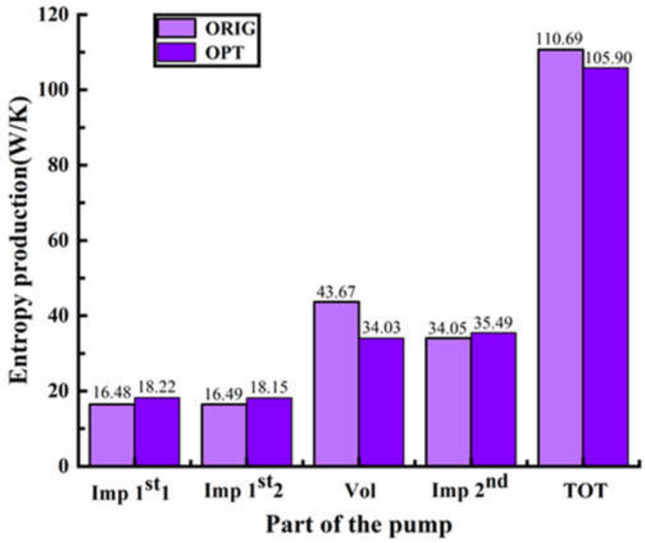


**Figure 13.** Comparison of entropy field values of components before and after optimization.

## 5. Conclusions

In this paper, the efficiency of a multi-stage double-suction pump under its design conditions was selected as the optimization target, and nine design parameters were used as the optimization variables. The LHS method was used to sample and screen the data, based on the improved RSM to optimize the efficiency. Finally, the MIGA was used for global optimization, and the hydraulic performance of the pump before and after optimization was compared and analyzed.

(1) The non-hierarchical RSM selected in this paper, namely, the fourth-order 40th-degree non-hierarchical polynomial, can effectively approximate the nonlinear relationship between the optimization target efficiency and the design variables. The fitted $R^2$ value was 0.919, which was significantly improved compared with the fourth-order hierarchical polynomial and met the accuracy requirements. The efficiency under the design case after the final numerical verification was increased by 3.717%.

(2) For the fourth-order 40th-degree hierarchical polynomial selected in this paper, the degree of influence of each variable on the efficiency can be obtained through the coefficients of each polynomial, among which the blade front cover inlet placement angle $\beta_{1s}$, the baffle starting position $\theta$, and the blade wrap angle $\varphi$ were found to have a greater impact on the efficiency, while other variables were found to have less impact.

(3) The internal flow of the optimized double volute was well improved, eliminating the large-area vortex phenomenon in the low-pressure area at the outlet of the volute and the impact loss at the separation tongue, and the overall velocity inside the volute was reduced, improving the outlet, the speed at the exit is increased, so that kinetic energy is converted into pressure energy to a greater extent, and the energy loss is reduced.

(4) By comparing and analyzing the entropy production value of each component before and after optimization, it is concluded that the total entropy production of the

pump was reduced by 4.79 (W/K) compared with that before optimization, and the optimized double volute entropy production was reduced by 9.64 (W/K), which is mainly due to the reduction in the wall surface entropy generation and dissipation value in the double volute, which effectively reduces the energy loss of the pump and improves the overall operating performance of the pump.

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