

1 Contextual accuracy assessments for InSAR methods
2 using synthetic data

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4 **Abstract**

InSAR and associated analytic methods enable relative surface deformation measurements from low Earth orbit with a potential accuracy of centimeters to millimeters. However, assessing the actual accuracy of individual points can be quite difficult. The analytic methods are complicated enough that naïve analytic error propagation is infeasible, and, in many settings, InSAR practitioners lack sufficient ground truth to assess results. Phase noise due to partial decorrelation from changes in the scattering properties of the ground is a prominent source of accuracy loss. In this paper we present a method to assess the loss of precision due to this component of phase noise. The proposed method consists of generating synthetic data whose statistical properties match that of the actual input SAR data stacks, and then using the synthetic data for an ensemble calculation. The spread of the results of the ensemble calculation indicates the loss of precision. We show examples of the ensemble analysis at a mining operation in South Africa, and demonstrate the ability to assess the most reliable methods for particular points of interest using this ensemble analysis and the ability to filter out points based on the width of the spread of results.

5 *Keywords:* InSAR, deformation, synthetic data, ensemble methods,
6 uncertainty estimate, time series analysis

7 **1. Introduction**

8 Interferometric Synthetic Aperture Radar (InSAR) enables measurements
9 of surface deformation with a potential accuracy of centimeters to millime-
10 ters (Rosen et al., 2000). A complex valued Synthetic Aperture Radar (SAR)
11 image consists of pixels whose phase is determined by the scattering properties
12 on the Earth’s surface and the effective round trip distance between the satel-
13 lite and the surface. If the scattering properties change minimally, the phase

change between corresponding pixels in two co-registered complex SAR images will reflect changes in the effective round trip distance between the satellite and the surface and can detect surface displacements in the imaged area. Methods such as the Small Baseline Subset (SBAS) method (Berardino et al., 2002), the Permanent Scatterers (PS) method (Ferretti et al., 2001), and their variants can then be used to extract surface deformation from phase differences in a series of complex SAR images.

There are a number of potential sources of error that InSAR methods must address to extract the most accurate surface deformation. First, the effective round trip distance between the satellite and the surface includes effects from tropospheric moisture and free electrons in the ionosphere (Goldstein, 1995; Zebker et al., 1997). Second, successive passes do not revisit the same orbital paths exactly, which can lead to geometrical decorrelation (Rodriguez and Martin, 1992; Zebker and Villasenor, 1992). Third, differences in the actual round-trip distance between the satellite and the ground can be estimated using precise orbit data and a digital elevation model (DEM), but the residual errors are still larger than the deformation one seeks to measure. Fourth, the desired component of the phase is the round trip distance between the satellite and the ground modulo the wavelength, which is a few centimeters, mapped to $[-\pi, \pi)$, see Goldstein et al. (1988). The process of recovering an “absolute” phase from the observed or “wrapped” phase is called *unwrapping*, and without additional constraints the problem is ill-posed. Finally, the scattering properties of the ground change with time, resulting in temporal decorrelation, see Zebker and Villasenor (1992).

Many of these issues have been addressed to an acceptable degree. Tropospheric moisture and its contribution to phase, known as atmospheric phase screening (APS), have been studied extensively, e.g. Hanssen (2001, Chapter 6),

41 and Ferretti et al. (1999, Section 2), and a number of methods to address APS,
42 without using external inputs like weather models, have been presented (Murray
43 et al., 2019; Yu et al., 2017; Tymofyeyeva and Fialko, 2015). Absent real-time
44 models of tropospheric moisture, one can assume APS varies slowly in space
45 and any spatial variations in APS can be assumed to be uncorrelated between
46 revisits (Zebker et al., 1997). This allows one to separate a deformation sig-
47 nal that is consistent in time from APS. Ionospheric effects, which tend to be
48 more significant for L-band and longer wavelength systems, have been similarly
49 studied (Chen and Zebker, 2013; Fattahi et al., 2017; Liang et al., 2019). Geo-
50 metrical decorrelation has been largely addressed by better orbit control. Errors
51 in DEMs manifest as a phase contribution that correlates with the component of
52 orbit offsets that is perpendicular to the look direction. This effect is sufficiently
53 reliable to use InSAR to generate DEMs (Van Zyl, 2001; Zink et al., 2014).

54 Assessments of the phase contribution due to changes in scattering proper-
55 ties have largely focused on how to select "good" pixels to include in an InSAR
56 analysis, e.g. Ferretti et al. (2001). The phase of any single pixel changes sig-
57 nificantly between successive collections. This remains true even after removing
58 the estimated phase contribution associated with the round trip distance, since,
59 as noted above, the combined uncertainty in the orbit data and DEM exceeds
60 half a wavelength. This inability to access the "phase on the ground" can be
61 addressed in several ways. In Ferretti et al. (2001) the authors present an ele-
62 gant argument relating phase noise to temporal variations in amplitude, which
63 is not affected by APS or orbit variation. This allows one to identify likely
64 permanent scatterers (PS), i.e. pixels where scattering is dominated by a single
65 stable feature. These PS pixels are included in the InSAR analysis. The draw-
66 back to this method is that it often selects too few pixels outside of urban areas.
67 Indeed, the method presented in Ferretti et al. (2001) includes additional steps

to include more pixels that are consistent with the initial PS pixels, but which themselves do not have the desired amplitude statistics. Alternatively, because many pervasive phase contributions vary slowly in space, one can choose a spatial window to determine if the phase changes are at least consistent within that spatial window. At the simplest level one can estimate the interferometric phase difference between two SAR collections as shown in Eqs. 1.13 and 1.14 in Ferretti et al. (2007a). Here one approximates the expectation value of the phase change for pixel (Eq. 1.13) by assuming the statistics are sufficiently uniform within the averaging window (Eq. 1.14). There are more sophisticated methods that incorporate a stack of coregistered complex SAR images to estimate a consistent time series for the phase, e.g. Guarnieri and Tebaldini (2008); Ansari et al. (2018). We refer to phase quality between two SAR collections as *coherence*, and the phase quality for a stack of (more than two) SAR collections *temporal coherence*. These methods to estimate the quality of interferometric phase, and, in the cases of Ferretti et al. (2007a, Eq. 1.14), Guarnieri and Tebaldini (2008), and Ansari et al. (2018), the phase itself, are based on the idea that phase noise should be considered as a statistical process and these methods can estimate the parameters of that process.

These coherence estimation methods can also be used to estimate uncertainty in phase (Bamler and Just, 1993; Jong-Sen Lee et al., 1994, Figs. 2,3). One could assume that uncertainty in the wrapped phase is the same as the uncertainty in the unwrapped phase, and finally, accumulate uncertainty according to the chosen interferogram network. While this approach can give a rough approximation, there are two shortcomings worth noting. First, unwrapping is necessarily non-linear, and the uncertainty in the unwrapped phase will likely not be the same as the uncertainty in the wrapped phase. As an aside, it is the authors' experience that the iterative and conditional logic in unwrapping schemes makes

a naïve error propagation infeasible. Second, the historical uncertainty is often correlated in a manner that cannot be captured by a single number such as temporal coherence, or even by coherence between, say, successive SAR scenes. For example, coherence between fall and spring SAR scenes may be high, while snow causes scene-to-scene coherence involving scenes collected in winter months to be low. Indeed, *phase-linking* InSAR methods (cf. Guarnieri and Tebaldini (2008); Ansari et al. (2018)) use a sample correlation matrix (SCM) to reconstruct a consistent wrapped phase from all $N(N - 1)/2$ non-redundant interferometric pairs. This suggests that models of uncertainties should capture correlations in those uncertainties.

This current work proposes a novel method to estimate uncertainty, more specifically precision, in deformation results based on uncertainty in phase on a point-by-point basis. The method is described in detail in Section 3. The two main ideas behind this method are, 1, we create synthetic data whose statistics match the statistics of the input data, then, 2, estimate the uncertainty based on the standard deviation of an ensemble calculation using these synthetic data. This method can then assess any deformation retrieval method by measuring the spread in results in the ensemble of deformations retrieved, and can allow the estimation of uncertainty between any two test and reference points. We demonstrate this with an example from a mining site in South Africa in Section 4.

2. Background

Previous studies have attempted to assess the accuracy of InSAR derived deformation using GPS/GNSS data, e.g. Ferretti et al. (2007b); Zebker (2021); Lee et al. (2005); Jiang and Lohman (2021); Armaş et al. (2016), leveling measurements, e.g. Yang et al. (2016); Marinkovic et al. (2007); Luo et al. (2017),

or indirect methods such as water level, e.g. Wdowinski et al. (2004); Lu and Kwoun (2008) or precipitation, e.g. Palomino-Ángel et al. (2022). However, some of these methods cannot provide a direct comparison to deformation signals and therefore make it difficult to estimate accuracy or precision, and the uncertainty of other methods such as GPS/GNSS and leveling data are comparable if not greater than what is anticipated for InSAR measurements, e.g. You (2006). Further, these sources of validation are often temporally or spatially sparse, which does not allow one to estimate the accuracy of all points or epochs used in the InSAR surface deformation measurement and limits the ability to detect where the signal is most reliable and where it may be less accurate. Simulation has been used to assess particular phase contributions, e.g. Yunjun et al. (2019) provides a means to simulate atmospheric phase errors.

The method presented here is most similar to Agram and Simons (2015) where the authors present a thorough model of the correlations in uncertainties. The matrix that describes these correlations is quite large, and the current work does not so much "build on" but "remove from" that work, with the goal of establishing a more computational tractable method.

3. Method

3.1. Overview

Let N denote the number of complex SAR scenes. We choose a kernel k in the spatial domain and compute the Sample Correlation Matrix (SCM) for each pixel. Recall the r, c entry for the SCM (associated with time epochs r and c) corresponding to pixel i is

$$SCM_{rc}^i = \frac{\langle d_r^i; d_c^i \rangle_k}{\|d_r^i\|_k \|d_c^i\|_k}. \quad (1)$$

where

- 145 • d_a^b is the co-registered complex SAR data at epoch a for pixel b ,
- 146 • $\langle f; g \rangle_k$ is the inner product generated by integrating the convolution of
- 147 the product $f\bar{g}$ with the kernel k and
- 148 • $\|\cdot\|_k$ is the induced L^2 norm, i.e. $\sqrt{\langle \cdot; \cdot \rangle_k}$.

149 To generate sample data for pixel i , we notionally draw ϕ_0 from $(SCM^i)^{1/2}Z$
 150 where Z is a random column-vector variable of N i.i.d. entries each drawn from
 151 the complex normal distribution $\mathcal{CN}(0, 1)$. We let

$$\phi = \phi_0/|\phi_0|, \quad (2)$$

152 which we treat as the synthetic phase. The synthetic data for pixel i is then the
 153 Hadamard or element-wise product of amplitude vector of the input SAR data
 154 at pixel i and ϕ .

155 In addition to stacks of coregistered complex SAR scenes, InSAR analysis
 156 pipelines require a number of ancillary data such as orbit data and collection
 157 dates. We use these data from the input stack. With this, we can create an
 158 ensemble of synthetic data that have the same form as the input data, and
 159 any InSAR analytic pipeline can run on both real and synthetic data without
 160 change.

161 Running an InSAR analysis pipeline on an ensemble of such stacks generates
 162 a range of deformation results, which helps us understand the likely spread of
 163 deformation results due to decorrelation, and provides an estimate of precision
 164 per pixel and epoch.

165 3.2. Properties

166 There are a number properties of this method worth discussing.

167 *The SCM may not be positive semidefinite.* Since the SCM is Hermitian,
 168 we compute $(SCM^i)^{1/2}$ using an eigenvalue decomposition, taking the square

169 roots of the eigenvalues. If we encounter any negative eigenvalues we set them
 170 to zero and proceed.

171 *The phase of the SCM is reflected in the synthetic data.* To see this we
 172 consider the case of the SCM for a pixel for two epochs. The SCM will have the
 173 following form

$$SCM = \begin{bmatrix} 1 & z \\ z^* & 1 \end{bmatrix}, \quad (3)$$

174 where the modulus of z is the coherence and the argument of z is the estimated
 175 interferometric phase, both of which will depend on our choice of kernel. The
 176 eigenvalues and corresponding eigenvectors are $1+|z|$, $\frac{1}{\sqrt{2}}[\exp(i\theta), 1]^T$ and $1-|z|$,
 177 $\frac{1}{\sqrt{2}}[-\exp(i\theta), 1]^T$, where θ is chosen such that $\exp(i\theta) = \frac{z}{|z|}$. We can compute
 178 the square root of the SCM as

$$SCM^{1/2} = \frac{1}{2} \begin{bmatrix} U & V \exp(i\theta) \\ V \exp(-i\theta) & U \end{bmatrix}, \quad (4)$$

179 where

- 180 • $U = \sqrt{1+|z|} + \sqrt{1-|z|}$ and
- 181 • $V = \sqrt{1+|z|} - \sqrt{1-|z|}$.

182 For two i.i.d. random complex Gaussian variables Z_1 and Z_2 , we can compute
 183 the synthetic phase at the two epochs as

$$\begin{pmatrix} P_1 \\ P_2 \end{pmatrix} \equiv SCM^{1/2} \begin{pmatrix} Z_1 \\ Z_2 \end{pmatrix} = \frac{1}{2} \begin{pmatrix} U Z_1 + V \exp(i\theta) Z_2 \\ V \exp(-i\theta) Z_1 + U Z_2 \end{pmatrix} \quad (5)$$

184 We are interested in the relative phase of P_1 and P_2 so we consider

$$\begin{aligned}
 P_1 \overline{P_2} = \frac{1}{4} \exp(i\theta) (& \\
 & UV|Z_1|^2 \\
 & + U^2 Z_1 \overline{Z_2} \exp(-i\theta) \\
 & + V^2 Z_2 \overline{Z_1} \exp(i\theta) \\
 & + UV|Z_2|^2 \\
 &)
 \end{aligned} \tag{6}$$

185 The expectation value of this is

$$\frac{1}{2} |z| \mathbb{E}(|Z_1|^2) \exp(i\theta), \tag{7}$$

186 and has phase θ . The standard deviation of the phase of $P_1 \overline{P_2}$ as a function of
 187 $\gamma = |z|$ derived from numerical experiment is shown in Figure 1, and has the
 188 expected form (Bamler and Just, 1993).

189 For this reason, phase contributions from sources whose length scales exceed
 190 the width of k – typically APS, deformation and gross DEM errors – will be re-
 191 produced in the synthetic data (Guarnieri and Tebaldini, 2007), and one should
 192 not add these phase contributions in via additional simulated data.

193 *We are using the SCM as a covariance matrix.* It is natural to consider
 194 using the covariance matrix itself, which will only fail to be Hermitian positive
 195 semi-definite in the case of roundoff error. The covariance matrix would be
 196 appropriate if we were attempting to estimate the uncertainty in, and create
 197 synthetic versions of, both the amplitude and phase by using a windowed average
 198 specified by k . Our goal, however, is to create synthetic data that allows us to

199 estimate the uncertainty in phase estimations obtained by spatial averaging.¹.

200 *One must choose k .* The choice of k impacts the effective resolution of the
 201 synthetic data. One should choose k large enough to capture the uncertainty,
 202 but small enough to resolve the features of interest.

203 *Such synthetic data are only appropriate for kernel-based phase estimation.*

204 This process of generating synthetic data assumes that phase statistics are nearly
 205 uniform on a small scale, and that the kernel k can recover these phase statistics.

206 This assumption does not hold for a single permanent scatterer surrounded
 207 by noisy pixels. Consequently, if one uses amplitude dispersion to detect PSs
 208 within a noisy surrounding environment, these synthetic data will not repre-
 209 sent the uncertainty in the phase recovered from pixels selected by amplitude
 210 dispersion. The method we propose here will construct synthetic data using
 211 the statistics from a windowed average, the synthetic version of a single PS
 212 surrounded by noisy pixels will reflect the entire neighborhood of that PS, and
 213 will not capture the quality of that single pixel on its own. Note that, because
 214 the synthetic data uses the original amplitudes, the amplitude dispersion cal-
 215 culation will agree exactly for both synthetic and real data, while the synthetic
 216 phase for the pixel will likely be less reliable than the corresponding real phase.

217 One can create synthetic data for single pixels, essentially following Ferretti
 218 et al. (2001). One creates a Rice distribution that matches the observed am-
 219 plitude dispersion, selects the phase from the samples and pairs that with the
 220 input amplitude. While substantially simpler than our method based on the
 221 SCM, it suffers the drawback that the phase statistics are assumed to be fixed
 222 in time, as the estimate] is parametrized by a single input statistic. In this case,
 223 e.g., seasonal noise will not be captured. Further, such a method will, without

¹We attempted to use the actual covariance matrix and found that the resulting synthetic data had coherence that was much lower than the coherence of the real data.

additional inputs, produce synthetic data with mean zero phase.

4. Example

As an example, we apply our analytic methods to a mining operation in South Africa, shown in Figure 2. We use a stack of 31 images collected by Sentinel-1 track 131 between November of 2019 and November of 2020. The images were coregistered to a UTM grid with resolution of 2.5m East-West by 10m North-South. We chose the anisotropic resolution of the UTM grid to approximate the anisotropy of range and azimuth resolutions in an IW-TOPS mode collection.

We compare coherence between two scenes (Figure 3) and the magnitude of the SCM for an arbitrary pixel (Figure 4) derived from real data and synthetic data. In both cases it is evident that the coherence for the real data is slightly higher than it is for the synthetic data. We show this discrepancy in Figure 5. This plot shows the absolute values of SCM entries for synthetic data plotted against the absolute value for corresponding entries for real data. We've binned the data into narrow intervals of real coherence and computed the standard deviation of the synthetic coherence.

There are two notable features in Figure 5. First, the largest difference between coherence of real and synthetic data occurs roughly when the coherence of the real data is 0.7. At this point the coherence of the synthetic data has a mean of 0.56. Our hypothesis is that our noise model fails to capture correlations between nearby pixels that increase coherence. Second, at very low coherence the synthetic coherence is higher than real coherence. We believe this is caused partly because noise perturbs the SCM from being purely diagonal, but primarily because the coherence is bounded below by zero, and the mean is likely not entirely appropriate for determining a "representative" synthetic

250 coherence.

251 We anticipate that the first issue – that our model fails to capture corre-
 252 lations between nearby pixels – will have two effects. First, we believe that
 253 ensemble studies will likely overestimate the uncertainties, and second we might
 254 see systematic biases in the mean of the ensemble results. For this reason, we
 255 use ensemble results primarily as a tool to estimate uncertainties in particular
 256 deformation retrieval methods, and take that as a likely upper bound on the
 257 uncertainty.

258 We apply our ensemble methods to two deformation retrieval methods. The
 259 first, which we will call method 1, begins by using a variant of the recursive
 260 phase estimation scheme described in Ansari et al. (2018). Pixels are included
 261 based on temporal coherence, in particular the ability to find a rank-one approx-
 262 imation of the SCM per ministack. We then form a mesh of all spatial links less
 263 than a certain radius. For each link we apply the LAMBDA method described
 264 in Kampes and Hanssen (2004), rejecting links based on their temporal coher-
 265 ence. We reconstruct point data from link data using the method described
 266 in Gonzalez et al. (2011). The second method (method 2) estimates phase us-
 267 ing the *phase-linking* method described in Guarnieri and Tebaldini (2008). As
 268 with the first method, pixels are include based on temporal coherence. Follow-
 269 ing Pepe and Lanari (2006), we use a minimum cost-flow method to establish
 270 consistent link values in temporal-perpendicular baseline-space, and then use
 271 a minimum cost-flow method on the Delaunay triangulation of good points to
 272 recover unwrapped phase per point. With both methods we look for historically
 273 anomalous jumps of 2π , which we treat as unwrapping errors and attempt to
 274 remove. No effort has been made to identify and remove APS, nor have we
 275 applied any temporal smoothing.

276 We’ve run each method on the real data, and on an ensemble of thirty syn-

277 thetic stacks of data. These sixty-two deformation retrieval computations were
 278 performed on the Descartes Labs Platform (Beneke et al., 2017). Deformation
 279 histories for the test points relative to the reference point, as shown in Figure 2,
 280 are presented in Figures 6, 7, 8 and 9. The orange crosses in these figures show
 281 deformation histories derived only from actual complex SAR data, while the
 282 blue circles with error bars show the ensemble mean and standard deviation of
 283 deformation histories derived only from synthetic data.

284 In Figures 6 and 7 the deformation at the green point relative to the orange
 285 point retrieved by the two methods from the actual data (orange crosses) are
 286 largely in agreement. The standard deviations of the ensemble results (blue dots
 287 and error bars) for method 1 are smaller than those of method 2, suggesting
 288 that *for this pair of test and reference points* method 1 is less sensitive to phase
 289 noise.

290 In Figure 8 the deformation at the blue point relative to the orange point
 291 retrieved from the actual data by method 1 (orange crosses) suggests almost
 292 no deformation, however the ensemble mean standard deviation (blue dots and
 293 error bars) suggest that this method *for this pair of test and reference points* is
 294 very sensitive to the modeled phase noise. In Figure 9 method 2 shows roughly
 295 50mm of deformation and *for this pair of test and reference points* is not nearly
 296 so sensitive to phase noise.

297 5. Discussion

298 InSAR results can vary based on the deformation retrieval method, e.g. (Hu
 299 et al., 2016; Osmanoglu et al., 2016; Parizzi and Brcic, 2010; Yang et al., 2016,
 300 Figs. 8,9), and even individual methods likely have accuracy and uncertainty
 301 that varies in space and time. This can make it difficult to determine which
 302 points in the results are actually reliable, or to determine the best deformation

303 retrieval method to use for a particular area of interest.

304 The primary value in this ensemble calculation is that it allows us to deter-
305 mine the method that is least sensitive to phase noise and therefore the best
306 method to choose for a particular set of points or area of interest. For instance,
307 we see that the estimated uncertainty is relatively small for the green test point
308 relative to the orange reference point for both method 1 and 2, and while method
309 1 may be less sensitive to phase noise, both methods have uncertainties within
310 the range we can expect for InSAR measurements (Rosen et al., 2000). However,
311 the relative uncertainty between the blue test point and the orange reference
312 point for method 1 is quite high, indicating this method is very sensitive to
313 phase noise for this test and reference pair combination, and therefore would
314 not be the best deformation retrieval method to choose for recovering relative
315 deformation between this pair of points.

316 The other main benefit in using the presented approach is that, because
317 this method estimates uncertainty point-by-point and epoch-by-epoch, one can
318 filter points based on the magnitude of their estimated uncertainty, which al-
319 lows confidence that the extracted deformation at the remaining points is likely
320 insensitive to phase noise and is likely more reliable. In the above example, if
321 method 1 was selected and the orange point chosen as our reference, we would
322 exclude the blue test point due to its high sensitivity to the modeled phase
323 noise, but would preserve the green test point due to its lower uncertainty.

324 As we can see in the second-to-last epoch of Figures 6 and 7, there are likely
325 other sources of phase changes beyond expected deformation and estimated
326 phase noise. These other sources add temporal "jitter" to the signal that exceeds
327 the standard deviations. This "jitter" is correlated between the methods, and
328 is captured in both the ensemble mean and the actual retrieved deformation.
329 Further, the standard deviation doesn't grow in time at these events. From this

330 we conclude that this jitter is caused by some other physical phase contribution,
331 possibly APS or maybe spatially varying dielectric changes in the surface in
332 response to other environmental effects.

333 As we discussed in Section 3, we treat a correlation matrix as a covariance
334 matrix, and take the square root of positive semi-definite approximation of this
335 correlation matrix. While these approximations may not be entirely accurate,
336 they appear to best match the real coherence data, and enable us to achieve
337 beneficial insights into the reliability of each point we analyze. However, this
338 method does not perfectly capture coherence using the correlation matrix, which
339 may cause us to over estimate our uncertainty in some cases, so we take the
340 standard deviation of the spread as the upper bound on the uncertainty.

341 As for the computational and storage costs, aside from generating the syn-
342 thetic data, we must run the entire deformation retrieval process tens of times
343 to generate the actual retrieved deformation and the ensemble analysis, which
344 can each be computationally-expensive, depending on the size and resolution of
345 the area. We also must store each of these results before actually calculating
346 the ensemble mean and standard deviation, which adds additional storage and
347 computation costs. Because the standard deviations varies with choice of ref-
348 erence point, there is not a way to compute the standard deviation "up front".
349 Instead, when reviewing the results, we have all ensemble results available and
350 re-compute the standard deviations for the test point if we move the reference
351 point. Although this does add some additional computation and storage costs,
352 it also allows for the flexibility assess the best reference point, and to compare
353 relative differences in standard deviation for points that are near to each other
354 spatially, which can help eliminate noisy points.

355 6. Conclusions

356 We've developed a method for generating synthetic SAR data stacks based
357 on real SAR data stacks. The coherence and estimated phase statistics of the
358 synthetic data appears to match those same properties of the real data. This
359 allows us to perform ensemble studies to assess the sensitivity of particular
360 deformation retrieval method to phase noise. The proposed ensemble analysis
361 can provide a point-by-point, epoch-by-epoch, indication of reliability of the
362 result.

363 Futher research includes a more thorough understanding of possible biases
364 in the simulated synthetic phase.

365 7. Acknowledgements

366 The authors would like to thank Mike Warren and Scott Arko for maintain-
367 ing Descartes Labs Sentinel-1 data pipeline and for helpful discussions relating
368 to InSAR. This work contains modified Copernicus Sentinel data 2019-2020,
369 processed by ESA.

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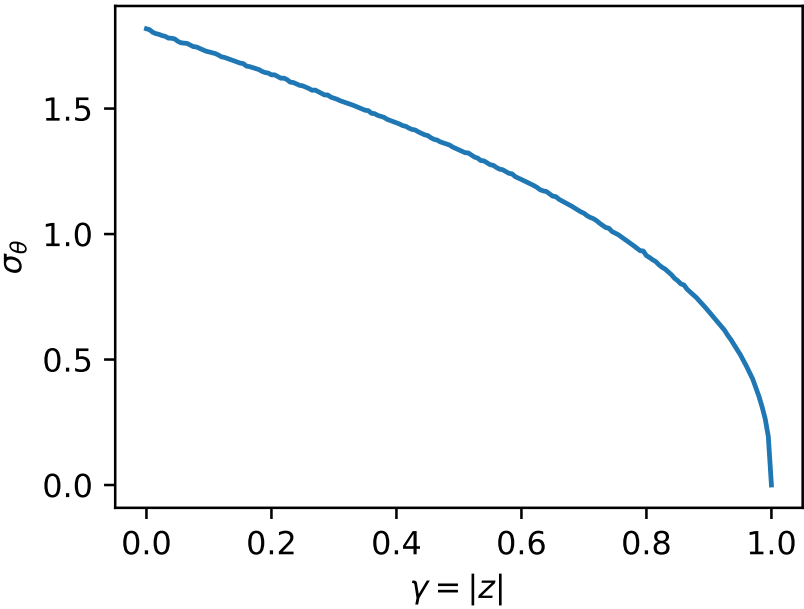


Figure 1: Numerical experiments showing the standard deviation of phase of $P_1 \overline{P_2}$ as a function of $\gamma = |z|$.

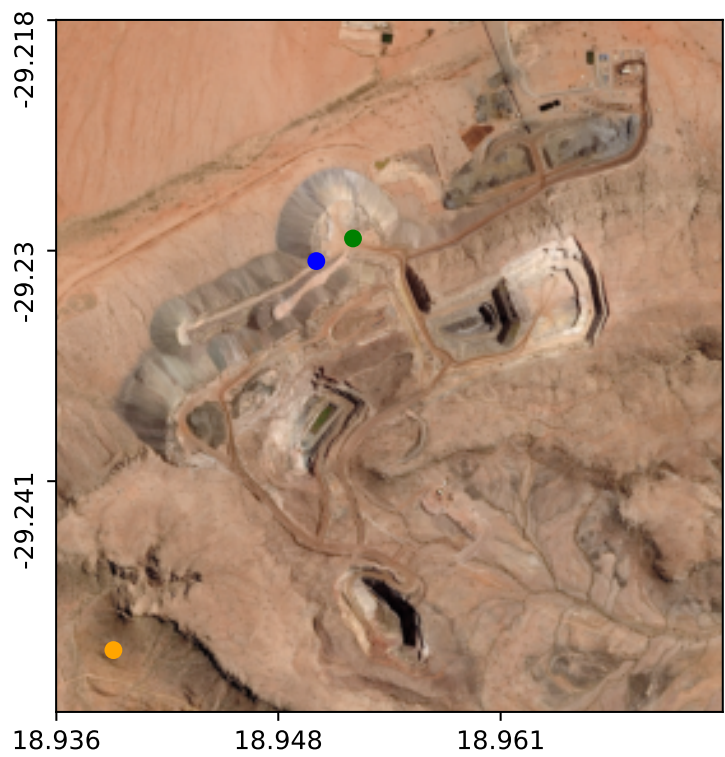


Figure 2: Our area of interest with the reference point (orange) and test points (green and blue).

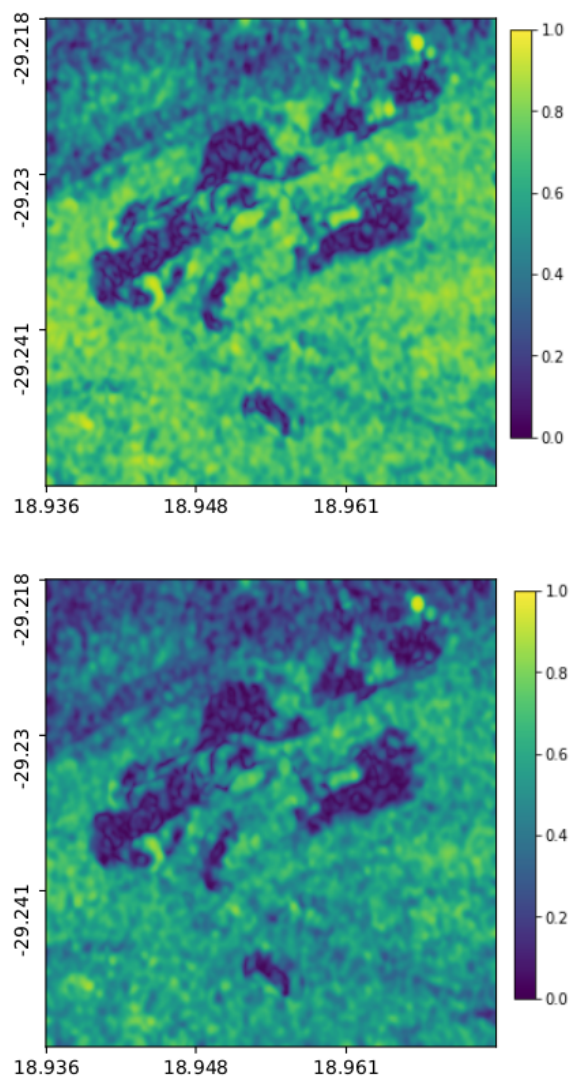


Figure 3: Coherence between the tenth and twentieth scene for real input data (top) and synthetic data (bottom).

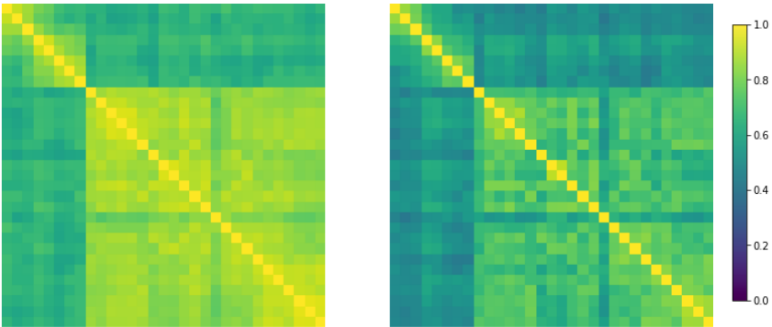


Figure 4: Absolute value of the sample correlation matrix for an arbitrary pixel using real data (left) and synthetic data (right).

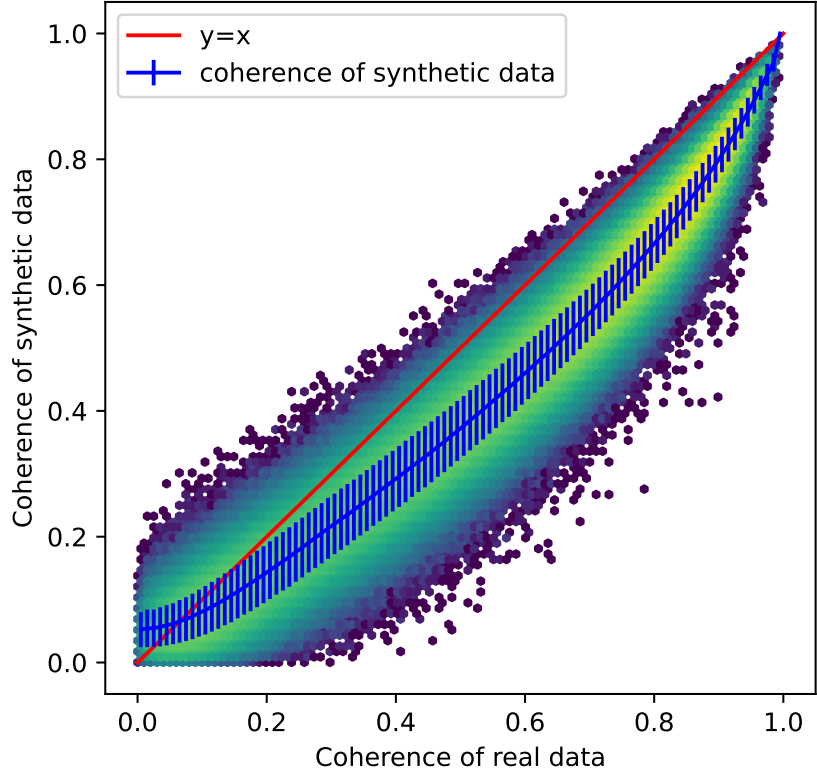


Figure 5: A heat plot of coherence from synthetic data vs. coherence from real data. The red line is $y = x$ and the blue line is the mean synthetic coherence, with an error bar showing the standard deviation of values.

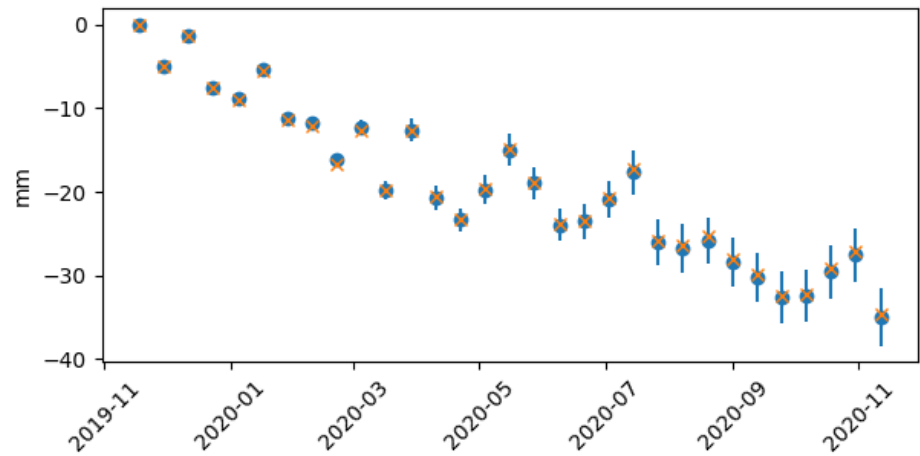


Figure 6: Recovered deformation histories of the green test point relative to the orange reference point as shown in Figure 2 for method 1.

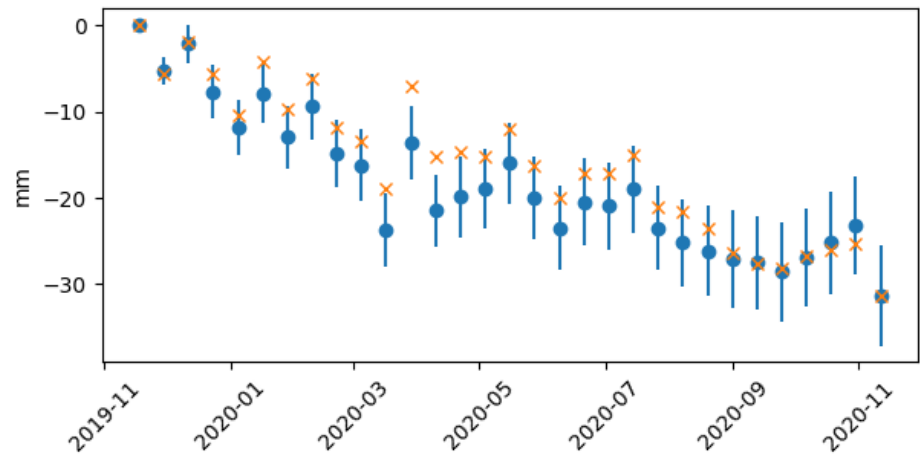


Figure 7: Recovered deformation histories of the green test point relative to the orange reference point as shown in Figure 2 for method 2.

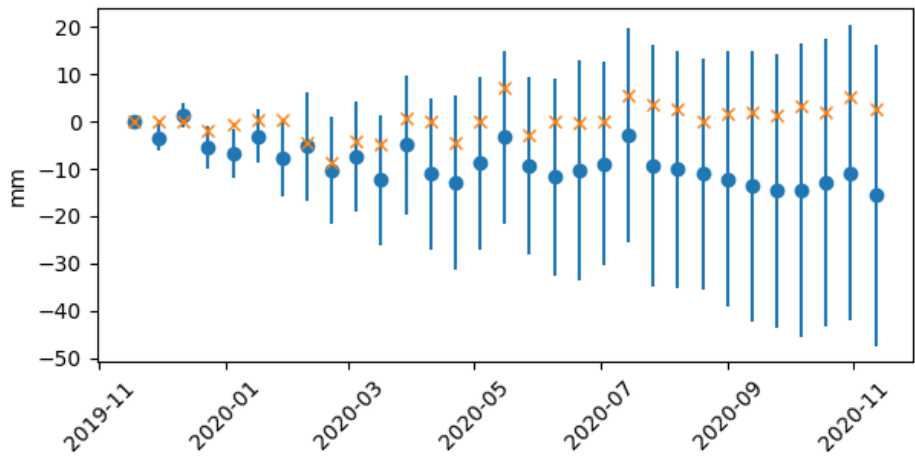


Figure 8: Recovered deformation histories of the blue test point relative to the orange reference point as shown in Figure 2 for method 1.

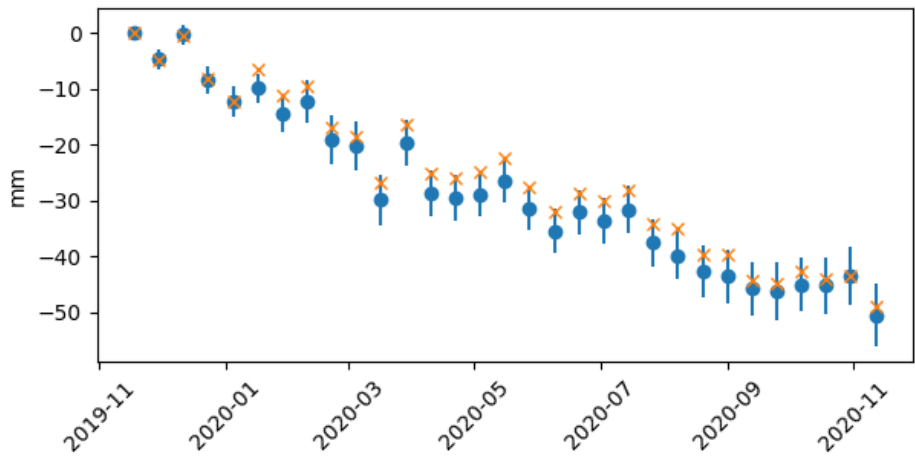


Figure 9: Recovered deformation histories of the blue test point relative to the orange reference point as shown in Figure 2 for method 2.