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Article

Atmospheric Carbon Dioxide: A Fresh Look at Exchange Flux, Isotopic Carbon, Fossil Fuel Emissions and Residence Time

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Abstract: This paper presents a new implementation of a 2-box absolute flow model, enabling the calculation of CO₂ transfer between the atmosphere and a mixing reservoir representing the combined effect of the terrestrial and ocean regions. The model uses published values of anthropogenic CO₂ fossil fuel emissions (CO₂ff), atmospheric CO₂ mixing ratio and nuclear weapons bomb yields, to compute atmospheric $\delta^{13}\text{C}$ and $\Delta^{14}\text{C}$ time-series. The level of agreement between the calculated values and the accepted values for $\delta^{13}\text{C}$ is within $\pm 0.05\text{‰}$, and for $\Delta^{14}\text{C}$ is within $\pm 3\text{‰}$, spanning 200 years. The model contains only seven internal parameters; these are varied to optimize the fit, leading to reasonable parameter values, indicating the validity of the method. In addition the model shows how a rising CO₂ exchange influx fluctuating with surface temperature is compatible with a nett atmospheric CO₂ sink. The study demonstrates that a single CO₂ residence time applies to both the ¹⁴C bomb pulse and the anthropogenic flux, contrary to the conventional view that "bomb radiocarbon and anthropogenic CO₂ do not behave identically." [Joos 1994 Nature 370, 181–182]. It is shown that the difference in behaviour, claimed to be due to seawater chemistry, is not significant in practice since fluctuating bulk carbon dominates fluctuating isotopic ratio. A new analysis of the airborne fraction indicates that the assumption of a time-dependent exchange influx results in a similar residence time for bulk CO₂ as for ¹⁴C in the nuclear bomb pulse, reinforcing the view that the ¹²C, ¹³C and ¹⁴C carbon isotopic forms of CO₂ behave similarly in the carbon cycle.

Keywords: anthropogenic emissions; Revelle factor; exchange flux; bomb pulse; carbon cycle; airborne fraction

1. Introduction

The absolute (gross) size of the atmospheric CO₂ exchange flux between the atmosphere and the land/ocean are crucial in understanding the carbon cycle and estimating the effects of anthropogenic CO₂ fossil fuel emissions (CO₂ff). As the Earth rotates daily and orbits the sun annually, the land and oceanic reservoirs cyclically "inhale" and "exhale" CO₂. These are the exchange fluxes, caused by corresponding changes in temperature, seawater solubility, photosynthesis, plant decay and respiration, resulting in a continuous cycle of absorption and emission. Within this complicated pattern of two-way fluxes, absorption occurs at one geographical location while simultaneously emitting at another. (IPCC Ed., 2001) This paper presents a means of determining the gross exchange flux. Measuring the gross exchange flux is more difficult than the task of finding its nett value, since the gross value does not contribute to the global carbon budget. Current estimates by the Global Carbon Budget (GCB) are for the land exchange flux 130 GtC yr⁻¹, and for the ocean flux 80 GtC yr⁻¹ (Friedlingstein et. al., 2022). GCB refers its data source as being IPCC Canadell 2021 (IPCC Ed., 2021) who indicate that for the ocean, the gross figures originate from Figure 1 of Sarmiento & Gruber 2002 with applied corrections. Earlier GCB reports refer to IPCC (Ed) 2013 which states "Individual gross fluxes and their changes since the beginning of the Industrial Era have typical uncertainties of **more than 20%**" (author's highlight).

In the 1970s attempts were made to model the carbon cycle and exchange flux using box models (e.g. Oeschger et al., 1975). Subsequently, General Circulation Models (GCMs) became common, since they could create global geographical distribution maps of the carbon-related processes despite requiring more computer resources. Although GCMs have been helpful in understanding the complexity of the global carbon cycle, box models, with their top-down simplification, offer specific advantages over the open-ended micro-accounting of GCMs. Such simpler models are "better testable" (Popper, 1934) and can help identify errors in formulation by focusing on the most significant factors. This paper presents a 2-box model that computes ^{14}C and ^{13}C isotopic ratios and differs from other box models in several ways. Firstly, it uses atmospheric CO_2 levels as input rather than output. Secondly, it employs absolute flow instead of net flow. Lastly, it incorporates a factor describing the proportion of fossil fuel emissions that are directly absorbed by sinks before dilution in the atmosphere takes place. The model accurately calculates $^{14}\text{CO}_2$ and $^{13}\text{CO}_2$ isotopic ratios spanning over 200 years, including the bomb pulse. Yet, notably, it uses the same residence time and reservoir mixing properties for ^{14}C , ^{13}C and ^{12}C .

In addition to uniquely providing estimates of the magnitude of the exchange flux, the method also challenges the widely held view that "*the atmospheric impulse response function of an isotopic perturbation decays much more rapidly than the impulse response function for bulk carbon*" (Heimann 1993). See Discussion Section 5.

1.1. Radiocarbon

The isotopic abundance of ^{13}C and ^{14}C present within atmospheric CO_2 is accurately known globally, with records going back hundreds of years. These isotopes have been measured in samples from tree rings (Stuiver & Quay 1981), ice cores (Rubino et al., 2013), and the atmosphere (Stuiver et al., 1998; D6). While ^{13}C is radioactively stable and forms around one percent of atmospheric carbon, ^{14}C undergoes radioactive decay with a half-life of 5700 ± 30 years (Kutschera 2013), and comprises about 1 part in 10^{12} . There is virtually no presence of ^{14}C in fossil fuel since it has all radioactively decayed. The $^{14}\text{C}/^{12}\text{C}$ ratio decreased between 1800 and 1960 because of dilution from rising fossil fuel emissions which are ^{14}C depleted, a process known as "Suess dilution" (Suess 1955). In 1960, atmospheric nuclear weapons testing began, causing a doubling of the absolute ^{14}C level within ten years. Since 1975 the level of ^{14}C has decreased (D6), partially due to washout from the exchange flow which contains a somewhat lower level of ^{14}C , but also because of "Suess dilution". Turning now to ^{13}C , its presence in fossil fuels is reduced when compared with the atmosphere, but only by around 2% (Stuiver and Polach, 1977). This small reduction arises from fractionation, which means the larger less mobile isotopic CO_2 molecules are less likely to become embedded in leaves and fossil carbon. Measurements show that the atmospheric $^{13}\text{C}/^{12}\text{C}$ ratio has decreased during the past 200-year period (D4), again partially due to dilution by fossil fuel carbon which contains the lower level of ^{13}C , and partially because of washout from the exchange flow. See Section 4.2.

2. CO_2 Absolute Flow Finite Reservoir Model

The model calculates the movement of CO_2 and its carbon isotopes between the atmosphere and a mixing reservoir, by calibration against isotopic measurements. The size of this mixing reservoir, and the exchange flux are "effective" values, combining the effect of the ocean and terrestrial origins. The concept of effective values is commonplace in the fields of engineering, applied science and economics. For example, economic inflation represents, in a single figure, the average of many different price increases on many items. Productivity measures the effective value created by each hour of work. Effective resistance in electronic engineering or hydraulics gives the overall resistance of a complex system to current or fluid flow. In each of these examples a single value is used to account for a property which is distributed throughout a complex system. The model presented is able to determine the effective values without requiring a detailed audit of the underlying constituent components, unlike other methods. The model consists of two boxes which are considered well-mixed, Figure 1. At each annual iteration, the isotopic mixing ratio within the reservoir and

atmosphere changes because of incoming ^{13}C and ^{14}C . This change is calculated by the dilution formulae in Section 2.1. Using a similar notation as Tans et al.(1993) and Cawley (2011) we have

$$\frac{dC}{dT} = F_a + F_i - F_e \quad (1)$$

where C is the atmospheric carbon mass, F_a is the anthropogenic input flux from fossil fuel emissions, F_i is the environmental flux flowing into the atmosphere and F_e is the environmental flux exiting the atmosphere. It is essential that F_i and F_e are kept separate because inflow and outflow are occurring in different places and they are needed to calculate the isotopic flows. Summing these components to form a nett flow would lose track of the separate isotopic mixing occurring across the globe.

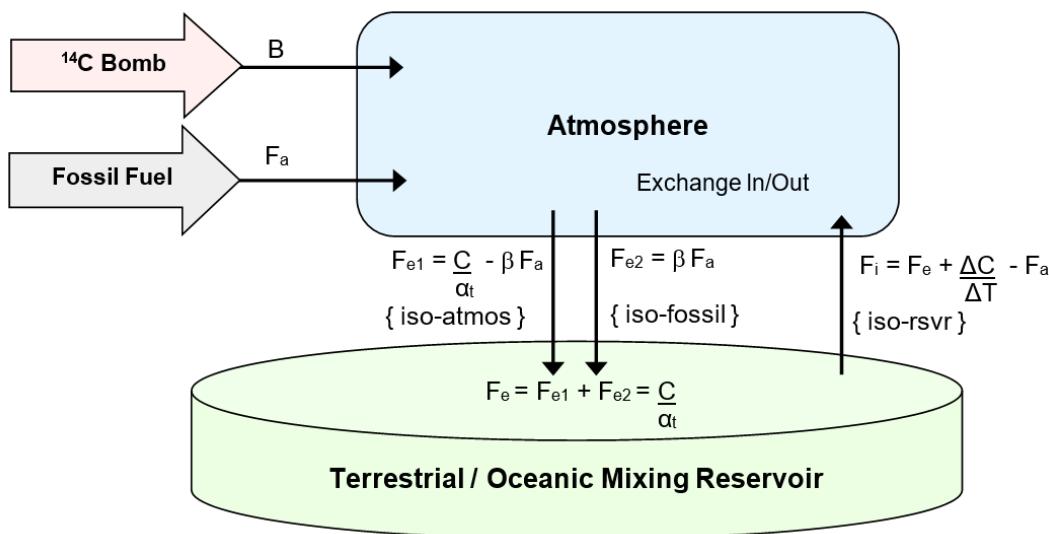


Figure 1. CO_2 Finite Reservoir Two-Box Model. The model describes the CO_2 flux between the reservoir and atmosphere, including its isotopic carbon contents. Anthropogenic fossil fuel CO_2 ff F_a , enters the atmosphere and a portion β exits without mixing. The remaining portion, $(1-\beta)$ mixes into the atmosphere. Otherwise, full isotopic mixing is assumed to occur within each of the two storage regions, atmosphere and reservoir. At each iteration, the CO_2 flux outflow, F_e is proportional to atmospheric CO_2 , $C(t)$; its isotopic level corresponds to a mixture of atmospheric and fossil content. The CO_2 influx, F_i is determined by using equation (1) below. Atomic weapons ^{14}C $B(t)$ directly enters the atmosphere with the timing and amounts determined by historical records of individual atmospheric atomic weapons testing, and bomb yield.

We first need an expression which connects the atmospheric CO_2 mass C , to the outflow; F_e is an obvious choice, being

$$F_e = aC \quad (2)$$

Equation(2) meets the common sense requirements that the absolute exchange exit flux varies in proportion to C , and if C were to be zero, the exit flux, F_e would also be zero. Substituting (2) into (1), we obtain an ordinary differential equation (ODE) for absolute inflow as

$$F_i = aC + \frac{dC}{dT} - F_a \quad (3)$$

The formula also describes the analogous situation of the flow of water from a bucket with a hole of size related to a , and two input taps (faucets) one of which represents the environmental exchange inflow, F_i and the other represents fossil fuel inflow, F_a . In practice, for use with data series, we replace dC/dt with $\Delta C/\Delta T$ where ΔC is the difference of successive carbon atmospheric mass and ΔT is the time difference, leading directly to the implementation equations shown in Section 2.4. The formulation differs from those used by other authors in two important respects. First, the incoming flux, F_i is not assumed constant and is calculated at each iteration by "balancing the books". Second, F_e is calculated by direct proportionality to the atmospheric CO_2 mass. As a consequence the model is not a predictive model of CO_2 atmospheric level. However, a great advantage of this formulation is that it enables the inflow to be calculated from measured data, eliminating the need to provide a

bottom-up audit of individual CO₂ sources. Furthermore it provides estimates of CO₂ inflow and CO₂ isotopic ratios, and it computes a value for the CO₂ residence time constant. This constant applies equally to all three carbon species ¹⁴CO₂, ¹³CO₂ and ¹²CO₂. See Section 5.0 for further discussion.

The model uses a bomb yield multiplication factor, Y_b, as one of its seven optimisation parameters, to calculate the nuclear bomb yield of ¹⁴C each year, B₁₄[i]. The bomb yield in megatons is obtained from records of atomic weapons atmospheric test detonations [D5]. A simple time delay of one year was used to allow for atmospheric mixing. The initial values i.e. $\delta^{13}\text{C}_{\text{init}}$, $\Delta^{14}\text{C}_{\text{init}}$, are also optimisation parameters within the model and determine the initial level of the curves in Figures 3 & 4, while the parameter $\delta^{13}\text{C}_{\text{FF}}$ determines the curve slopes in Figure 5.

2.1. Mass-Balance and Isotopic Dilution

If a number of gases are mixed but do not react, and each component gas, i, has a mass M_i, then from the law of conservation of mass (also known as Mass Balance)

$$M_T = \sum M_i$$

If within each component gas there is a specific molecule present in a ratio to its mass, R_i, then, since that specific molecule is conserved, the resulting mixture (e.g. paint mixing) having mass M_T has a ratio R_T given by

$$R_T = \sum R_i M_i / M_T \quad (4)$$

Equation(4) also applies to the situation of mixing of isotopic gases. For practical reasons associated with historical measurements of radioactivity, studies of isotopic mixtures normally utilize the ratio of the sample activity, A_s to a standard sample activity A_{abs}, this ratio being known as the relative specific activity, A. Its value is then offset by one to provide the radioactivity scale, δ written (see Strenstrom 2011) in the form

$$\delta = (A_s / A_{\text{abs}}) - 1 = A - 1 \quad (5)$$

In this scale, although δ increases linearly with A_s, its value is offset to be zero at the value of the absolute standard (often chosen to approximate background level), as for example in the cases of $\delta^{13}\text{C}$ and $\delta^{14}\text{C}$. Applying (5) and substituting A_{Si} for each component R_i of the mixture, leads after some algebraic manipulation, to

$$\delta_T = \sum (\delta_i M_i) / M_T \quad (6)$$

The similarity with equation(4) shows that although δ is defined by an offset ratio scale we can still directly apply equation(6) to calculate the resulting value of δ_T for the mixture. The above scheme forms the basis for the system of equations A₁₄[i], R₁₄[i], A_{FF}[i] and R_{FF}[i] shown in the implementation Section 2.4 below.

2.2. Fractionation

The model generates values of $\delta^{14}\text{C}$, but we require $\Delta^{14}\text{C}$ for comparison with observed $\Delta^{14}\text{C}$ values. $\Delta^{14}\text{C}$ incorporates a fractionation correction to "translate the measured activity to the activity the sample would have had if it had been wood" (Strenstrom 2011). The correction formulae, which are themselves a function of $\delta^{13}\text{C}$, may be derived from equations by Strenstrom 2011 using equations 25, 28, 38, or Stuiver & Polach 1977 p356, 360, Table 1, where A_{SN} is the normalised specific sample activity, A_s is the specific activity of the atmosphere or reservoir, A_{abs} is the absolute specific standard, $\delta^{13}\text{C}_M$ is the ¹³C sample measurement, and $\delta^{13}\text{C}_w$ is the ¹³C standard for wood, giving

$$\delta^{14}\text{C} = A_s / A_{\text{abs}} - 1$$

$$\Delta^{14}\text{C} = A_{\text{SN}} / A_{\text{abs}} - 1$$

$$A_{\text{SN}} = A_s [(1 + \delta^{13}\text{C}_w) / (1 + \delta^{13}\text{C}_M)]^2$$

Eliminating A_s, A_{abs} and A_{SN} gives

$$\Delta^{14}\text{C} = [1 + \delta^{14}\text{C}] \cdot [(1 + \delta^{13}\text{C}_W) / (1 + \delta^{13}\text{C}_M)]^2 - 1$$

The above formula was used to convert values of $\delta^{14}\text{C}$ computed by the model to $\Delta^{14}\text{C}$. The standard value of $\delta^{13}\text{C}$ for wood was taken as 25‰, while $\delta^{13}\text{C}_M$ was taken from the model. The correction was small, approximately 10‰ when the bomb pulse has a peak value of 800‰, decreasing to 0.03‰ when $\delta^{14}\text{C}$ approaches its background value of zero.

2.3. Age Correction

The radioactive decay of ^{14}C over the period from 1820 to 2020 is approximately 2%, since its half-life is 5700 ± 30 years (Kutschera, W., 2013). However, this relatively small value is even less significant when one considers that the steady production of stratospheric ^{14}C approximately balances the amount of ^{14}C decay, since the two are in quasi-equilibrium. Therefore, neither ^{14}C decay nor natural stratospheric ^{14}C production were included in the model.

2.4. Implementation

Using equations 1-3 for the amount of CO_2 in the atmosphere C , its removal time α_t , the atmosphere outflow F_e , the inflow F_i , and the amount of CO_2 in the reservoir R , with the square brackets '[i]' referring to the value at each iteration i , we have

$$F_e[i] = C[i] / \alpha_t, F_{e2}[i] = \beta F_a[i], F_{e1}[i] = F_e[i] - \beta F_a[i]$$

$$F_i[i] = F_e[i] + C[i] - C[i-1] - F_a[i-1], R[i] = R[i-1] + F_e[i] - F_i[i]$$

The ratio of ^{14}C to C in the atmosphere is denoted by A_{14} and in the reservoir by R_{14} and is calculated as follows. The new amount is given by the previous amount added to the input amounts with the output amount being subtracted, all divided by the total mass (see equation 4). If necessary we include the annual ^{14}C production due to atomic weapons bomb input, B_{14} . Hence we obtain

$$A_{14}[i] = (A_{14}[i-1] \cdot C[i-1] + F_i[i-1] \cdot R_{14}[i-1] - F_e[i-1] \cdot A_{14}[i-1] + B_{14}[i-1]) / C[i]$$

$$R_{14}[i] = (R_{14}[i-1] \cdot R[i-1] - F_i[i-1] \cdot R_{14}[i-1] + F_{e1}[i-1] \cdot A_{14}[i-1]) / R[i]$$

Similarly, the relative atmospheric and reservoir fossil fuel content A_{FF} and R_{FF} , are calculated on a scale of 0 to 1 using the values at the previous iteration levels, where A refers to the atmosphere, and R the reservoir and subscripts FL means fossil fuel level, and NL means natural non-fossil level,

$$A_{FF}[i] = (A_{FF}[i-1] \cdot C[i-1] + F_i[i-1] \cdot R_{FF}[i-1] - F_{e1}[i-1] \cdot A_{FF}[i-1] - F_{e2}[i-1] + F_a[i-1]) / C[i]$$

$$R_{FF}[i] = (R_{FF}[i-1] \cdot R[i-1] - F_i[i-1] \cdot R_{FF}[i-1] + F_{e1}[i-1] \cdot A_{FF}[i-1] + F_{e2}[i-1]) / R[i]$$

$$A_{FL}[i] = C[i] \cdot A_{FF}[i], A_{NL}[i] = C[i] - A_{FF}[i]$$

$$R_{FL}[i] = R[i] \cdot R_{FF}[i], R_{NL}[i] = R[i] - R_{FF}[i]$$

The offset isotopic ratiometric measure for ^{13}C is $\delta^{13}\text{C}$ and that for ^{14}C is $\Delta^{14}\text{C}$. These are calculated from the iterative series by:-

$$\delta^{13}\text{C} = \delta^{13}\text{C}_{FF} \cdot A_{FF} + \delta^{13}\text{C}_N (1 - A_{FF}), \Delta^{14}\text{C} = (A_{14} - 1)$$

Initial Conditions: $F_i[0] = F_e[0], A_{14}[0] = 1, R_{14}[0] = 1, A_{FF}[0] = 0, R_{FF}[0] = 0$

It is not necessary to embed a formula for attenuation factor, as discussed by Stuiver & Quay (1981), or for Suess dilution (Suess 1955) because they are implicitly represented.

3. Method

The input data between 1750 and 2020 was selected from the appropriate data source listing (D1-D6) and entered into a spreadsheet. The cell formulae were set to correspond to those equations in Section 2.4

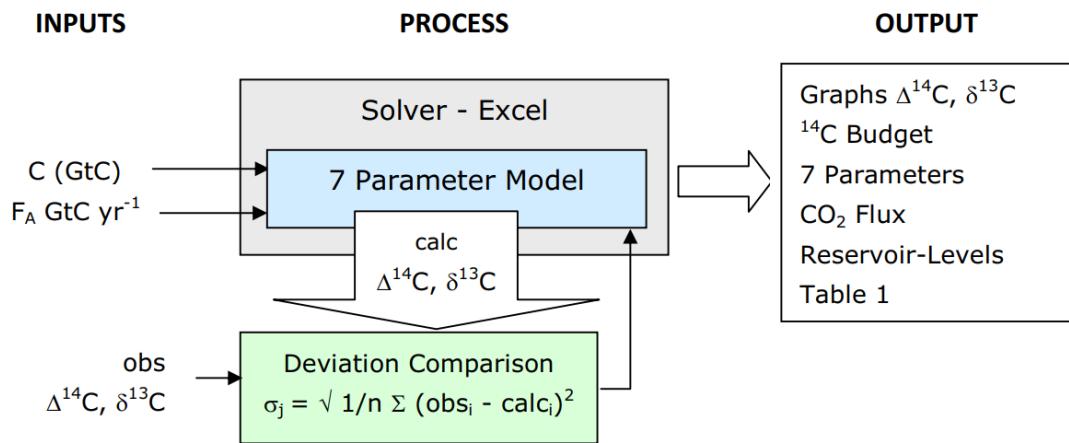


Figure 2. Solution Optimisation. The 7 parameter model and the deviation comparison formulae are implemented as cell formulae within Microsoft Excel. The deviation comparison formulae compares the measured time-series values of atmospheric $\Delta^{14}\text{C}$ and $\delta^{13}\text{C}$ with the calculated values. The Add-In Solver was used to numerically minimise the standard deviation by varying the values of the 7 parameters, providing the above outputs shown below in this study.

The optimisation-solution process is shown in Figure 2 above. The standard deviation between observed and predicted values, σ_1 , σ_2 were determined for $\Delta^{14}\text{C}$ and $\delta^{13}\text{C}$ respectively. The values of $\Delta^{14}\text{C}$ between 1950 and 1968 were excluded from the summation because of the bomb pulse transient. The square root of the product of the standard deviations gives the overall geometric mean standard deviation, σ_T , hence

$$\sigma_j = \sqrt{1/n \sum (obs_i - calc_i)^2} \quad (7)$$

$$\sigma_T = (\sigma_1 \times \sigma_2)^{1/2} \quad (8)$$

The solver system was set to optimise the 7 parameters shown in Table 1 using the default solver options. The optimal solution was found by minimizing the discrepancy, σ_T using the standard "Solver" function of Microsoft Excel. The model has been tested using Microsoft Excel 2002 Version 10 and Microsoft Excel for Microsoft 365 MSO Version 2306 producing identical results.

4. Results

The results are in three main categories; the optimised seven parameters, the isotopic series, and the flux time series.

4.1. Parameter Values

Table 1 lists all seven parameter values along with their estimated errors. The error values were derived by varying each parameter up and down, while keeping the other parameter values constant, until the overall geometric mean standard deviation, σ_T approximately doubled. The error figure presented in the table is the mean of the two variations.

Table 1. Solved 7 Parameter Values.

Parameter	Sym.	Value	err.
Residence time (years)	α_t	14.21	1.7
Reservoir Size rel. to Atmosphere 1750	RCO2	6.18	1.4
CO2ff Unmixed Uptake Factor	β	0.43	0.1
Atomic Bomb Yield: $^{14}\text{C}/\text{MT}$ ‰	Yb	1.61	0.1
Pre-industrial ^{14}C ‰	$\Delta^{14}\text{C}_{\text{init}}$	-3.2	10
Pre-industrial ^{13}C ‰	$\delta^{13}\text{C}_{\text{init}}$	-6.7	0.2
Fossil fuel ^{13}C ‰	$\delta^{13}\text{C}_{\text{ff}}$	-20.1	4

1) Residence Time

This compares favourably with Revelle & Suess (1957) of approximately 10 years, and Arnold (1957) of 10-20 years. It has subsequently been stated that "No single lifetime can be defined for CO₂." (IPCC (Ed.): 2001, Table 1 p38). This study finds to the contrary.

2) Reservoir Size rel. to Atmosphere (1750)

The value for reservoir size are reported in IPCC Ed., (2013) in GtC as vegetation 450 to 650, soils 1500 to 2400, permafrost 1700, and surface ocean 900, corresponding to a range of 4550 to 5650. This study finds a reservoir size in GtC of $6.18 \times 589 = 3640$ GtC increasing to 3809 GtC in 2020 with errors of +/- 22%.

3) CO₂ Relative Unmixed Uptake

This study determines the quantity of CO₂ff entering the sink without mixing in the atmosphere. Factor β accounts for the unmixed uptake, corresponding to that portion of the CO₂ff emissions which directly enter the terrestrial/oceanic mixing reservoir. The effect of entry of pure undiluted CO₂ff is a lowering of the ^{14}C and ^{13}C curves. The model finds that around 43% of CO₂ff exits before mixing while 57% becomes fully mixed. A recent study has revealed that plants near nuclear power stations can uptake significant quantities of $^{14}\text{CO}_2$ (Naegler & Levin 2006). A study of urban grasses near a major highway revealed plants were composed of up to 13% of fossil-fuel carbon (Lichtfouse et al, 2005; Ota et al 2016). According to Kuderer et al (2018), "*The ^{14}C signals from such point sources are well detectable in plant samples*". Since most fossil fuel emission sources are located near the land or ocean surface, it is reasonable to propose that a significant portion of CO₂ fossil fuel emissions are absorbed before fully mixing in the atmosphere.

4) Nuclear Bomb Yield

The model returned a value of $1.6 \pm 0.1\text{‰}$ of ^{14}C per MT for the nuclear bomb yield, with a total bomb yield of 440MT. For comparison, we convert the figures to ^{14}C atoms per MT bomb yield using formulae derived from Svetlik (2010) and Strensom (2011). The weight of carbon atoms, A_{CO2} in the atmosphere in 1950, when the CO₂ mixing-ratio was 312.8 ppm, was 664.43 GtC. The number of carbon atoms is given in terms of the molecular weight of naturally abundant carbon, MC = 12.01, and Avogadro's number N_A = 6.022×10^{23} by

$$NC = A_{\text{CO2}} \times N_A / MC.$$

The $^{14}\text{C}/\text{C}$ ratio R ^{14}C can be obtained from the specific activity, $\alpha = 0.226$ Bq per gram, ^{14}C half life T_{1/2} = 5730 years = 1.807×10^{11} s, molar weight of ^{14}C is MC₁₄ = 14, and ln(2) = 0.6931, with

$$R^{14}\text{C} = \alpha T_{1/2} M_{\text{C}14} / (N_A \times \ln(2)).$$

The number of ^{14}C atoms N ^{14}C created is given by the product of the two expressions giving

$$N^{14}\text{C} = NC \times R^{14}\text{C} = (A_{\text{CO2}} / MC) \times (\alpha T_{1/2} M_{\text{C}14} / \ln(2)) = 4.6 \times 10^{28}.$$

Hence the number of ^{14}C atoms per MT of bomb yield is $0.0016 \times 4.6 \times 10^{28} = 7.4 \times 10^{25}$, comparing favourably with reported values by Hesshaimer et al. (1994) of 1 to 2×10^{26} atoms per MT. The total bomb yield of 440MT gives a figure of 3.23×10^{28} atoms per MT, which compares with 6.1×10^{28} atoms per MT by Naegler & Levin (2006). The difference may be due to ^{14}C which is stored but does not mix in the biosphere or because of variation in bomb type and design.

5) Isotopic Pre-Industrial Levels and Fossil Fuel Content

- a) ^{14}C shows good agreement with Intcal20[D6] for 1820 of 0.7‰. For comparison the maximum level between 1820-2020 is 800‰.
- b) ^{13}C initial value, shows excellent agreement with Rubino (2013) 1820 of $\delta^{13}\text{C} = 6.7\text{‰}$.
- c) $\delta^{13}\text{C}$ for CO₂ff is slightly low as compared to figures reported by Stuiver & Polach (1977) for coal of $\delta^{13}\text{C} = -23\text{‰}$.

4.2. Isotopic Time Series

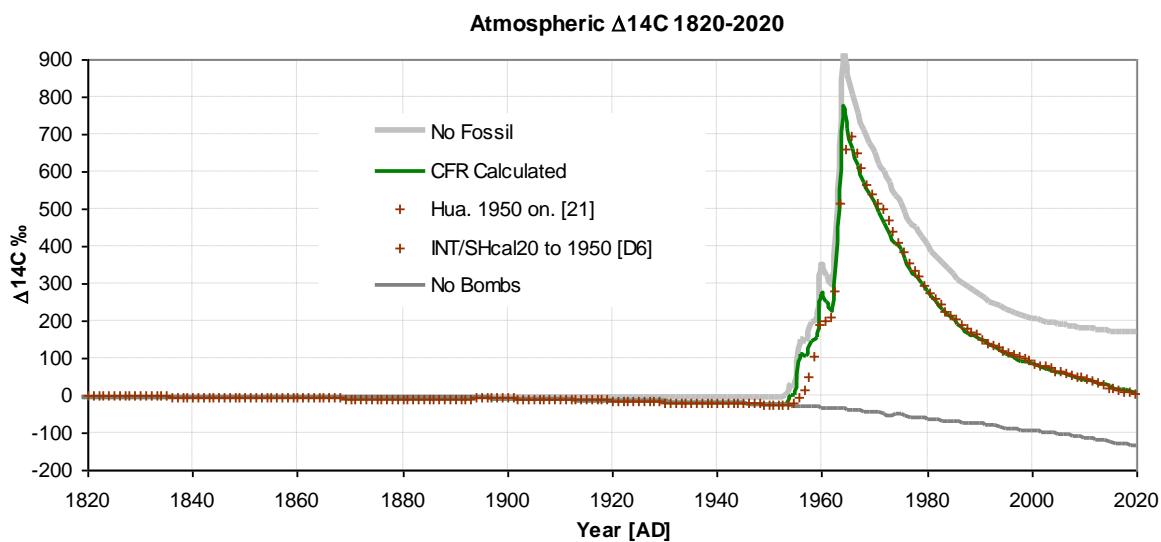


Figure 3. Atmospheric $\Delta^{14}\text{C}$ 1820-2020. showing 130 values before 1950 [D6] and 70 values from 1968 onwards (Hua et al. 2022). Standard deviation between observed and predicted is $\sigma = 3.01\text{‰}$. For "no bombs" and "no fossil" see text.

Figures 3 & 4 display the actual and modelled $\Delta^{14}\text{C}$ values. The total number of observed data points used was 340, resulting in an overall standard deviation of 0.39‰. The quality of the fit is excellent, with a standard deviation of 3‰, corresponding to 0.4% of the range. In Figure 3, the slight decrease from 1820 to 1940 is due to Suess dilution, but the magnitude of the dilution is somewhat reduced by reservoir inflow, resulting in excellent agreement with the observations. Figure 4 shows the bomb pulse plotted as $\Delta^{14}\text{C}$. Initially, the pulse shape is a decaying exponential but, subsequently, due to Suess dilution and the re-emergence of ^{14}C from the mixing reservoir, the $\Delta^{14}\text{C}$ decay shape becomes more linear. Despite these complicating factors, the predicted and observed values are in excellent agreement, indicating that the iteration formulae accurately handle the dilution and mixing processes. Figures 3 and 4 also show two hypothetical scenarios: "no bombs" and "no fossil". The "no bombs" scenario represents how the decrease would have continued without nuclear atmospheric weapons testing, while "no fossils" shows the situation without fossil fuel emissions. A recent work published by Graven et al. 2020 using a model originally produced by M. Heimann and R. Keeling and rewritten for Matlab, gave very similar results for "no bombs" and "no fossil" scenarios, further vindicating the model presented here and its approach.

Figure 5 shows the variation of $\delta^{13}\text{C}$ over 200 years, with predicted values showing a standard deviation of 0.05‰ from those observed, corresponding to 2.5% of the range. The steady reduction in $\delta^{13}\text{C}$ is caused by Suess dilution as fossil fuel emissions contain a lower level of ^{13}C , and by the re-emission of CO₂ from the reservoir. Again, this shape is accurately described by the iteration formulae.

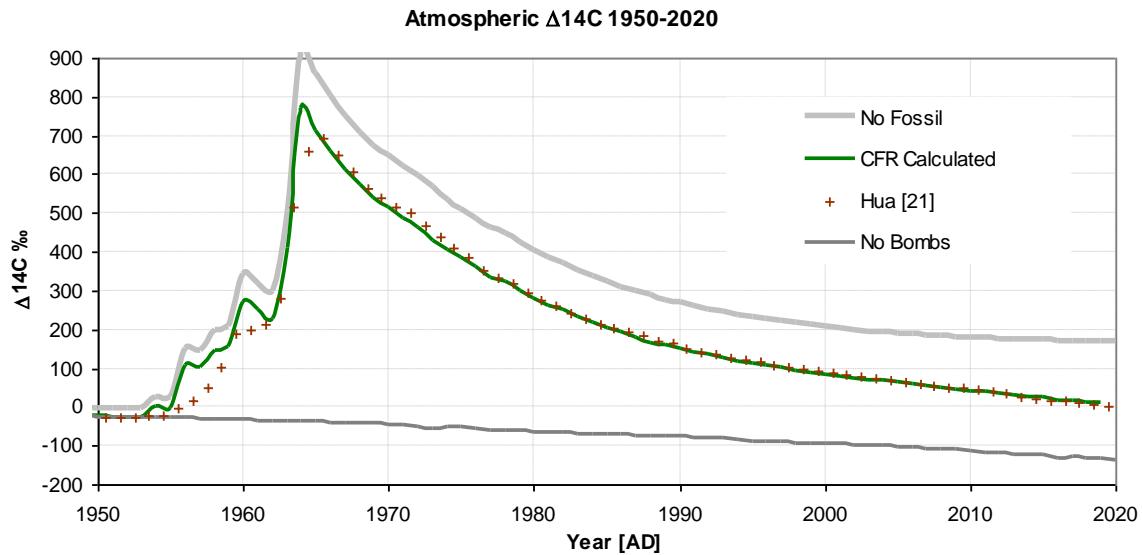


Figure 4. Atmospheric $\Delta^{14}\text{C}$ between 1950 and 2020 showing the "bomb pulse" to 2020. For "no bombs" and "no fossil" see text.

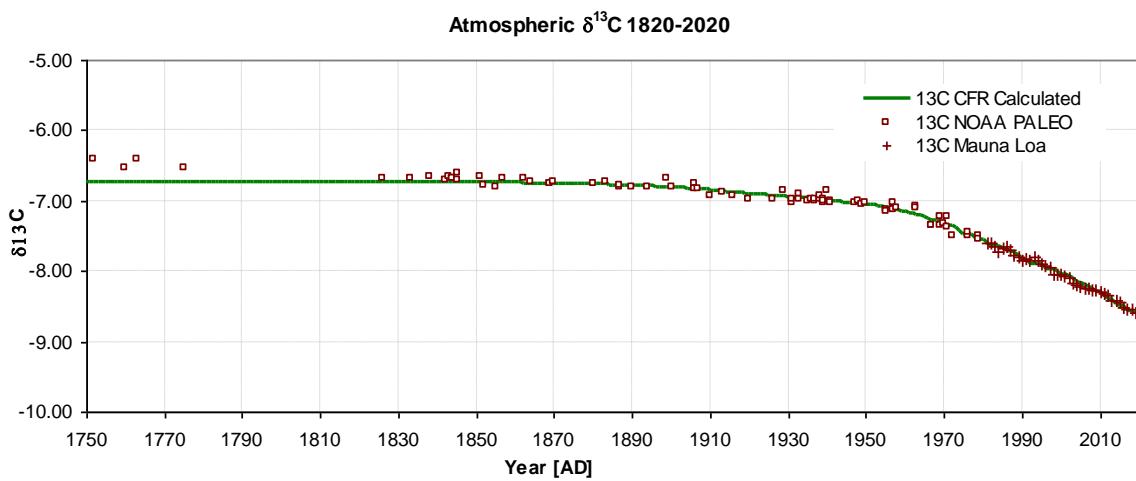


Figure 5. Atmospheric $\delta^{13}\text{C}$ 1820-2020. Model calculated values: solid green, Observed NOAA Paleo [D4]: squares, Mauna Loa [D2]: crosses, $\sigma_{1820-2020} = 0.05\text{‰}$.

Inspection of the curve in Figure 4 reveals that atmospheric $\Delta^{14}\text{C}$ is decreasing to a value below its original background level. This decrease deviates from an exponential decay due to three main factors. a) Incoming fossil fuel emissions (free of ^{14}C) are diluting the atmosphere, this is commonly referred to as "Suess Dilution" (Suess 1955). b) Some of that fossil fuel emissions in the atmosphere is being washed out by the two way exchange flow. c) As time progresses, some of the incoming exchange flow has an increasing level of ^{14}C , because bomb ^{14}C has accumulated in the reservoir and is now being re-introduced to the atmosphere. If the effect (a) were the sole cause of the deviation of the bomb curve shape from a pure exponential decay, the curve would, by now, be well below the initial background. However, effect (b), the washout of fossil fuel emissions, raises the curve. This was noticed many years ago and termed by Stuiver and Quay (1981) the "attenuation factor". Effect (c) raises the tail slightly, further improving the fit in later years to the current level shown in Figure 4. Keeping track of all of these factors in a "back of the envelope" calculation is complex. The absolute flow model described in this study calculates at each iteration ^{13}C and ^{14}C in the reservoir and atmosphere (see Section 2.4). It does not need to explicitly consider Suess dilution, the Stuiver

attenuation factor or flow-back since these emerge implicitly from the calculation embedded within the implementation equations (2.4)

4.3. Absolute $^{14}\text{CO}_2$, Activity Concentration

The activity concentration of ^{14}C , which is its absolute amount rather than its ratio with C, is displayed in Figure 6. The graph shows how the activity concentration of ^{14}C in the atmosphere and the reservoir changes over time. The total amount of ^{14}C in the system is represented by the blue curve, while that in the atmosphere is shown in green. In 1960, atmospheric nuclear weapons testing caused a sudden step increase in the amount of ^{14}C . The atmospheric activity concentration shows the bomb pulse decaying to a minimum around 2000[AD] but then rising slightly again, this has been experimentally determined by Svetlik (2010). Some authors (e.g. Levin et al. 2010) have suggested ^{14}C industrial pollution may be a partial cause of the increase. However, this study accurately reproduces the ^{14}C curve without such a requirement. In this study, the increase in activity concentration can be attributed to $^{14}\text{CO}_2$ re-emerging from the mixing reservoir, while at the same time causing a slight drop in the reservoir level.

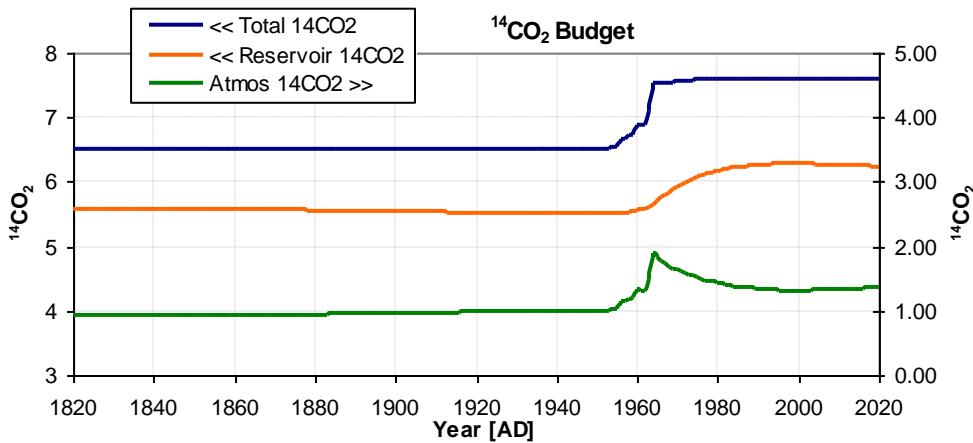


Figure 6. $^{14}\text{CO}_2$ Budget. Model derived absolute $^{14}\text{CO}_2$ (also known as activity concentration) for the reservoir (orange), the atmosphere (green), and their total (blue) in units relative to the $^{14}\text{CO}_2$ content of the atmosphere in 1950. The total, being the sum of reservoir plus atmosphere, shows the near step function in $^{14}\text{CO}_2$ created by nuclear atmospheric weapons testing. After the initial fall of atmospheric content, the small rise since 2000 is confirmed by the results of Svetlik (2010), see text.

4.4. Flux Time Series

The model calculates the mean gross exchange flux in 2019 as 60 GtC yr^{-1} , these are approximately 27% of the total IPCC figures (IPCC Ed., 2021 Fig 5.12) of 217 GtC yr^{-1} , and more closely resemble the ocean figures alone of 54 GtC yr^{-1} . Why is there such a discrepancy? We propose here that it is related to the definition of exchange flux. In the real world, if the nett value of a significant two-way exchange flux is zero, it can only have an effect if it has mixed isotopically en route to and from the reservoir. If there is no isotopic mixing, the returned flux is identical to the original absorbed flux both in its bulk and isotopic content; its effect is therefore insignificant. Thus a portion of the exchange flux has no effect. To determine the "effective exchange flux", the model finds the size of the flux which when mixed in both the reservoir and atmosphere, minimises the discrepancy between its computed ^{13}C and ^{14}C curves and the experimentally determined values. This flux is different from the exchange flux of IPCC 2021, which includes cyclical flows of isotopically identical i.e. unmixed gas. This issue also influences the calculation of residence time, which is defined by "atmospheric mass/removal rate", and has been quoted by many in the field (Harvey 2000). The calculation provides a figure of ~4 years, being obtained by dividing the atmospheric size of 871 GtC by the gross exchange flux of IPCC 2016 i.e. 217 GtC yr^{-1} . However, using the model-determined

exchange flux of 60 GtC yr⁻¹, we obtain 871/60 which yields a residence lifetime of ~14 years, similar to the observed decay of the bomb pulse. Because isotopic mixing occurs more readily within the ocean, it is therefore also unsurprising that the exchange figure from the model more closely resembles the ocean exchange figures rather than the terrestrial portion. See Discussion.

4.5. Influx and Temperature

The absolute inflow, computed by the model, is plotted from 1960 to 2020 in Figure 8. Essentially this is an expanded version of the green plot on Figure 7. The same graph also shows global temperature (D7) plotted on a suitable vertical scale, revealing that many of the temperature excursions match CO₂ inflow excursions. The correlation coefficient between the two plots is 0.94. Although correlation cannot be considered as causation, it is difficult to conceive of a mechanism whereby a global temperature change could be influenced by the rate of CO₂ inflow, rather than its absolute atmospheric mixing ratio. An obvious temperature-dependant mechanism for CO₂ gas release in the ocean is the decrease of solubility with temperature. On land the balance between productivity, respiration, decay and temperature has been discussed by Melillo et al. 2011. According to the model, from 1960, annual atmospheric inflow has increased by 12.7 GtC yr⁻¹ while annual outflow has increased by 13.7 GtC yr⁻¹.

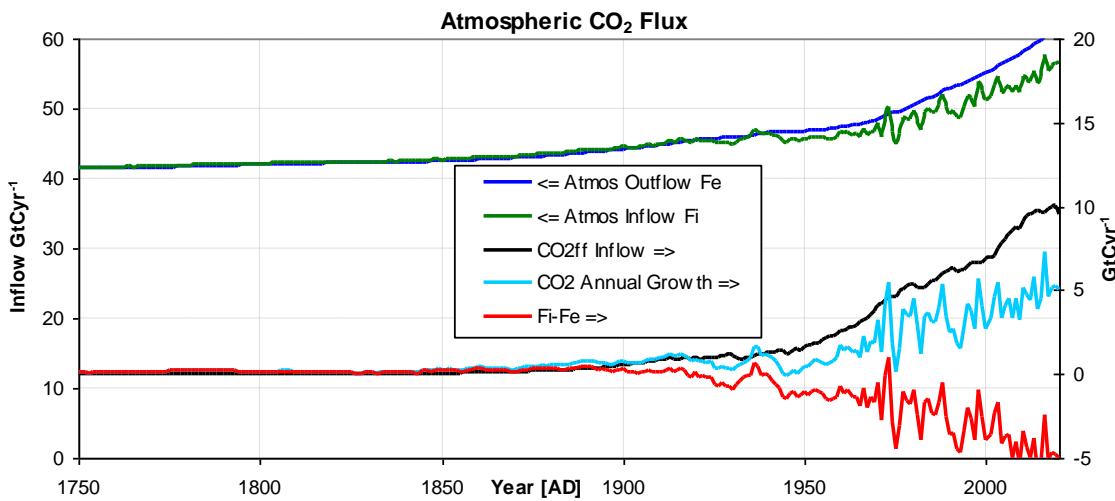


Figure 7. Output from the model showing:- Atmospheric Outflow (blue), Atmospheric Inflow(green), CO₂ff Inflow (black) CO₂ Atmospheric growth (light blue), and Fi-Fe, (red). The arrow on the legend indicates the relevant axis.

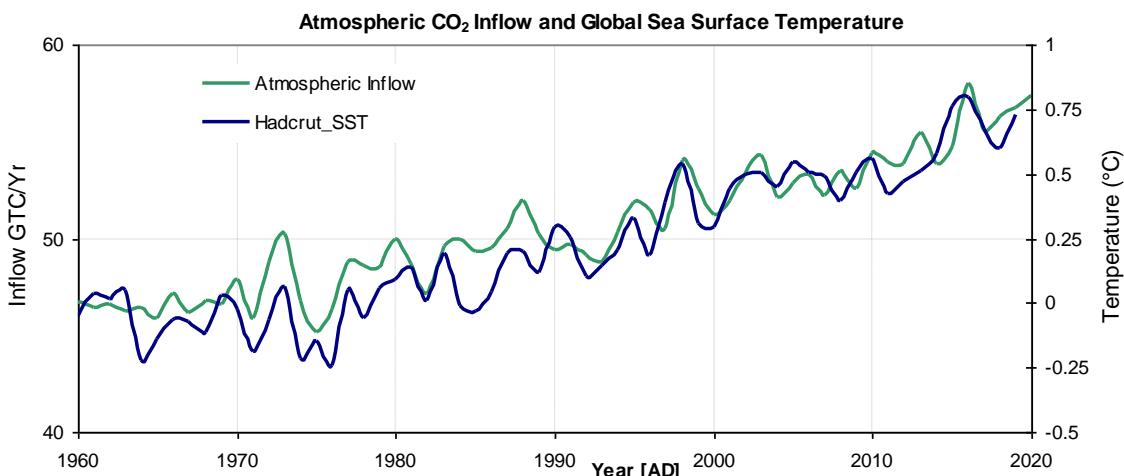


Figure 8. CO₂ inflow (green) and observed global sea-surface temperature (blue) indicating a high degree of correlation of 0.94.

4.6. Cumulative Flux & Levels 1750-2020

Figure 9 shows the cumulative CO₂ flux and CO₂ levels over a period 270 years, during which, 453 GtC of CO₂ff was supplied. Out of this, 282 GtC remains in the atmosphere, while 171 GtC is stored in the reservoir. However, the level of fossil sourced CO₂ increased by 120 GtC in the atmosphere and by 333 GtC in the reservoir, also totalling 453 GtC. Although these figures may seem inconsistent, there is no contradiction. The bulk figures indicate the amount of CO₂ without accounting for its isotopic composition. Fossil level is based upon isotopic content, which depends upon dilution caused by the size of the exchange flux in and out and its source and destination isotopic levels. The relative growth of the atmospheric CO₂ level since 1750 is 48%. The relative level of fossil fuel emissions within the atmosphere in 2020, isotopically tracked is 14% (See Rice 2022). In terms of bulk flow, there is a net outflux of 171 GtC from the atmosphere. Figure 7 shows that while both exchange outflow F_e (blue) and inflow F_i (green) are increasing, the outflow has increased more, confirming the nett uptake of CO₂ from the atmosphere by the terrestrial and ocean reservoirs.

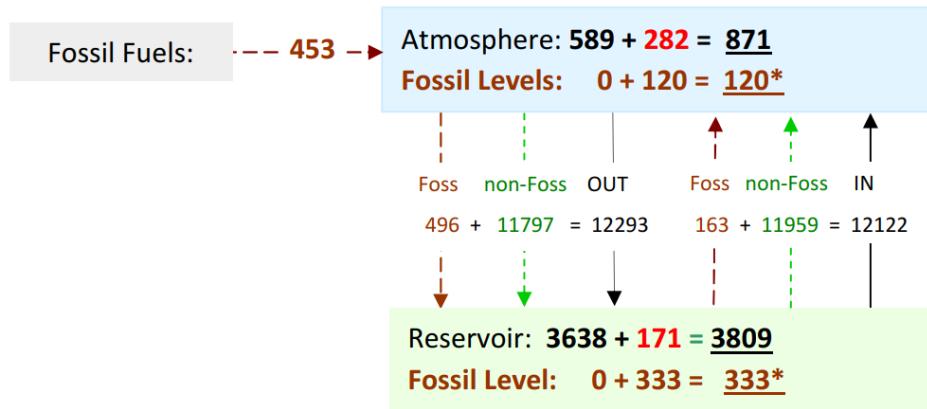


Figure 9. Cumulative CO₂ flux, between atmosphere and reservoirs levels over the period 1750-2020. Fossil fuel flux and fossil levels (identified by composition) are shown in brown, non-fossil flux are in green while total variation are shown in black with growth being shown in red. Final values (in 2020) are underlined. * As determined by isotopic tracking.

5. Discussion

This section provides further evidence challenging the current consensus view regarding CO₂ residence time summarised below. Comparing the oceanic uptake of isotopic CO₂ with the uptake of excess of CO₂ "Siegenthaler and Oeschger (1987) reported that, "The isotopic perturbation vanishes faster than the CO₂ excess". The concept was discussed by Heimann in 1993 "the atmospheric impulse response function of an isotopic perturbation decays much more rapidly than the impulse response function for bulk carbon." Joos in 1994 reported "bomb radiocarbon and anthropogenic CO₂ do not behave identically." Houghton 1996, IPCC explained "an atmospheric perturbation in the isotopic ratio disappears much faster than the perturbation in the number of 14C atoms". Tans 2022 explains "The Revelle factor does not apply to isotopic equilibration because a 12CO₂ molecule is replaced by a 13CO₂ or vice versa. As a result, an isotopic anomaly disappears from the atmosphere more quickly than a total CO₂ anomaly." We term here this concept as the "Revelle Bypass" for brevity. The following two sections provide further evidence that bomb radiocarbon and anthropogenic CO₂ *do* behave similarly within the atmosphere and carbon cycle, supporting the same clear implications derived from the model earlier in this paper, refuting the "Revelle Bypass". This position is not taken lightly, the author believes it is a necessary correction to climate science.

5.1. Isotopic Behaviour

Following Tans' approach (Tans 1994, 2022) we see that a change in atmospheric ^{14}C concentration can arise in two ways; first from a change in concentration of atmospheric CO_2 even though the $^{14}\text{C}/\text{C}$ ratio remains constant (term A), and second, from a change in $^{14}\text{C}/\text{C}$ ratio while atmospheric CO_2 remains constant (term B). Hence, we have

$$\begin{array}{c} \text{A} \quad \text{B} \\ \hline \end{array}$$

$$\text{F14C} = \frac{d(\text{CR})}{dt} = \text{R} \cdot \frac{d\text{C}}{dt} + \text{C} \cdot \frac{d\text{R}}{dt}$$

The author believes Tans' application of this equation to the real world is flawed since Tans only derives the second term. However, the first term is highly significant. In the real world, there is no short-term daily equilibrium in CO_2 , since it fluctuates diurnally as the Earth rotates, and the terrestrial/oceanic reservoirs cyclically and continuously exchange large quantities of CO_2 . This CO_2 flux brings with it quantities of ^{14}C and ^{12}C , which constitute the first term A. The second term B, is driven by fluctuations in the $^{14}\text{C}/^{12}\text{C}$ ratio, such as from decaying vegetable matter which has previously preferentially absorbed ^{12}C during photosynthesis. We can better estimate the magnitude of the contribution of variations of either term, by dividing both sides by CR giving

$$\begin{array}{c} \text{A} \quad \text{B} \\ \hline \end{array}$$

$$\frac{d(\text{CR})}{\text{CR}} = \frac{d\text{C}}{\text{C}} + \frac{d\text{R}}{\text{R}}$$

where R is the isotopic ratio, and C is the amount of bulk carbon. The Revelle factor applies only to term A because it describes a change in bulk carbon; in doing so it applies to the various isotopes equally. A comparison of the local values of the relative sizes of $d\text{C}/\text{C}$ and $d\text{R}/\text{R}$ shows term A greatly dominates B. This can be determined by examination of a number of studies (e.g. Bishcof 1960, Dai et al. 2009, Faassen et al. 2022, Leinweber et al., 2009, Olsen et al. 2004, Palonen et al. 2018, Takahashi et al. 2002). The local variation $d\text{C}/\text{C}$ was estimated from the above references as being of the order of ≈ 6 ppm/400 ppm (≈ 0.015) per hour, while the variation $d\text{R}/\text{R}$ for ^{14}C estimated at most as around $\approx 8\%$ (≈ 0.008) per hour and for $^{13}\text{C} \approx 0.2\%$ (≈ 0.0002) per hour. Hence $|A|$ significantly dominates $|B|$. Therefore, it is irrelevant whether or not B has an ability to bypass carbonate chemistry (i.e. "Revelle Bypass") as claimed by many, since A is the dominant flux, and term A does not have the ability to preferentially bypass seawater chemistry for ^{14}C . Therefore in practice term A dominates B and both ^{12}C and ^{14}C are equally subject to the effects of seawater carbonate chemistry.

5.2. Airborne Fraction

The airborne fraction is defined as the ratio of atmospheric CO_2 growth to total CO_2 anthropogenic emissions (Friedlingstein 2022). Its relatively stable value of approximately one half was remarked upon by Broeker in 1975. It has been suggested that, since the rise in anthropogenic emissions is approximately exponential, this indicates that absorption of atmospheric CO_2 is governed by an ordinary differential equation (ODE) (Cawley 2011). We provide below the mathematical derivation of airborne fraction, AF from an ODE containing the bulk residence time, t_a and exponential rate-of-rise t_s of anthropogenic emissions, where $t_a = 1/a$ and $t_s = 1/s$. If fossil fuel emissions increase exponentially, F_a in equation (3) may be replaced with e^{st} as

$$\frac{d\text{C}}{dt} = -a \text{C} + F_i + e^{st} \quad (13)$$

The particular solution for (13) is given by many authors, (Gilbert Strang MIT 2015), and shows the resulting response of CO_2 level (sometime called the "adjustment time") comprises the sum of two time exponentials as

$$\text{C} = (e^{st} - e^{-at})/(s+a) + C_0 \quad (14)$$

The solution can be verified by substitution of (14) into (13), a step which shows that $aC_0 = F_i$ where C_0 represents the equilibrium value. The instantaneous airborne fraction, being defined as the ratio of the rate of rise of atmospheric CO_2 to the assumed exponentially increasing CO_2 input rate, can be found by dividing by e^{st} , which, after some manipulation, gives

$$\text{AF} = \frac{\frac{d\text{C}}{dt}}{e^{st}} = \frac{s}{s+a} + a e^{-(s+a)t}/(s+a) + (F_i(t) - aC_0) e^{-st} \quad (15)$$

The first term represents the value to which the airborne fraction tends; the second represents a transient with negative exponent which rapidly diminishes, and the third represents the out of equilibrium contribution. If F_i is constant it balances the outgoing exchange flux, aC_0 setting the third term to zero and the first term of (15) is the value to which the AF settles. Putting $t_a = 1/a$ and $t_s = 1/s$ we can then write

$$AF \Rightarrow s/(s+a), \quad a \Rightarrow s(1/AF - 1), \quad t_a \Rightarrow t_s/(1/AF - 1) \quad (16)$$

The formulae in (16) apply if F_i is constant and if the transient has settled; it also applies to cumulative values. Typical values are $AF=0.45$, $t_s=53$ years, (Cawley 2011) giving t_a a value 43 years. The difference in value between this and the bomb pulse decay time (by at least a factor of 3), has been quoted as supporting evidence that the "Revelle Bypass" applies (Harvey 2000). However if F_i is not constant, as is the case in practice, the calculation of AF must use equation (15) rather than the simplified form equation (16), returning a different value. IPCC 2021, in AR6 figure 5.12, shows values of F_i from 1750 rising for land by 25.6 GtC yr^{-1} and for ocean 23.0 GtC yr^{-1} . Similar rising values of F_i are also shown by the model presented in this study, which reports its rise from 41.5 GtC yr^{-1} to 56.7 GtC yr^{-1} , see Figure 8. Applying this rising sequence of F_i to equation (13) with a time constant of 14.2 years yields the resulting time-series shown graphically Figure 10 below. Thus the time constant differs significantly from the value of 43 years which was obtained when F_i was incorrectly assumed to be constant. The longer time constant therefore does not indicate that the "Revelle Bypass" applies, but rather indicates it is an incorrect result caused by ignoring the rising value of F_i . The equation was also solved using an exponential fit to the fossil fuel emissions rather than the emissions themselves giving the similar plot shown in green. The similarity of the plots shows there is no need to invoke the "Revelle Bypass" in order to account for differing time constants.

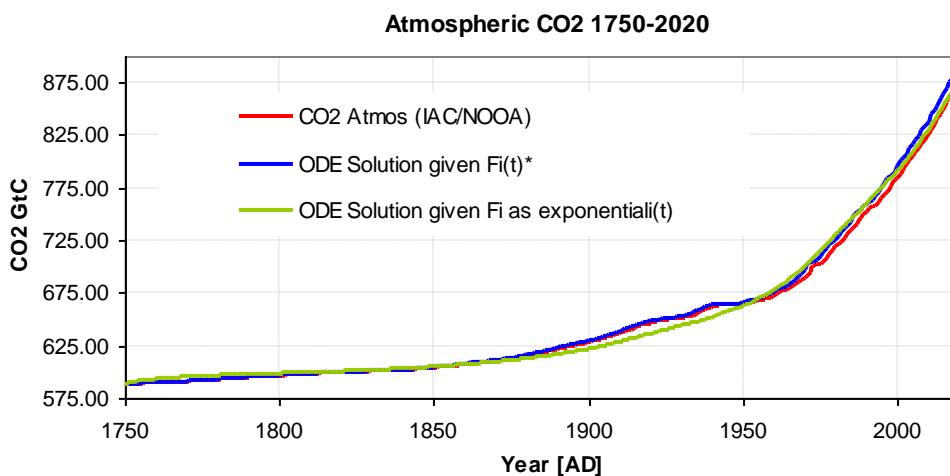


Figure 10. Atmospheric CO₂ as calculated using equation (13) with F_i as derived by model (blue) and as derived from exponential fit (green), both using a time constant of 14.2 years. The rising value of F_i results in an excellent fit using a time constant of just 14.2 years (as indeed should be the case because this is the inverse problem of the model itself). The atmospheric CO₂ has been shifted by 2 years for clarity, otherwise it is barely visible.

In addition to resolving any inconsistency in the value of the time constant, the rising nature of F_i also resolves the confusion regarding a single CO₂ lifetime. For example, IPCC tabulates CO₂ lifetime as "5 to 200 yr" with note "c" stating "No single lifetime can be defined for CO₂ because of the different rates of uptake by different removal processes" (IPCC Ed., 2001, Table 1 p38). Thus, even though there may be different removal processes, it is entirely possible to determine the system behaviour using a single effective residence time for CO₂.

6. Conclusion

This study describes a new approach to modelling the carbon cycle, which is implemented in a two-box CO₂ absolute flow model.

- The model accurately describes the values of $\delta^{14}\text{C}$ and $\delta^{13}\text{C}$ over 200 years, yet without assuming that bomb radiocarbon and anthropogenic carbon behave differently.
- The study shows that the claimed "Revelle Bypass" does not occur in practice because bulk exchange flows dominate isotopic ratio flows. In practice, there is therefore no significant difference in the behaviour of different isotopes, apart from fractionation.
- The study applies the airborne fraction to the calculation of residence time, with modifications to consider and include the presence of a temporal variation in bulk CO₂ exchange inflow. This modification yields the same residence time for bulk CO₂ exchange inflow as for the ¹⁴C bomb pulse, showing the "Revelle Bypass" has been the cause of this confusion.
- Furthermore, the study shows that it is possible for there to be a rising influx of CO₂ from the ocean/land reservoir, and at the same time, a net sink of atmospheric CO₂.

The study overturns a key assumption of the carbon cycle, namely that "bomb radiocarbon and anthropogenic CO₂ do not behave identically." (Joos 1994). This has profound implications for our understanding of the exchange flux and climate change.

The model is available on <https://geomatix.net/downloads/iso-uptake.htm>

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Symbol Table and Acronyms

t_a	Residence time (years)
RCO2	Relative Reservoir Size
β	Rel. proportion of CO ₂ ff not mixing in atmosphere
Y_b	Bomb Yield in megatons
$\Delta^{14}\text{C}_{\text{init}}$	Initial value of $\Delta^{14}\text{C}$ at start of iteration
$\delta^{13}\text{C}_{\text{init}}, \delta^{13}\text{C}_{\text{ff}}$	The value of $\delta^{13}\text{C}$: initial isotopic ratio, fossil fuels
a	$1/t_a \text{ yr}^{-1}$
A, A_{abs}, A_s	Relative, absolute standard, and specific ¹⁴ C activity
$A_{14}[], R_{14}[]$	¹⁴ C/C ratio: atmosphere, reservoir
AF	Airborne Fraction
$A_{\text{FF}}[],$	Atmospheric fossil fuel content
$A_{\text{FL}}[], A_{\text{NL}}[]$	Atmospheric fossil level, Natural (non-fossil) level (0-1)
B	Listed annual bomb yield (Mega Tonnes)
C	Atmospheric carbon mass, GtC
C_0	Equilibrium value of C, GtC
CO ₂ ff	Anthropogenic fossil fuel CO ₂ emissions, GtC
$F^{14}\text{C}$	¹⁴ C Carbon Flux, GtC yr ⁻¹
F_a, F_e, F_i	Atmospheric CO ₂ flux: Anthropogenic, exiting, going in GtC yr ⁻¹
GCM	General Circulation Models

GtC	Gigatonnes Carbon: equals 10^9 tonnes of carbon
M_T, M_i	Mass of mixture, Mass of portion i
R, R^{14}	Isotopic ratio, $^{14}\text{C}/\text{C}$ ratio
R_T, R_i	Ratio of a specific molecule in mixture, T and portion i
s	The exponential rate of rise of fossil fuel emissions
t_a	Time constant associated with atmospheric absorption, y_r
t_s	Time constant associated with CO ₂ ff rise, y_r
$\Delta^{14}\text{C}$	An offset age & fractionation corrected ratio of $^{14}\text{C}/\text{C}$ †
ΔC	Change in C at each iteration, GtC
ΔT	Change in time at each iteration, y_r
δ	Isotopic ratio relative to a standard
$\delta^{13}\text{C}$	An offset measure of $^{13}\text{C}/\text{C}$ ratio relative to a standard
$\delta^{13}\text{C}_F, \delta^{13}\text{C}_N$	$\delta^{13}\text{C}$: For Fossil CO ₂ , Natural (non-fossil CO ₂)
$\delta^{13}\text{C}_M, \delta^{13}\text{C}_W$	$\delta^{13}\text{C}$ for a Measurement, for Wood
$\delta^{14}\text{C}$	An offset measure of $^{14}\text{C}/\text{C}$ ratio relative to a standard
$\sigma_{\text{Total}}, \sigma_1, \sigma_2$	Standard deviation of fit of time series: Total, 1, 2
(t)	Denotes a function value at time, t
[i]	Denotes the value at each iteration, i

Bold-shaded indicates the seven internal optimized parameters.

† $^{14}\text{C}/^{12}\text{C}$ ratio rel. to hypoth. value of atmosphere ^{14}C in 1950 (Stuiver & Polach, 1977).

References

Arnold, J.R., 1957. The Distribution of Carbon-14 in Nature. *Tellus* 9, 28–32. <https://doi.org/10.1111/j.2153-3490.1957.tb01850.x>

Bischof, W., 1960. Periodical Variations of the Atmospheric CO₂ -content in Scandinavia. *Tellus* 12, 216–226. <https://doi.org/10.1111/j.2153-3490.1960.tb01303.x>

Broecker, W.S., 1975. Climatic Change: Are We on the Brink of a Pronounced Global Warming? *Science* 189, 460–463. <https://doi.org/10.1126/science.189.4201.460>

Cawley, G.C., 2011. On the Atmospheric Residence Time of Anthropogenically Sourced Carbon Dioxide. *Energy Fuels* 25, 5503–5513. <https://doi.org/10.1021/ef200914u>

Dai, M., Lu, Z., Zhai, W., Chen, B., Cao, Z., Zhou, K., Cai, W.-J., Chenc, C.-T.A., 2009. Diurnal variations of surface seawater pCO₂ in contrasting coastal environments. *Limnol. Oceanogr.* 54, 735–745. <https://doi.org/10.4319/lo.2009.54.3.0735>

Faassen, K.A.P., Nguyen, L.N.T., Broekema, E.R., Kers, B.A.M., Mammarella, I., Vesala, T., Pickers, P.A., Manning, A.C., Vilà-Guerau De Arellano, J., Meijer, H.A.J., Peters, W., Luijkx, I.T., 2022. Diurnal variability of atmospheric O₂, CO₂ and their exchange ratio above a boreal forest in southern Finland (preprint). *Gases/Field Measurements/Troposphere/Physics (physical properties and processes)*. <https://doi.org/10.5194/acp-2022-504>

Friedlingstein, P. et al., 2022: Global Carbon Budget 2022, *Earth Syst. Sci. Data*, 14, 4811–4900, <https://doi.org/10.5194/essd-14-4811-2022>

Gilbert Strang, Massachusetts Institute of Technology, MIT Open Courseware, Differential Equations, Response to Exponential Input, <http://ocw.mit.edu/RES-18-009F15>

Graven, H., Keeling, R.F., Rogelj, J., 2020. Changes to Carbon Isotopes in Atmospheric CO₂ Over the Industrial Era and Into the Future. *Global Biogeochemical Cycles* 34, e2019GB006170. <https://doi.org/10.1029/2019GB006170>

Harvey, L.D.D., 2000. Global warming: the hard science, 1. publ. ed, Pearson education. Prentice Hall, Harlow Munich. Routledge, (2018) ISBN 0582-38167-3

Heimann Martin, 1993. The Global Carbon Cycle In The Climate System. in *Modelling Oceanic Climate Interactions*, NATO ASI Series VolIII, Springer-Verlag, Heidelberg, 1993

Hesshaimer, V., Heimann, M., Levin, I., 1994. Radiocarbon evidence for a smaller oceanic carbon dioxide sink than previously believed. *Nature* 370, 201–203. <https://doi.org/10.1038/370201a0>

Houghton, J.T., Intergovernmental Panel on Climate Change (Eds.), 1996. Climate change 1995: the science of climate change. Cambridge University Press, Cambridge ; New York.

Hua, Q., Turnbull, J.C., Santos, G.M., Rakowski, A.Z., Ancapichún, S., De Pol-Holz, R., Hammer, S., Lehman, S.J., Levin, I., Miller, J.B., Palmer, J.G., Turney, C.S.M., 2022. ATMOSPHERIC RADIOCARBON FOR THE PERIOD 1950–2019. *Radiocarbon* 64, 723–745. <https://doi.org/10.1017/RDC.2021.95>

Intergovernmental Panel On Climate Change (Ed.) Climate Change 2001: "The Scientific Basis". Third Assesment Report, IPCC, Cambridge University Press.

Intergovernmental Panel On Climate Change (Ed.), 2013. Carbon and Other Biogeochemical Cycles, in: Climate Change 2013 – The Physical Science Basis. Cambridge University Press, pp. 465–570. <https://doi.org/10.1017/CBO9781107415324.015>

Intergovernmental Panel On Climate Change, 2021. Global Carbon and other biogeochemical Cycles and Feedbacks in Climate Change 2021 – The Physical Science Basis: Working Group I Contribution to the Sixth Assessment Report of the Intergovernmental Panel on Climate Change, 1st ed. Cambridge University Press. <https://doi.org/10.1017/9781009157896>

Joos, F., 1994. Imbalance in the budget. *Nature* 370, 181–182. <https://doi.org/10.1038/370181a0>

Kuderer, Matthias, Samuel Hammer, and Ingeborg Levin, 2018 "The Influence of 14CO₂ Releases from Regional Nuclear Facilities at the Heidelberg 14CO₂ Sampling Site (1986–2014)." *Atmospheric Chemistry and Physics* 18, no. 11 (June 6, 2018): 7951–59. <https://doi.org/10.5194/acp-18-7951-2018>.

Kutschera, W., 2013. Applications of accelerator mass spectrometry. *International Journal of Mass Spectrometry* 349–350, 203–218. <https://doi.org/10.1016/j.ijms.2013.05.023>

Leinweber, A., Gruber, N., Frenzel, H., Friederich, G. E., and Chavez, F. P., 2009, Diurnal carbon cycling in the surface ocean and lower atmosphere of Santa Monica Bay, California, *Geophys. Res. Lett.*, 36, L08601, doi:10.1029/2008GL037018.

Levin, I., Naegler, T., Kromer, B., Diehl, M., Francy, R.J., Gomez-Pelaez, A.J., Steele, L.P., Wagenbach, D., Weller, R., Worthy, D.E., 2010. Observations and modelling of the global distribution and long-term trend of atmospheric 14CO₂. *Tellus B: Chemical and Physical Meteorology* 62, 26. <https://doi.org/10.1111/j.1600-0889.2009.00446.x>

Lichtfouse, E., Lichtfouse, M., Kashgarian, M., Bol, R., 2005. 14C of grasses as an indicator of fossil fuel CO₂ pollution. *Environ Chem Lett* 3, 78–81. <https://doi.org/10.1007/s10311-005-0100-4>

Melillo, J.M., Butler, S., Johnson, J., Mohan, J., Steudler, P., Lux, H., Burrows, E., Bowles, F., Smith, R., Scott, L., Vario, C., Hill, T., Burton, A., Zhou, Y.-M., Tang, J., 2011. Soil warming, carbon–nitrogen interactions, and forest carbon budgets. *Proc. Natl. Acad. Sci. U.S.A.* 108, 9508–9512. <https://doi.org/10.1073/pnas.1018189108>

Naegler, T., Levin, I., 2006. Closing the global radiocarbon budget 1945–2005. *J. Geophys. Res.* 111, D12311. <https://doi.org/10.1029/2005JD006758>

Ota, M., Katata, G., Nagai, H., Terada, H., 2016. Impacts of C-uptake by plants on the spatial distribution of 14 C accumulated in vegetation around a nuclear facility—Application of a sophisticated land surface 14 C model to the Rokkasho reprocessing plant, Japan. *Journal of Environmental Radioactivity* 162–163, 189–204. <https://doi.org/10.1016/j.jenvrad.2016.05.032>

Oeschger, H., Siegenthaler, U., Schotterer, U., Gugelmann, A., 1975. A box diffusion model to study the carbon dioxide exchange in nature. *Tellus A: Dynamic Meteorology and Oceanography* 27, 168. <https://doi.org/10.3402/tellusa.v27i2.9900>

Olsen, A., Omar, A. M., Stuart-Menteth, A. C., and Triñanes, J. A., 2004, Diurnal variations of surface ocean *pCO₂* and sea-air CO₂ flux evaluated using remotely sensed data, *Geophys. Res. Lett.*, 31, L20304, doi:10.1029/2004GL020583.

Palonen, V., Pumpanen, J., Kulmala, L., Levin, I., Heinonsalo, J., Vesala, T., 2018. Seasonal and Diurnal Variations in Atmospheric and Soil Air 14 CO₂ in a Boreal Scots Pine Forest. *Radiocarbon* 60, 283–297. <https://doi.org/10.1017/RDC.2017.95>

Popper, Karl. 1934. Logik der Forschung [The Logic of Scientific Discovery] (2nd ed.). Reprint 1962. La lógica de la investigación científica. Tecnos, Madrid. Reprint 1992. London: Routledge. pp. 121–132. (1992) ISBN 978-84-309-0711-3.

Revelle, R., Suess, H.E., 1957. Carbon Dioxide Exchange Between Atmosphere and Ocean and the Question of an Increase of Atmospheric CO₂ during the Past Decades. *Tellus* 9, 18–27. <https://doi.org/10.3402/tellusa.v9i1.9075>

Rice, Ken; Schade, Gunnar W.; Maslin, Mark A. Comment on "World Atmospheric CO₂, Its 14C Specific Activity, Non-fossil Component, Anthropogenic Fossil Component, and Emissions (1750–2018)" (Skrable et al. 2022) *Health Physics* 123(1):p 28-30, July 2022. | DOI: 10.1097/HP.0000000000001582

Rubino, M., Etheridge, D.M., Trudinger, C.M., Allison, C.E., Battle, M.O., Langenfelds, R.L., Steele, L.P., Curran, M., Bender, M., White, J.W.C., Jenk, T.M., Blunier, T., Francey, R.J., 2013. A revised 1000 year atmospheric δ 13 C-CO₂ record from Law Dome and South Pole, Antarctica: 1000 YEARS OF ATMOSPHERIC δ 13 C-CO₂. *J. Geophys. Res. Atmos.* 118, 8482–8499. <https://doi.org/10.1002/jgrd.50668>

Sarmiento, Jorge L., Gruber Nicolas., 2002. Sinks for Anthropogenic Carbon. *Physics Today* August 2002.

Siegenthaler, U., Oeschger, H., 1987. Biospheric CO₂ emissions during the past 200 years reconstructed by deconvolution of ice core data. *Tellus B: Chemical and Physical Meteorology* 39, 140. <https://doi.org/10.3402/tellusb.v39i1-2.15331>

Stenström, K., Skog, G., Georgiadou, E., Genberg, J. & Mellström, A. 2011. A guide to radiocarbon units and calculations. LUNFD6(NFFR-3111)/1-17/(2011), Lund University,

Stuiver, M., Polach, H.A., 1977. Discussion Reporting of ¹⁴C Data. *Radiocarbon* 19, 355–363. <https://doi.org/10.1017/S0033822200003672>

Stuiver, M., Quay, P.D., 1981. Atmospheric ¹⁴C changes resulting from fossil fuel CO₂ release and cosmic ray flux variability. *Earth and Planetary Science Letters* 53, 349–362. [https://doi.org/10.1016/0012-821X\(81\)90040-6](https://doi.org/10.1016/0012-821X(81)90040-6)

Stuiver, M., Reimer, P.J., Bard, E., Beck, J.W., Burr, G.S., Hughen, K.A., Kromer, B., McCormac, G., Van Der Plicht, J., Spurk, M., 1998. INTCAL98 Radiocarbon Age Calibration, 24,000–0 cal BP. *Radiocarbon* 40, 1041–1083. <https://doi.org/10.1017/S0033822200019123>

Suess, H.E., 1955. Radiocarbon Concentration in Modern Wood. *Science* 122, 415–417. <https://doi.org/10.1126/science.122.3166.415.b>

Svetlik, I., Povinec, P.P., Molnár, M., Meinhardt, F., Michálek, V., Simon, J., Svingor, É., 2010. Estimation of Long-Term Trends in the Tropospheric ¹⁴CO₂ Activity Concentration. *Radiocarbon* 52, 815–822. <https://doi.org/10.1017/S0033822200045835>

Takahashi, H.A., Konohira, E., Hiyama, T., Minami, M., Nakamura, T., Yoshida, N., 2002. Diurnal variation of CO₂ concentration, Δ 14C and δ 13C in an urban forest: estimate of the anthropogenic and biogenic CO₂ contributions. *Tellus B: Chemical and Physical Meteorology* 54, 97. <https://doi.org/10.3402/tellusb.v54i2.16651>

Tans, P.P., Berry, J.A., Keeling, R.F., 1993. Oceanic ¹³C/ ¹²C observations: A new window on ocean CO₂ uptake. *Global Biogeochemical Cycles* 7, 353–368. <https://doi.org/10.1029/93GB00053>

Tans, P., 2022. Reminiscing On The Use And Abuse Of ¹⁴C And ¹³C In Atmospheric CO₂. *Radiocarbon* 64, 747–760. <https://doi.org/10.1017/RDC.2022.7>

Data References

D1. Institute for Atmospheric and Climate Science (IAC), CO₂ Mean Global AD0 to AD2014 ftp://data.iac.ethz.ch/CMIP6/input4MIPs/UoM/GHGConc/CMIP/yr/atmos/UoM-CMIP-1-0/GHGConc/gr3-GMNHSH/v20160701/mole_fraction_of_carbon_dioxide_in_air_input4MIPs_GHGConcentrations_CMIP_UoM-CMIP-1-1-0_gr3-GMNHSH_0000-2014.csv

D2. NOAA GML. Accessed 04-March-2022. https://gml.noaa.gov/ccgg/trends/gl_data.html File: https://gml.noaa.gov/webdata/ccgg/trends/co2/co2_annmean_gl.txt

D3. Global Carbon Budget: [National_Carbon_Emissions_2021v0.4.xlsx](https://www.globalcarbonbudget.org/) Historical Budget, Global Fossil Emissions Visited 04 March 2022. Friedlingstein et al (2021),

D4. World Data Service for Paleoclimatology, Boulder and NOAA Paleoclimatology Program, National Centers for Environmental Information (NCEI) <https://www1.ncdc.noaa.gov/pub/data/paleo/icecore/antarctica/law/law2018d13c-co2.txt>, <https://doi.org/10.25919/5bfc29ff807fb>

D5. UNSCEAR: United Nations Scientific Committee on the Effects of Atomic Radiation 2000 Report To The General Assembly. Volume I: Sources. Annex C: Exposures To The Public From Man-Made Sources Of Radiation 207 Sources And Effects Of Ionizing Radiation. Table 4. Annual Fission And Fusion Yields.

D6. Calib: INTCAL20/SGCAL20. Stuiver, M. et al, 2022 CALIB 8.2 [WWW program] at <http://calib.org>, accessed 2022-3-4 Rev 8.1.0 intcal20.14c, shcal20.14c

D7. HADCRUT4, <https://www.metoffice.gov.uk/hadobs/>

D8. hadcrut4 /data/current/time_series/ HadCRUT.4.6.0.0.monthly_ns_avg.txt, hadsst4/data/csv/ HadSST.4.0.0.0_monthly_GLOBE.csv

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