

Article

Use of the Analytic Hierarchy Process and selected methods in the managerial decision-making process in the context of sustainable development

Jana Stofkova ¹, Matej Krejnos ¹, Peter Malega ², Katarina Repkova Stofkova ^{1*}, Vladimira Binasova³

¹ Department of Communication, University of Zilina, Univerzitna 1, 010 26 Zilina, Slovakia; krejnos1@stud.uniza.sk (M.K.); jana.stofkova@uniza.sk (J.S.); katarina.repkova@uniza.sk (K.R.S.);

² Department of Industrial and Digital Engineering, Technical university of Kosice, Park Komenského 9, 040 02 Košice, Slovakia; peter.malega@tuke.sk (P.M.)

³ Department of Industrial Engineering, University of Zilina, Univerzitna 1, 010 26 Zilina, Slovakia; vladimira.binasova@fstroj.uniza.sk (V.B.)

* Correspondence: katarina.repkova@uniza.sk Tel.: 421 917 144 757 (K.R.S.)

Abstract: This article describes the Analytic Hierarchy Process (AHP) method which can be calculated by several methods. The AHP method is essential for the managerial decision-making to recognize which method is efficient for the calculation and to determine the proper order of criteria. In the article there are included methods that can be used in order to calculate the matrix in the AHP process for setting criteria such as Geometric mean, Arithmetic mean, Row sum of the adjusted the Saaty matrix, Reverse sums of the Saaty matrix columns and Row sums of the Saaty matrix. The paper is focused on the accuracy of the methods used. The results show that the most accurate method is the Saaty method, and the second most accurate method is the geometric mean in order to determine the ranking. This method is easier to use when the same order is achieved as in the Saaty method, followed by Geometric mean, which is favourable for fast and easy determination of alternatives using in the AHP process. The survey carried out among managers graduated from Universities and Colleges in Slovakia showed that the respondents considered the Saaty method as the most complex and also the most difficult method and the geometric mean average method the simplest method. 44% of respondents stated that they are capable to use a program to calculate the AHP. 46% of respondents said they had experience with some method related to the managerial decision-making process. 72% of managers regarded as important to manage some method for decision-making in their managerial position.

Keywords: AHP process; Saaty method; Eigenvalue; Excel program; managerial decision-making process

1. Introduction

The situation on world markets in companies or other institutions places high demands on the quality of managerial decision-making. An incorrect decision in any area can cause losses and sometimes even lead to the liquidation of businesses. Therefore, in preparing managers and other essential positions for the 21st century, it is crucial to return the importance of mathematics and its applications, for example, in decision-making processes. This approach brings benefits to both smaller and larger companies.

The decision-making process is considered as one of the key aspects for entrepreneurs and managers. For quality decision-making, it is necessary to choose a suitable method, which can be involved into decision-making. Also, the decision must be a logical process, which results in selecting of the most appropriate alternative. The selection of the

solution must be conscious and purposeful and the chosen solutions need to be of high quality due to their significant impacts on the functioning of the part of the organization, in which it takes place. [1]

The AHP method has many applications in different fields and combinations of different methods. Therefore, it is helpful to understand several options for calculating the AHP, from more complex to more straightforward methods.

The article describes the procedure of using the Saaty method and compares the results with other more uncomplicated methods. Therefore, this method is accessible even for users with a basic knowledge of mathematics to obtain comparable results, which is a great benefit.

The Saaty method can be used in managerial decision-making in several criteria and improves the analytical hierarchical process while helping to find the optimal alternative for the person with decision-making authority. [2,3]

Professor Saaty described the method of multicriteria decision-making, which is now called by his name or the analytical hierarchical process (AHP) method. Dr. Thomas L. Saaty developed the method in 1970 at the Wharton School of the University of Pennsylvania in the USA. [4]

By the decision-making people are usually influenced by their personality, environment, social and political background when deciding between options. It means that they make decisions based on their knowledge and experience, they analyze the risks and benefits of their decision. Decision-making can be freed from subjective influences by evaluating each alternative. It is easier to compare alternatives than to try to calculate their preferences. At the same time, the comparison has to be within the permissible range of consistency. The hierarchical analytical process (AHP) method includes both comparison and evaluation. [4]

Multicriteria is a critical element of economic, social, political, military, and other decision-making. Decision-making processes are problem-solving processes with more than one possible solution. The process of solving a multicriteria decision-making task is a procedure by which the solver determines the optimal state of the system concerning more than one criterion considered. This procedure is called multicriteria optimization. [3,5]

The Saaty method is used for the analysis and solution of decision-making tasks, with the help of which the solver selects the alternative that best meets the setting goal. The manager determines the alternatives and criteria as follows, using a pairwise comparison, compares the criteria and alternatives with each other and determines the preferences and the weight of the given preference. [6,7,8]

The pairwise comparison method is also called the Saaty method. The mentioned method is used for multicriteria decision-making. [8,9]

The Analytic Hierarchy Process (AHP) method is accepted as the traditional method for determining the weighting of the suitability assessment. The authors used the AHP, and coefficient of variation (AHP-CV) combined weight method to evaluate the suitability of urban green space better. Several authors use this method with another method to improve results. The research has shown that it can be extensively used in the related fields of planning concerning land-use, green infrastructure, and transportation and tourism. [10]

By promoting sustainable and transparent management, people in various economies could build and implement sustainable development practices and incorporate them into business activities. Sustainable development management integrates and connects relevant various sections of economics and science with the most relevance for sustainable development.

Table 1. Current research problems solved by the AHP method.

Reference	The main purpose of the study	Criteria considered	Methodology
Ranji, Parashkoochi and Zamani et al (2022) [11]	Agricultural area	Agronomic, Technical, Economic, Environmental	AHP
F. Chan and H. Chan (2010) [12]	The selection of suppliers in the fastchanging fashion market	Delivery, Quality, Assurance of Supply, Flexibility, Costs.	AHP
Kaymaz, Birinci and Kizilkan (2022)[13]	Sustainable development goals assessment of Erzurum province	Strengths, Weaknesses, Opportunities, Threats	AHP-SWOT
Imran, Agha, Ahmed et al. (2020)[14]	Simultaneous Customers and Supplier's Prioritization	Economic, Social, Environment	Fuzzy-AHP
Nikhil, Danumah, Saha et al. (2021)[15]	Forest Fire Risk Zone Mapping	Land cover types, slope angle, aspect, topographic wetness index, distance from the settlement, distance from the road, distance from the tourist spot and so on	GIS and AHP
Ayyildiz, Gumus (2021)[16]	Hazardous material transportation: an application in Istanbul	Road, Environment, Traffic, Vehicle, Material	Pythagorean Fuzzy AHP
Ayyildiz, Gumus (2020)[17]	Petrol station location selection problem	Financial, Environmental, Opportunities, Supplier	Fuzzy AHP
A limited number of experts whose opinions were used to determine the priorities of the criteria determined by the AHP method.			
Certain limitations were encountered during the preparation of the studies	<p>Experts avoided in-person meetings for binary comparisons in criteria priority determination due to the Covid-19 pandemic. These circumstances caused fewer consulted experts to be less than the desired number.</p> <p>The operation research models have some limitations using insufficient information, and it is challenging to incorporate the expert decision-making. Therefore, evaluation models are based on data analysis along with expert opinion.</p>		

Source: own processing

The most frequent experiments using the AHP method are related to semi-agriculture, forest fire risk zone, pumping station determination and others. Other methods, such as GIS, Fuzzy or MAGDM, help solve common decision-making problems. [18] However, the researchers also encountered limitations, such as the limited number of experts whose views were used to prioritise of the set the criteria. The AHP method is often used in combination with another method enabling its extension into different fields.

2. Materials and Methods

The first method is a survey among managers graduated from universities and colleges in Slovakia. In addition to this method, the AHP process and various methods of its calculation were analyzed in order to find out the most accurate and the simplest method of calculation of AHP process.

In addition to this, a survey was performed on a sample of 346 managers with the aim to detect the managerial experience with mathematic methods supporting the decision-making in important issues. The survey was carried out by an electronic inquiry in the period from March to April 2022. All respondents were managers graduated from Colleges or Universities in Slovakia and worked at different levels of management, mainly primary and middle management, but also at top management. More details are about the survey are listed in Table 2.

Table 2. Sample characteristics.

Characteristics	Slovakia (N=346)
Age (years)	
Means	36.5
Gender	
Male (%)	61.6%
Female (%)	38.4 %
Graduation	
High school (%)	41.1 %
University (%)	56.9 %
PhD (%)	2 %

Source: own processing

Table 2 shows that men represented the largest group in the survey. In terms of education, the largest group of managers graduated from Universities, followed by Colleges and at least the PhD. graduated were involved in the survey. The results of this survey are discussed in the section "Results of the survey".

AHP process

The AHP - Analytical Hierarchical Process method follows three steps in decision-making process, such as 1) structure of the model; 2) comparison of the criteria, alternatives and calculation of the weights; and 3) synthesis of priorities. The basic feature of the AHP method is the representation of the complete decision-making issue as a specific hierarchical structure. Within the hierarchical structure, we distinguish the tree view with several levels representing the individual parts of decision-making process. Each of them includes several elements. The top level of the hierarchy always contains only one element, which is the objective of the evaluation. A standard example of the AHP is a three-level hierarchy; for a more comprehensive picture, they can also be assigned under criteria. The method is based on the assumption of evaluation consistency; it also arises from the hypothesis that inconsistency occurs mainly in evaluations between alternatives of seemingly minor importance to the decision making manager. [19]

Hierarchical structure

Hierarchical structures can be divided into two types with criteria and alternatives and with criteria below the criteria and alternatives. Subsequently, it is necessary to recalculate the individual weights between the sub-criteria. The structure of the AHP process is displayed in Figure 1.

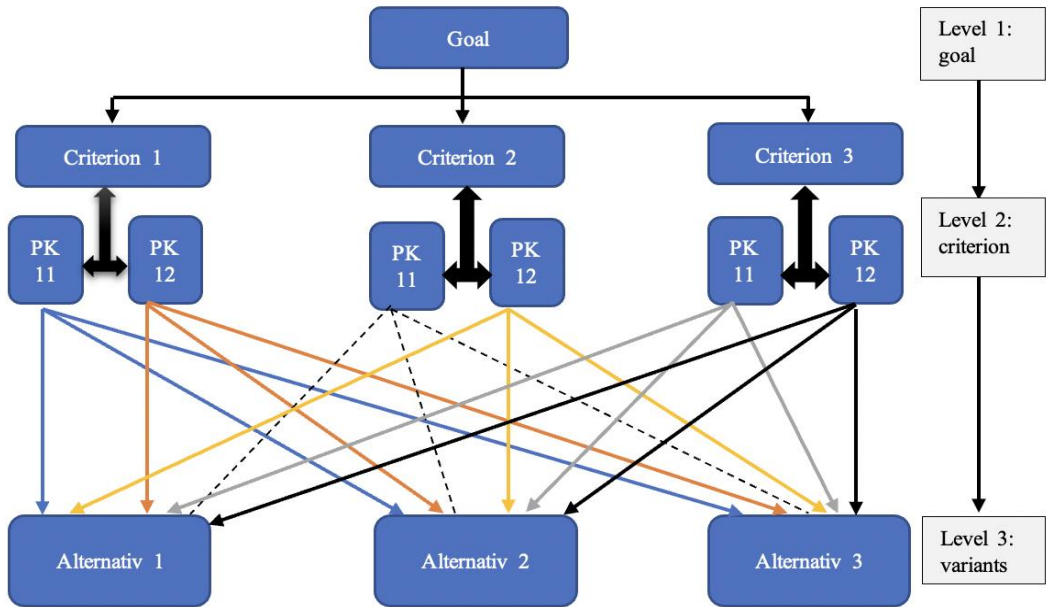


Figure 1. The structure of the AHP method.

The level 1 is the goal of the analysis. The level 2 is a multi-criterion consisting of several criteria or sub-criteria. PK – sub-criterion.

- Level 1 – the goal of the analysis - arrangement of alternatives.
- Level 2 - evaluation criteria - evaluation of the importance of the criteria.
- Level 3 - assessment of alternatives - evaluated by the importance of alternatives.

The first step in the AHP process is a pairwise comparison between each criterion, possibly sub-criterion. Next, the individual criteria have to be assessed based on the scales enlisted in the following table. Table 3. shows the determination of the evaluation of the individual criteria with the preference rate from which the criteria are selected for the construction of the matrix. An example of comparison scales is listed in the Table 3.

Table 3. The Saaty scale.

Scales	Degree of preference
1	The criteria are equally important
3	Medium importance of one factor over another
5	Strong or essential
7	Crucial importance
9	Extremely important
2,4,6,8	Intermediate values

Source: own processing

The comparison results for each pair of factors were described as an integer value from 1 (the same importance) to 9 (extremely different importance), where a higher number means that the selected factor is more important than the other compared factor. Table 4 can help identify individual scales.

Table 4. Choosing the values in the Saaty scale.

Factor	Factor weighting score																	Factor
	More important							Equally important			Less important							
X1	9	8	7	6	5	4	3	2	1	2	3	4	5	6	7	8	9	X1
X2	9	8	7	6	5	4	3	2	1	2	3	4	5	6	7	8	9	X2
X3	9	8	7	6	5	4	3	2	1	2	3	4	5	6	7	8	9	X3

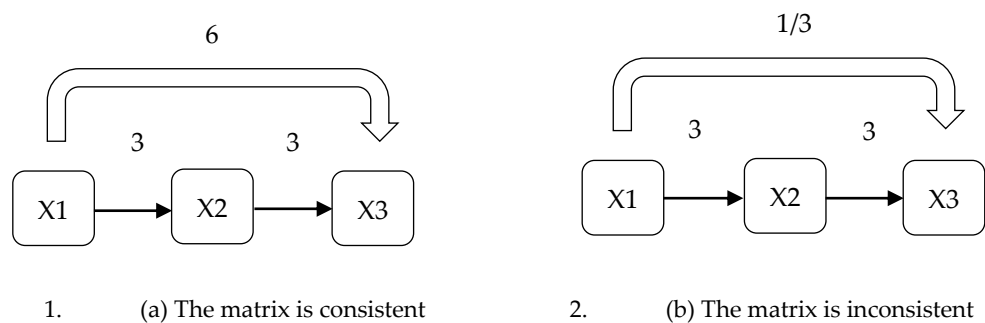
Source: own processing

$$(A) = \begin{matrix} & \begin{matrix} X1 & X2 & X3 & \dots & Xn \end{matrix} \\ \begin{matrix} X1 \\ X2 \\ X3 \\ \dots \\ Xn \end{matrix} & \begin{pmatrix} 1 & 4 & 5 & \dots & 5 \\ 1/4 & 1 & 1/2 & \dots & 3 \\ 1/5 & 2 & 1 & \dots & 7 \\ \dots & \dots & \dots & 1 & \dots \\ 1/5 & 1/3 & 1/7 & \dots & 1 \end{pmatrix} \end{matrix}$$

After determining the individual weights, it is necessary to create a matrix. The matrix (A) shows the breakdown of the individual weights.

Consistency index

Before calculating, it is necessary to check the table to ensure that it is sufficiently consistent and that there are no discrepancies within the pairwise comparison of the individual criteria. The consistency can be explained in the following comparison. If criterion X2 is three times more important than criterion X3 and X1 is three times more important than X2, then X1 is six times more significant than X3. In this case, the matrix is consistent. Figure 2 and 3 show the mentioned principle in visual form. The figure on the left shows the matrix consistency and the figure on the right shows the matrix inconsistency.

**Figure 2.** Principle of matrix consistency and inconsistency.

If X1 is only 1/3 more significant than X3 then the matrix will not be consistent. We verify the consistency using the matrix consistency index and calculate it according to the formula.

$$CI = \frac{\lambda_{\max} - x}{x - 1} \quad (1) \quad CR = \frac{CI}{RI} \quad (2)$$

CI - consistency index; Saaty matrix is sufficiently consistent if $CI < 0.1$

X - is the number of criteria.

λ_{\max} - specifies the largest correct number of the matrix.

CR - consistency ratio, consistency value.

RI - table value based on the number of criteria, so-called random index.

Methods of calculation of the Saaty method

The Saaty method: Eigenvalue and eigenvector. For comparison and reliability of individual calculation methods, calculations will be performed on one type of example by all selected methods. We can divide the individual calculations on the Saaty procedure, which is implemented using eigenvector and eigenvalue, i.e., custom vector matrix. The result must be standardized. The left-hand side represents matrix-vector multiplication, but the right-hand side is scalar-vector multiplication. Firstly, we can rewrite that right-hand side as some kind of matrix-vector multiplication using a matrix which has the effect of scaling any vector by a factor of λ . Using the unit matrix, we obtain the final formula (4).

$$A.v = \lambda_{\max}.v \quad (3)$$

$$\begin{bmatrix} \lambda & 0 & 0 \\ 0 & \lambda & 0 \\ 0 & 0 & \lambda \end{bmatrix} \quad \left| \quad \lambda \cdot \underbrace{\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}}_I \right.$$

$$(A - \lambda_{\max}.I).v = 0 \quad (4)$$

v - custom vector, so-called eigenvector

λ_{\max} - the corresponding eigenvalue

v. λ_{\max} - vector scale

A - decision matrix

I - unit matrix

This method serves as a basis for comparison with other calculation methods and for determining the accuracy of other methods. Of all these approaches, this is the most complex and the most suitable one. Due to its complexity, it is used in specialized decision support programs such as Super decision, Expert choice Comparison and Priority Estimation tool.

The following table summarizes the individual calculation methods and the equations that will be implemented. In addition, there are several methods for calculating the matrix, which are demonstrated in Table 5. In other words, Table 5 shows methods for calculating in different ways.

Table 5. Methods for calculating the Saaty decision matrix.

Method description	Geometric mean	Arithmetic mean	Row sum of the adjusted Saaty matrix	Reverse sums of Saaty matrix columns	Row sums of the Saaty matrix
Method of calculation	The method is based on calculating the geometric mean of the individual rows of the decision matrix.	This is the simplest method of calculation. It consists of averaging the matrix's individual rows that need to be standardised.	The method consists of 2 steps. The first step is to modify the decision matrix by dividing each column by the sum of the columns. In the second step, a row-wise summation is performed.	The method is based on the principle that the preference intensity vector is calculated as the inverse of the columns of the matrix.	The method of calculation is based on a simple sum of the rows of the decision matrix.
Equation	$v_i' = \sqrt[x]{\prod S_{ij}}$	$v_i' = \sum \frac{S_i}{x}$	$v_i' = \sum \frac{S_{ij}}{T_j}$	$v_i' = \frac{1}{\sum S_{ij}}$	$v_i' = \sum S_{ij}$
The result must be standardized	yes	yes	yes	yes	yes
Explanation of acronyms	v_i - unstandardized vector of preference intensities S_{ij} - elements of the decision matrix T_j - sum of elements of the j-th column				

Source: own processing.

The calculations will be performed by the Saaty method and compiled with the following methods – geometric, arithmetic mean, row sum of the adjusted the Saaty matrix, inverted sums of the columns of the Saaty matrix and the row sum of the Saaty matrix. The results need to be standardized to achieve the correct result.

Evaluation of alternatives is discussed below.
To evaluate the criteria and partial alternatives, the overall evaluation of alternatives H_j from another method are overall calculated.

$$H^j = \sum_{i=1}^n v_i * h_i^j \quad (5)$$

H_j - overall rating of the alternatives.
 h_{ji} - partial rating of the j-th alternative in relation to the i-th criterion.
 v_i - criteria weights

A higher value determines the better fulfillment of the target H_j . For example, a value after standardization of 0.543 meets better our requirement than a value of 0.034, i.e. $0.543 > 0.034$. The optimal alternative is the one with the highest overall rating.
The values for setting the criteria were determined based on the measurement carried out for selecting routes for the carrier. Criteria A, B, C, D, E represent time, route length, costs, type of transport and type of route.
The determination of the weights was carried out according to academic experience at the university, and the criteria are used in the calculations and serve as the basis for the Saaty matrix.

3. Results

Comparison of calculation methods for the AHP process is one of the main goals of this paper. There is designed an example of the matrix for calculations with selected methods in the Table 3. The matrix contains 5 criteria from A to E. The matrix has a size of 5x5 elements. In order to compare with other methods, it is necessary to perform the calculation by the Saaty method. Matrix (A) shows the criteria embedded in the matrix.

$$(A) = \begin{matrix} & \begin{matrix} A & B & C & D & E \end{matrix} \\ \begin{matrix} A \\ B \\ C \\ D \\ E \end{matrix} & \begin{pmatrix} 1 & 2 & 4 & 1/2 & 1/3 \\ 1/2 & 1 & 3 & 1/4 & 1/6 \\ 1/4 & 1/3 & 1 & 1/5 & 1/8 \\ 2 & 4 & 5 & 1 & 1 \\ 3 & 6 & 8 & 1 & 1 \end{pmatrix} \end{matrix}$$

The Saaty method

The calculation will be performed based on (3) $A.v = \lambda_{\max}.v$. Saaty procedure was calculated using Excel and its functions. To the created symmetric Matrix A it is necessary to design a single-unit Matrix B. Figure 3 shows Matrix A and Matrix B, which are used for calculation in the next steps.

	A	B	C	D	E	F	G	H
102		Eigenvector						
103		A	B	C	D	E		
104	A	1	2	4	1/2	1/3		
105	B	1/2	1	3	1/4	1/6		
106	C	1/4	1/3	1	1/5	1/8		MatrixA
107	D	2	4	5	1	1		
108	E	3	6	8	1	1		
109		A	B	C	D	E		
110	A	1	0	0	0	0		
111	B	0	1	0	0	0		
112	C	0	0	1	0	0		MatrixB
113	D	0	0	0	1	0		
114	E	0	0	0	0	1		
115								

Figure 3. Matrix A and Matrix B.

In order to obtain the lambda sign, it is necessary to determine the first estimation of the lambda, which can be any number, e.g. 999. Then we use formula (4) and multiply the matrixes with the estimated lambda. Based on this matrix, we compute the determinant of the matrix using the MDETERM function. From the menu of MS Excel, one shall click Tools-Goal Seek and the Goal Seek dialog window shows up. After (set cell: determinant; to value=0; by changing cell: λ_{\max}). The lambda matrix is 5.102357 (see Figure 4).

B119								
	A	B	C	D	E	F	G	H
118		A	B	C	D	E		
119	A	-4 1/9	2	4	1/2	1/3		(A- λ_{\max} .I).v = 0
120	B	1/2	-4 1/9	3	1/4	1/6		
121	C	1/4	1/3	-4 1/9	1/5	1/8		MDETERM
122	D	2	4	5	-4 1/9	1		-9,235E-06
123	E	3	6	8	1	-4 1/9		λ_{\max}
124								5,102357342

Figure 4. Calculation based on formula (4).

The consistency matrix is calculated using relations (1) and (2) and reported in Table 4. To calculate the matrix vectors, we use the matrix from Figure 5 and the Sumproduct function shown in Figure 5.

G126 \times \checkmark f_x =SUMPRODUCT(B126:F126;\$B\$131:\$F\$131)								
	A	B	C	D	E	F	G	H
	A	B	C	D	E		(function) Sumproduct	
125								
126	A	-4,102	2,000	4,000	0,500	0,333	15,56	
127	B	0,500	-4,102	3,000	0,250	0,167	3,13	
128	C	0,250	0,333	-4,102	0,200	0,125	-9,97	
129	D	2,000	4,000	5,000	-4,102	1,000	13,59	
130	E	3,000	6,000	8,000	1,000	-4,102	22,49	
131	Vectors	1,000	2,000	3,000	4,000	5,000	Sum=	15,000
132	Normalization	0,06667	0,13333	0,20000	0,26667	0,33333	Sum=	1

Figure 5. Principle of matrix consistency and inconsistency.

It is unnecessary to choose random vectors for example 1, 2, 3, 4, 5. To calculate the matrix vectors, a manager uses the matrix from Figure 5 and clicks the Sumproduct function, as shown in Figure 6. Vectors can be selected in any value for example 1, 2, 3, 4, 5. From the MS Excel menu, one shall click solver parameters and choose the values as shown in Figure 6.

Figure 6. Solver function in Excel program.

G126 \times \checkmark f_x =SUMPRODUCT(B126:F126;\$B\$131:\$F\$131)								
	A	B	C	D	E	F	G	H
	A	B	C	D	E		(function) Sumproduct	
125								
126	A	-4,102	2,000	4,000	0,500	0,333	0,00	
127	B	0,500	-4,102	3,000	0,250	0,167	0,00	
128	C	0,250	0,333	-4,102	0,200	0,125	0,00	
129	D	2,000	4,000	5,000	-4,102	1,000	0,00	
130	E	3,000	6,000	8,000	1,000	-4,102	0,00	
131	Vectors	1,932	1,073	0,546	3,870	4,990	Sum=	12,411
132	Normalization	0,15570	0,08646	0,04397	0,31181	0,40207	Sum=	1

Figure 7. The results and standardization of vectors.

Based on the results of the solver parameters and in order to calculate the vectors of the matrix, it is important to use the matrix $(A - \lambda_{\max} I) \cdot v = 0$ and use the Sumproduct function, as shown in Figure 6. It is unnecessary to choose random vectors determined in figure 5. From the MS Excel menu, one shall click solver parameters and choose the values as shown in Figure 6. Finally, the results of the vector are unstandardised and it is necessary to standardise the vector, such as $(1.932/12.411=0.15570)$, etc. Figure 7 shows vector values after standardization $v=(0.15570; 0.08646; 0.04397; 0.31181; 0.40207)$. Table 6 shows the truncated matrix calculation results by means of the Saaty method.

Table 6. Results of the Saaty method.

Alternative	Values	Order	$\lambda_{\max} = 5,10235$ N=5 RI= 1,21 CI = 0,02110
A	0,15570	3	
B	0,08646	4	
C	0,04397	5	
D	0,31181	2	
E	0,40207	1	
\sum			

Source: own processing

Tables 7 and 8 below summarize all the results. Especially, table 7 shows the results of the other methods.

Table 7. Results after calculation and ranking.

Methods	Geometric mean	Arithmetic means	Row sum of the adjusted Saaty matrix	Reverse sums of Saaty matrix columns	Row sums of Saaty matrix
Used formula	$v_i' = \sqrt[x]{\prod s_{ij}}$	$v_i' = \sum \frac{s_i}{x}$	$v_i' = \sum \frac{s_{ij}}{T_j}$	$v_i' = \frac{1}{\sum s_{ij}}$	$v_i' = \sum s_{ij}$
The result have to be standardized	yes	yes	yes	yes	yes

Source: own processing

Table 8. Results of calculations.

Methods	Geometric mean	Arithmetic means	Row sum of the adjusted Saaty matrix	Reverse sums of Saaty matrix columns	Row sums of the Saaty matrix	Ranking
Alternatives	Values					-
A	0.15768	0.16789	0.16453	0.14954	0.16789	3
B	0.08550	0.10538	0.09417	0.07570	0.10538	4
C	0.04330	0.04090	0.04436	0.04807	0.04090	5
D	0.31131	0.27862	0.29334	0.34216	0.27862	2
E	0.40221	0.40722	0.40359	0.38453	0.40722	1
λ_{\max}	5.10190	5.16337	5.16337	5.14357	5.16337	-
CI	0.02105	0.03375	0.02371	0.02966	0.03375	-

Source: own processing

The results were standardized to obtain relevant data for comparison. The results show that the ranking of the individual alternatives is very similar, provided that the matrix is sufficiently consistent under the assumption that we can achieve the same ranking

of alternatives. Furthermore, we can detect deviations in the values between the individual types of calculation. The arithmetic mean and the row sum method of the Saaty matrix display the same results.

Table 9. Comparison of all methods with the Saaty method.

Method		Geometric mean	Arithmetic mean	Row sum of the adjusted Saaty matrix	Reverse sums of Saaty matrix columns	Row sums of the Saaty matrix
Deviation from Saaty method	Average of calculation results	0.00103	0.01684	0.00885	0.01284	0.01684
	Maximum	0.00198	0.03319	0.01846	0.03036	0.03319
	Minimum	0.00050	0.00307	0.00040	0.00410	0.00307
	λ_{\max}	0.00046	0.06101	0.01238	0.04122	0.06101
	CI	0.00010	0.01261	0.00256	0.00852	0.01261

Source: own processing

From the table mentioned above, we can see the differences in the individual calculations (Table 9). The results show that the most accurate method after the Saaty method is the geometric mean method (see Table 10.), where we can notice the slightest deviation. On the other hand, the exact methods are Row sum of the adjusted the Saaty matrix and Arithmetic mean. However, these methods are the easiest to be calculated and can be performed without major mathematical calculations. Deviations occurred in all methods, but the most accurate method is the Saaty method, and accurate results were also obtained by using the geometric mean.

Table 10. Ranking of methods according to their order based on results.

Methods		Order	Equations
Ranking based on the accuracy of the method	Saaty method	-	$(A - \lambda_{\max} \cdot I) \cdot v = 0$
	Geometric mean	1	$v_i' = \sqrt[x]{\prod s_{ij}}$
	Row sum of the adjusted Saaty matrix	2	$v_i' = \sum \frac{s_{ij}}{T_j}$
	Reverse sums of the Saaty matrix columns	3	$v_i' = \frac{1}{\sum s_{ij}}$
	Arithmetic mean	4	$v_i' = \sum \frac{s_i}{x}$
	Row sums of the Saaty matrix		$v_i' = \sum s_{ij}$

Source: own processing.

To verify the correctness of the calculations and the order of the individual methods, we calculated the random matrix (B).

$$(B) = \begin{matrix} & \begin{matrix} A & B & C & D & E \end{matrix} \\ \begin{matrix} A \\ B \\ C \\ D \\ E \end{matrix} & \begin{pmatrix} 1 & 3 & 6 & 7 & 6 \\ 1/3 & 1 & 3 & 4 & 2 \\ 1/6 & 1/3 & 1 & 4 & 2 \\ 1/7 & 1/4 & 1/4 & 1 & 2 \\ 1/6 & 1/2 & 1/2 & 1/2 & 1 \end{pmatrix} \end{matrix}$$

From the calculation results, we can detect the same result as for the matrix (A), implying that even when repeatedly calculated with a different matrix the results are equal. Values of matrix B with Saaty method: where (A=0.52081 B=0.21760 C=0.12776 D=0.06675 E=0.06704) $\lambda_{\max} = 5.39447$ CI=0,08150. The results of the other methods are shown in Table 11.

Table 11. Matrix B control statement.

Methods	Geometric mean	Arithmetic means	Row sum of the adjusted Saaty matrix	Reverse sums of Saaty matrix columns	Row sums of the Saaty matrix	Ranking
Alternatives	Values					-
A	0.53494	0.48791	0.53135	0.56383	0.48791	1
B	0.21495	0.21913	0.22517	0.20089	0.21913	2
C	0.12106	0.15910	0.12358	0.09493	0.15910	3
D	0.06327	0.07722	0.05261	0.06185	0.07722	5
E	0.06578	0.05664	0.06729	0.07850	0.05664	4
λ_{\max}	5.39038	5.52760	5.43108	5.50335	5.52760	-
CI	0.09759	0.13190	0.10777	0.12584	0.13190	-

Source: own processing.

The results of the survey among managers

The survey performed among managers on different levels of management showed that the most complex method of the AHP process in managerial decision-making is the Saaty method, see Figure 8. Most managers regarded this method as a more difficult way of calculation comparing with other methods.

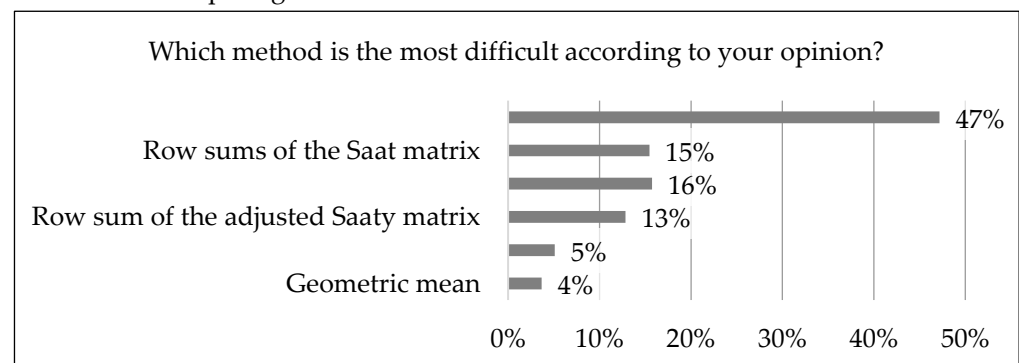


Figure 8. The difficulty requirements of the studied methods within the AHP process from the point of view of managers.

Therefore, it can be stated that the other methods are less demanding. Managers considered the arithmetic and the geometric mean as the least difficult.

Table 12. Further questions of the survey concerning decision-making process among managers.
Source: own processing

	Questions of the survey	Definitely yes	Rather yes	Maybe	Rather no	Definitely no
1	Is the Saaty method your choice when the same outputs are achieved?	15%	4%	2%	9%	69%
2	Could you use some program to calculate the Saaty method?	35%	9%	29%	18%	9%
3	Do you have some experience with any method for decision making?	46%	-	18%	-	35%
4	Do you think that it is important to know and control/understand, use a method for decision making in management positions?	72%	16%	5%	3%	4%

Evaluation of the results

In the first question most respondents (69%) marked the option “definitely no”. According to managers if the result of the Saaty method achieves the correct calculation and ranking of specific method, the selected method of calculation is not important. In the second question 44% of respondents answered that they are capable to use a program to calculate the AHP. There are several software products, but their main limitation is price or subscription. In the third question 46% of respondents said they had experience with some method related to the managerial decision-making process. In the fourth question 72% of the respondents think it is important to know and manage some method for decision-making in managerial positions.

From the survey results, we can conclude that the outcomes confirmed our assumptions when using the specific calculating methods. Certain methods are simpler, and this is especially appreciated by college and university graduates who do not have experience with more complex types of calculations or software.

In order to assess which procedure is the most accurate, it was necessary to calculate the matrix using the Saaty method, or Eigenvalue. Moreover, we tested all the procedures that could be used to calculate the matrix and determine the order of the criteria. The results showed that the second most accurate calculation method is the geometric mean method. A positive result is that all the procedures showed the same order of criteria. Furthermore, this fact opens the question for further research, how many criteria are necessary for the different calculation methods to show different rankings. From the results of scientific works, we can conclude the AHP method has a wide application in various fields, from IT to management.

The decision-making via the AHP method, for instance the personnel department could take the decide on selecting the applicants, because it is necessary to determine the criteria of the applicants in order to select the most appropriate one. The method must be combined with human decision. The next example, where it can be used, is the logistic department, when a manager is about/needs to select the best supplier for the company.

Criteria can be the size of the company, the number of trucks, and the use of Just in Time, etc. The AHP can be used wherever a high-quality decision based on criteria is necessary.

4. Discussion

Another positive finding is that even a person with less knowledge of mathematics can use the AHP method to achieve the same order of elements as in the most demanding method, which is the Saaty method.

In addition, this document provides a complete framework and all calculation methods for setting the criteria in the AHP method. The reader can choose, which method of calculation suits him, based on his mathematic skills and his work goals to achieve the desired result.

Decision-making is a key element for all businesses. Decision-making is essential for companies to make a good decision for leading the company in the present and future. The results from the AHP method can improve the company's performance.

Certain limitations were encountered during the preparation of the studies, such as the limited number of experts to determine individual weights for selected areas, which, however, depends on the area addressed. In addition to these limitations, for many alternatives, it is recommended to use software that calculates many alternatives.

With sufficient knowledge of mathematics and the use of spreadsheet programs, it is possible to achieve a quality decision with a well-chosen method. [20,21]

This method not only supports and qualifies the decisions but also enables the decision-makers to justify their selection and simulates possible results. As we need the most accurate and possible decision, it needs to be consistent and coherent with organizational results. [22,23] It implies that this method can be utilized in the cases/situations/occasions where the management or any individual needs to be involved into the decision-making process, which needs to be supported by the adequate results. The AHP method can be used in personal, logistic, and economic departments. Interestingly, the AHP method can also be used to determine the degree of cave damage. In this case, the AHP method helps to identify major deteriorations or judge conservation orders, etc. [24]

Another use of this method is in agriculture area, where the authors evaluated four criteria including "agronomic", "technical", "economic" and "environmental" as the criteria affecting the selection of rice weeder machine. The AHP method is very useful in all areas, where we can use it. Hence, it is important to know which method for calculation is accurate in order to set right order of criteria.[25] Another research reported that inconsistency in judgment and hence in pairwise comparison matrix of the AHP is the most significant issue to be addressed. Besides, analytical hierarchy processing has been widely applied on various case studies and numerous applications. [26]

The authors state that Cost-benefit analysis and AHP are utilized by the Government and public administration for appraising competing alternatives with positive and negative social implications. The AHP is resistant to rank reversal between ratio and difference methods of aggregation. The authors like Harkar and Vargas argued that the AHP is based on sound theoretical foundation and is useful for diverse decision-making scenarios. [27] According to Chai and other authors, the AHP method enables to assign a value representing the preference degree for a given alternative to each additional alternative. [28] Gupta, Jadhav and Sonar noted that the AHP is the most widely used method for software evaluation. [29,30] Bolpur used the SWOT-AHP-Fuzzy AHP model for formulation and prioritization of ecotourism strategies. The AHP method can be used in combination with other method or methods. [31] The AHP provides a structured way to analyze complex decision problems and deal with tangible and non-tangible criteria. On the other hand, the AHP provides practical tools for calculating criteria weights and ranking failure modes. [32] The AHP has shown advantages for decision-making when the factors are difficult to measure. Additionally, the AHP is a valid social science research method and is extensively used not only in business management decision-making processes, but in various areas of information systems research as well. [33] Daengsi, Sirawongphatsara and Pornpongtechavanich, used the AHP as an easier decision-making technique to be

used to evaluate the considered criteria. [34] Amandeep, Mohammad and Yadav stated that the AHP is a structured technique for dealing with complex decisions based on mathematics and psychology. Besides, the AHP gives a complex framework for structuring a decision problem from different areas. [1] Guimarães, Leal and Mendes have applied this new AHP approach in their article. [35] Sakhardande and Gaonkar reported that consistency in pairwise comparisons has been a major hurdle in solving large matrices. [36] The AHP process is a systematic method that simplifies complex problems establishing a hierarchical structure between factors. [37] Ishizaka and Lustis outlined that a high level of agreement between the different scaling techniques and the number of ranking contradictions increases with the dimension of the matrix and the inconsistencies. [38] The AHP has also been applied to supplier and vendor selection, according to Tam and Tummala. Despite its broad applicability, the AHP method suffers from a notable drawback: it requires many comparisons to make a decision. [39]

5. Conclusions

The decision-making process is considered one of the key aspects for entrepreneurs and managers. For quality decision-making, it is necessary to choose an appropriate method, which can be involved into decision-making and has to be accurate due to its significant impacts on the functioning of the organization.

Therefore, the accuracy of the calculations in managerial decision-making process have to be as precise as possible. The paper compares several methods based on two matrices. From the calculation results, we can detect the same result as for the matrix (A), implying that even when repeatedly calculated with a different matrix the results are equal. Values of matrix B with Saaty method: where ($A=0.52081$ $B=0.21760$ $C=0.12776$ $D=0.06675$ $E=0.06704$) $\lambda_{\max} = 5.39447$ $CI=0.08150$.

The results of these calculations are very close and have negligible deviations. Due to the findings of the research the most accurate method according to the determination criteria the Saaty method was identified as the most accurate method and subsequently the method of geometric mean.

In conclusion, it is important to emphasize that decision-making via the AHP process in many fields of use, e.g. in industry companies Criteria can be the size of the company, the number of trucks, and the use of Just in Time, etc. The AHP can be used wherever a high-quality decision based on criteria is necessary. The method must be combined with human decision.

This paper defines the current research problems addressed by the AHP method. Moreover, the AHP method is often used in combination with other methods such as fuzzy, SWOT, etc. Therefore, the acquaintance of this method and several variants of its calculations form a good basis for its expansion.

There were some limitations in the research, such as the limited number of experts to determine the individual weights for the selected areas, but this depends on the selected area. The paper contains the procedure of using the Saaty method through the Excel program.

According to managers if the result of the Saaty method achieves the correct calculation and ranking of specific method, the selected method of calculation is not important.

In the second question 44% of respondents answered that they are capable to use a program to calculate the AHP. There are several software products, but their main limitation is price or subscription. 46% of respondents said they had experience with some method related to the managerial decision-making process. 72% of the respondents think it is important to know and manage some method for decision-making in managerial positions. From the survey results, we can conclude that the outcomes confirmed our assumptions when using the specific calculating methods. Certain methods are simpler, and this is especially appreciated by managers who do not have experience with more complex types of calculations or software.

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References

1. Saaty, R.W. The Analytic Hierarchy Process-What It Is and How It Is Used. *Math. Model.* **1987**, *9*, 176, doi:10.1016/0270-0255(87)90473-8.
2. Yaraghi, N.; Tabesh, P.; Guan, P.; Zhuang, J. Comparison of AHP and Monte Carlo AHP under Different Levels of Uncertainty. *IEEE Trans. Eng. Manag.* **2015**, *62*, 122–132, doi:10.1109/TEM.2014.2360082.
3. Ramík, J. *Analytický Hierarchický Proces (AHP) a Jeho Využití v Malém a Středním Podnikání*; Print Book: Czech.; Slezská univerzita Obchodně podnikatelská fakulta v Karviné: V Opavě, 2000; ISBN 9788072480883.
4. Saaty, T.L. How to Make a Decision: The Analytic Hierarchy Process. *Interfaces (Providence)*. **1994**, *24*, 19–43, doi:10.1287/INTE.24.6.19.
5. Gavalec, M.; Ramík, J.; Zimmermann, K. Decision Making and Optimization: Special Matrices and Their Applications in Economics and Management. **2014**, 225.
6. Golden, B.L.; Wasil, E.A.; Harker, P.T. *Analytic Hierarchy Process: Applications and Studies*; Springer-Verlag Berlin and Heidelberg GmbH & Co. KG, 1989; ISBN 9783642502460.
7. Ocelíková Elena *Multikriteriálne Rozhodovanie*; Elfa Kosice, 2004; ISBN 8089066283.
8. Saaty, T.; Vargas, L. Models, Methods, Concepts & Applications of the Analytic Hierarchy Process. ... -Driven Demand Oper. *Manag. Model.* **2012**, 1–20, doi:10.1007/978-1-4614-3597-6.
9. Čestnější, A. *Manažérske Rozhodovanie*; 2nd ed.; Univerzita Komenského Bratislava, 2004; ISBN 8022319422.
10. Li, Z.; Fan, Z.; Shen, S. Urban Green Space Suitability Evaluation Based on the AHP-CV Combined Weight Method: A Case Study of Fuping County, China. *Sustain.* **2018**, *Vol. 10*, Page 2656 **2018**, *10*, 2656, doi:10.3390/SU10082656.
11. Bandichode, S.; Bhumes, M.; Sofia, P.; Umesh, S.; Mayur, W. Modality of Particle Swarm Optimization to Improve Decision Making Process of Analytical Hierarchy Processing *. *HELIX* **2018**, *8*, 3888–3891, doi:10.29042/2018-3888-3891.
12. Majumdar, A.; Tiwari, M.K.; Agarwal, A.; Prajapat, K. A New Case of Rank Reversal in Analytic Hierarchy Process Due to Aggregation of Cost and Benefit Criteria. *Oper. Res. Perspect.* **2021**, *8*, 100185, doi:10.1016/J.ERP.2021.100185.
13. Chai, J.; Liu, J.N.K.; Ngai, E.W.T. Application of Decision-Making Techniques in Supplier Selection: A Systematic Review of Literature. *Expert Syst. Appl.* **2013**, *40*, 3872–3885, doi:10.1016/J.ESWA.2012.12.040.
14. Verma, R.; Gupta, A.; Singh, K. A Critical Study and Comparison of Manufacturing Simulation Softwares Using Analytic Hierarchy Process. *J. Eng. Sci. Technol.* **2010**, *5*, 108–129.
15. Jadhav, A.S.; Sonar, R.M. Framework for Evaluation and Selection of the Software Packages: A Hybrid Knowledge Based System Approach. *J. Syst. Softw.* **2011**, *84*, 1394–1407, doi:10.1016/J.JSS.2011.03.034.
16. Sahani, N. Application of Hybrid SWOT-AHP-FuzzyAHP Model for Formulation and Prioritization of Ecotourism Strategies in Western Himalaya, India. *Int. J. Geoh Heritage Park.* **2021**, *9*, 349–362, doi:10.1016/J.IJGEOP.2021.08.001.
17. Tahri, M.; Kašpar, J.; Madsen, A.L.; Modlinger, R.; Zabihi, K.; Marušák, R.; Vacik, H. Comparative Study of Fuzzy-AHP and BBN for Spatially-Explicit Prediction of Bark Beetle Predisposition. *Environ. Model. Softw.* **2022**, *147*, 105233, doi:10.1016/J.ENVSOFT.2021.105233.
18. Yoo, S.K.; Kim, B.Y. A Decision-Making Model for Adopting a Cloud Computing System. *Sustain.* **2018**, *Vol. 10*, Page 2952 **2018**, *10*, 2952, doi:10.3390/SU10082952.
19. Daengsi, T.; Sirawongphatsara, P.; Pornpongtechavanich, P. QoE Modeling Associated with QoS Impairment Parameters in 5G Networks Using AHP Decision Making Technique. *2021 Int. Conf. Decis. Aid Sci. Appl. DASA 2021* **2021**, 550–552, doi:10.1109/DASA53625.2021.9682366.
20. Amandeep; Mohammad, F.; Yadav, V. Automatic Decision Making for Multi-Criteria Load Balancing in Cloud Environment Using AHP. *Int. Conf. Comput. Commun. Autom. ICCCA 2015* **2015**, 569–576, doi:10.1109/CCAA.2015.7148473.
21. Guimarães, A.M.C.; Leal, J.E.; Mendes, P. Discrete-Event Simulation Software Selection for Manufacturing Based on the Maturity Model. *Comput. Ind.* **2018**, *103*, 14–27, doi:10.1016/J.COMPIND.2018.09.005.
22. Sakhardande, M.J.; Prabhu Gaonkar, R.S. On Solving Large Data Matrix Problems in Fuzzy AHP. *Expert Syst. Appl.* **2022**, *194*, 116488, doi:10.1016/J.ESWA.2021.116488.
23. Yang, Y.Z.; Yen, C.H. An AHP Method with Cosmetic Design Application. *Proc. - 2019 Int. Conf. Virtual Real. Intell. Syst. ICVRIS 2019* **2019**, 346–349, doi:10.1109/ICVRIS.2019.00090.
24. Ishizaka, A.; Lusti, M. How to Derive Priorities in AHP: A Comparative Study. *Cent. Eur. J. Oper. Res.* **2006**, *14*, 387–400, doi:10.1007/s10100-006-0012-9.

25. Leal, J.E. AHP-Express: A Simplified Version of the Analytical Hierarchy Process Method. *MethodsX* **2020**, *7*, 100748, doi:10.1016/J.MEX.2019.11.021.
26. Ranji, A.; Parashkoohi, M.G.; Zamani, D.M.; Ghahderijani, M. Evaluation of Agronomic, Technical, Economic, and Environmental Issues by Analytic Hierarchy Process for Rice Weeding Machine. *Energy Reports* **2022**, *8*, 774–783, doi:10.1016/J.EGYR.2021.12.028.
27. Chan, F.T.S.; Chan, H.K. An AHP Model for Selection of Suppliers in the Fast Changing Fashion Market. *Int. J. Adv. Manuf. Technol.* **2010**, *51*, 1195–1207, doi:10.1007/s00170-010-2683-6.
28. Kaymaz, Ç.K.; Birinci, S.; Kızılkın, Y. Sustainable Development Goals Assessment of Erzurum Province with SWOT-AHP Analysis. *Environ. Dev. Sustain.* **2022**, *24*, 2986–3012, doi:10.1007/s10668-021-01584-w.
29. Imran, M.; Agha, M.H.; Ahmed, W.; Sarkar, B.; Ramzan, M.B. Simultaneous Customers and Supplier's Prioritization: An AHP-Based Fuzzy Inference Decision Support System (AHP-FIDSS). *Int. J. Fuzzy Syst.* **2020**, *22*, 2625–2651, doi:10.1007/s40815-020-00977-9.
30. Nikhil, S.; Danumah, J.H.; Saha, S.; Prasad, M.K.; Rajaneesh, A.; Mammen, P.C.; Ajin, R.S.; Kuriakose, S.L. Application of GIS and AHP Method in Forest Fire Risk Zone Mapping: A Study of the Parambikulam Tiger Reserve, Kerala, India. *J. Geovisualization Spat. Anal.* **2021**, *5*, 14, doi:10.1007/s41651-021-00082-x.
31. Ayyildiz, E.; Gumus, A.T. Correction to: Pythagorean Fuzzy AHP Based Risk Assessment Methodology for Hazardous Material Transportation: An Application in Istanbul. *Environ. Sci. Pollut. Res.* **2022**, *29*, 27607, doi:10.1007/s11356-022-19045-w.
32. Ayyildiz, E.; Taskin Gumus, A. A Novel Spherical Fuzzy AHP-Integrated Spherical WASPAS Methodology for Petrol Station Location Selection Problem: A Real Case Study for İstanbul. *Environ. Sci. Pollut. Res.* **2020**, *27*, 36109–36120, doi:10.1007/s11356-020-09640-0.
33. Akram, M.; Khan, A.; Ahmad, U. Extended MULTIMOORA Method Based on 2-Tuple Linguistic Pythagorean Fuzzy Sets for Multi-Attribute Group Decision-Making. *Granul. Comput.* **2022**, doi:10.1007/s41066-022-00330-5.
34. Tam, M.C.Y.; Tummala, V.M.R. An Application of the AHP in Vendor Selection of a Telecommunications System. *Omega* **2001**, *29*, 171–182, doi:10.1016/S0305-0483(00)00039-6.
35. Fernando, E.; Siagian, P. Proposal to Use the Analytic Hierarchy Process Method Evaluate Bank Credit Submissions. *Procedia Comput. Sci.* **2021**, *179*, 232–241, doi:10.1016/J.PROCS.2021.01.002.
36. Using the Analytic Hierarchy Process (Ahp) to Select and Prioritize Projects in a Portfolio Available online: <https://www.pmi.org/learning/library/analytic-hierarchy-process-prioritize-projects-6608> (accessed on 28 February 2022).
37. Horňáková, N.; Jurík, L.; Hrablík Chovanová, H.; Cagánová, D.; Babčanová, D. AHP Method Application in Selection of Appropriate Material Handling Equipment in Selected Industrial Enterprise. *Wirel. Networks* **2019**, *27*, 1683–1691, doi:10.1007/S11276-019-02050-2.
38. Yao, X.; Zhao, F. A Quantitative Evaluation Based on an Analytic Hierarchy Process for the Deterioration Degree of the Guangyuan Thousand-Buddha Grotto from the Tang Dynasty in Sichuan, China. *Herit. Sci.* **2022**, *10*, 1–18, doi:10.1186/S40494-022-00655-Z/TABLES/14.
39. Gui, M.; Argentin, G. Digital Skills of Internet Natives: Different Forms of Digital Literacy in a Random Sample of Northern Italian High School Students. *New Media Soc.* **2011**, *13*, 963–980, doi:10.1177/1461444810389751.