 doi:10.20944/preprints202205.0257.v1

Gravitational mass of the rarefied cloud of relativistic material particles

Wladimir Belayev

Abstract

A cloud of relativistic material particles is considered, for which the distortions of length and time caused by the presence of the Lorentz factor an order of magnitude greater than curvature of space-time by gravity. The gravitational mass of the cloud is found in the region where its size is insignificant. It is established that the coefficient of dependence of the cloud mass on the total rest mass of its particles includes a complete elliptic integral of the 2nd kind.

Keywords: Relativistic material particles; active gravitational mass; Lorentz transformations; isotropic Schwarzschild metric; geodesics equations.

1 Introduction

The weak principle of equivalence, which Einstein had specifically built into his general theory of relativity, identifies passive gravitational mass with inertial mass, and these masses were identified with the active gravitational mass of matter [1]. The mass energy is determined according to the special theory of relativity and is equal to the energy of an inertial mass. This served as the basis for the introduction of the hydrodynamic tensor $T_i^j = (c^2\rho + p)u^j u_i - \delta_i^j p$ with density ρ and pressure p of frictionless adiabatic fluid as a source of matter gravitation in field equations. In [2, 3] it is stated that the identification of inertial and active gravitational masses is erroneous, and the energy-momentum tensor T_i^j should be determined by the density of the active gravitational mass and the potential of the scalar field. In present

Preprints (www.preprints.org) | NOT PEER-REVIEWED | Posted: 19 May 2022 doi:10.20944/preprints202205.0257.v1

paper the active gravitational mass of a rarefied cloud of relativistic material particles was obtained based on the properties of Lorentz transformations and the geometry of Schwarzschild space-time.

2 Weakly gravitating gas cloud

We study a weakly gravitating gas cloud consisting of identical particles with a rest mass m chaotically moving with the same absolute value of velocity v in a certain frame of reference K'. It is assumed that at time $t' = 0$ the distance δr between particles can be neglected to determine the gravity created by this cloud in the considered area. The rarefaction of the gas is determined by the condition

$$\alpha_M / \delta r << v^2 / c^2, \tag{1}$$

where $\alpha_M = \frac{2\gamma M}{c^2}$ with cloud gravitational mass M and gravitational constant γ.

Statistically, the cloud can be represented as a set of systems consisting of two particles A and B, which move in opposite directions in the coordinate frame $K' = (t', x', y', z')$ with the velocities v and $-v$. The weak gravitational field of one particle is described approximately [4] in associated coordinates $K = (t, x, y, z)$ by linearised isotropic Schwarzschild metric

$$ds^2 = c^2 \left(1 - \frac{\alpha}{\bar{r}}\right) dt^2 - \left(1 + \frac{\alpha}{\bar{r}}\right)(dx^2 + dy^2 + dz^2) \tag{2}$$

with $\bar{r} = \sqrt{x^2 + y^2 + z^2}$ and $\alpha = \frac{2\gamma m}{c^2}$.

3 Applying Lorentz transformations to the Schwarzschild metric

Condition (1) means that the distortions of length and time caused by the presence of the Lorentz factor $\frac{1}{\sqrt{1-\tilde{\beta}^2}}$ with $\tilde{\beta} = \frac{\tilde{v}}{c}$ will be an order of magnitude greater than curvature of space-time by gravity. Therefore, the influence of gravity on the Lorentz transformations

$$t = \frac{t' + \frac{\tilde{\beta}}{c} x'}{\sqrt{1 - \tilde{\beta}^2}}, \quad x = \frac{x' + \tilde{v} t'}{\sqrt{1 - \tilde{\beta}^2}}, \quad y = y', \quad z = z' \tag{3}$$

at

$$\tilde{v} = v \tag{4}$$

or

$$\tilde{v} = -v \tag{5}$$

is insignificant and they can be applied to the metric (2). Transformation of coordinates with

$$\bar{r}' = \sqrt{\left(\frac{x' + \tilde{v}t'}{\sqrt{1-\tilde{\beta}^2}}\right)^2 + y'^2 + z'^2} \tag{6}$$

yields

$$ds^2 = c^2\left(1 - \frac{1+\tilde{\beta}^2}{1-\tilde{\beta}^2}\frac{\alpha}{\bar{r}'}\right)dt'^2 - \frac{4\tilde{v}}{1-\tilde{\beta}^2}\frac{\alpha}{\bar{r}'}dt'dx' - \tag{7}$$

$$\left(1 + \frac{1+\tilde{\beta}^2}{1-\tilde{\beta}^2}\frac{\alpha}{\bar{r}'}\right)dx'^2 - \left(1 + \frac{\alpha}{\bar{r}'}\right)(dy'^2 + dz'^2). \tag{8}$$

4 Two body system

In associated with bodies reference frames K_A, K_B the gravity of each of them separately is described in the corresponding frame by the metric (2). Let us pass from these coordinate systems to K', using the Lorentz transformations for velocities (4), (5).

If we represent metric coefficients in the form

$$g_{ij} = \eta_{ij} + h_{ij}, \tag{9}$$

where η_{ij} correspond to the Minkovsky metric, then with weak gravity, [5] the ratio

$$h_{ij} \approx \sum_n h_{ij}^n \tag{10}$$

is performed for the total field created by n subsystems with metric coefficients

$$g_{ij}^n = \eta_{ij} + h_{ij}^n. \tag{11}$$

Summing the fields obtained after substitutions (4) and (5) into the metric (8), we find approximate path interval in the vicinity of $t' = 0$ in a two-body system

$$ds^2 = c^2\left(1 - \frac{1+\beta^2}{1-\beta^2}\frac{\alpha_1}{\bar{r}'}\right)dt'^2 - \left(1 + \frac{1+\beta^2}{1-\beta^2}\frac{\alpha_1}{\bar{r}'}\right)dx'^2 - \tag{12}$$

Preprints (www.preprints.org) | NOT PEER-REVIEWED | Posted: 19 May 2022 doi:10.20944/preprints202205.0257.v1

$$\left(1+\frac{\alpha_1}{\bar{r}'}\right)\left(dy'^2+dz'^2\right) \tag{13}$$

at $\alpha_1 = 2\alpha$ and $\beta = \frac{v}{c}$.

The equations of geodesics

$$\frac{du^i}{ds} + \Gamma^i_{kl} u^k u^l = 0, \tag{14}$$

with Christoffel symbols $\Gamma^l_{ij} = \frac{1}{2} g^{lk} \left(\frac{\partial g_{jk}}{\partial x^i} + \frac{\partial g_{ik}}{\partial x^j} - \frac{\partial g_{ij}}{\partial x^k}\right)$ are used to search for the acceleration of a massive particle in described by metric (13) gravitational field. For spatial coordinates of particle at rest they turn out to be

$$\frac{du^k}{ds} = -\frac{1}{2} g^{kk} \frac{\partial g_{11}}{\partial x^k} \left(u^1\right)^2 \tag{15}$$

with indices $k = 2, 3, 4$. These equations, disregarding small quantities of a larger order, yield coordinate accelerations

$$\ddot{x}' = -\frac{1}{2} \frac{c^2 x'}{\sqrt{1-\beta^2}} \frac{1+\beta^2}{1-\beta^2} \frac{\alpha_1}{(\bar{r}')^3}, \tag{16}$$

$$\ddot{y}' = -\frac{1}{2} c^2 y' \frac{1+\beta^2}{1-\beta^2} \frac{\alpha_1}{(\bar{r}')^3}, \tag{17}$$

$$\ddot{z}' = -\frac{1}{2} c^2 z' \frac{1+\beta^2}{1-\beta^2} \frac{\alpha_1}{(\bar{r}')^3}. \tag{18}$$

5 The gravity mass of gas cloud

The absolute value of the particle acceleration

$$a' = \sqrt{(\ddot{x}')^2 + (\ddot{y}')^2 + (\ddot{z}')^2} \tag{19}$$

will be

$$a' = \frac{1+\beta^2}{2(1-\beta^2)} \frac{c^2 \alpha_1}{(\bar{r}')^3} \sqrt{\frac{(x')^2}{1-\beta^2} + (y')^2 + (z')^2}. \tag{20}$$

In spherical coordinate frame $(t', \bar{r}', \varphi, \theta)$ defined by transformations

$$x' = \bar{r}' \cos\varphi, \quad y' = \bar{r}' \sin\varphi \cos\theta, \quad z' = \bar{r}' \sin\varphi \sin\theta \tag{21}$$

Preprints (www.preprints.org) | NOT PEER-REVIEWED | Posted: 19 May 2022 doi:10.20944/preprints202205.0257.v1

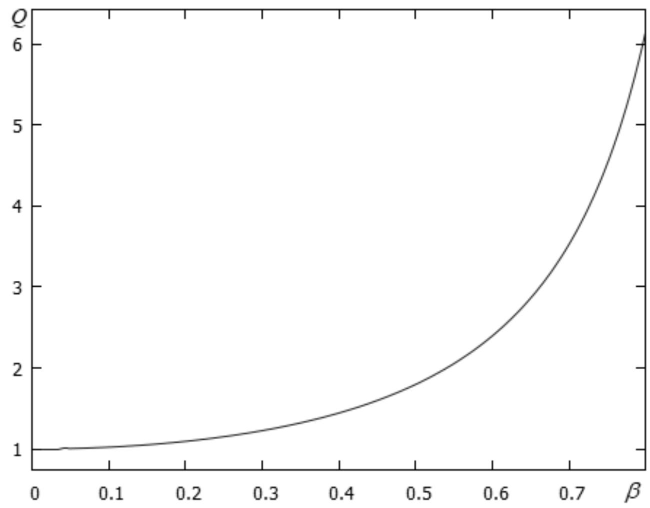

Figure 1: The dependence of the normalized gravitational mass of the cloud Q on the value β.

we obtain

$$a' = \frac{1+\beta^2}{2(1-\beta^2)^{3/2}} \frac{c^2\alpha_1}{(\bar{r}')^2} \sqrt{1-\beta^2 \sin^2\varphi}. \tag{22}$$

The value $\bar{r}'$ is the distance to the gas cloud. Parameter a' corresponds to the gravitational mass of a system consisting of a pair of particles:

$$m_2 = 2m \frac{1+\beta^2}{(1-\beta^2)^{3/2}} \sqrt{1-\beta^2 \sin^2\varphi}. \tag{23}$$

For each pair of particles, the direction of the axes of the coordinate system is chosen so that the axis X' is orthogonal to the line of their motion. Assuming a uniform distribution of the directions of motion of pairs of particles over the corners, the average gravitational mass of a pair of particles in a cloud will be

$$\bar{m}_2 = \frac{2}{\pi} \int\limits_0^{\pi/2} m_2 d\varphi. \tag{24}$$

It determines the gravitational mass of a cloud consisting of n particles

$$M = nm\frac{2}{\pi}\frac{1+\beta^2}{\left(1-\beta^2\right)^{3/2}}E\left(\beta\right), \tag{25}$$

where E is complete elliptic integral of the 2nd kind. Fig. 1 shows how the ratio $Q = \frac{M}{nm}$ changes with increasing particles velocity.

6 Conclusions

The erroneous assumption of the equality of inertial and active gravitational masses follows from the analysis of a system of two bodies moving towards each other. The obtained gravitational mass of the gas cloud does not confirm the concept of the mass density, included in the hydrodynamic tensor, which is accepted as the density of the gravity source.

References

[1] A. Einstein, *Ann. der Physik* **49** (1916) 769.

[2] H. G. Ellis, *IJMPD* **21** 11 (2012) 1242022, arXiv:1205.5552 [physics].

[3] H. G. Ellis, *IJMPD* **24** 08 (2015) 1550069, gr-qc/0701012.

[4] G. C. McVittie, *General Relativity and Cosmology*, (Chapman and Hall Ltd., London, 1956).

[5] L. D. Landau, E. M. Lifshitz, *The Classical Theory of Fields*, (Butterworth-Heinemann, Oxford, 2000).