

On the Consistent Meaning of Local Realism

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Abstract

We present a pragmatic analysis of the different meanings assigned to the term “local realism” in the context of the empirical violations of Bell-type inequalities since its inception in the late 1970s. We point out that most of them are inconsistent and arise from a deeply ingrained prejudice that originated in the celebrated 1935 paper by Einstein-Podolski-Rosen. We highlight the correct connotation that arises once we discard unnecessary metaphysics.

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1 Introduction

Despite their relevance in the fields of quantum information and quantum foundations, physicists and philosophers of science still hotly debate the interpretation of the Bell theorem and its associated Bell inequality. There seems to be no universal agreement regarding what are their main hypotheses and consequences of the experimentally confirmed Bell inequality violations.

The term local realism, however, has become almost universal. A poll carried out in 2011 [1] reported that 64% of the surveyed scientists interpret the observed violations of Bell’s inequalities as the untenability of local realism. But some proponents of quantum nonlocality strongly resist the expression of local realism [2–5], while those who accept its use do not always agree on its precise meaning.

We analyze whether the experimental Bell-type inequalities violations can falsify local realism according to its usual assigned meanings since its appearance in the late 1970s.

We avoid a direct discussion of quantum nonlocality. That discussion constitutes a different matter and would only obfuscate our analysis. However, references to nonlocality in some instances will eventually be unavoidable.

2 Locality and Realism

The compound word local realism is the logical conjunction of the locality and realism hypotheses. The meaning of the locality assumption is mostly uncontroversial. But physicists and philosophers sustaining that the Bell theorem is a quantum nonlocality theorem strongly resist the realism hypothesis [3, 4, 6–8]. Here we analyze the usually assigned meanings of the term realism and whether Bell-type experiments can falsify it. We point out that most of them are inconsistent, but there is a correct way to understand it once we discard unnecessary metaphysical traits.

We do not intend a deep and sophisticated philosophical analysis but a rather pragmatic one, appropriate for a factual science.

2.1 Realism and Preexisting Values

We can trace the origin of local realism to an influential review article by Clauser and Shimony in 1978 [9]. Although the authors did not use the term local realism,¹ they introduced the expression “local realistic theories” and defined realism as

Realism is a philosophical view, according to which external reality is assumed to exist and have definite properties, whether or not they are observed by someone.

This view seems the most commonly accepted one among physicists [11–15]. An equivalent way of expressing it is

[Realism is] the assumption that measurement outcomes are well defined prior to and independent of the measurements. [12]

The idea is particularly convenient to present the macroscopic classical view that measurements merely disclose preexisting values as opposed to quantum superposition and collapse. This metaphysical form of realism is a consequence of the *Einstein, Podolski, Rosen* (EPR) [16] argument for quantum incompleteness. Physicist Abraham Pais, the famous Einstein biographer, reportedly said:

The EPR paper was the only slip Einstein made. [17]

But we disagree. Einstein did not write the EPR paper. He soon dismissed the EPR realism criterium replacing it with his far better separation principle argument [18]. Bohr also criticized and rejected EPR’s criterium of physical reality [19].

One problem with that kind of realism is that it is unfalsifiable or, at least, the Bell inequality cannot falsify it. Indeed, in 1964 Bell [20] employed a similar EPR reasoning to conclude that locality required a deterministic hidden variable model of quantum mechanics. It is essential to realize that Bell’s reasoning led him to determinism, which only means that we can predict the results of measurements with certainty but does not imply the existence of unobserved properties unless, of course, we accept the metaphysical EPR definition of physical reality.

¹Travis Norsen [2] traced the origin of the local realism expression to an article by d’Espagnat in 1979 [10].

Bell's insight allowed him to mathematically prove the incompatibility of non-conspiratorial² local hidden variables with quantum mechanics, also opening the possibility of empirically testing the plausibility of completing quantum mechanics.

However, the Bell inequality is silent regarding the metaphysical speculation about the existence or nonexistence of unobserved physical properties. Indeed, the correct inequality derivation exclusively relies on the results of actually performed experiments (cf. [21, 22], also Appendix 5.2). Hence, it cannot falsify whether those results existed or not previous to measurements.

Since the claim that experimental violations of Bell-type inequalities rule out pre-existing properties is a widespread prejudice, we review a more formal counter-argument in Appendix 4.

2.2 Realism and Determinism

As we established in section 2.1, Bell-type inequalities cannot falsify the pre-existence of physical properties. Hence, perhaps what Bell-type experiments indeed falsify is determinism.

The 1964 Bell theorem relies on a deterministic hidden-variable model. Given that quantum mechanics is essentially not deterministic, a possible interpretation for local realism could be local determinism. Thus, it is possible to identify realism with determinism and claim that experiments rule out local determinism.

That could have been a possible correct interpretation until 1974. In that year, Clauser and Horne [23] enlarged the scope of Bell-type inequalities to not deterministic hidden variables models. The date of the appearance of such models can even be pushed back to 1971 by Bell himself [24].³

Since the expression “local realism” only appeared in the late 1970s, it never could have been correctly interpreted as local determinism.

2.3 Realism and Counterfactual Definiteness

Counterfactual definiteness has two usual different meanings:

²Non-conspiratorial means that the hidden variables model respects the statistical independence hypothesis.

³Only in a footnote, Bell observed that his model could be interpreted as not deterministic.

- (a) Any real or hypothetical experiment gives a definite result. [9, 25]
- (b) Results of experiments exist and are physically real independently of being performed and of our observation. [26, 27]

The rejection of (a) signifies there are experiments with no results. Unless considering experimental inefficiencies, rejecting (a) is not plausible, so we can safely assume it. It does not mean that repeating a measurement yields the same result or that we can predict outcomes with certainty, so it does not imply determinism. Since assuming (a) is so natural, to avoid confusion, we implicitly accept it without calling it counterfactual definiteness.

The case of (b), however, is very different. Instead of being natural and self-evident is highly metaphysical and artificial. It assumes the physical existence of ghostly entities that we neither observe nor measure. Even worse, those ghostly entities purportedly allow us to make falsifiable and meaningful predictions. Henceforth, we shall understand counterfactual definiteness according to this second meaning.

We point out we should not confuse the counterfactual definiteness hypothesis with counterfactual reasoning. While counterfactual reasoning is a rational means of inference, counterfactual definiteness seems to be a case of *incredible metaphysical extravaganza*, borrowing Van Fraassen's expression [28]. In our opinion, it is puzzling to base scientific inferences on such superstition. Probably the idea is motivated by the EPR definition of "elements of physical reality". Fortunately, contrary to widespread beliefs [29–33], the original Bell inequality formulation requires neither counterfactual definiteness nor counterfactual reasoning [22].

The first to introduce counterfactual reasoning related to the Bell theorem was Stapp in 1971 [34]. Stapp, however, contrary to Bell, did not pursue a falsifiable argument against local hidden variables. He intended to prove quantum nonlocality by producing a mathematical contradiction between quantum mechanics and relativistic locality.

Stapp's program of counterfactuals for proving quantum nonlocality is quite different from Bell's arguments against local hidden variables. Not distinguishing both different approaches has resulted in much confusion and incorrect arguments. We present an analysis of this endemic problem in Appendix 5.

The obvious conclusion is that whatever Bell-type experiments falsify, it cannot be the ghostly existence of not performed experiments. Thus the

claim that the empirical violations of Bell-type inequalities disprove local realism because they disprove counterfactual definiteness is devoid of logical and physical sense.

2.4 Realism and Hidden-Variables

Although we avoid discussing the contentious issue of quantum nonlocality, we point out that an unbiased reading of Bell's papers [20, 35, 36] shows that he interpreted his inequalities as ruling out non-conspiratorial local hidden variables and not as ruling out orthodox quantum mechanics' locality. Of course, he believed that quantum mechanics is nonlocal but did not claim his inequality proves it [37].

Our point is that the only way to make sense of realism in the expression of local realism is to interpret realism as meaning non-conspiratorial hidden variables. Hence, when we interpret realism as hidden variables plus statistical independence, it is correct to say that Bell-type experiments falsify it. However, hidden variables frequently are incorrectly assumed to imply preexisting properties, for instance:

The non-locality theorems of Bell and followers assume, as further condition, the necessity of assigning non-measured observables pre-existing values (hidden variables). [38]

As we observed before, hidden variables – even in their deterministic version – do not require preexisting physical properties.

Thus the correct way to interpret local realism when claimed to be falsified by Bell-type experiments is

$$\text{Local Realism} \equiv \text{Non-Conspiratorial Local Hidden Variables} \quad (1)$$

The remarkable fact Bell-type experiments falsify is that nature does not admit a local hidden variable model that respects the statistical independence hypothesis. However, they have no direct implications regarding the preexistence of physical properties.

3 Conclusions

Most experimental violations of Bell-type inequalities report having ruled out local realism [12, 14, 39, 40], considering local realism as locality plus

the assumption of preexisting properties in the EPR sense of elements of physical reality, meaning that properties exist independently of observation. However, the realism assumption they report to have falsified is untestable by Bell-type inequalities. Those remarkable experiments indeed prove what Bell insistently explained from 1964 [20] to 1990, the year of his death:

Quantum mechanics cannot be embedded in a locally causal theory. [36]

Implicit in the former assertion is that “locally causal theory” means local hidden variable theory. Although some interpret it as proof of quantum nonlocality, its unambiguous meaning is that a local hidden variable theory (either deterministic or stochastic) cannot reproduce the experimental findings when statistical independence is assumed.

Along with Bell, we believe there are good reasons to sustain the nonlocal character of orthodox quantum mechanics,⁴ but the Bell inequality is not one of them.

In 1964 John Bell turned a previously metaphysical conjecture, namely, the possibility of local hidden variables, into a concrete physical problem decidable in the experimental arena. However, experiments cannot falsify what they usually report they have proved, confirming Einstein’s dictum:

It is the theory which decides what we can observe. [43]

Thus, an adequate interpretation of experiments requires a careful analysis of the more subtle points of the theoretical foundations. A similar problem occurred in the 19th century when interpreting the Michelson-Morley experiment. Physicists believed the failure to detect the aether wind resulted from a dynamical length contraction effect until Einstein proved the ether hypothesis was unnecessary. Those experiments indeed falsified the existence of the luminiferous aether, not the dynamical effects of an aether wind.

Although the shut up and calculate philosophy may disregard our considerations as irrelevant, the elimination of ambiguities and inaccuracies from scientific parlance may justify them.

⁴See, for instance, [41]. Also, Harrigan and Spekkens [42] explain how Einstein (and we would add Bell [35]) argued for quantum nonlocality. A critical analysis on quantum nonlocality is presented in [37].

APPENDIX

4 Apparatuses' Influence on Measurements

We can include the influence of the measuring devices in the derivation of the Bell inequality. Their explicit inclusion makes it more evident that the Bell inequality cannot falsify pre-existing values. On the contrary, we can surmise the inclusion of such influences is according to the orthodox Copenhagen dictum that observation creates such values. Of course, all about pre-existence is mere speculation, but the explicit inclusion of apparatuses' hidden variables may help us get rid of such prejudices.

In 1971 Bell showed [24] how to include the uncontrollable influences of the measuring devices in the derivation of his inequality. That is according to Bohr's views:

Indeed the *finite interaction between object and measuring agencies* conditioned by the very existence of the quantum of action entails – because of the impossibility of controlling the reaction of the object on the measuring instruments if these are to serve their purpose – the necessity of a final renunciation of the classical ideal of causality and a radical revision of our attitude towards the problem of physical reality. [19]

In [24], Bell explained, “If we average first over these instruments variables”; without giving further details. Reference [44] contains a more thorough explanation. Here we present a slightly different approach. We start with the expression for the joint probability of a stochastic local non-conspiratorial hidden variable model

$$P(A, B | x, y) = \int_{\Lambda} P(A | x, \lambda) P(B | y, \lambda) P(\lambda) d\lambda \quad (2)$$

In (2), $A, B \in \{-1, 1\}$ are the results, and x, y are the measurement settings. The hidden variables λ are the local common causes, and Λ is the space of these variables. We include the apparatuses' effects assuming hidden variables $\xi \in \Lambda_1$ for Alice's measuring device and $\eta \in \Lambda_2$ for Bob's. After the inclusion of these variables, the probabilities for the measurement results respectively become

$$P(A | x, \lambda, \xi) \quad \text{and} \quad P(B | y, \lambda, \eta)$$

The λ represents common causes lying at the causal past of the measuring events while ξ and η describe the local instruments. Therefore all these variables are independent of each other and are independently distributed. Putting $P_\lambda = P(\lambda)$, $P_\xi(x) = P(\xi, x)$, $P_\eta(y) = P(\eta, y)$

$$P(A, B | x, y) = \int_{\Lambda} P_\lambda d\lambda \int_{\Lambda_1} P_\xi(x) d\xi \int_{\Lambda_2} P_\eta(y) d\eta P(A | x, \lambda, \xi) P(B | y, \lambda, \eta) \quad (3)$$

The fact that $P(\lambda)$ is independent of x and y is a consequence of the non-conspiratorial hypothesis. Integrating first over the instruments variables ξ and η

$$\int_{\Lambda_1} P_\xi(x) P(A | x, \lambda, \xi) d\xi = \bar{P}(A | x, \lambda) \quad (4)$$

$$\int_{\Lambda_2} P_\eta(y) P(B | y, \lambda, \eta) d\eta = \bar{P}(B | y, \lambda) \quad (5)$$

Taking (4) and (5) into (3) we have

$$P(A, B | x, y) = \int_{\Lambda} \bar{P}(A | x, \lambda) \bar{P}(B | y, \lambda) P_\lambda d\lambda \quad (6)$$

Since $\bar{P}(A | x, \lambda)$ and $\bar{P}(B | y, \lambda)$ range between 0 and 1, the derivation of the Bell inequality goes through as usual.

5 On Stapp's Counterfactual Arguments

Here we discuss the differences between Stapp's counterfactual inequality and Bell's hidden variables. Stapp admired the Bell theorem. In 1975, he famously said

Bell's theorem is the most profound discovery of science. [45]

Indeed, proving that a previously metaphysical speculation that would allow a classical down-to-earth interpretation of quantum mechanics is accessible to direct experimental tests is not a minor feat.

However, he sensed that the presence of hidden variables was an undesired characteristic for claiming quantum nonlocality. He was aware of the difference between the proof of quantum nonlocality and a no-go theorem for local hidden variables.

Stapp's program to prove quantum nonlocality is quite different from Bell's. Remarkably, much of the physical community has missed that Bell did not base his quantum nonlocality arguments on his inequality. Bell and Einstein's arguments for quantum nonlocality are indeed very similar [37]. The purpose of the Bell theorem, and the Bell inequality, is to prove that we cannot supplement quantum mechanics with additional variables to render a local theory. In the third line of his 1964 introduction, Bell wrote:

These additional variables were to restore to the theory causality and locality. [20]

But he did not say such variables were supposed to prove the nonlocal character of orthodox quantum mechanics. Assuming that quantum mechanics is nonlocal, Bell proved we cannot turn it local by adding hidden variables. From a logical and conceptual stance, that is light-years away from proving quantum nonlocality.

On the other hand, Stapp also was largely misunderstood. Stapp intended a theoretical argument for quantum nonlocality, while the Bell theorem is a directly falsifiable no-go theorem for local hidden variables.

From a historical and conceptual point of view, it is relevant to follow Stapp's arguments because the failure to recognize the different nature between Stapp's and Bell's inequalities resulted in widespread confusion when conflating Stapp's and Bell's methods.

Stapp began his long crusade to prove quantum nonlocality in 1971. From 1971 to 1997, he used an inequality without hidden variables [34,46,47]. Then from 1997 to 2004 [48–50], Stapp turned to modal logic. Finally, in 2004, he recognized the problems with his modal logic approach and devised a reasoning based on Hardy's 1993 paper [51].

Stapp's method in the period from 1971 to 1997 is particularly relevant. It still influences the formulation of the Bell theorem to this day, notwithstanding the different nature of the Bell inequality.

Here we shall review Stapp's reasoning during that period and remark on the differences between his inequality and Bell's.

5.1 Counterfactual derivation without hidden variables

Stapp assumed a Bell-like experimental scenario. As usual, Alice and Bob each receive a particle in the singlet-state

$$|\psi\rangle = \frac{1}{\sqrt{2}}(|+\rangle|-\rangle - |-\rangle|+\rangle) \quad (7)$$

After measuring in one of two possible directions $i \in \{1, 2\}$, Alice finds the result $A_i \in \{-1, +1\}$. Analogously for Bob, $B_k \in \{-1, +1\}$, $k \in \{1, 2\}$.

Similarly to the case of the *Clauser, Horne, Shimony, Holt* (CHSH) [52] form of the Bell inequality, Stapp considers the four possible experiments and their results

$$A_1B_1, A'_1B_2, A_2B'_1, A'_2B'_2 \quad (8)$$

Stapp interprets those results according to the following counterfactual rule:

Of these eight numbers only two can be compared directly to experiment. The other six correspond to the three alternative experiments that could have been performed but were not. [53]

The counterfactual rule plus the locality condition impose some restrictions on the possible values of (8). Suppose A_1B_1 are the measured values Alice and Bob find, then had Bob measured in direction 2 instead of 1, he would have found a different result, say B_2 . Locality demands Alice's previous result A_1 cannot change because of Bob's different choice. So, the second term A'_1B_2 in (8) becomes A_1B_2 .

In a similar way, from the actual result A_1B_1 , we can infer that if it was Alice who decided to change her measurement direction to 2, Bob's result would not have changed and we find for the third term that $A_2B'_1 = A_2B_1$. Finally, had both Alice and Bob changed their settings, they would have found the same values as if the other one did not change hers finding that $A'_2B'_2 = A_2B_2$.

Thus, the counterfactual interpretation plus the locality condition reduces the eight possible numbers in (8) to only four

$$A_1B_1, A_1B_2, A_2B_1, A_2B_2 \quad (9)$$

Let us assume a series of actual experiments performed with settings (1, 1)

$$\lim_{N \rightarrow \infty} \frac{1}{N} \sum_k A_1^{(k)} B_1^{(k)} = E^*(1, 1) \quad (10)$$

Based on the results of the actual experiments as given by (10), Stapp reasoned about what would have happened in the other three different series of hypothetical experiments, given the locality constraint (9) on the possible results in each run k of the experiment

$$\lim \frac{1}{N} \sum_k A_1^{(k)} B_2^{(k)} = E^*(1, 2) \quad (11)$$

$$\lim \frac{1}{N} \sum_k A_2^{(k)} B_1^{(k)} = E^*(2, 1) \quad (12)$$

$$\lim \frac{1}{N} \sum_k A_2^{(k)} B_2^{(k)} = E^*(2, 2) \quad (13)$$

The only necessary assumptions to write down (11), (12), and (13) are locality and the possibility of those experiments or that they were a real possibility. There is no need for determinism or the infamous counterfactual definiteness hypothesis. Indeed we do not need the physical existence of the counterfactual results, much less the simultaneous existence of those experiments. Neither do we need the no conspiracy hypothesis or statistical independence. That hypothesis only is necessary in the presence of hidden variables.

Note the irrelevance of the odd ideas usually invoked, such as incompatible experiments and simultaneous existence of conjugate magnitudes. The four series of experiments (10), (11), (12), and (13) are not supposed to exist simultaneously. They are independent possibilities; if ever, only one eventually takes place. There is nothing in those experiments that is inconsistent with quantum mechanics. The best proof of that is that quantum mechanics also makes precise predictions for those series of experiments, say $E(1, 1)$, $E(1, 2)$, $E(2, 1)$, $E(2, 2)$.

Stapp proved that for certain settings, the values imposed by locality on the statistical correlations $E^*(1, 1)$, $E^*(1, 2)$, $E^*(2, 1)$, $E^*(2, 2)$ are incompatible with the quantum mechanical predictions $E(1, 1)$, $E(1, 2)$, $E(2, 1)$, $E(2, 2)$ of those correlations. For instance, in [53] and [46], he showed that incompatibility arriving at the Stapp inequality

$$\sqrt{2} \leq 1 \quad (14)$$

Stapp's argument is simple and direct, albeit he received many criticisms [9, 54–56]. In most cases, he responded to those criticisms by trying to explain misinterpretations and unnecessary complications introduced by such

misunderstandings [57–60]. Even very eminent physicists [9] seem to have missed the simple reasoning implied by (14). It is about the mathematical incompatibility of the predictions for different series of experiments, experiments that are not supposed to have meshed together in incompatible or inconsistent forms. In our opinion, Stapp’s argument, as we reproduce it here, is unassailable. Probably the only weak point is that it relies on counterfactual reasoning:

Although philosophers contend that counterfactual concepts pervade science, and are needed for science, the significance of results based on the use of counterfactuals remains somewhat shaky in the minds of most quantum physicists. [61]

Indeed, sometimes it is claimed that we cannot apply counterfactual reasoning to quantum mechanics. But that is just an unjustified prejudice, at least for the case that Stapp considered. Quantum mechanics makes precise predictions for all four independent series of experiments irrespective of whether they are real or hypothetical, so the factual or counterfactual nature of such experiments cannot invalidate the logical inference obtained from them:

Quantum mechanical predictions are incompatible with locality.

However, we shall never be able to directly falsify the predictions (11), (12), and (13). Indeed, we do not even know the values of $A_2^{(k)}$ and $B_2^{(k)}$, $k \in \{1, \dots, N\}$. But whatever those values, quantum mechanics statistical predictions are incompatible with the locality constraints. Thus, although quantum mechanics is no-signaling, there is a clear sense in which it implies a sort of nonlocality. Whether it is a sort of nonlocality requiring action-at-a-distance is a controversial matter.

5.2 Comparison of Stapp’s and Bell’s formulations

From (10), (11), (12), and (13) we can also arrive at a Bell-CHSH inequality

$$| \langle A_1 B_1 \rangle - \langle A_1 B_2 \rangle + \langle A_2 B_1 \rangle + \langle A_2 B_2 \rangle | \leq 2 \quad (15)$$

To the best of our knowledge, the first to apply Stapp’s counterfactual method to obtain a Bell-CHSH inequality was Eberhard⁵ in 1976 [62].

⁵Eberhard unnecessarily assumed counterfactual definiteness and free will.

Thus, we have two different formulations of the Bell-CHSH inequality based on quite different hypothesis. The original Bell-CHSH inequality as derived by Bell and by Clauser, Hall, Shimony, and Holt is based on:

- 1) Hidden variables.
- 2) Local causality or the screening off condition.
- 3) Statistical independence.

It is relevant to note that the hidden variables approach does not use counterfactual reasoning [22]. As originally formulated, it is a no-go theorem for local hidden variables. Some physicists and philosophers vigorously resist this interpretation of the Bell-CHSH inequality [5–8]. According to them, the list of hypotheses should reduce to 2) and 3) because hidden variables are a consequence of locality. In our opinion, Bell wisely avoided that approach [37]. He preferred to argue for quantum nonlocality differently and only introduced his inequality to prove that a local completion is not possible unless we violate statistical independence.

On the other hand, if we use Stapp's counterfactual method to derive the Bell-CHSH inequality, the interpretation is quite different. Stapp's reasoning allows us to arrive at (15) assuming only locality and is directly applicable to quantum mechanics.

However, from an empirical point of view, the counterfactual (15) is not very useful. While the hidden variable approach is a constraint on every four series of actual experiments, therefore is directly falsifiable, the counterfactual Bell-CHSH inequality only establishes the eventual existence of four such series satisfying it. So, the value of the counterfactual Bell-CHSH inequality is purely theoretical.

Stapp's formulation is not very informative for those who accept quantum nonlocality for different reasons, such as Bell and Einstein. For them, the relevant significance of the Bell inequality is that we cannot fix quantum nonlocality and a local completion is impossible unless we accept superdeterminism.

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