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Article

Impulsive control and synchronization for fractional-order hyper-chaotic financial system

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Abstract: This paper reports a new global Mittag-Leffler synchronization criterion with regard to fractional-order hyper-chaotic financial systems by designing the suitable impulsive control and the state feedback controller. The significance of this impulsive synchronization lies in the fact that the backward economic system can synchronize asymptotically with the advanced economic system under the effective impulse macroeconomic management means. Matlab LMI-toolbox is utilized to deduce the feasible solution in numerical example, which shows the effectiveness of the proposed methods. It is worth mentioning that the LMI-based criterion usually requires the activation function of the system to be Lipschitz, but the activation function in this paper is fixed and truly nonlinear, which cannot be assumed to be Lipschitz continuous. This is another mathematical difficulty overcome in this paper.

Keywords: Mittag-Leffler stability; Caputo fractional-order derivative; Non-Lipschitz continuity; Hyper-chaotic financial system; Mittag-Leffler function

MSC: 34D06, 34A37

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1. Introduction

Hyperchaotic financial mathematical model includes the information of average profit margin, which simulates the actual complex and changeable financial market better, and has attracted much attention of researchers ([1-5]). As what has been pointed out in [1,6], an extension of fractality concepts in the investigation of economic systems has been used. In the field of mathematical applications to physics and engineering, numerous researchers have adopted fractional calculus as an effective method to model and simulate various nonlinear system ([1,7,8]). For example, in [1], a fractional-order hyperchaotic financial system was investigated and the adaptive control scheme for synchronization and chaos suppression of the fractional-order economic systems was proposed. However, impulsive control is not considered in [1]. In fact, impulsive control is always one of the means of macroeconomic management ([2,3]). In [2], a synchronization criterion of hyperchaotic financial system was derived by using impulsive control method and differential mean value theorem. The authors in [3] utilized impulse, Lapalcian semigroup and fixed point theorems to stabilize globally hyperchaotic financial system. But the results in [2,3] are only applicable to integer order differential equation models, and the effective methods used in [2,3] are not suitable to fractional-order hyperchaotic financial mathematical model. Indeed, fractional-order financial system always involves Mittag-Leffler stability, which is different from those of [2,3]. Besides, Riemann-Liouville fractional-order derivative was studied in [1], and so in this paper we consider Caputo fractional-order derivative. To overcome the above-mentioned mathematical difficulties, we shall design a suitable controller and utilize impulsive control to achieve drive-response synchronization. The

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significance of this impulsive synchronization lies in the fact that the backward economic system can synchronize asymptotically with the advanced economic system under the effective impulse macroeconomic management means.

This article has the following novelties:

⋄ This paper offers an original definition (see Definition 4) of chaotic financial systems' synchronization, in which the boundedness of the interest rate and the investment demand conforms to financial reality.

⋄ Because the activation function of chaotic financial system in this paper is non-Lipschitz continuous, many lemmas given in the previous fractional-order neural network literature cannot be applied to this paper. The authors overcome the mathematical difficulties caused by true nonlinearity and obtains the LMI-based synchronization criterion for the first time.

⋄ Because the LMI-based synchronization criterion of fractional-order chaotic financial system is obtained in this paper, the mathematical methods in this paper are different from those of existing literature involved in synchronization of fractional-order chaotic financial systems (see,e.g. [13]).

2. Preliminaries

Generally speaking, there are three common fractional derivatives, including the Grunwald-Letnikov fractional derivative, Riemann-Liouville fractional derivative, as well as the Caputo fractional derivative.

In this paper, the Caputo fractional derivative is considered.

Definition 1.([9]) The fractional integral of order χ for a function $u(t) \in C[[0,+\infty),\mathbb{R}]$ is defined by

$$D^{-\chi}u(t) = \frac{1}{\Gamma(\chi)}\int_0^t (t-s)^{\chi-1}u(s)ds,$$

where $\chi > 0$, and $\Gamma(\chi) = \int_0^{+\infty} e^{-t}t^{\chi-1}dt$.

Definition 2.([9]) The Caputo fractional derivative of order χ for a function $u(t) \in C^{n+1}[[0,+\infty),\mathbb{R}]$ (the set of all n-order continuous differentiable functions on $[0,+\infty]$) is defined as

$$D^{\chi}u(t) = \frac{1}{\Gamma(n-\chi)}\int_0^t (t-s)^{n-\chi-1}u^{(n)}(s)ds,$$

where $\chi > 0$, n is the first integer greater than χ, that is, $n-1 < \chi < n$, and the Laplace transform of $D^{\chi}u(t)$ is given as

$$\mathcal{L}\{D^{\chi}u(t)\} = s^{\chi}\mathcal{H}(s) - \sum_{k=0}^{n-1} s^{\chi-k-1}u^{(k)}(0),$$

where $\mathcal{H}(s) = \mathcal{L}\{u(t)\}$ stands for the Laplace transform of $u(t)$. In particular, when $0 < \chi < 1$, one obtains that

$$D^{\chi}u(t) = \frac{1}{\Gamma(1-\chi)}\int_0^t (t-s)^{-\chi}u'(s)ds,$$

$$\mathcal{L}\{D^{\chi}u(t)\} = s^{\chi}\mathcal{H}(s) - s^{\chi-1}u(0).$$

Definition 3.([9]) The one-parameter and the two-parameter Mittag-Leffler function are defined as

$$E_{\chi}(w) = \sum_{k=0}^{\infty}\frac{w^k}{\Gamma(\chi k+1)}, \quad \chi > 0,\ w \in \mathbb{C},$$

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$$E_{\chi,\beta}(w) = \sum_{k=0}^{\infty} \frac{w^k}{\Gamma(\chi k + \beta)}, \quad \chi > 0,\, \beta > 0,\, w \in \mathbb{C},$$

respectively, and the Laplace transform of the two-parameter Mittag-Leffler function is

$$\mathcal{L}[t^{\beta-1}E_{\chi,\beta}(-\gamma t^{\chi})] = \frac{s^{\chi-\beta}}{s^{\chi}+\gamma}, \quad t \geqslant 0,\, Re(s) > |\gamma|^{\frac{1}{\chi}},$$

where $\gamma \in \mathbb{R}$ and $Re(s)$ denotes the real parts of s.

Consider the following fractional-order hyper-chaotic financial system as the drive system:

$$\begin{cases} D^{\alpha}x = -x \cdot a + (y \cdot x + u + z), \\ D^{\alpha}y = 1 - y \cdot b - x^2, \\ D^{\alpha}z = -z \cdot c - x, \\ D^{\alpha}u = -u \cdot k - x \cdot d \cdot y, \end{cases} \tag{1}$$

which can be written in compact form

$$D^{\alpha}X(t) = AX(t) + f(X(t)), \quad t \geqslant 0, \tag{2}$$

where $X = (x_1, x_2, x_3, x_4)^T = (x, y, z, u)^T$,

$$A = \begin{pmatrix} -a & 0 & 1 & 1 \\ 0 & -b & 0 & 0 \\ -1 & 0 & -c & 0 \\ 0 & 0 & 0 & -k \end{pmatrix}, \quad f(X) = (f_1(X), f_2(X), f_3(X), f_4(X))^T = \begin{pmatrix} x_1x_2 \\ -x_1^2+1 \\ 0 \\ -dx_1x_2, \end{pmatrix}. \tag{3}$$

and the initial value is of

$$X(0) = X_0. \tag{4}$$

Here, the parameters a, b, c, d and variables x, y, z, u can be referenced in detail in the literature ([1-5]),

Consider the corresponding response system as follows,

$$\begin{cases} D^{\alpha}Y(t) = AY(t) + f(Y(t)) + W(t), & t \geqslant 0,\, t \neq t_k, \\ Y(t_k^+) = Y(t_k^-) + g(Y(t_k) - X(t_k)), & k = 1, 2, \cdots \\ Y(0) = Y_0, \end{cases} \tag{5}$$

where $W(t) = (w_1(t), w_2(t), w_3(t), w_4(t))^T$, and each $w_i(t)$ is a controller to be designed. $0 < t_1 < t_2 < \cdots$, and each $t_k (k \in \mathbb{Z}_+)$ represents a fixed impulsive instant, and $Y(t_k^-) = \lim\limits_{t \to t_k^-} Y(t) = Y(t_k)$, $Y(t_k^+) = \lim\limits_{t \to t_k^+} Y(t)$, for all $k = 1, 2, \cdots$. Here, $Y = (y_1, y_2, y_3, y_4)$.

Set $e(t) = Y(t) - X(t)$, then the systems (1) and (5) synchronization error model is

$$\begin{cases} D^{\alpha}e(t) = Ae(t) + F(e(t)) + W(t), & t \geqslant 0,\, t \neq t_k, \\ e(t_k^+) = e(t_k^-) + g(e(t_k)), & k = 1, 2, \cdots \\ e(0) = Y_0 - X_0, \end{cases} \tag{6}$$

where

$$F(e) = f(Y) - f(X). \tag{7}$$

Design the following state feedback controller

$$W(t) = M(X(t) - Y(t)), \tag{8}$$

where M is a constant matrix.

In our study, the following assumption conditions are useful:
(H1) $|x_i(t)| \leqslant m_i$, and $|y_i(t)| \leqslant m_i$, for all $i = 1, 2$.
(H2) $g(s) = Ls$, where $s \in \mathbb{R}^4$, and L is a constant matrix.

Definition 4. The system (5) is said to achieve global Mittag-Leffler synchronization with the system (2) if there exist two positive constants $0 < K_1, K_2 < 1$ such that for any initial values $X_0 = (x_{01}, x_{02}, x_{03}, x_{04})$ and $Y_0 = (y_{01}, y_{02}, y_{03}, y_{04})$ with $|x_{0i}| \leqslant K_i m_i$ and $|y_{0i}| \leqslant K_i m_i$, $i = 1, 2$, the null solution of the system (6) is global Mittag-Leffler stable.

The following lemma about Caputo fractional-order derivative with one-dimensional variable is common in many literatures ([1,7,14,16,17])

Lemma 1. Let $\xi(t) \in \mathbb{R}^1$ be a continuous and derivable function. Then, for any $t \geqslant 0$,

$$\frac{1}{2} D^\alpha \xi^2(t) \leqslant \xi(t) D^\alpha \xi(t), \quad \forall\, \alpha \in (0, 1).$$

3. Main result

In this section, we present the LMI-based synchronization criterion for chaotic financial systems:

Theorem 1. Assume that conditions (H1)-(H2) hold. If there exists a positive definite symmetric matrix P and positive real number ε_1 such that

$$PA + A^T P - PM - M^T P + \varepsilon_1^{-1} I + \varepsilon_1 H^T H < 0, \tag{9}$$

$$(I + L)^T P (I + L) < P, \tag{10}$$

then the system (5) achieves global Mittag-Leffler synchronization with the system (2), where I is the identity matrix, and

$$H = \begin{pmatrix} m_2 & m_1 & 0 & 0 \\ m_1 + m_2 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ dm_2 & dm_1 & 0 & 0, \end{pmatrix}.$$

Proof. Firstly, it follows from (H1) and (3) that

$$|F(e(t))| \leqslant H|e(t)|,$$

Consider the Lyapunov function as follows,

$$V(t, e(t)) = e^T(t) P e(t),$$

where P is a positive definite symmetric matrix. Below, in order to derive a common result on Captuto fractional derive and quadratic forms, we might temporarily assume that P is a positive definite symmetric matrix with n-dimension, and e is a vector with n-dimension.

Then there is a congruent transformation such that

$$Q^{-1} P Q = \begin{pmatrix} \lambda_1 & 0 & \cdots & 0 & 0 \\ 0 & \lambda_2 & 0 & \cdots & 0 \\ & & \ddots & & \\ 0 & 0 & 0 & \cdots & \lambda_n \end{pmatrix},$$

where Q is the n-dimensional matrix with $Q^T = Q^{-1}$, and each λ_i is a positive real nember for $i = 1, 2, \cdots, n$.

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Hence,

$$
\begin{aligned}
P =& Q\begin{pmatrix} \lambda_1 & 0 & \cdots & 0 & 0 \\ 0 & \lambda_2 & 0 & \cdots & 0 \\ & & \ddots & & \\ 0 & 0 & 0 & \cdots & \lambda_n \end{pmatrix} Q^{-1} \\
=& Q\begin{pmatrix} \sqrt{\lambda_1} & 0 & \cdots & 0 & 0 \\ 0 & \sqrt{\lambda_2} & 0 & \cdots & 0 \\ & & \ddots & & \\ 0 & 0 & 0 & \cdots & \sqrt{\lambda_n} \end{pmatrix} Q^T Q \begin{pmatrix} \sqrt{\lambda_1} & 0 & \cdots & 0 & 0 \\ 0 & \sqrt{\lambda_2} & 0 & \cdots & 0 \\ & & \ddots & & \\ 0 & 0 & 0 & \cdots & \sqrt{\lambda_n} \end{pmatrix} Q^T \\
=& B^T B,
\end{aligned}
$$

where B is a a positive definite symmetric matrix with

$$
B = Q\begin{pmatrix} \sqrt{\lambda_1} & 0 & \cdots & 0 & 0 \\ 0 & \sqrt{\lambda_2} & 0 & \cdots & 0 \\ & & \ddots & & \\ 0 & 0 & 0 & \cdots & \sqrt{\lambda_n} \end{pmatrix} Q^T.
$$

Let $v = Be$, then

$$
V(t, e(t)) = e^T(t)Pe(t) = v^T(t)v(t),
$$

where

$$
V(t, e(t)) = (v_1(t), v_2(t), \cdots, v_n(t))^T
$$

Lemma 1 and $v = Be$ yield

$$
\begin{aligned}
D^\alpha V(t, e(t)) =& D^\alpha\left(\sum_{i=1}^{n} v_i^2(t)\right) \\
=& \frac{1}{\Gamma(1-\alpha)} \int_0^t \frac{(\sum\limits_{i=1}^{n} v_i^2(t))'}{(t-s)^\alpha} \\
=& \sum_{i=1}^{n} \frac{1}{\Gamma(1-\alpha)} \int_0^t \frac{(v_i^2(t))'}{(t-s)^\alpha} \\
=& \sum_{i=1}^{n} D^\alpha(v_i^2(t)) \\
\leqslant& 2e^T(t)PD^\alpha e(t).
\end{aligned}
$$

then

$$
\begin{aligned}
D^\alpha V(t, e(t)) \leqslant& e^T(t)\left(PA + A^T P - PM - M^T P\right)e(t) + [e^T(t)F(e(t)) + F^T(e(t))e(t)] \\
\leqslant& e^T(t)\left(PA + A^T P - PM - M^T P + \varepsilon_1^{-1} I + \varepsilon_1 H^T H\right)e(t) \\
\leqslant& qV(t, e(t)), \quad t \geqslant 0,\, t \neq t_k,
\end{aligned}
\tag{11}
$$

where

$$
q = \lambda_{\max}\left(PA + A^T P - PM - M^T P + \varepsilon_1^{-1} I + \varepsilon_1 H^T H\right) < 0.
$$

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Obviously, there exists a function $\nu(t, e(t)) = D^{\alpha} V(t, e(t))$, and so there is a function $\mu(t) \geqslant 0$ such that

$$\mu(t) + \nu(t, e(t)) = qV(t, e(t)), \quad t \geqslant 0,\, t \neq t_k, \tag{12}$$

Denote by $*$ the convolution operator, then it follows by taking the Laplace transform and inverse Laplace transform of (12) that

$$V(t, e(t)) = V(0, e(0)) E_{\alpha}(qt^{\alpha}) - \mu(s) * (t^{\alpha-1} E_{\alpha,\alpha}(qt^{\alpha})), \quad t \geqslant 0,\, t \neq t_k, \tag{13}$$

which together with $\mu \geqslant 0$ means

$$V(t, e(t)) \leqslant V(0, e(0)) E_{\alpha}(qt^{\alpha}), \quad \forall\, t \in (t_{k-1}, t_k]. \tag{14}$$

On the other hand, (14) and (10) yield

$$\begin{aligned} V(t_k^+, e(t_k^+)) =& [(I+L)e(t_k)]^T P[(I+L)e(t_k)] \\ =& e^T(t_k)(I+L)^T P(I+L)e(t_k) \\ \leqslant& e^T(t_k) P e(t_k) \leqslant V(0, e(0)) E_{\alpha}(qt_k^{\alpha}), \quad \forall\, k = 1, 2, \cdots, \end{aligned} \tag{15}$$

which together with (14) implies

$$\lambda_{\min}(P) \|e(t)\|^2 \leqslant V(t, e(t)) \leqslant V(0, e(0)) E_{\alpha}(qt^{\alpha}), \quad \forall\, t \geqslant 0,$$

or

$$\|e(t)\| \leqslant \sqrt{\frac{1}{\lambda_{\min}(P)} V(0, e(0)) E_{\alpha}(qt^{\alpha})}, \quad \forall\, t \geqslant 0.$$

In consideration of $q < 0$, the above inequality means that the null solution of System (6) is globally Mittag-Leffler stable. This completes the proof.

Remark 1. The LMI-based synchronization criterion of Theorem 1 is different from that of [13, Theorem 1], which illuminates that the mathematical methods used in this paper are different from those of [13].

4. Numerical example

Below, we give a numerical example on synchronization of fractional-order ($\alpha = 0.95$) financial systems :

Example 1. Equip System (2) and System (5) with the following data

$$L = \begin{pmatrix} -0.0111 & 0.0001 & 0 & 0 \\ 0.0001 & -0.0112 & 0 & 0 \\ 0 & 0 & -0.0113 & 0 \\ 0 & 0 & 0 & -0.0111 \end{pmatrix}, M = \begin{pmatrix} 2.11 & 0.0001 & 0 & 0 \\ 0.0001 & 2.2 & 0 & 0 \\ 0 & 0 & 2.3 & 0 \\ 0 & 0 & 0 & 2.1 \end{pmatrix}.$$

Let $a = 0.9, b = 0.2, c = 1.5, d = 0.2, k = 0.17, m_1 = 8, m_2 = 9$, then

$$A = \begin{pmatrix} -0.9 & 0 & 1 & 1 \\ 0 & -0.2 & 0 & 0 \\ -1 & 0 & -1.5 & 0 \\ 0 & 0 & 0 & -0.17 \end{pmatrix}, \quad f(X) = (f_1(X), f_2(X), f_3(X), f_4(X))^T = \begin{pmatrix} x_1 x_2 \\ -x_1^2 + 1 \\ 0 \\ -0.2 x_1 x_2, \end{pmatrix}.$$

$$H = \begin{pmatrix} 9 & 8 & 0 & 0 \\ 17 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 1.8 & 1.6 & 0 & 0 \end{pmatrix}.$$

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Using MATLAB LMI-toolbox to solve LMI conditions (9) and (10) results in the following feasibility data:

$$P = \begin{pmatrix} 301.5313 & 0 & 0 & 0.0011 \\ 0 & 289.7811 & 0 & 0 \\ 0 & 0 & 313.9712 & 0 \\ 0.0011 & 0 & 0 & 276.7876 \end{pmatrix}, \quad \varepsilon_1 = 0.5013$$

According to Theorem 1, the system (5) is global Mittag-Leffler synchronization with the system (2) (see Fig.1-7).

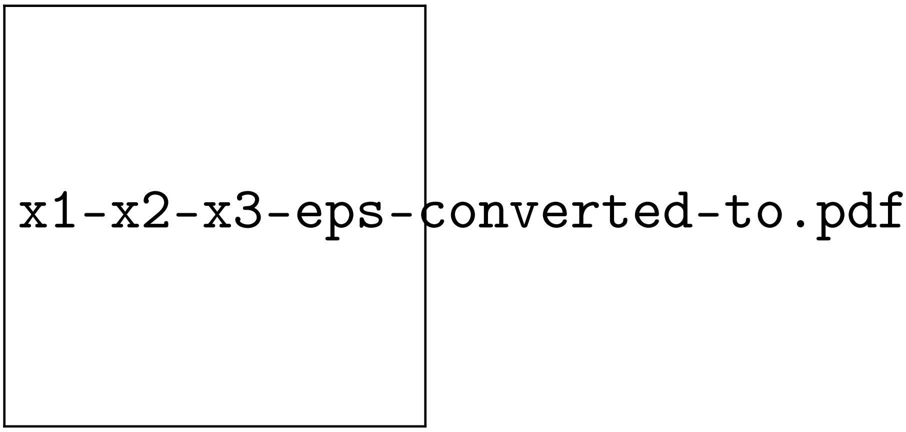

Fig. 1: 3D view synchronization for fractional-order 0.95

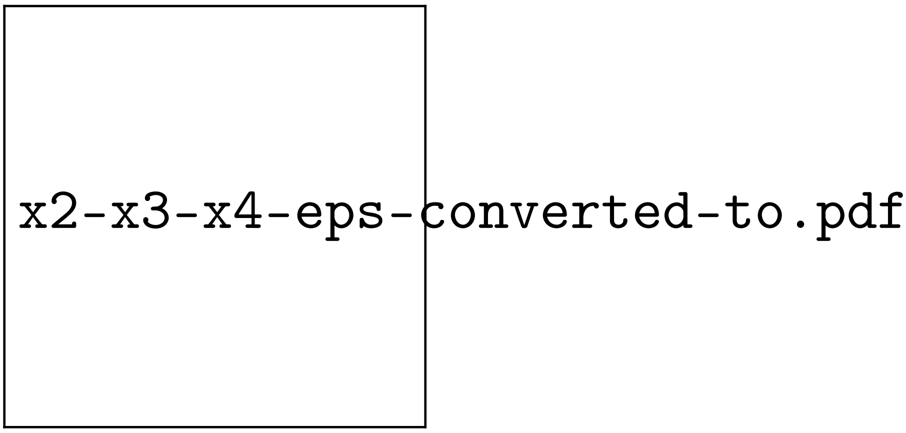

Fig. 2: 3D view synchronization for fractional-order 0.95

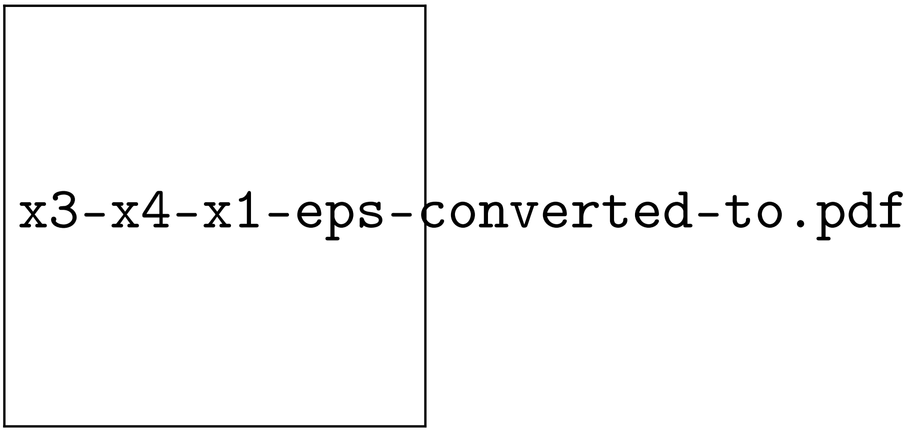

Fig. 3: 3D view synchronization for fractional-order 0.95

u1-eps-converted-to.pdf

Fig. 4: Computer simulations of synchronization between x_1 and y_1 for fractional-order 0.95

u2-eps-converted-to.pdf

Fig. 5: Computer simulations of synchronization between x_2 and y_2 for fractional-order 0.95

u3-eps-converted-to.pdf

Fig. 6: Computer simulations of synchronization between x_3 and y_3 for fractional-order 0.95

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Fig. 7: Computer simulations of synchronization between x_4 and y_4 for fractional-order 0.95

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Remark 2. Numerical simulation shows the effectiveness of Theorem 1. Indeed, we can see it from Fig.1-3 that System (5) is global Mittag-Leffler synchronization with System (2). Besides, we can also see it from Fig.4-7 that $|x_1(t)| < 8$ and $|y_1(t)| < 8$, $|x_2(t)| < 9$ and $|y_2(t)| < 9$. And hence, the condition (H1) holds.

Finally, we point out that Theorem 1 is also suitable to the case of integer order, for example, integer order $\alpha = 1$.

Example 2. Set $\alpha = 1$ in the systems (5) and (2). Equip System (2) and System (5) with the following data

$$L = \begin{pmatrix} -0.0122 & 0 & 0 & 0 \\ 0 & -0.0123 & 0 & 0 \\ 0 & 0 & -0.0113 & 0 \\ 0 & 0 & 0 & -0.0111 \end{pmatrix}, M = \begin{pmatrix} 2.11 & 0 & 0 & 0 \\ 0 & 2.2 & 0 & 0 \\ 0 & 0 & 2.3 & 0 \\ 0 & 0 & 0 & 2.1 \end{pmatrix}.$$

$$PA + A^T P - PM - M^T P + \varepsilon_1^{-1} I + \varepsilon_1 H^T H < 0, \tag{9}$$

$$(I+L)^T P(I+L) < P, \tag{10}$$

Let $a = 0.6, b = 0.3, c = 0.8, d = 0.1, k = 0.11, m_1 = 8, m_2 = 9$, then

$$A = \begin{pmatrix} -0.6 & 0 & 1 & 1 \\ 0 & -0.3 & 0 & 0 \\ -1 & 0 & -0.8 & 0 \\ 0 & 0 & 0 & -0.11 \end{pmatrix}, \quad f(X) = (f_1(X), f_2(X), f_3(X), f_4(X))^T = \begin{pmatrix} x_1 x_2 \\ -x_1^2 + 1 \\ 0 \\ -0.1 x_1 x_2, \end{pmatrix}.$$

$$H = \begin{pmatrix} 9 & 8 & 0 & 0 \\ 17 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0.9 & 0.8 & 0 & 0 \end{pmatrix}.$$

Using MATLAB LMI-toolbox to solve LMI conditions (9) and (10) results in the following feasibility data:

$$P = \begin{pmatrix} 398.5313 & 0 & 0 & 0.0003 \\ 0 & 301.3316 & 0 & 0 \\ 0 & 0 & 321.8983 & 0 \\ 0.0003 & 0 & 0 & 331.9879 \end{pmatrix}, \quad \varepsilon_1 = 0.4996$$

According to Theorem 1, the system (5) is global Mittag-Leffler synchronization with the system (2) (see Fig.8-11).

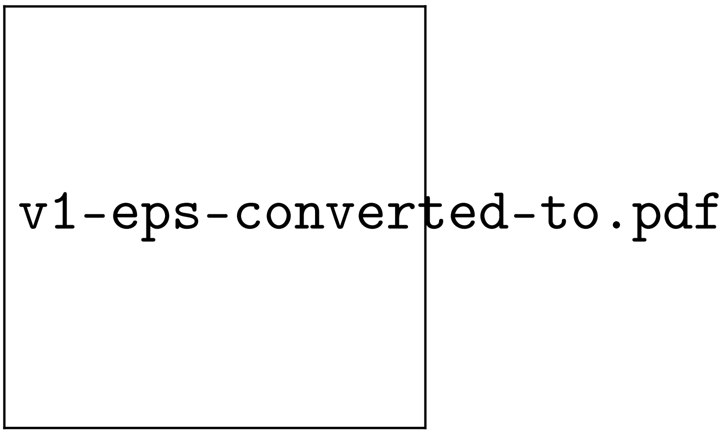

Fig. 8: Computer simulations of synchronization between x_1 and y_1 for $\alpha = 1$

v2-eps-converted-to.pdf

Fig. 9: Computer simulations of synchronization between x_2 and y_2 for $\alpha = 1$

v3-eps-converted-to.pdf

Fig. 10: Computer simulations of synchronization between x_3 and y_3 for $\alpha = 1$

v4-eps-converted-to.pdf

Fig. 11: Computer simulations of synchronization between x_4 and y_4 for $\alpha = 1$

Remark 3. Fig.8-11 illustrate that the synchronization speeds of the financial systems in Example 2 are faster than those in Example 1, for the data $a = 0.6, b = 0.3, c = 0.8, d = 0.1, k = 0.11$ are different from those of Example 1. In fact, the corresponding data in Example 1 made financial systems chaos (see [1,3-5,13]).

5. Conclusions

Inspired by recent literature related to fractional-order models or chaotic systems ([14-19]), the authors design the suitable impulsive control and the state feedback controller to make fractional-order hyper-chaotic financial systems global Mittag-Leffler synchronization, and use computer Matlab LMI-toolbox to verify the effectiveness of newly-obtained criterion. Both theoretical and numerical examples show that as long as the impulsive macroeconomic management measures are appropriate, the backward economic system can gradually synchronize with the advanced economic system. Finally, the impulsive control involving time delay and the impulsive control under trigger event mechanism still need to be studied for the mathematical model of macroeconomics([10-12,20]). It is an interesting problem how to establish a reasonable model in line with the principles of Macroeconomics.

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