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Analytical Derivations of Critical Temperatures in Every Type of Fe-based Superconductors Based on Two Electrons' Rotational Model

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Abstract
In this study, Fe-based superconductors (SCs) are described with a model showing a novel attractive force. First, we describe this novel force using an analog from electromagnetism. From electromagnetism, it was found that this force is a Lorentz force in which two electrons orbit around a Fe-ion with the same velocity. Then, we consider a wave function of an electron. Consequently, due to the property of the proposed attractive force and the quantum-field Hamiltonian, we clarified that the Bardeen–Cooper–Schrieffer (BCS) ground state can be reused. Afterward, attractive force energy was calculated according to the presented model. In Fe-based SCs, a structure transition is essential. Considering that the derived attractive force energy is relatively large, we employ the expanded T_c-equation from the BCS theory that includes the structure transition effect and the derived attractive force energy. In addition, we succeed in analytically reproducing T_c-dome diagrams in various types of Fe-based SCs. Moreover, we discuss the universal property a general SC should have as well as the quantum critical point.

Keywords:
Fe-based superconductor; critical temperature; transition temperature; attractive Lorentz force; T_c-dome diagram; structure transition; quantum critical point

1. Introduction

It has been over a century since the first superconductor (SC) discovery. During this period, some significant progress has been made. For instance, the Bardeen–Cooper–Schrieffer (BCS) theory [1], which claims that a Cooper pair and phonon pairing are mandatory to form superconductivity, has been proposed.

Moreover, in the 1980s, ceramic cuprates [2], whose critical temperatures are higher than that of LN_2, were found. These SCs are essential because the superconducting mechanism differs from the BCS theory. Several experimental and theoretical studies have been conducted; particularly, in our previous study [3], we succeeded in analytically describing this mechanism.

Moreover, MgB_2 metal-based SC [4], whose critical temperature is 39 K, was presented; in 2008, Kamihara et al. [5] discovered Fe-based SCs whose critical temperature is around 50 K. Recently, H-based SCs [6], whose critical temperature is very high and near room temperature but requires very large pressures (around 250 GPa), have been devised.

MgB_2- and H-based SCs are considered the phonon paring; thus, their mechanisms are not difficult to clarify. However, several scholars claim that the mechanism of Fe-based SCs is considered to be difficult to approach because Fe is a ferromagnetic element. Based on common sense, if a ferromagnetic element is included in a compound, that compound cannot exhibit superconductivity.

Among Fe-based SCs, various types of SCs, such as 11-, 122-, and 1111-types, exist. Moreover, to clarify their mechanisms, many experiments have been performed [7-9]. Mainly, experiments were performed to measure the T_c-dome diagrams, pair-symmetry, and magnetic moment of Fe. Notably, the measure of Fe magnetic moment significantly differs from that of the theoretical calculation. However, as discussed below, our pairing model well-explains this fact. Moreover, importantly experiments revealed that a structure transition appears below around 160 K [10-12]. For the paring

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symmetry, it is not currently determined, but it is among a subspecies of s-wave symmetry [13].

Several theoretical studies have been conducted. Among them, the spin-density wave theory [14] and the orbital model system [15] are well-known. However, even with the support of a Hubbard-like model or dynamic mean field theory, currently, the theoretical mechanism has not been well-established. Particularly, based on the proposed mechanisms, a T_c-dome has not been reproduced.

Nevertheless, describing the correct theory is urgent because it might be a guide to finding a higher critical temperature SC. Motivated by this, we analytically calculate T_c-domes in various types of Fe-based SCs in this study. Starting with a completely novel paring mechanism, the "expanded BCS theory," wherein the structure transition effect is incorporated, succeeds to reproduce T_c-dome quantitatively. Moreover, in the Discussion section, some insights into general and Fe-based SCs are discussed. We emphasize that our analytical calculations were consistently performed with no fitting or numerical calculations.

We briefly discuss the mechanism of paring. As discussed in our previous study [3], when two electrons have zero relative velocity (i.e., the same velocity), the electrons experience an attractive force from the analogy of electromagnetism. There are various forms to show this attractive force, but we propose a model in which two electrons orbit around a Fe-ion with the same velocity. Because the movements induce a current, the orbits itself has magnetic field energy and magnetic flux. We claim that this is the identity of the electron nematic phase [16], and this is the the reason why the experimental magnetic moment differs from that of the calculations [17].

According to the proposed model, we derived attractive force energy V. This type of paring has rotational symmetry and thus a subspecies of s-wave is applied. After considering a wave function and the property of the attractive force, it is allowed to reuse the BCS ground state and its critical temperature equation. However, this equation incorporates the structure transition effect in addition to the derived attractive force energy by the proposed model. Thus, we refer to this equation as the "expanded BCS equation." Employing this derived T_c -equation, we succeeded in reproducing various types of T_c-dome diagrams. Although Fe-based SCs have various tapes and various T_c-domes, we use the same T_c-equation (i.e., the expanded BCS equation) that is related to the structural transition and that quantitatively expresses the various type of T_c-domes, implying that our established T_c-equation can be considered valid and the value of the attractive force energy V in it is valid.

Now, we present the content of this paper. First, we review the attractive force origin in an SC. Then, according to the proposed model, the attractive force energy V is derived. Considering the property of the attractive force and a wave function, we reconsider the validity to reuse the BCS ground state. After incorporating the above attractive force energy and the structure transition effect, we derive the critical temperature equation (expanded BCS equation). In the Method section, we describe a concrete method of T_c-dome calculations. In the Result section, we show the consistency between our analytical calculations and the experimental T_c-dome diagrams in terms of various types of Fe-based SCs. In the Discussion section, the universal property of an SC and the quantum critical point of a Fe-based SC are discussed. Finally, we summarize the entire contents in the Conclusion section.

2. Theory

2.1. Attractive force

In general, to understand an SC is to clarify the force F that works between two carrier electrons. As mentioned previously [3], this force is one from magnetic fields when the relative velocity between the two electrons is zero. As shown in Fig. 1, when considering parallel current leads whose current directions are the same, electromagnetism clarifies that these current leads experience an attractive force F between them. Then, as shown in Fig. 2, even when these current leads are made to be shortened, the attractive force F still exists. Therefore, as shown in Figs. 3 and 4, even when this shortening results in a wavelength of an electron carrier, the attractive force F still works between two

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electrons. In fact, this implies that, when the relative velocity between two electrons is zero, a Lorentz force results in an interactive attractive force F between the two electrons.

Although Figs. 3 and 4 imply the movement which has a center-of-mass (CoM) motion (i.e., d-wave symmetry of a Cooper pair), Fig. 5 indicates s-wave symmetry whose CoM motion does not exist in the thermal equilibrium. Moreover, Fig. 5 is an example in which the attractive force F is generated. In this figure, each electron takes rotations but the relative momentum is zero. We claim that this figure is an s-wave symmetry because of the rotational symmetry.

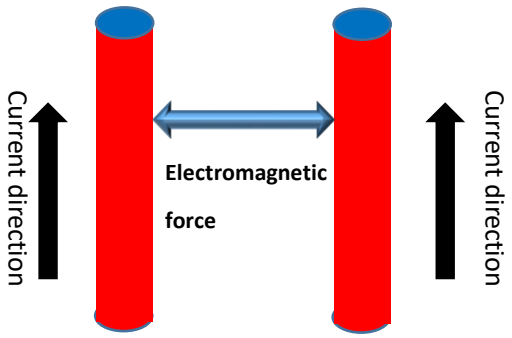

Fig. 1 Schematic of parallel current leads. In this figure, two current leads are set by the directions as parallel. Notably, electromagnetism describes that there is an attractive force between the two current leads.

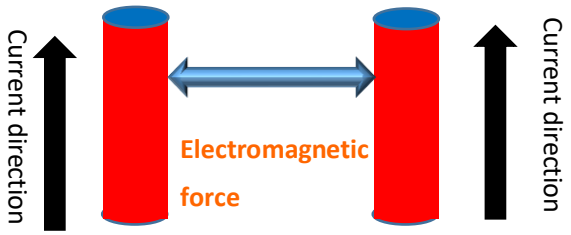

Fig. 2 Schematic of shortened current leads. Compared with Fig. 1, the lengths of current leads are shortened. Nonetheless, the attractive electromagnetic force still exists

Fig. 3 Schematic of the attractive force that works between two electrons. Compared with Fig. 2, the lengths of the current leads are further shortened to a wavelength of an electron. However, provided the velocity $\vec{v}$ is equal, the electromagnetic force (i.e., the Lorentz force) still exists.

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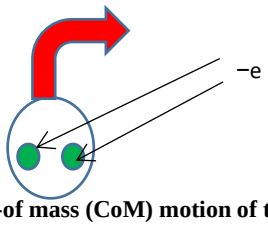

Fig. 4 Schematic of the center-of mass (CoM) motion of the pair.

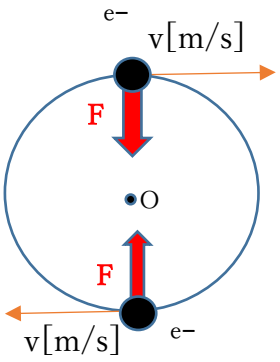

Fig. 5 Schematic of an s-wave symmetry. The velocity v of two electrons is the same, forming a circle. As mentioned, this case gives the situation that the two electrons experience an attractive force *F*. Notably, Fig. 3 indicates a d-wave symmetry whose velocity of the CoM motion is not zero and has the anisotropic property, whereas an s-wave symmetry provides zero velocity of the CoM motion macroscopically in the thermal equilibrium and rotational symmetry.

2.2. Model to discuss Fe-based SCs

As shown in Fig. 6, there is a Fe-ion that has a magnetic moment $\vec{\mu}$ and two carrier electrons have rotational movements along the Fe-ion due to the existence of the magnetic moment $\vec{\mu}$. Notably, the two electrons have up and down spins, respectively. As mentioned previously, the two electrons can have almost the same rotational velocity, implying that the two electrons experience the attractive Lorentz force. We herein define this paring as a subspecies of an s-wave symmetry. Notably, similar to an s-wave, this model has rotational symmetry but does not have the anisotropic property. We do not consider an "accidental node [13]." When the two electrons take rotations, the currents are induced; thus, self-magnetic fields must be generated, implying that the original magnetic moment $\vec{\mu}$ must be modified to be reduced. This fact was measured in experiments [17].

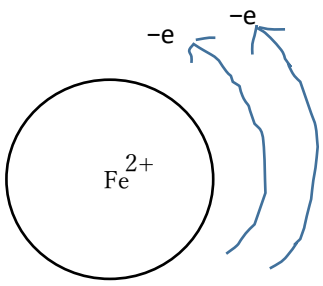

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Fig. 6 Schematic of paring around a Fe-ion. The velocities of the two electrons are almost equal. Thus, as mentioned in the text, between the two electrons, the attractive force *F* appears. Similar to the case of an s-wave symmetry, the CoM ordinate does not move macroscopically in the thermal equilibrium, particularly it has rotational symmetry. Thus, it is allowed to refer to this type of paring as a subspecies of an s-wave symmetry.

2.3. Wave function

Now, let us consider the wave function of a Cooper pair.
For the existence of the aforementioned Lorentz force *F*, the wave function of a pair must be

$$\psi_0 = |\psi_0|\exp(jk_r\xi), \quad (1)$$

where j, k_r, and ξ denote the imaginary unit, relative wave number (i.e., the relative momentum), and the coherence of a Cooper pair, respectively.
Notably, the relative wave number is almost zero:

$$k_r \approx 0 \equiv \varepsilon. \quad (2)$$

Thus,

$$\psi_0 = \alpha\exp(j\varepsilon x_1) \times \alpha\exp(-j\varepsilon x_2), \quad (3)$$

where x_1 and x_2 denote each position of an electron among a pair. Each wave function in eq. (3) implies up- and down-spin electrons, respectively. The above implies that each electron is well approximated by a plane wave. Note that, because the proposed attractive force functions only in momentum space and considering the Fourier transformation of the Hamiltoninan of one body, the Hamiltonian of one body in the momentum space cannot determine the wave function.

Moreover, when considering the normalization,

$$\int|\psi_0|^2 dV = 1. \quad (4)$$

This is approximated as follows:

$$|\psi_0|^2\xi^3 = 1, \quad (5)$$

and

$$\xi^3 \equiv \frac{1}{\varepsilon^3}. \quad (6)$$

Thus,

$$|\psi_0|^2 \approx \varepsilon^3. \quad (7)$$

Considering the value ε is almost zero, the uncertainty relation holds and coherence ξ cannot be determined given probability. Actually, as mentioned later, $|\psi_0|^2$ is minute.

According to the approximation of plane waves and subspecies of s-wave, for many-body interactions, the following BCS theory picture can hold.

i) $V_{kk'} \approx -V$: Because the proposed attractive force is dependent on the momentum, this force functions only in momentum space. Moreover, as mentioend, because each electron can be approxamted by a plane wave, the matrix element of the interaction in the quantum-field Hamiltoninan must be a constant.

ii) The BCS ground state: Considering i) and that each electron is a plane wave, the above quantum-field Hamiltoninan inevitably results in the BCS ground state. Thus, for many-body interactions, the BCS ground state is applied.

From the above, a critical temperature T_c will be considered later, based on the BCS theory calculation [1]

2.4. Calculations of the attractive force and T_c-equation

Now, let us calculate the interaction attractive force between two electrons. In the principle model in Fig. 6, the two electrons have an orbital angular momentum $\vec{l}$. Using the spin angular momentum $\vec{s}$, the following condition is assumed:

$$2|\vec{s}| = |\vec{l}|. \quad (8)$$

From the definition of an orbital angular momentum $\vec{l}$, we have

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$$l = pr = mva_B, \quad (9)$$

where p, r, m, v, and a_B denote the momentum, radius, mass of an electron, velocity, and Bohr radius, respectively.

Thus,

$$\mathrm{v} = \frac{l}{ma_B} = \frac{1}{ma_B} \cdot 2 \cdot \frac{1}{2}\hbar, \quad (10)$$

where eq. (8) was used.

Then, we consider the density of probability flow with the plane wave approximation as follows:

$$\mathrm{j} = \mathrm{e}|\psi|^2 \frac{\hbar k_r}{m}, \quad (11)$$

where e denotes the charge of an electron, k_r denotes the wave number, and ψ denotes the wave function in which the volume integral is given as

$$\int |\psi|^2 \, dV = 1. \quad (12)$$

This normalization equation is approximated as

$$|\psi|^2 \cdot \frac{4}{3}\pi\xi^3 = 1, \quad (13)$$

where ξ denotes the coherence of a Cooper pair.

When the momentum p is considered $\hbar k$, the density of probability flux becomes

$$\mathrm{j} = \mathrm{e}|\psi|^2 v = e\frac{3}{4\pi\xi^3} \cdot 2 \cdot \frac{1}{2}\hbar \cdot \frac{1}{ma_B}. \quad (14)$$

Considering the Fe-ion to be a sphere and the radius a_B is sufficiently small, the current I is approximately

$$\mathrm{I} \approx \pi a_B^2 j. \quad (15)$$

Thus,

$$\mathrm{I} = \mathrm{e}\frac{3}{4\pi\xi^3} \cdot 2 \cdot \frac{1}{2}\hbar\frac{1}{ma_B} \cdot \pi a_B^2. \quad (16)$$

That is,

$$\mathrm{I} = \mathrm{e}\frac{3}{4\pi\xi^3}\hbar\frac{\pi a_B}{m}. \quad (17)$$

From Ampere's law, we have

$$2\pi\xi\mathrm{B} = \mu_F I. \quad (18)$$

That is,

$$\mathrm{B} = \frac{\mu_F}{2\pi\xi} I, \quad (19)$$

where B and μ_F denote the interactive magnetic field and the microscopic magnetic permeability, respectively.

In this study, the Lorentz force F that is the identity to form a Cooper pair is given by

$$\mathrm{F} = \mathrm{evB}. \quad (20)$$

That is,

$$\mathrm{F} = \mathrm{e}\left(\frac{1}{ma_B} \cdot 2 \cdot \frac{1}{2}\hbar\right)\left(\frac{\mu_F}{2\pi\xi} e \frac{3}{4\pi\xi^3}\hbar\frac{\pi a_B}{m}\right). \quad (21)$$

Thus,

$$\mathrm{F} = \frac{3e^2\hbar^2}{8\pi\xi^4}\frac{\mu_F}{m^2}. \quad (22)$$

Hence, its energy V is derived according to

$$\mathrm{V} \approx -\mathrm{F}\xi. \quad (23)$$

Therefore, to conclude,

$$\mathrm{V} \approx -\frac{3e^2\hbar^2}{8\pi\xi^3}\frac{\mu_F}{m^2}. \quad (24)$$

Notably, from the magnetic property of a Fe-ion, μ_F is determined as the number 1.0.

As mentioned previously, considering the normalization,

$$\mathrm{V} \approx -\frac{3e^2\hbar^2}{8\pi\xi^3}\frac{1}{m^2} \approx -\frac{3e^2\hbar^2}{8\pi}\frac{1}{m^2}|\psi_0|^2 \equiv -\frac{3e^2\hbar^2}{8\pi}\frac{1}{m^2}n_e, \quad (25)$$

where n_e denotes the microscopic variable carrier concentration. Notably, as aforementioned, the values of n_e are minute, and μ_F is substituted by the number 1.0. However, this value of μ_F is allowed to be varied to 0.1.

Because we do not consider the two-dimensional anisotropic property, the state density D is given by

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$$D = \frac{1}{2\pi^2}\left(\frac{2m}{\hbar^2}\right)^{\frac{3}{2}}\sqrt{E_F}, \quad (26)$$

where E_F denotes the Fermi energy, which we set to 2.2 eV in this study.

Notably, many Fe-based SCs have a structure transition below around 160 K [7-9]. This structure transition implies the lattice constant is linearly reduced with doping. Considering the above and a conclusion from the BCS theory [1], we derive the following critical temperature T_c:

$$T_c = (\theta_D - n\langle\theta_0\rangle)\exp\left(-\frac{1}{D|V|}\right), \quad (27)$$

where θ_D denotes the original Debye temperatures and n is a natural integer, which varies, interlocking with the variety of doping carrier concentrations n_e. Moreover, $\langle\theta_0\rangle$ implies the average Debye temperature that reflects the structure transition. Notably, for each n value, θ_0 is different. However, the above equation proposes the average of θ_0. In our calculations, within low dopes, the approximation of a weak combination (i.e., $D|V| \ll 1$) is well-applied. Nevertheless, in higher dopes, the values of $D|V|$ becomes relatively large. To take a consistent equation of T_c over all dope realms, a strong combination approximation (i.e., $D|V| \gg 1$) cannot be applied. Rather, an equation of T_c must not be significantly different, and the consistent and average expression of T_c over all doping realms proposes the following equation that is taken from the aforementioned equation of T_c:

$$T_c = (\theta_D - n\langle\theta_0\rangle)\exp\left(-\frac{1}{D|V|}\right) \approx (\theta_D - n\langle\theta_0\rangle)\left[1 - \frac{1}{D|V|}\right]. \quad (28)$$

2.5. Debye temperatures

Notably, the original Debye temperatures θ_D in Fe-based SCs are relatively small. Let us consider the reason;

The two electrons have the orbital angular momentum l but this is independent for the thermal oscillation of a Fe-ion. Moreover, there is a Coulomb interaction between the Fe-ion and two electrons. In this case, the following balance approximately holds:

(The thermal oscillation energy) ≈ (The Coulomb interaction energy)

Thus, the schematic illustrated in Fig. 7 does not occur, implying that the thermal oscillations of the Fe-ion become relatively small, which indicates the Debye energy $\hbar\omega_D$ takes small energy. From this study, we infer that the original Debye temperature only in the superconducting state becomes 1/10 times that of the normal state. Thus, for example, the original Debye temperature θ_D is given as around 10–50 K.

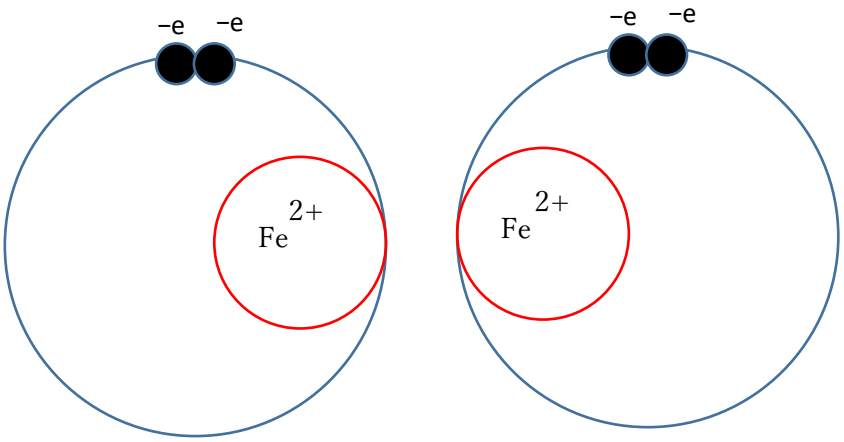

Fig. 7 Prohibited cases for the oscillation of a Fe-ion. In the Fe-based SCs, the Coulomb interaction between the Fe-ion and the two paired electrons is almost equal to the thermal oscillation of the ion. Thus, the left and right panels in this figure are prohibited.

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3. Method

First, we use the MS software Excel.
In calculations, mainly T_c-equation is used:

$$T_c = (\theta_D - n\langle\theta_0\rangle)[1 - \frac{1}{D|V|}]. \quad (28)$$

In this equation and as an example, the original Debye temperature θ_D is given around 10–50 K. Moreover, D is the state density described as follows:

$$D = \frac{1}{2\pi^2}(\frac{2m}{\hbar^2})^{\frac{3}{2}}\sqrt{E_F}, \quad (26)$$

where m, $\hbar$, and E_F denote the mass of an electron, the Planck constant, and Fermi energy, respectively. As an example, E_F is given as 2.2 eV every time in the calculations. For the attractive force energy $|V|$, the following equation is employed:

$$V \approx -\frac{3e^2\hbar^2}{8\pi}\frac{1}{m^2}n_e, \quad (25)$$

where e and n_e denote the charge of an electron and variable local carrier concentration, respectively. For variable integer n for the above T_c-equation (28), the number 1 to 500 is prepared in the software Excel. Moreover, concentration n_e is made to be interlocked with variable integer n, i.e., n_e is given 0.1 to 50.0. Notably, as shown later, the Debye temperature $\langle\theta_0\rangle$ related to the structure transition will be given, corresponding to each type of Fe-based SCs.
Finally, when showing the graphs, we make the relation between concentration n_e and replacement quantity x to be a constant, as follows:

$$n_e\alpha = x, \quad (29)$$

where α denotes the converting constant with the unit, and this constant is kept constant whenever showing the graphs. For example, the converting constant α is set to be 0.1 in this paper.

4. Result

As a result of the theory, we succeeded to reproduce the experimental T_c-domes (Figs. 8–10).

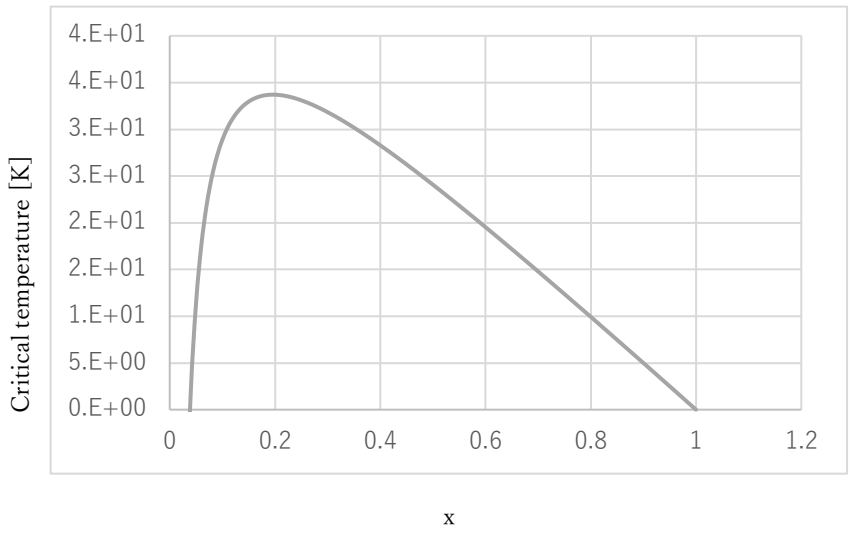

Fig. 8 Analytical calculation of a critical temperature dome in a Fe-based SC. In this figure, $\theta_D = 52.2$ K and $\langle\theta_0\rangle$ = 0.01 K, for example. Notably, the compared experiment is in [18].

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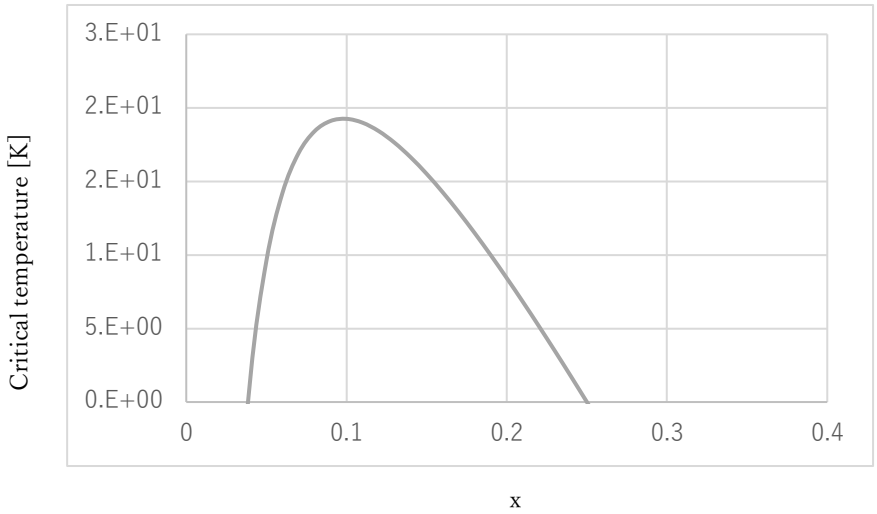

Fig. 9 Analytical calculation of critical temperatures of a Fe-based SC. In this substance, $\theta_D = 52.2$ K and $\langle\theta_0\rangle$= 0.04 K are given, for example. Notably, the compared experiment is in [18].

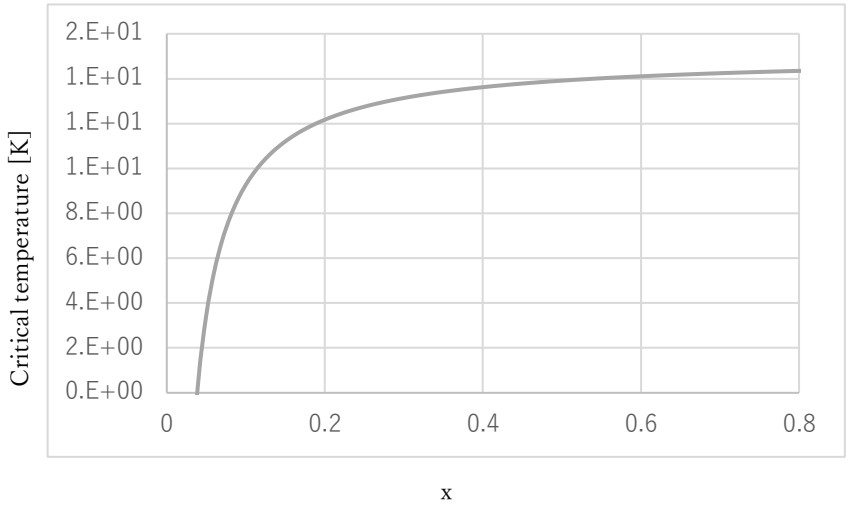

Fig. 10 Analytical calculation of critical temperatures of a Fe-based SC (11-type). $\theta_D = 15$ K, for example. Notably, this type of Fe-based SC contains the very weak structural transition, i.e., $\langle\theta_0\rangle \approx 0$ consistently. Notably, the compared experiment is in [19].

5. Discussion

5.1. Validity of results in this paper

In general, Fe-based SCs have various tapes and various T_c-domes. However, we employed the same T_c-equation (Eq. (28)), varying only the Debye temperatures for the valid realms. Consequently, we succeeded in quantitatively expressing various types of T_c-domes, implying that our employed T_c-equation is valid and our value of the attractive force energy V is also valid.

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5.2. Why do Fe-based SCs have a large critical temperature, and what is the logic of derivation of T_c?

In the proposed electrons' rotational model, the attractive force energy $|V|$ becomes relatively large when the magnetic permeability μ_F is sufficiently large. This fact simply resulted in the relatively high critical temperature. Let us review the logic of deriving T_c-equation. First, from the model that two electrons orbit around a Fe-ion, the interactive energy was discussed. Then, considering the property of the attractive force, each wave function can be approximated as a plane wave. Considering the properties of the attractive force and the quantum-field Hamiltonian, it was found that the BCS ground state was applied. Thus, according to the BCS theory's calculation, the critical temperature was derived but the structure transition element and the attractive force energy V were incorporated into it.

5.3. Isotope property

The critical temperatures T_c in general metal SCs are

$$T_c = \theta_D \exp(-\frac{1}{D|V|}). \quad (30)$$

Because θ_D implies the phonon energy, an isotope effect appears due to the variety of θ_D.
In turn, our derived T_c is still

$$T_c = (\theta_D - n\langle\theta_0\rangle)[1 - \frac{1}{D|V|}]. \quad (28)$$

Thus, although θ_D is varied, $n\langle\theta_0\rangle$ would cancel this variation. Thus, to conclude, we claim that it is difficult to measure an isotope effect in Fe-based SCs.

5.4. Why do the ceramic cuprates exclude the BCS picture?

We believe that it is common and universal for SCs to have the attractive force that comes from the fact that the relative momentum is zero. However, why do ceramic cuprates not follow the BCS theory? This is because there is a pseudogap temperature T^* and transition temperature T_0 at which the anomaly metal phase appears. In our previous study [3], we succeeded in describing the above transitions in addition to T_c, with full consideration of many-body interaction. According to our previous study [3], the key to all in the ceramic cuprates is the existence of a macroscopic Boson that is created by the rotation of a hole due to the two-dimensional CuO_2 surface and due to the conservation of angular momentum.

5.5. The double count of the phonon effect in the general BCS theory equation

As shown in Fig. 11 and from the exclusion principle, a normal metal has two electrons with up-and down-spins, which have the same momentum k (i.e., the same velocity). These two electrons experience the attractive force due to the zero relative velocity with each other, although they take a very long relative distance. We claim that the attractive force system is universal for every SC. However, why is the BCS theory claiming the attractive force is a phonon? The critical temperature T_c in the BCS theory is still

$$T_c = \theta_D \exp(-\frac{1}{D|V|}). \quad (30)$$

In this equation, the element of a phonon is already included in the Debye temperature θ_D, which results in an isotope effect. If we counted $|V|$ as the phonon interaction, we would take the double count in terms of a phonon. Instead, again, we claim that while θ_D denotes the Debye temperature relating to a phonon, $|V|$ must be another attractive force. Because the two electrons depicted in Fig. 11 experience the attractive force, a general metal state should be just before the superconductivity state.

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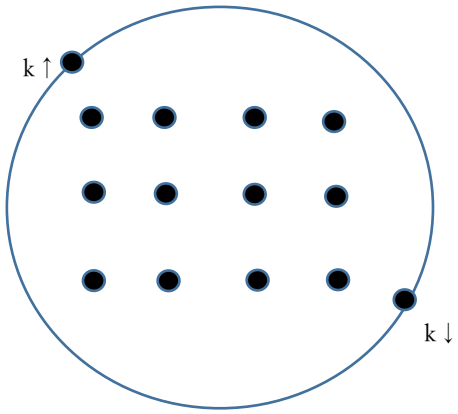

Fig. 11 Schematic of a Cooper pair in a normal metal. Notably, each black dot implies an electron. From the exclusion principle, a metal has the wave-number-k electron whose spin is up and the same wave-number-k electron whose spin is down. The wave function of each electron is well-approximated as a plane wave. Although the two electrons have a long distance between which many other electrons exist, they experience the attractive Lorentz force due to the zero relative velocity (i.e., the momentum). Many scholars claim that the magnetic interaction among a Cooper pair is spin, but that cannot create the long distance interaction over which many other electrons locate. We claim that the identity of the force to create a Cooper pair in general SCs is universally a Lorentz attractive force that works between two electrons whose relative velocity is zero.

5.6. The quantum critical point in 122-type of the Fe-based SC

Many scholars claim that 122-type has the non-Fermi liquid properties at the optimum doping. Let us consider this critical point.

Our T_c is again

$$T_c = (\theta_D - n\langle\theta_0\rangle)[1 - \frac{1}{D|V|}], \quad (28)$$

where

$$V = -\frac{3e^2\hbar^2}{8\pi m^2}n_e, \quad (25)$$

$$D = \frac{1}{2\pi^2}(\frac{2m}{\hbar^2})^{\frac{3}{2}}\sqrt{E_F}. \quad (26)$$

Considering n is proportional to n_e and the proportional constant is given by β, the critical temperature becomes

$$T_c = (\theta_D - \beta n_e\langle\theta_0\rangle)[1 - \frac{1}{D}\frac{8\pi m^2}{3e^2\hbar^2}\frac{1}{n_e}]. \quad (30)$$

Then, the following extreme provides another phase transition:

$$\frac{dT_c}{dn_e} \equiv 0. \quad (31)$$

Thus,

$$n_e \equiv n_{e0} = \sqrt{\frac{1}{D}\frac{8\pi m^2}{3e^2\hbar^2}\frac{\theta_{D0}}{\beta\langle\theta_0\rangle}}. \quad (32)$$

In the phase transition, β is determined such that the normal structure transition disappears. That is, $\beta\langle\theta_0\rangle \equiv 1$. Moreover, θ_{D0} in eq. (32) is the transition point in terms of the original Debye temaprautre. Note that, considering *Sec.2.5*, at the Debye temperatures smaller than this transition point, a Cooper pair takes a local center-of-mass motion, which results in the electron nematic phase [16]. That is, the Cooper pair becomes independent for a Fe-ion. This is because, due to the small oscillation enegy of the Fe-ion, the corresponding wavelength of the oscillation becomes large, which results in the fact that the oscillation wavelength of the Fe-ion dominates over the orbit radius of the Cooper pair. This independent Cooper pair can be the analogy of a macroscopic Boson in [3], and thus the anomaly phase appears temporally. Moreover, $\frac{\theta_{D0}}{\beta\langle\theta_0\rangle}$ in eq. (32) must be almost constant consistently as long as eq. (32) for n_{e0} drived by the extreme is not valiable. Given that $\theta_{D0} \equiv 17.4$ K, n_{e0} is calculated

approximately,
$n_{e0} = 3.3$.
That is,
$x = 0.33$.
This implies that, at a doping point such as in the vicinity of the optimum doping, another transition occurs that loses temporally the normal structure transitions and in which the anomaly phase appears [20].

5.7. The significance of this paper

Although many experiments revealed many facts related to Fe-based SCs, the mechanism has not yet been clarified. According to our literature reviews, the analytical calculation of T_c is inadequate. Starting with the concept that two electrons orbit around a Fe-ion due to the magnetic moment, we revealed the origin of the attractive force, and combining it with the concept of the structure transition, we analytically derived T_c-equation that succeeded in describing T_c over all doping realms for various type of Fe-based SCs, implying that the basic mechanism of Fe-based SCs has been clarified. Importantly, this employed T_c-equation well explains the quantum critical point and non-Fermi liquid properties at this point. Notably, we employed no fitting or numerical calculations. For the ceramic cuprates, as aforementioned, in our previous study [3], we revealed purely analytically the three transition temperatures over doping. Thus, combined with this fact, we have now succeeded in describing the mechanism of the typical high-temperature SCs such as the ceramic cuprates and Fe-based SCs.

6. Conclusion

In this study, we succeeded in describing analytically T_c-domes in Fe-based SCs, not depending on fitting or numerical calculations. First, starting with a model that two electrons orbit with the same velocity around a Fe-ion due to the existence of a magnetic moment, a novel but universal Lorentz attractive force was derived. Incorporating this attractive force and the effect of the structural transition to the BCS theory's T_c-equation, critical temperatures were derived, which agree well with experiments. Notably, we did not consider the hole doping in the 122-type, which we would consider as a future study.

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Additional information

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