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Proposed method of combining continuum mechanics with Einstein Field Equations

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Abstract. The article proposes some improvements to the relativistic continuum mechanics which introduce the relationship between density tensors and the curvature of spacetime. The resulting formulation of a symmetric stress-energy tensor for a system with an electromagnetic field, leads to the solution of Einstein Field Equations indicating a relationship between the electromagnetic field tensor and the metric tensor. In flat Minkowski spacetime, the vanishing four-divergence of this stress-energy tensor expresses relativistic Cauchy's momentum equation, leading to the emergence of new force densities which can be further developed and parameterized to obtain known interactions.

Keywords: General relativity, Cosmology, Continuum mechanics, Fluid dynamics, Field theory, Electrodynamics, Hamiltonian mechanics.

1. Introduction

Currently, field phenomena in physics is described in many ways, e.g., [1], [2], [3], [4], [5], [6], but in most field theories [7], the field is still something additional to the spacetime - not natural consequence of spacetime existence. There is also still no consensus regarding the description of the electromagnetic field in curved spacetime and the relationship of this description with GR. Stress-energy tensor for a system with electromagnetic field, derived from the widely accepted Lagrangian density for electromagnetic field, is not symmetrical and attempts are still being made to link the description of such a system with the GR, e.g. [8], [9], [10].

Much theoretical work was also done to combine the equations of GR and fluid dynamics, e.g. [11], [12], [13], [14], [15], however, so far the general solutions connecting these two branches of physics are unknown. There are also some unresolved problems, e.g. with the dependence of four-velocity on four-position. In relativistic electrodynamics, it is assumed that the four-velocity is independent of the four-position, while a large number of fluid dynamics equations operate on velocity gradients, such as Navier-Stokes equations [16] and many others.

The motivation of this article was to find a general solution to the Einstein Field Equations that would explain electrodynamics in curved spacetime, allow for generalization to other fields and be consistent with the equations of the continuum mechanics. The article may also be considered as the voice in still present scientific discussion about foundations of electromagnetism and its relation to spacetime geometry and spacetime itself, discussed e.g. in [17], [18], [19], [20], [21] and [22].

In the first part of the article, the consequences of Hamiltonian mechanics for electrodynamics were considered. The conclusions were then used to make a minor tweak to relativistic continuum mechanics equations. Finally, the stress-energy tensor was proposed for a system containing an electromagnetic field, and then it was used to analyze the transformation to curvilinear coordinates and its relation to Einstein Field Equations.

The author uses the Einstein summation convention, metric signature $(+,-,-,-)$ and some standard definitions: t denotes coordinate time, τ denotes test body proper-time, m denotes test body rest mass, q denotes test body charge, S denotes Hamilton's principal function (action), L denotes Lagrangian, $\mathcal{L}$ denotes Lagrangian density, H denotes Hamiltonian.

The author also uses some standard four-vector definitions: U^α for four-velocities, P^α for four-momentums, F^α for four-forces, $\mathbb{F}^{\alpha\beta}$ for electromagnetic tensors, A^α for four-accelerations, $\mathbb{A}^\alpha \equiv \left(\frac{\phi}{c}, \vec{\mathbb{A}}\right)$ for electromagnetic four-potentials, J^α for four-currents,

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$H^\alpha \equiv \left(\frac{H}{c}, \vec{p_h}\right)$ for generalized, canonical four-momentums.

2. From Hamiltonian mechanics to geometry of spacetime

One may start discussion considering Lagrangian and Hamiltonian mechanics [23] in flat Minkowski spacetime. Using Hamilton–Jacobi equations, one may express generalized canonical four-momentum H^α as a function of Hamilton's principal function S [24] as follows

$$H^\alpha \equiv \left(\frac{H}{c}, \vec{p_h}\right) = -\partial^\alpha S \tag{2.1}$$

For a system containing only electromagnetic field above takes form of

$$H^\alpha = P^\alpha + q\mathbb{A}^\alpha \tag{2.2}$$

where $\mathbb{A}^\alpha$ is the electromagnetic four-potential. This equation yields the relativistic Lagrangian [24] (minimal coupling) for the electromagnetic field

$$-L = \frac{1}{\gamma} \cdot U_\alpha H^\alpha = mc^2\frac{1}{\gamma} + q(\phi - \vec{u}\vec{\mathbb{A}}) \tag{2.3}$$

It is known relativistic version of Lagrangian and Hamiltonian for electromagnetism, however, there is something that was missed what is oversight of important consequences. Taking four-gradient on (2.2) for both indexes and subtracting from each other, one obtains

$$\partial^\beta P^\alpha - \partial^\alpha P^\beta = q(\partial^\alpha \mathbb{A}^\beta - \partial^\beta \mathbb{A}^\alpha) \tag{2.4}$$

Element related to H^α vanished, since $H^\alpha = -\partial^\alpha S$ and from calculus rules for any scalar S there is

$$\partial^\beta\partial^\alpha S - \partial^\alpha\partial^\beta S = 0 \tag{2.5}$$

what is fundamental rule behind gauge fixing [25] for electromagnetic field.

It is important to emphasize that the above reasoning and eq. (2.4) rules out the conviction, that four-momentum is independent of four-position. It is worth noting, that there are many concepts of continuum mechanics that depend on velocity gradients. An example would be Cauchy stress tensor, deviatoric stress tensor [26] or vorticity [27], which is a term from dynamical theory of fluids that describes the rotation of a fluid element, usually denoted as ω and defined as

$$\vec{\omega} \equiv \nabla \times \vec{u} \tag{2.6}$$

Velocity gradients and velocity gradient tensors are important concepts of fluid dynamics [28], [29], [30] thus velocity independent of the four-position would create significant problems in continuum mechanics. It would be difficult to combine continuum mechanics with GR, discarding the key elements of continuum mechanics.

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Therefore, for further discussion, conclusions from the continuum mechanics will be adopted and it will be assumed that four-velocity depends on four-position. As it will be shown soon, such an assumption (after a minor amendment) does not cause problems for GR and classical mechanics, and in fact leads to the integration of these branches of physics.

Analyzing (2.4) from the gauge theory perspective, in considered system (system containing only electromagnetic field), four-momentum P^α is just some chosen gauge for the electromagnetic four-potential. Electromagnetic filed tensor $\mathbb{F}^{\alpha\beta}$ for such system in flat Minkowski spacetime may then be expressed equivalently as

$$\mathbb{F}^{\alpha\beta} = \partial^\alpha \mathbb{A}^\beta - \partial^\beta \mathbb{A}^\alpha = \frac{1}{q}\left(\partial^\beta P^\alpha - \partial^\alpha P^\beta\right) \tag{2.7}$$

what produces the Lorentz force F^α by

$$F^\alpha = U_\beta \partial^\beta P^\alpha = q U_\beta \, \mathbb{F}^{\alpha\beta} \tag{2.8}$$

since the Minkowski metric property gives

$$U_\beta P^\beta = mc^2 \quad \rightarrow \quad U_\beta \partial^\alpha P^\beta = 0 \tag{2.9}$$

Adding other fields to the system by adding to (2.2) successive four-potential $\mathbb{A}^\alpha_i$ and related constants q_i of i-fields (marked with the i index), would generalize the force equation (2.8) to the form of

$$F^\alpha = U_\beta \partial^\beta P^\alpha = \sum_i q_i \, U_\beta \left(\partial^\alpha \mathbb{A}^\beta_i - \partial^\beta \mathbb{A}^\alpha_i\right) \tag{2.10}$$

The four-current J^α issue remains to be clarified, where

$$\mu_o J^\alpha \equiv \partial_\beta \, \mathbb{F}^{\alpha\beta} \tag{2.11}$$

and where μ_o represents the permeability of free space. The continuity equation requires that $\partial_\alpha J^\alpha = 0$. Denoting ρ_o as constant charge density, it becomes clear that the classical equation $J^\alpha = \rho_o U^\alpha$ should be verified. Four-divergence of four-velocity does not have to vanish, since U^α is dependent on four-position. One may assume, that it does not vanish and note some inconsistency in the classical calculation of the zero element of the four-current.

Analogous to the method of calculating the energy density in the stress-energy tensor [31], it should be noted that both the charge q and the volume V are subject to Lorentz contraction effects ($q \rightarrow q\gamma$ and $V \rightarrow V\frac{1}{\gamma}$). For the constant density of charge ρ_o, contraction of the volume alone would change the density as follows

$$\rho = \rho_o \gamma \tag{2.12}$$

For this reason, the total effect due to the increase of the charge and volume contraction drives to the four-current given by equation

$$J^\alpha = \rho \, U^\alpha = \rho_o \gamma \, U^\alpha \tag{2.13}$$

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Calculating the vanishing four-divergence of above, keeping in mind that γ is a function of the four-position only, one obtains

$$\partial_\alpha U^\alpha = -\frac{d\gamma}{dt} \tag{2.14}$$

The above reasoning would remain correct for any density in motion, providing a continuity equation for any density flux in flat Minkowski spacetime. However, it has much more significant implications. If one would like to perceive the effects of the existence of a field in flat spacetime, as some form of spacetime curvature in curvilinear coordinates, then the four-divergence of U^α should vanish in curvilinear coordinates, where the four-acceleration is replaced by the curvature of spacetime and geodesics. Therefore,

$$U^\alpha{}_{;\alpha} = 0 \quad \rightarrow \quad \Gamma^\alpha{}_{\alpha\beta} U^\beta = \frac{d\gamma}{dt} \tag{2.15}$$

where $\Gamma^\alpha{}_{\alpha\beta}$ represents Christoffel symbols of the second kind. This leads to another observation. In flat Minkowski spacetime, four-divergence of the following tensor does not vanish

$$\partial_\alpha \, U^\alpha U^\beta = -\frac{d\gamma}{dt} U^\beta + A^\beta = \left(0\,,\, \vec{a}\gamma^2\right) \tag{2.16}$$

where $\vec{a} \equiv \frac{d\vec{u}}{dt}$ is the classic acceleration. Its disappearance in curved spacetime thus leads immediately to the following conclusion

$$U^\alpha U^\beta{}_{;\alpha} = 0 \quad \rightarrow \quad \Gamma^\alpha{}_{\alpha\mu} U^\mu U^\beta + \Gamma^\beta{}_{\alpha\mu} U^\alpha U^\mu = -\left(0\,,\, \vec{a}\gamma^2\right) \tag{2.17}$$

what, taking into account (2.15), yields

$$\Gamma^\beta{}_{\alpha\mu} U^\alpha U^\mu = -A^\beta \tag{2.18}$$

This is the expected result, making that intrinsic covariant derivative of four-velocity vanishes in curved spacetime.

$$\frac{D\,U^\beta}{D\,\tau} = \frac{dU^\beta}{d\tau} + \Gamma^\beta{}_{\alpha\mu} U^\alpha U^\mu = 0 \tag{2.19}$$

Finally, introducing ϱ_o as a constant mass density, according to above findings, the total force density f^β acting in the system should be defined as follows

$$f^\beta \equiv \varrho A^\beta = \varrho_o \gamma A^\beta = \partial_\alpha \, \varrho \, U^\alpha U^\beta \tag{2.20}$$

which is in line with the main assumption behind the derivation of the Navier–Stokes equations, making it possible to derive their relativistic counterpart.

Analyzing all above on the transition to curved spacetime for

$$\partial_\alpha \, \varrho \, U^\alpha U^\beta = f^\beta \quad \rightarrow \quad \varrho \, U^\alpha U^\beta{}_{;\alpha} = 0 \tag{2.21}$$

it can be seen, that there must be relationship between the density tensors and some tensors describing the curvature of spacetime with vanishing covariant four-divergence, which opens the way to linking the continuum mechanics with GR.

Above reasoning opens the possibility of perceiving the presence of any field as some spacetime curvature and vice versa, where eq. (2.10) opens the way to the inclusion of other fields.

It is also possible to propose a solution where some interactions are the result of fluid dynamics, as presented in the next chapter.

3. Results

Returning back to flat Minkowski spacetime, one may analyze the implications of the previous chapter for the stress-energy tensors.

One could build a stress-energy tensor for a system with an electromagnetic field, based on the density tensor $\varrho\, U^\alpha U^\beta$ in such a way, that the vanishing four-divergence of the stress-energy tensor would result from a cancellation of force densities.

The density of force due to electromagnetism f^α_{EM} is given by

$$f^\alpha_{EM} \equiv J_\beta \mathbb{F}^{\alpha\beta} \tag{3.1}$$

This force density may be calculated as the four-divergence of classic [32] stress–energy tensor for electromagnetic field $\eta^{\alpha\beta}\,\frac{1}{4\mu_o}\mathbb{F}^{\gamma\mu}\,\mathbb{F}_{\gamma\mu} - \frac{1}{\mu_o}\mathbb{F}^{\alpha}{}_{\gamma}\,\mathbb{F}^{\beta\gamma}$ where $\eta^{\alpha\beta}$ represents Minkowski metric tensor.

For universality, one may introduce a definition of the stress-energy tensor for the electromagnetic field, in the form independent of the metric tensor $g^{\alpha\beta}$, which in the Minkowski spacetime will turn into the above classic one. Such a tensor will be denoted as $\Upsilon^{\alpha\beta}$ and defined as follows

$$\Upsilon^{\alpha\beta} \equiv \Lambda_g\, g^{\alpha\beta} - \frac{1}{\mu_o}\mathbb{F}^{\alpha\mu}\, g_{\mu\gamma}\, \mathbb{F}^{\beta\gamma} \tag{3.2}$$

where Λ_g is a scalar value with the dimension of energy density, related to the Lorentz invariant of the electromagnetic field tensor in the metric (subscript g)

$$\Lambda_g \equiv \frac{1}{4\mu_o}\mathbb{F}^{\alpha\mu}\, g_{\mu\gamma}\, \mathbb{F}^{\beta\gamma} g_{\alpha\beta} \tag{3.3}$$

Assuming that the only field in the system is an electromagnetic field and remaining in the Minkowski spacetime ($g^{\alpha\beta} \to \eta^{\alpha\beta}$), the below equation brings conservation of linear momentum and energy by electromagnetic interactions

$$\partial_\beta \left(\varrho\, U^\alpha U^\beta - \Upsilon^{\alpha\beta}\right) = f^\alpha - f^\alpha_{EM} \tag{3.4}$$

so this expression fits well as an expression to describe vanishing four-divergence of the stress-energy tensor for the whole system.

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However, it is known that there are other forces in the system, such as gravity, weak interactions and strong interactions. One may then propose the following definition of the stress-energy tensor $T^{\alpha\beta}$ for the whole system, including new force densities that will provide prototypes for the missing forces and allow for further development and parameterization

$$T^{\alpha\beta} \equiv \varrho\, U^\alpha U^\beta - \left(1 + \frac{c^2 \varrho}{\Lambda_g}\right) \Upsilon^{\alpha\beta} \tag{3.5}$$

As will be shown shortly, this will ensure compliance with the Cauchy momentum equation [33], what is known issue in EFE [34].

Vanishing four-divergence of tensor $T^{\alpha\beta}$ would create two additional density of forces:

$$\partial_\beta T^{\alpha\beta} = 0 \quad \rightarrow \quad f^\alpha - f^\alpha_{EM} - f^\alpha_{sw} - f^\alpha_{gr} = 0 \tag{3.6}$$

where

$$f^\alpha_{sw} \equiv \frac{c^2 \varrho}{\Lambda_g} f^\alpha_{EM} = \frac{c^2 \varrho}{\Lambda_g} \partial_\beta\, \Upsilon^{\alpha\beta} \tag{3.7}$$

$$f^\alpha_{gr} \equiv \Upsilon^{\alpha\beta}\, \partial_\beta\, \frac{c^2 \varrho}{\Lambda_g} = \partial^\alpha\, c^2 \varrho - \frac{1}{\mu_o} \mathbb{F}^\alpha{}_\gamma\, \mathbb{F}^{\beta\gamma}\, \partial_\beta\, \frac{c^2 \varrho}{\Lambda_g} \tag{3.8}$$

In the above picture, element f^α_{gr} seems to be related to the density of the gravitational force. It is not defined as interaction between bodies. This contraction of the electromagnetic stress–energy tensor expresses the phenomenon of bending the light path by gradient of energy density.

The relationship of this force density with the Einstein curvature tensor will be confirmed later in the article.

Element f^α_{sw} seems to be related to the density of weak interactions and strong interactions, linking both phenomena with additional electromagnetic force density moderated by the density of energy. On small scales with high energy density, the density of this force will be extremely great; one may recognize it as a strong interaction property. On larger scales with small energy density, this force will be extremely weak; one may recognize it as a weak interaction property.

The relation between strong forces and gravity has already been noted by the double copy theory [35], [36], [37]. Due to the lack of equations describing the weak and strong fields in classical field theory, confirmation of the proposed relationship of these fields with energy density must take place on the basis of quantum theories, where equation (3.7) is a quantitative prediction that can be verified or expanded with additional components in the stress-energy tensor.

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The proposed stress-energy tensors $T^{\alpha\beta}$ and $\Upsilon^{\alpha\beta}$ allows the replacement of force densities in flat Minkowski spacetime with the corresponding metric tensor in curved spacetime. It requires a metric tensor $g^{\alpha\beta}$ defined as

$$\Lambda_g \cdot g^{\alpha\beta} \equiv \frac{1}{\mu_o}\mathbb{F}^{\alpha\mu}\, g_{\mu\gamma}\, \mathbb{F}^{\beta\gamma} \tag{3.9}$$

Such definition eliminates the whole electromagnetic stress-energy tensor $\Upsilon^{\alpha\beta} \to 0$, but this tensor is no longer needed. Equation (3.5) for curvilinear coordinates reduces to the postulate of General Relativity

$$T_{\alpha\beta} = \varrho\, U_\alpha U_\beta \tag{3.10}$$

where the stress-energy tensor $T_{\alpha\beta}$ determines the curvature of spacetime, the relation between the metric tensor and electromagnetic field tensor is given by eq. (3.9) and instead of fields and forces one obtains curved spacetime. The vanishing covariant four-divergence of (3.9) ensures that there is no force density (acceleration) resulting from the electromagnetic field and all movement takes place according to geodetic.

The question then arises about the relationship of the above equation with the main GR equation. One may thus express the energy density of the universe by the tensor $R_{\alpha\beta}$ describing perfect fluid, where the difference between pressure p and energy density is equal to the doubled invariant of the electromagnetic field tensor Λ_g

$$R_{\alpha\beta} \equiv \frac{1}{c^2}\left([p - 2\Lambda_g] + p\right) U_\alpha U_\beta - p\, g_{\alpha\beta} \tag{3.11}$$

Next, one may define trace of this tensor by scalar R

$$R \equiv R_{\alpha\beta}\, g^{\alpha\beta} = -2p - 2\Lambda_g \tag{3.12}$$

thus

$$R_{\alpha\beta} - \frac{1}{2}R\, g_{\alpha\beta} - \Lambda_g\, g_{\alpha\beta} = \frac{2}{c^2}\left(p - \Lambda_g\right) U_\alpha U_\beta \tag{3.13}$$

Both sides of the equation should have a vanishing four-divergence in the considered metric and left side is apparently proportional to the Einstein tensor. Therefore, one may expect that $R_{\alpha\beta}$ is a Ricci tensor with an accuracy of some constant and comparing the above to (3.10), one may expect that both equations are proportional to the accuracy of some constant.

Introducing $G_{\alpha\beta}$ as Einstein curvature tensor, one may propose the following relation

$$\frac{c^4}{8\pi G}\, G_{\alpha\beta} - \frac{1}{2}\Lambda_g\, g_{\alpha\beta} = \frac{1}{c^2}\left(p - \Lambda_g\right) U_\alpha U_\beta = \varrho\, U_\alpha U_\beta \tag{3.14}$$

In the above solution, cosmological constant Λ is related to the invariant of electromagnetic field tensor

$$\Lambda = -\frac{4\pi G}{c^4}\Lambda_g \tag{3.15}$$

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and it would mean, that vacuum energy that has been sought for years [38] is related to the energy of the electromagnetic field that fills the entire space. It would also lead to the conclusion, that cosmological constant Λ should be taken into account in the calculation of the metric, as they propose, inter alia, authors in [39].

It can also be seen, that in above picture the energy density, which is measured as $\varrho c^2 = p - \Lambda_g$, is only the surplus of the pressure over the vacum energy density. The total energy density taking into account the vacuum energy density is present in the tensor $R_{\alpha\beta}$ in (3.11) and is equal to $[\varrho c^2 - \Lambda_g]$.

The covariant four-divergence of the tensor $R_{\alpha\beta}$ in curved spacetime is related to gravitational force density ($\partial^\alpha \varrho c^2 = \partial^\alpha p$) derived in (3.8) and is reset by the four-divergence of component related to its trace $\frac{1}{2}R\, g_{\alpha\beta}$ in Einstein tensor. Therefore, in curved spacetime, Einstein tensor has vanishing four-divergence and is indeed related to the force density (3.8) and corresponding metric tensor.

It is worth noting, that in curved spacetime, Einstein tensor may also be interpreted as stress-energy tensor describing perfect fluid, however this time, vacuum energy density acts as pressure

$$\frac{c^4}{4\pi G}\, G_{\alpha\beta} = \frac{1}{c^2}\left(\left[\varrho c^2 + p\right] - \Lambda_g\right) U_\alpha U_\beta + \Lambda_g\, g_{\alpha\beta} \tag{3.16}$$

By analyzing above and equations (3.12) and (3.14) one may notice, that

$$G_{\alpha\beta} = 0 \quad \rightarrow \quad p = -\Lambda_g \quad \rightarrow \quad R = 0 \quad \rightarrow \quad R_{\alpha\beta} = 0 \tag{3.17}$$

thus this way one obtains Schwarzschild and Kerr vacuum solutions.

In flat Minkowski spacetime ($g^{\alpha\beta} \rightarrow \eta^{\alpha\beta}$), one may now simplify eq. (3.5) to the following form:

$$T^{\alpha\beta} = \varrho\, U^\alpha U^\beta - p \cdot \frac{\Upsilon^{\alpha\beta}}{\Lambda_g} \tag{3.18}$$

Vanishing four-divergence of the above, expresses the four-dimensional relativistic Cauchy momentum equation (convective form). To see it, one may introduce tensor $\Pi^{\alpha\beta}$ defined as

$$\Pi^{\alpha\beta} \equiv c^2 \varrho\, \frac{\mathbb{F}^{\alpha}{}_{\gamma}\, \mathbb{F}^{\gamma\beta}}{\mu_o \Lambda_g} \tag{3.19}$$

Density of electromagnetic force may be expressed as

$$f^\alpha_{EM} = \partial_\beta\, \Lambda_g\, \frac{\mathbb{F}^{\alpha}{}_{\gamma}\, \mathbb{F}^{\gamma\beta}}{\mu_o \Lambda_g} \tag{3.20}$$

and taking four-divergence on (3.18), after easy rearrangement of elements, one obtains

$$f^\alpha = \partial^\alpha p + f^\alpha_{EM} + \partial_\beta \Pi^{\alpha\beta} \tag{3.21}$$

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This equation expresses convective form of the relativistic Cauchy momentum equation, where $\Pi^{\alpha\beta}$ acts as a four-dimensional deviatoric stress tensor in the mentioned fluid. According to present knowledge in the subject, deviatoric stress tensor depends only on velocity gradients [40], and indeed, in the relativistic version thanks to (2.7), it may be expressed as

$$\Pi^{\alpha\beta} = c^2 \varrho \cdot \frac{\mathbb{Z}^{\alpha}{}_{\gamma}\, \mathbb{Z}^{\gamma\beta}}{\frac{1}{4}\mathbb{Z}^{\mu\nu}\, \mathbb{Z}_{\mu\nu}} \quad where \quad \mathbb{Z}^{\mu\nu} \equiv \partial^{\mu} U^{\nu} - \partial^{\nu} U^{\mu} \tag{3.22}$$

since all other constants from (2.7) cancel out.

In the presented solution, the only real force density in the system is the electromagnetic force density, where prototypes of gravity, strong and weak interactions are consequences of fluid dynamics.

The above also explains the origin of the metric tensor (3.9) for curvilinear coordinates. Adopting the metric tensor in such a way, to eliminate deviatoric stress

$$g^{\alpha\beta} \equiv -\frac{1}{c^2 \varrho} \Pi^{\alpha\beta} \tag{3.23}$$

one indeed makes mentioned fluid perfect and all forces disappear, what should be kept when introducing other, additional fields to the above solution.

4. Discussion

The presented solution creates a coherent picture in which spacetime is in fact a way of perceiving the electromagnetic field. It allows for further development, adding additional fields, different parameterization and simple transformation between Minkowski spacetime and curvilinear reference systems. It should be noted that the proposed solution does not question the correctness of the currently existing, well-established physical theories, but rather leads to their integration, opening up a new field for further research.

In curved spacetime, the main equation of the proposed solution (3.14) expresses the Einstein Field Equations; in flat Minkowski spacetime vanishing four-divergence of the stress-energy tensor turns out to be relativistic Cauchy momentum equation (which is the expected relationship); four-current is still given by ρU^{α} and four-momentum density by ϱU^{α}. The γ factor introduced in equations (2.13) and (2.20) is actually expected to keep the continuum mechanics consistent with the Lorentz transformation.

In above picture, in Minkowski flat spacetime, gravitational force density prototype is explained on the basis of classical field theory as the bending of the light path by the gradient of energy density what is a simple analogy to curving the spacetime. It

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allows for development and further study of this approach in search of quantum gravity. The force density (3.7) that occurs naturally in the equation, interpreted here as weak and strong interactions prototype, creates a new area of research to confirm the above approach or for further analysis of weak and strong interactions based on classical field theory by developing the proposed solution.

Finally, the cosmological constant Λ in above solution is certainly not "Einstein's greatest mistake", but appears to be a measure for the value of invariant of the electromagnetic field filling entire space, what is a surprisingly natural explanation to the vacuum energy problem. It also may be further parameterized and extended with invariants of other fields.

5. Statements

Data sharing is not applicable to this article, as no datasets were generated or analysed during the current study.

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