Relativistic Dilation and Contraction of the Probabilities of Quantum States of Light at Angular Incidence

Daniel Souza Cardoso[1]; José Rafael Bordin[2]

[1]Instituto Federal Sul Rio-grandense, Campus CaVG, Pelotas, Rio Grande do Sul, Brasil.
[1]danielcardoso@cavg.ifsul.edu.br
[2]Departamento de Física, Instituto de Física e Matemática, Universidade Federal de Pelotas. Pelotas, Brasil
[2]jrbordin@ufpel.edu.br

ABSTRACT

The transport and entanglement of photons is becoming prominent in optics applied to information and quantum computing, where the angular momentum of light stands out in the exchange and inversion of quantized states, with prospects for several technological applications, such as the transport and storage of quantum information. In order to contribute to the understanding of quantized states in photon-matter interaction, we describe a quantized state equation in multidimensional Hilbert space for the diagnosis of OAM states, where probabilities arise in a relativistic setting. It was found that the classical-relativistic variability of the probabilities constitutes a resultant capable of describing the quantized states of light, where the state variable is the variation of the angular momentum of the photon, capable of estimating the orbital angular momentum inversion points at angular incidence. It was found that the chances of finding the quantized states of light at angular incidence can be treated by purely relativistic probabilities, explaining that when both states have equal chances of being found and the angular momentum variation is zero, the source-observer synchronizations occur at the step of increasing relativistic regime of the photon dynamics. We found that the relativistic effect from the perspective of the source referential is able to alter the chances of an event occurring, dilating and contracting the probabilities of finding a quantized state of light at angular incidence.

Keywords: Relativistic probability; relativistic synchronization; classical relativistic variability; quantized states of light; polarization inversion; light resonance curve; relativistic photon ignition; angular momentum; moment of inertia; relativistic constant; relativistic energy wave; photon-matter interaction

1 ORCID: https://orcid.org/0000-0001-7558-7641
2 ORCID: https://orcid.org/0000-0002-8025-6529

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1. INTRODUCTION

In face of a backdrop of advances in quantum information teleportation processes [22], [31], [29], [2] involving entanglement processes in optical cavities [27], [4], the properties of quantized states and their modulations stand out as a basis for quantum information [38], [5]. In this sense, Barreiro et al. [3] presented an encoding for quantum communication considering two photons entangled in orbital (OAM) and spin angular moments (SAM), with linear optics, polarizing each input photon from a given ±OAM state and subsequently passing to an orbital state where the inversion is controlled by spin, constituting a NOT logic gate.

Some variables such as angular momentum stand out in state transmission processes in photon-matter interaction either by the response of matter. In the scope of ultra fast magnetic switching of a molecular sample, Tauchert et al. [34], performed an analysis of demagnetization under ultrafast femtosecond pulses, finding that the variation of magnetization ΔM is associated with the variation of angular momentum ΔL. For the authors, some or all angular momentum associated with spin before the interaction presents itself in the form of spin motion of the lattice atoms within hundreds of femtoseconds after photon-matter interaction, characterizing phonons with circular polarization.

Circular polarization exhibits points of reversals in the angular range of incidence [24], [14], and rotation of the transverse structure. In agreement with Sekiguchi et al. [32], the geometric aspects of photon polarization are found in spin polarization, where in both absorption and emission there is polarization entanglement, where the quantas associated with the two polarizations can represent quantum bits (qubits). Hiekkamäki et al. [18] found experimentally that the angular sensitivity of two photons to the rotation of the transverse structure is greater in comparison to a single photon, where the difference increases with the number of photons and with increasing modulation of the OAM. The variation of the geometric phase is capable of inverting the cross section of any beam, as discussed in the rotation of a dove prism. [20].

The polarization reversal points are very close to the Brewster angles [14], [37], [21], [28], similar to observed for the classical-relativistic synchronization angles where the OAM inversions are found [8], [7]. Note that these are special regions of incidence, since they comprise processes where the polarized light does not seem to escape from the region where the OAM inversions take place, which in turn offers a transmission path that is not able to change its angular momentum state from the perspective of the source referential, as we will discuss in this paper.

From the perspective of transmission, at reflection inhibition at the Brewster angle, the process can be characterized in the successive absorptions and emissions of electric dipoles oriented along the direction parallel to the direction expected for thereflection to occur. In [26], the authors argue that the inhibition of radiation on reflection is associated with interference between electric and magnetic dipoles. In this perspective, to keep the trajectory of a photon in a homogeneous refringent medium requires a long-range ordering of the directional properties of the electric dipoles, where in the perspective of Paniagua-Domíngues et al. [26] the process proceeds by a succession of inductions of electric dipoles within the material in response to the electromagnetic wave. According to Götte et al. [15], the angles of incidence at which the reflection is zero are not polarization angles.

Recently, we described the relativistic effects of photon-matter interaction in the transition between two refringent media [8], without dealing with intermediate processes in the transmission. We demonstrated through a conservative process for a single photon, that when transiting between media it is subject to the Abraham torque, which in turn acts as a relativistic ignition device for the photon, which is subject to a second torque, Minkowisk, which imposes a relativistic trajectory that arises with an angular delay, precisely at the points of classical-relativistic synchronization where occurs the reversal of the OAM [8].

In another analysis, we find that the delay is associated with the relativistic increase in translational inertia explaining that the Minkowisk torque, due to the difficulties of changing its directional properties, will only be able to impose its trajectory with decreasing relativistic translational inertia, from synchronization points [7].

Considering the importance of the transmission of quantized states in the photon-matter interaction for different uses, we will check the chances of finding each state front the geometric properties in the transmission, weighting the relativistic effects that do not arise when in the exclusive scope of quantum theory.

The polarization will be treated in the framework of the description of quantized OAM states in Hilbert space, in an analog to Poynting [19] who considered the mechanical description of a beam consisting of a row of particles in a spiral motion, centered on the propagation axis, to describe the circular polarization of light in transmission.

Considering that the transmission of the angular momentum variation ΔL features prominently in entanglement processes and quantum teleportation, we conducted a description in the angular momentum variation domain, characterizing the possible states associated with the photon in the transmission, considering the relativistic effects under angular incidence [8].

2. METHODOLOGY

In this analysis a photon is characterized by the displacement mass with inertial properties of translation and rotation [7]. When transiting between two media it is subject to two torques, Abraham's and Minkowisk's, conditioned to a relativistic motion in which the relativistic trajectory appears with delay [8]. In this context, classical and relativistic properties can be related according to our previous works [8], [7], where the difference between classical and relativistic quantities are described by classical-relativistic variability, for different variables for example:

$$[\Delta\theta]_{relativistic} = [1-\gamma(\theta_1, n_{12})][d\theta]_{observer} \quad , \tag{1}$$

where the relativistic angular constant [8], [7]:

$$\gamma(\theta_1, n_{12}) = \frac{sen\,\theta_1}{\sqrt{1-n_{12}^2}} \quad . \tag{2}$$

Classical-relativistic probability variability and relativistic probability estimates are discussed in multidimensional Hilbert space for the diagnosis of the quantized state of ΔL at angular incidence and OAM reversal points, with transposition to discussion of the polarization of light.

We considered the variation of angular momentum from the perspective of the source referential [8]:

$$\Delta L(\theta) = \frac{2\hbar(1-n_{12}^2)}{\sqrt{1-n_{12}^2} - sen\,\theta} \quad . \tag{3}$$

In section 3.1 it is verified the transposition of probabilities between the source-observer referential, dealing with the chances of finding a well-defined state in the source referential while the chances in the observer referential are 50% for each of the quantized states considered. In section 3.2, we considered the use of relativistic probabilities for the diagnosis of the quantized states of light at angular incidence, considering the physical properties of the medium and statistical behavior of angular momentum, considering the polarization effects. In section 3.3, it was sought to verify the behavior of the superposition of states in the synchronization angles as a function of the thermal properties of the refringent medium.

To analyze the variations of the points of state superpositions as a function of the thermodynamic properties of the refringent medium, the estimates of the refractive index as a function of temperature considered the model presented by Djurišić and Stanić [11] for a wavelength of 589.3 nm, disregarding the error margin, according to the expression:

$$n(T) = 1{,}33455 - 0{,}0000553132\,T - 0{,}00000112008\,T^2 \tag{4}$$

In treating the refractive index of air as a function of temperature, Walker's model was adopted [36], given by:

$$n(T,p)=1+\frac{0{,}0002928}{1+0{,}0036\,T}\frac{p}{76}, \quad (5)$$

disregarding the margin of error inherent in the model [36] assuming a constant atmospheric pressure of 76 cm Hg.

3. DEVELOPMENT AND DISCUSSION

In the treatment of the angular momentum and moment of inertia of the photon presented in the previous works [8], [7] we discussed the points of reversal of direction for both moments at angular incidence, from the perspective of the referential of the source. In turn, the resonance curves known in the literature present points of reversal of polarization in the vicinity of the Brewster angle [15], [9], [1]. Naturally these quantized state inversions have relevance in processes involving matter manipulation by optical tweezers, entanglement processes and quantum teleportation, among others, in the more recent scenario of optical applications [16], [12], [6]. The reversal of direction of ΔL in agreement with Cardoso [8], can be represented in the alternation of well-defined states $|\circlearrowright\rangle$ e $|\circlearrowleft\rangle$ arising from the[3] torques of Abraham and Minkowisk respectively [8] so that for one or two photons we can consider a wave with arbitrary polarization in the superposition of two states:

$$|\Delta L\rangle = a|\circlearrowright\rangle + b|\circlearrowleft\rangle \quad , \quad (6)$$

of which the linear combination of these states constitute a qubit in the scope of quantum computing and information, in a treatment analogous to the literature [25], [33].

Considering the angular ranges of incidence and synchronizations [8] in an analog to Poynting [19] associating the state $|\Delta L\rangle$ to the polarization of the light, we can polarize the pulse with operators comprised in the range $\theta_- \leq \theta_{sync} < \theta_+$, so that we can write:

$$\hat{\theta}_+ |\Delta L\rangle = |\circlearrowleft\rangle \quad , \quad (7)$$

$$\hat{\theta}_- |\Delta L\rangle = |\circlearrowright\rangle \quad . \quad (8)$$

In Figura 1, we represented the generation of a depolarized wave consisting of two[4] photons with different polarizations in each arm of a polarizer designed to handle the operators, with characteristics of a prism with segments of the same constitution, which may represent conduction fibers. The individual polarizations of the photons consider the angular momentum states in the angular ranges around the source-observer synchronization angle [8] where the resulting

3 The reader should realize that it is not an exclusive reverse torque, which would cause an anomalous refraction with $\theta_2 > \theta_1$, but it is considered the action of two non-additive torques while the first one gives rise to the second one, where there is an inversion of the OAM keeping $\theta_2 < \theta_1$, in agreement with Cardoso [8], [7].

4 In this preliminary analysis we will consider only the photons represented by the colors red and purple. Section 3.3 discusses the third incidence (green).

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depolarized pulses in each arm experience inverse polarizations, according to the operators of eq.(s) (5) and (6).

In this perspective we can consider that the original pulses can be represented in the superposition of two or more states, agreeing with literature. In agreement with Mair [23], considering the conservation of the orbital angular momentum OAM, where the resultant of a pair of photons represents the OAM of the beam emitted by the pump and that in the multidimensional entangled state neither of the two photons has a well-defined OAM, where the simple measurement defines the OAM state of one photon and projects that of the second.

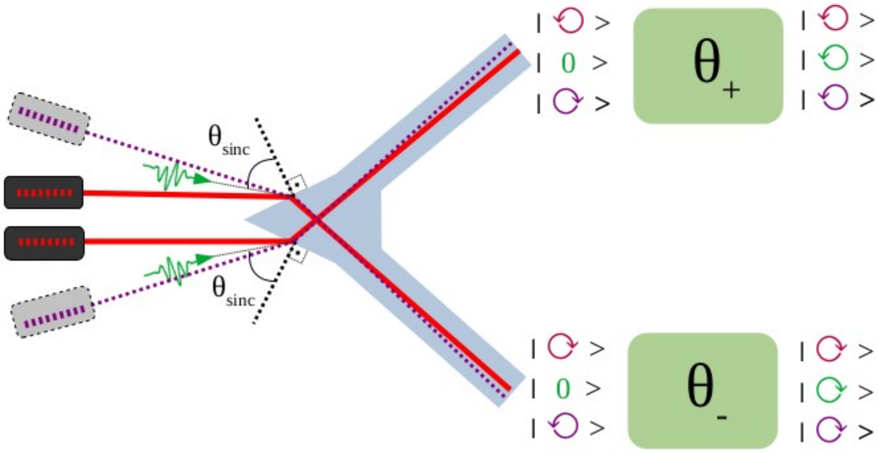

Figura 1- Polarization of light from the perspective of angular momentum variation [8]. The operator colors indicate a change in the refracting medium. The spacing between conduction fiber and input port characterized by operators are merely illustrative. (Source: authors)

3.1 RELATIVE VARIABILITY OF THE PROBABILITIES OF QUANTIZED STATES AT ANGULAR INCIDENCE

While the amplitudes "a" and "b" can be found in the probabilities of finding each of these states, let us consider that in the referential of the observer the polarization of the photon in a state $|\phi\rangle$ where angular momentum is a quantum observable, will present equal probabilities for two possible states:

$$P(\circlearrowright \vee \circlearrowleft) = |\langle \circlearrowright \vee \circlearrowleft | \phi \rangle|^2 = \frac{1}{2} \quad , \tag{9}$$

where the amplitudes:

$$|\langle \circlearrowright \vee \circlearrowleft | \phi \rangle| = \frac{e^{\mp i\phi}}{\sqrt{2}} \quad . \tag{10}$$

It was considered that observer describes quantum probabilities according to eq. (9) and records complex amplitudes according to eq. (10), because the employment of classical probabilities for a quantum equation of state can imply a considerable increase in the margin of

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error in the estimates, given that quantum estimates differ when the theories have different bases, in real or complex numbers. Renou et al. [30], discusses some limits of the representations of complex quantum theory and real quantum theory, finding that the complex and real theories differ from their predictions in some scenarios in Hilbert space, demonstrating that the complex quantum theory has better fit in scenarios susceptible to entanglement.

In an analog to classical-relativistic variability [7], employing the estimates of quantum probabilities, we start to verify if the relativistic effect is able to change the chances of an event happening in the perspective of the transmission scenario under angular incidence.

Considering that in general we can in the angular range considered, the classical chances with geometric profile, knowing that after synchronization we find the states $|\circlearrowleft\rangle$ we would have a small amount:

$$P(\circlearrowleft)_{classic} = \frac{2\Delta\theta_+}{\pi} \quad , \tag{11}$$

which in turn is not a solution because it does not carry any information of the photon-matter interaction. In turn, eq. (9) allows us to establish a relationship from eq. (1) between the probabilities recorded in the referential of the source and observer:

$$[P(\circlearrowleft)_{observer} - P(\circlearrowleft)_{source}] = [1-\gamma(\theta_1, n_{12})][P(\circlearrowleft)]_{observer} \quad , \tag{12}$$

so by adjusting the terms we can find:

$$P(\circlearrowleft)_{source} = \gamma(\theta_1, n_{12}) P(\circlearrowleft)_{observer} \quad , \tag{13}$$

where considering eq. (9) we can write:

$$P(\circlearrowright)_{source} = 1 - \frac{1}{2}\gamma(\theta_1, n_{12}) \quad . \tag{14}$$

The result of eq. (s) (13) and (14) demonstrates that the chances of finding an inverse quantized state at angular incidence are dilated or contracted under relativistic effect from the perspective of the source referential. In section 3.3 we show that this result characterizes relativistic chances independently of the probabilities given by eq. (9) and the choice of state presented at eq. (9). The behavior of eq. (12) is discussed in section 3.2 for different refringent media as represented in Figura 2.

3.2 RELATIVISTIC PROBABILITIES OF THE QUANTIZED STATES OF THE OAM

We can consider that in the ranges θ_- and θ_+ there are numerous chances to obtain the states $|\circlearrowright\rangle$ and $|\circlearrowleft\rangle$ respectively, so that the mean values exist:

$$\langle \Delta L(\circlearrowright)\rangle = \sum_i P_i(\theta)\Delta L_i(\theta) \quad , \tag{15}$$

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$$\langle \Delta L(\circlearrowleft)\rangle = \sum_i (1-P_i(\theta))\Delta L_i(\theta) \quad , \tag{16}$$

where P(θ) and 1-P(θ) are the probabilities of finding the states $|\circlearrowright\rangle$ e $|\circlearrowleft\rangle$ respectively, associating each incidence with an expected value:

$$\langle \Delta L(\circlearrowright)\rangle_i = P(\theta)\Delta L(\theta) \quad , \tag{17}$$

$$\langle \Delta L(\circlearrowleft)\rangle_i = (1-P(\theta))\Delta L(\theta) \quad , \tag{18}$$

considering ΔL(θ) in both components, due to the fact that they are characterized by the same expression, eq. (3), which alternates the direction as a function of θ. In this way, we are assuming one expression for any θ.

Considering that there will be two averages, before and after the reversal of direction of ΔL(θ) in synchronization, we can consider the deviations from the mean:

$$d_{\theta_-} = E_i(\theta_-, n_{12}) - \langle \Delta L(\circlearrowright)\rangle_i \quad , \tag{19}$$

$$d_{\theta_+} = E_i(\theta_+, n_{12}) - \langle \Delta L(\circlearrowleft)\rangle_i \quad , \tag{20}$$

so that we can write:

$$\Delta d + E_i(\theta_+, n_{12}) = E_i(\theta_-, n_{12}) + \left[\langle \Delta L(\circlearrowleft)\rangle - \langle \Delta L(\circlearrowright)\rangle\right]_i \quad , \tag{21}$$

$E(\theta_1, n_{12})$ being the estimate of ΔL(θ), and considering again that while ΔL(θ) alternates direction as a function of θ_1 may assume $E(\theta_-, n_{12})$ and $E(\theta_+, n_{12})$, and weighting that the mean difference of the deviations may assume positive or negative values:

$$\Delta L(\theta) \pm |\Delta d| = \Delta L(\theta) + \left[\langle \Delta L(\circlearrowleft)\rangle - \langle \Delta L(\circlearrowright)\rangle\right]_i \quad , \tag{22}$$

so that we can find the average in the interval $\Delta L(\theta) - |\Delta d| \leq \langle \Delta L\rangle \leq \Delta L(\theta) + |\Delta d|$, where we can assume that the expected value of the "i" component of the angular momentum variation, considering eq.(s) (17), (18) and (22):

$$\langle \Delta L\rangle_i = E_i(\theta_1, n_{12}) + \left[\langle \Delta L(\circlearrowleft)\rangle - \langle \Delta L(\circlearrowright)\rangle\right]_i = 2[1-P(\theta)]\Delta L(\theta) \quad . \tag{23}$$

The expected value can be approximated by the mean value theorem in the range 0 to θ:

$$\langle \Delta L\rangle = \frac{1}{\theta}\left[\frac{2\hbar(1-n_{12}^2)}{\sqrt{1-n_{12}^2} - sen\,\theta} - \frac{2\hbar(1-n_{12}^2)}{\sqrt{1-n_{12}^2}}\right] \quad , \tag{24}$$

where we now interpret that for $\theta_0 = 0$ and $\theta = \theta_0 + d\theta$:

$$\int_0^\theta \langle \Delta L\rangle d\theta = \frac{2\hbar(1-n_{12}^2)}{\sqrt{1-n_{12}^2} - sen(\theta_0 + d\theta)} - \frac{2\hbar(1-n_{12}^2)}{\sqrt{1-n_{12}^2}} \quad , \tag{25}$$

so that the sum can express different intervals, large or small, representing the average in the interval, for example of the "i" component:

$$\frac{2\hbar(1-n_{12}^2)}{\sqrt{1-n_{12}^2} - sen\,\theta} - \frac{2\hbar(1-n_{12}^2)}{\sqrt{1-n_{12}^2}} = 2[1-P(\theta)]\Delta L(\theta) \quad , \tag{26}$$

where considering $\Delta L(\theta)$ according to eq. (3), so that the probability of finding the state $|\circlearrowright\rangle$, considering eq. (2):

$$P(\theta, n_{12}) = 1 - \frac{1}{2}\gamma(\theta_1, n_{12}) \quad . \tag{27}$$

At Figura 2, we can verify the chances of finding the state $|\circlearrowright\rangle$ state are predominant until synchronization, which runs with equal probabilities for the states $|\circlearrowright\rangle$ and $|\circlearrowleft\rangle$ which becomes predominant for incidences in the range θ_+ . In the perspective of the relativistic probabilities of eq. (27), the fact that the states $|\circlearrowright\rangle$ and $|\circlearrowleft\rangle$ have equal chances in the synchronizations (½) explains that $\Delta L = 0$ where there is no well-defined state.

We can rewrite eq. (6) by considering eq.(s) (26) and (27):

$$|\Delta L\rangle = \sqrt{P(\theta_1, n_{12})}\,|\circlearrowright\rangle + \sqrt{1 - P(\theta_1, n_{12})}\,|\circlearrowleft\rangle \quad , \tag{28}$$

From the perspective of relativistic probabilities, a photon has an arbitrary polarization at incidence according to eq. (28) and can be polarized into a well-defined state in the incidence ranges θ_+ and θ_- , where the superposition of two states with equal chances in eq. (28) linearizes the polarization, where no state is well defined.

Considering the inertia analysis [7] precisely with the reduction of the relativistic inertia of the photon in the angular incidence, there is an increase of $\gamma(\theta_1, n_{12})$ until the synchronization between source and observer occurs where $\gamma(\theta_1, n_{12}) = 1$ and the photon has greater ease in changing its directional properties, assuming the trajectory imposed by the Minkowski torque [8] that with increasing angular incidence gradually conditions the photon to a purely relativistic dynamics with $\gamma(\theta_1, n_{12}) \longrightarrow \gamma(n_{12})$.

In Section 3.3, we will find that assuming the synchronizations via Brewster pseudo angles facilitates the diagnosis of quantized states by thermodynamic control of the refringent medium.

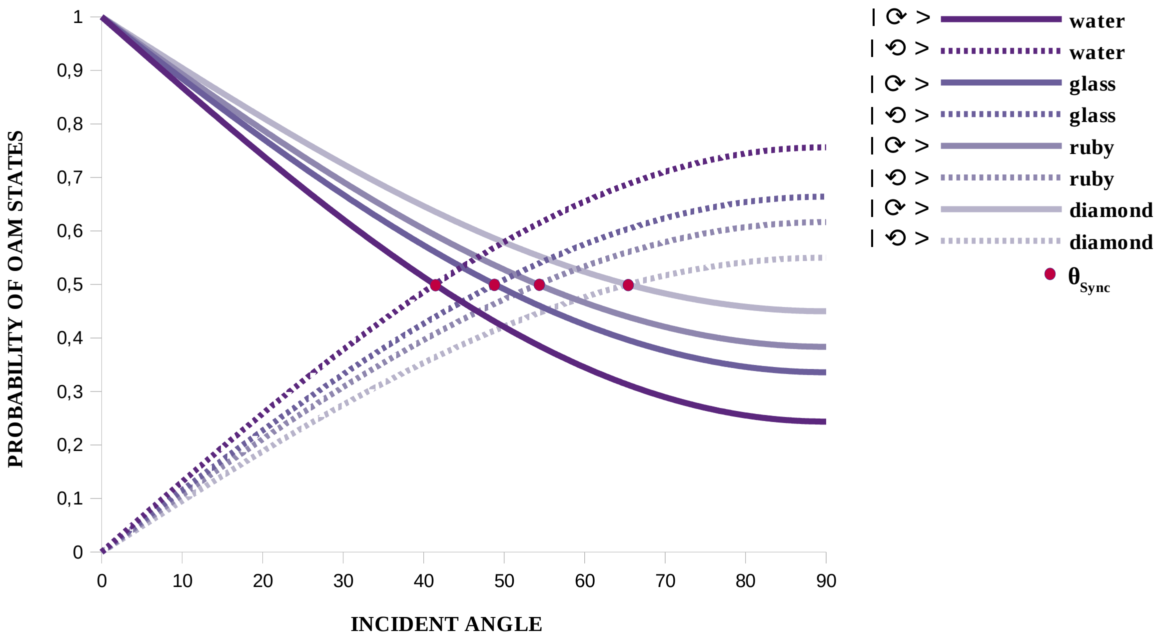

Figura 2- Relativistic probabilities of finding the quantized states of the OAM of light at angular incidence (Source: authors)

3.3 THERMAL DISPLACEMENT OF OVERLAPPING STATES IN THE BREWSTER PSEUDO-ANGLE

The region of incidence of synchronizations where we find the overlap of two states has potential employment in information transmission segments among others, whereas at these particular points ΔL is null. Note that the regions of synchronizations are not restricted to unusual relativistic scenarios, because the synchronization angles do not exist only in the scope of the relativistic energy wave [8], is possible to verify in experimental values for example for the air-water transition the compatibility with Brewster's angle, where Lin and Perlin [21] remove the reflection from a free surface considering that the minimum of the reflection coefficient lies at the Brewster angle of 42° for the water-water interface. To be more precise, the synchronizations θ_{sync} can directly assume the known approximation of the Brewster angle $\theta_{PBR} = acos(n_{12})$, [28], whereas in synchronizations:

$$\sqrt{1-n_{12}^2} = sen\,\theta_{sync} \quad , \qquad (29)$$

can be treated at Brewster's pseudo angle:

$$\theta_{sync} = acos(n_{12}) \quad , \qquad (30)$$

where $\gamma(\theta_{sync}, n_{12}) = 1$, indicating regions where the reflectance assumes its minima because θ_{sync} lies at a Pseudo Brewster angle, such that the minima can be found in a varied region $\Delta\theta_{PBR}$ in agreement with Pevtsovet al. [28], where the pseudo-Brewster angle considers the material's ability to absorb energy [37], [28]. It is noted that at synchronizations, the photon still remains in a relativistic dynamics since $\gamma(n_{12}) \neq 1$.

At Figura 1, we present an incidence precisely at θ_{sync} and characterize an output state $|0\rangle$ where ΔL is null, and subsequently polarize it into the observable. We can conceive of the superposition of two states at θ_{sync}, with equal polarization chances $|\circlearrowright \vee \circlearrowleft\rangle$ in the linear combination of eq. (28), composes a linear polarization of which the component parallel to the plane of incidence presents its minima at the pseudo-Brewster angle found in the source-observer synchronizations.

From the perspective of the angular ranges that compete each state, it is found that prior knowledge of θ_{sync} either through the pseudo-Brewster angle are shown to be important for determining the chances of each OAM state in angular incidence, for the treatment of probability density in front of normalization.

In this perspective we can predict the source-observer synchronization angles, θ_{sync} , as a function of the thermal properties of the refringent medium. We considered the refractive indices of water and air as a function of the thermal properties of the refringent medium according to eq.(s) (4) and (5).

At Figura 3, it is possible to identify the angles where we will not find a well defined state at each temperature of the temperature set considered in this analysis, because at the synchronization points there is an overlap where both states have equal chances. Naturally these estimates may fluctuate slightly according to the model adopted for the refractive indices, although the models adopted to exemplify the shifts of synchronizations with temperature according to eq. (4) and (5), are statistically well fitted.

Important to note that we found a maximum for θ_{sync} precisely at the melting point of water, displaced by 4°C from its point of highest density. Because this increase is associated with the increase in the estimate of the refractive index of water with a maximum in 0°C, we considered weighting of adopted model of Djurišić and Stanić [11], eq. (4).

The estimates in this work considered the extrapolation of the relation of Djurišić and Stanić [11] for negative temperatures, which can always increase the margin of error of the estimates the higher the extrapolation. In this regard, the model of eq. (4) was compared with the results of Harvey et al. [17] who reviewed the formulation of the refractive index of water where he found that the experimental values are slightly larger than those estimated for temperatures below 0°C and presented results, within the temperature and wavelength ranges comparable to those treated here, that are close to the estimates of eq. (4), agreeing with a maximum of refractive index of water at 0°C.

This behavior is explained in the context of the specific refractive index [10], [13], which according to G. Wilse Robison et al. [13], the thermal maximum of the refractive index of water displaced from its maximum density is associated with a mixture composed of two physical states, treated ices of the type[5] I and II, composing a liquid with transitions of the external separations between the nearest neighbors in the range of 3.5 and 4.5Å, respectively. According to the authors, at low temperatures the term associated with Ice, $f_I \cdot I_I$, is predominant in front $f_{II} \cdot I_{II}$ while the fraction of the composition related to Ice II, f_{II} , increases with temperature and is associated with increasing density, where the specific refractive index of the mixture $I = f_I \cdot I_I + f_{II} \cdot I_{II}$. The authors found that if treated as a single state the maximum refractive index will be found near 4°C, while in the mixture of two states it is near 0°C.

Although the authors [13] point out that a single state model based on thermodynamic properties conditions the maximum of the refractive index at 4°C, it seems clear that the models of Djurišić and Stanić [11] and Harvey et al. [17] present the maxima at 0°C in 1 atm. In this sense, new studies are indicated for the treatment of the maximum of the refractive index at water in 0°C.

We note the extrapolations of the air refractive index from eq.(5), precisely by the maxima of θ_{sync} in low temperatures. Although we did not find estimates that comprise the entire range of negative temperatures presented in this paper, we found in the literature that the gradual increase in the refractive index of air at low temperatures estimated by extrapolations from Walker's model [36] is consistent with estimates for dry air down to -30°C [35].

In the field of spectroscopy, the maxima of θ_{sync} found in a temperature range, such as the one treated in this work, may indicate important properties such as the combined states in matter, and may be treated in the scope of analysis of the pseudo-Brewster angle. However, an extension of the studies of single or combined states of matter is required, in the regions of the refractive index maxima in water and air, among other materials that the maxima can be found.

5 Ice I - ordinary ice with densities < 1 g/cm3 and specific refractive index I_I, with fraction of the mixture composition f_I .
Ice II - moderately dense densities > 1 g/cm3 and specific refractive index I_{II}, with fraction of the mixture composition f_{II} .

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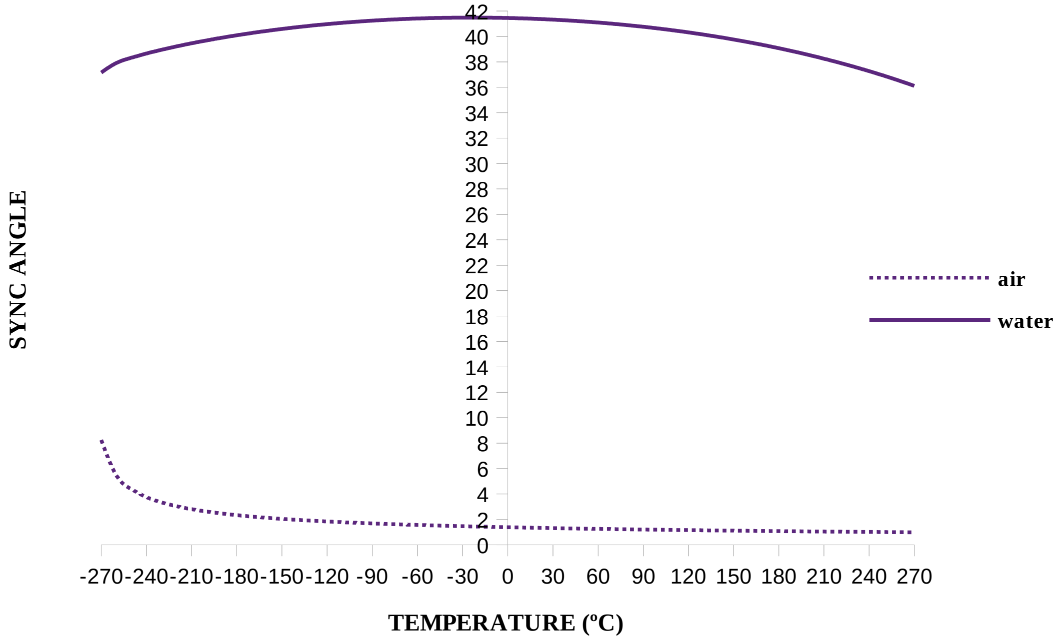

Figura 3 - Source-observer synchronization angles where there is state overlap as a function of temperature. (Source: authors)

4. CONSIDERATIONS

The introduction of the variation of angular momentum with relativistic angular delay as a state variable in Hilbert space, brings the descriptions of quantum states associated with the OAM of light closer to relativistic dynamics through probabilities and their variabilities. It was found that the classical-relativistic synchronization points are pseudo-Brewster angles, found in the regions of the reflectance minima, allowing us to verify that the relativistic estimates of the chances of finding a given state at angular incidence, prove adequate in the analysis of polarization processes and that the thermodynamic properties prove to be variables controlling the incidence points where we may or may not find a well-defined state.

The relative variability of the probabilities of finding each quantized state, in the analog of the classical-relativistic variabilities, presented a diagnosis of the quantized states of light in transmission processes in a scenario with relativistic effects, transposing the perception of the probabilities of the referential of the observer to that of the source, presenting agreement with the purely relativistic probabilities derived in this analysis that are independent of the probabilistic perception of the observer and yes of the variable adopted to characterize the polarizations, be it the angular momentum of the photon.

The purely relativistic probabilities were found to be fit in the estimation of quantized states of light at angular incidence, explaining that the variation of angular momentum is null at synchronizations while both states have equal chances of being found and that with increasing angular incidence the relativistic regime of photon dynamics increases.

Considering that the Theory of Relativity has a deterministic profile and not statistical, we should ponder that although we have dealt with the chances resulting from the classical-relativistic variability of the probabilities and the purely relativistic probabilities, one can verify that it is actually about the perception of the probabilities in the perspective of the referential of the source, so that we can consider that *"the theory of relativity is not probabilistic, but we note that the relativistic effect is able to alter the chances of an event occurring, dilating or contracting its probabilities"*.

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