

## Article

# Improved Multifactor Portfolio Optimization Method through Empirical Research in South Korea

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**Abstract:** Since financial institutions faced fatal scenario such as subprime mortgage crisis and COVID-19, the factor-based asset allocation methodology is noticed. Asset-only approach cannot manage the macro-factor risk. For instance, an institution which allocated assets by asset-only approach cannot deal with the inflation crisis. I review the problem of the traditional modern portfolio approach that is used by Korean financial institutions. For reasonable investment of institution, I suggest improved factor-based allocation approach. The first result of this paper is that the asset-only-based portfolio recorded lower performance than multi factor-based portfolio in macro factor crisis. Second, I discovered allocation model which can minimize correlation between liability and investment risk factors.

There are three steps in multi-macro factor-based asset allocation approach: First, discover macro factors and map asset classes to individual macro factor. Second, define liability account and mapping with considering income and pay out of institution. Third, minimize correlation of factor-based asset risk with liability volatility. Furthermore, using factor-based covariance made Pareto improvement and solve Home-bias problems.

**Keywords:** asset allocation; risk factor; risk exposure; macro-factor

## 1. Introduction

The important core of Strategy Asset Allocation(SAA) is improved Pareto efficiency. The purpose of allocation approach support to gain more return in same risk level or taking lower risk with same return level. After published work of Markowitz[1952] of modern portfolio theory, institutional asset allocation have developed from a risk-return profiling. In this two-dimensional space, institutions have made effort to search minimum variance with high expected return in the line called efficient frontier. The basic concept of modern portfolio theory is that each asset class has individual purpose and each other correlation. Under the concept, modern portfolio theory remains the most approach models for common mean-variance optimization related variants. Especially, for improving practical part of mean-variance approach, Jorion [1986] discussed about random matrix theory, Ben-Tal and Nemirovski[1998], Quaranta and Zaffaroni [2008] argued that robust optimization approach made better scenarios can be selected by investors. Furthermore, Black, Fischer[1972. 1992] suggested market-based solution approach that expected returns are derived by backing out market-implied returns using an estimate of the market size and risk under the assumption that all investors are reasonable. The point of Black-Litterman model is to compose prior distribution and likelihood distribution using Bayesian framework. In ante-post distribution, it supposes that total stock value weight of specific assets in America is GMVP described in MVO model. And it solves implied returns using a reserve optimization process. Tau is coefficient about confidence of implied return. There are many studies about lambda, but it is calculated by Sharpe ratio in almost cases. Implied return has meaning of prior distribution. Likelihood distribution is based on investor view. Investor (or analyst) have confidence about their view on specific assets as certain probability. Black-Litterman operates in these suppositions. Confidence of investor view and return of view are in likelihood distribution. It combines two

distributions with Bayesian framework. Likelihood distribution is investor view, prior distribution is implied return. Combining two distributions, it would make posterior distribution. Black-Litterman contributed reasonable estimation of expected return for double distribution than MVO model. And it is easy to make logical adjustments. Also, there is not corner solution in combined distribution. Many researchers studied adapting methodologies of the Black-Litterman model to Strategy Asset Allocation. He, Guangliang, and Robert Litterman[2002] studied new expected return and covariance which can be used in MVO models. Idzorek, Thomas[2005] summarized methodology of B-L in their article.

However, the asset-only approach makes investors to be unable to consider their business characteristic. For instance, pension fund must expense pension to qualified recipient. In the inflation, the income replacement rate of qualified recipient would be increased as much as same rate of inflation. Therefore, the pension fund should set their hurdle rate of managing fund for satisfying the income replacement rate or securing sound finance. On the other hand, some of fund called non-bank financial institutions have to consider business crisis such as guarantee of bankruptcy and default. These non-bank financial institutions should set investment strategies for tail-risk scenario such as COVID-19. That is the reason that institutional investors have to manage risk factors which affect spending and income. So, I reviewed in point of practice. Combined a macro-factor with Business-Index could increase return on investment when the spending was increased or income was decreased. Especially, Carsten Stendevad who worked in ATP said, "In the big picture, this gives us not only a better understanding of risk but also more investment flexibility".

Factor perspective is not a new approach. Sharpe [1964], Ross [1976], Rosenberg and Marathe [1976], Fama and French [1992, 1993] and Carhart [1997] already argued factor analysis in asset pricing and allocation. That is, factor-based portfolio can help not only choosing choice decision on the precise elements of market risk but also considering Business-Index such as income and spending. Stephen blyth [2016] introduced guidelines of factor allocation approach. And Goldman Sachs' Asl and Etula [2012] applicated first of a multi-factor model (robust portfolio optimization) on Strategic Asset Allocation Optimization of a new robust portfolio. Asl and Etula [2012] proposed the method has better risk-return characteristics for constructing a well-diversified portfolio. While former studies applied the multi-factor model within only asset class, Asl and Etula (2012) tried applying a multi-factor model to Strategic Asset Allocation, the estimated returns and risk across asset classes. Greenberg et al. [2016] presented one of the reasons why the portfolio allocation process was interrupted in a paradigm of modern asset pricing as "The standard mapping of risk factors to asset classes lack of procedure." and suggested a methodology for mapping given factors to asset types. Blyth et al. [2016] suggested FIFAA (flexible indeterminate factor-based asset allocation) and flexibly integrated into the Strategic Asset Allocation process with the reasonable view of Investment committee and board of directors with bottom-up analysis across the market. Bass et al. [2017] applied the factor-based SAA framework to the portfolios of representative institutional investors.

## 2. Factor-based approach into reference portfolio

As previously stated, I purposed model which combined factor-based approach and account indexes. The approach comprises the steps like Stephen blyth[2016]. First, select business ideas such as income, spending, and liability. Second, classify asset classes by investment committee. Third, measure macro-factor exposures on the investment universe and account index. Forth, optimize correlation exposure level of macro factors in asset-only and Business-Indexes.

For comparison multi-factor model with asset only optimization approach, I make imaginary portfolio and account of non-bank financial institution. The statement of profit and loss shown in Table 1.

**Table 1.** The imaginary statement of profit and loss.

		2016	2017	2018	2019	2020
Income	Issue of bonds	159,307	143,421	151,162	154,180	187,046
	Subscription deposit	190,685	183,270	170,708	162,074	211,701
	Collection bond	133,914	102,415	94,818	112,557	148,798
	Transferred	5,672	5,504	5,504	5,927	5,504
	Interest income	54,020	97,738	151,701	152,042	145,005
	Balance carried forward	130,791	132,947	119,321	159,154	304,978
	Total	674,390	665,294	693,213	745,935	1,003,031
Spending	Loan to PF	210,626	235,772	256,699	287,372	288,289
	Building residence	64,285	76,705	130,902	156,494	150,470
	Building Apt.	5,211	4,866	2,973	2,194	1,449
	Loan to individuals	113,216	127,056	88,095	94,124	99,122
	Improving residence	27,840	26,656	31,364	29,016	28,421
	Recovery business	74	489	3,365	5,544	8,827
	Redeeming debts	297,404	300,787	347,943	335,475	382,838
	Payment reserves	166,360	128,734	88,571	123,088	331,904
	Total	674,390	665,294	693,213	745,935	1,003,031

\* The data period is 2001 to 2020, and rate of change data is 2002 to 2020.

The imaginary institution is executing loan to project financing and building residence, apartment for people. All of buyer of real estate must buy bond which issued by imaginary institution as much as partially amount of transaction price. And people can buy house through a housing subscription account. The two subjects are income index of this institution. Since the income account and spending account are orthogonal, the financial projection is useless for discovering Business-Indexes. So, I consider two income factors as like national housing bond and collection bond (institution grant loan for building house and project financing). The Business-Indexes are analysed by 13 policy and economic variables shown in Table 2 for Principal Component Analysis.

**Table 2.** Statics of Business-Index and factors.

	Mean	Standard deviation	Observation
Default	-10.91%	27.66%	19
Unsold home	-2.66%	37.66%	19
Unsold home after building	1.61%	36.91%	19
Apartment price index	5.77%	8.09%	19
Apartment price index (Seoul)	4.26%	4.58%	19
Real estate trading volume	2.66%	14.67%	14
Real estate trading volume (Seoul)	-2.88%	3.45%	14
Mortgage rate	-4.87%	10.64%	19
Interest	-5.32%	9.99%	19
Total value of house	8.53%	4.04%	19
Construction house	-0.77%	20.66%	19
Issue MBS	-3.39%	33.38%	19
Supplying house rate	0.46%	0.41%	15
Collection bond	-1.18%	21.80%	19
Total income	10.20%	17.08%	19

<sup>1</sup> All of factors are computed by rate of change.

The result of PCA analysis proposed that it is affected as forth principal component. It is shown in Table 4. Default, unsold home and unsold home after building are negative with collection bond factor in first PC(Principal component). In the Second PC, Apartment price index, Real estate trading volume, Real estate trading volume proposed positive relative with collection bond. Mortgage rate and interest are also positive with collection bond. The PCA of total income index is similar with collection bond index, however the amount of issue MBS and construction house are positive relative with total income. Interest and mortgage rate proposed negative relative with index in third principal component. The result of analysis is shown in Table 3.

**Table 3.** Cross section of component coefficient.

		PRIN1	PRIN2	PRIN3	PRIN4
Default	Collection bond	-0.37748	0.072014	0.070139	-0.20942
	Total income	-0.29174	0.250459	-0.44256	-0.08862
Unsold home	Collection bond	-0.31274	-0.06965	-0.05255	-0.42014
	Total income	-0.3116	0.228226	-0.00852	0.416686
Unsold home after building	Collection bond	-0.35132	0.286805	-0.20502	0.174944
	Total income	-0.23254	0.422682	0.117238	-0.30219
Apartment price index	Collection bond	0.279398	0.364374	-0.02626	-0.09963
	Total income	0.386719	0.185964	0.109963	-0.1531
Apartment price index (Seoul)	Collection bond	0.327886	0.129019	0.273019	-0.37488
	Total income	0.414796	-0.05481	-0.24102	0.183889
Real estate trading volume	Collection bond	0.217801	0.365478	0.135423	-0.24102
	Total income	0.348211	0.225557	-0.29046	-0.06098
Real estate trading volume (Seoul)	Collection bond	0.057997	-0.30072	0.284126	0.60299
	Total income	0.072353	-0.36594	-0.1523	-0.20395
Mortgage rate	Collection bond	-0.16937	0.370992	0.412327	0.130978
	Total income	0.269494	0.357551	-0.02003	0.115649
Interest	Collection bond	-0.31242	0.196292	0.38183	0.100802
	Total income	0.105617	0.438884	-0.13463	0.298956
Total value of house	Collection bond	0.315224	0.23767	0.190931	0.19991
	Total income	0.443106	-0.06128	0.066037	-0.19827
Construction house	Collection bond	0.275548	-0.29644	0.371921	-0.11113
	Total income	0.138028	-0.15608	0.100773	0.680606
Issue MBS	Collection bond	0.30477	-0.01073	-0.48061	0.059048
	Total income	0.00031	0.009781	0.689675	0.057946
Supplying house rate	Collection bond	0.073751	0.462212	-0.23328	0.306202
	Total income	0.128264	0.368072	0.322309	-0.12645

Table 4. Scree table of PCA.

Collection bond					Total income			
	Eigenvalue	Difference	Proportion	Cumulative	Eigenvalue	Difference	Proportion	Cumulative
1	5.19032	2.47793	0.3993	0.3993	4.987878	2.031609	0.3837	0.3837
2	2.71239	0.849797	0.2086	0.6079	2.956269	1.473892	0.2274	0.6111
3	1.862593	0.850745	0.1433	0.7512	1.482377	0.170409	0.114	0.7251
4	1.011847	0.161016	0.0778	0.829	1.311968	0.492325	0.1009	0.826
5	0.850831	0.301954	0.0654	0.8945	0.819643	0.244145	0.063	0.8891
6	0.548877	0.148922	0.0422	0.9367	0.575498	0.245256	0.0443	0.9334
7	0.399956	0.166213	0.0308	0.9674	0.330243	0.017743	0.0254	0.9588
8	0.233743	0.138912	0.018	0.9854	0.3125	0.192163	0.024	0.9828
9	0.094831	0.017085	0.0073	0.9927	0.120337	0.040681	0.0093	0.9921
10	0.077746	0.068527	0.006	0.9987	0.079655	0.063654	0.0061	0.9982
11	0.009219	0.001571	0.0007	0.9994	0.016001	0.008369	0.0012	0.9994
12	0.007648	0.007648	0.0006	1	0.007631	0.007631	0.0006	1
13	0	0	0	1	0	0	0	1

<sup>1</sup> The end point is over 80% cumulative explanation ability.

Following previous procedure, Business-Indexes are affected as shown in Table 5. National housing bond affected positively by construction house, decreasing interest, increasing MBS issues, price, and volume.

Table 5. Effective table of Business-Index.

		Positive	Negative
Income	National housing bond	Construction house Issue MBS Price and volume	Unsold home Interest
	Housing subscription account		Construction house Supplying house Interest
	Collection bond	Price and volume Supplying house Interest	Default Unsold home

To analyze precise causality relationship of Business-Index and factors, I conduct Granger causality test. Trading volume and supplying house factors are insufficient number of data, in which I deleted these factors of causality test. And all time-series data of annual return are executed first differencing to secure stationarity. The test intimates that unsold home, apartment price and construction house impact to collect bond, similarly default and unsold home after building impact to total income of institution under the 95 percent confidence level. The result table is shown in Table 6. Therefore, I consider some factors as like default, unsold home, apartment price and construction house.

**Table 6.** Granger causality test of Business-Index.

Factors	Collection bond			Total income		
	DF	Chi-Square	Pr > ChiSq	DF	Chi-Square	Pr > ChiSq
Default	1	0.12	0.7339	1	7.48	0.0062
Unsold home	1	4.54	0.0331	1	0.43	0.51
Unsold home after building	1	3.66	0.0457	1	4.42	0.0356
Apartment price index	1	0.23	0.6342	1	2.55	0.1106
Apartment price index (Seoul)	1	2.72	0.009	1	0.6	0.4398
Mortgage rate	1	0.16	0.6877	1	0.01	0.9067
Interest	1	0	0.9737	1	0.01	0.9244
Total value of house	1	0.05	0.8229	1	0.02	0.8771
Construction house	1	3.02	0.082	1	1.3	0.2534
Issue MBS	1	0.02	0.9011	1	0.23	0.6325

Through result I gained by granger causality test and principal component analysis, economic factors which impact to income of institution are extracted. To map the macro factors on the Business-Index factors, I conducted multi-variate regression analysis of individual economic index and macro factors. It is shown in Table 7 and 8.

**Table 7.** Basic statics of macro factors.

Observation		Mean	Standard deviation	Info
KRCPI	19	2.21%	1.16%	Inflation of South Korea
KRGDP	19	3.56%	1.95%	Growth rate of the South Korea
KRRATE	19	0.40%	1.02%	Real interest rate of South Korea
USRATE	19	-0.64%	1.19%	Real interest rate of United States
DOLLAR	19	-1.37%	8.05%	The change rate of Dollar index
WGDP	19	3.44%	2.00%	Growth rate of the world economy
WCPI	19	3.74%	0.69%	Inflation of the world

<sup>1</sup> Macro factor's data are constructed by annual change rate. Inflation of South Korea, Growth rate of the South Korea and Real interest rate of South Korea are referred from Static system of the bank of Korea. The change rate of Dollar index is downloaded by CME. Growth rate of the world economy and Inflation of the world are downloaded by International Monetary Fund. Time series of dataset is from 2002 to 2020.



**Table 8.** Multi variate regression of Business-Index.

	KRCPI	KRGDP	KRRATE	USRATE	DOLLAR	WGDP	WCPI
Default	-0.55312 (-1.32)	0.185747 (0.50)	-0.92285 (-2.42)	0.408995 (1.55)	-0.07907 (-0.33)	0.113595 (0.32)	0.778058 (2.23)
Unsold home	0.23407 (0.6)	-0.49299 (-1.44)	-0.06102 (-0.17)	0.22545 (0.92)	-0.30506 (-1.38)	0.91354 (2.76)	-0.03728 (-0.12)
Apartment price index	0.28203 (0.85)	0.76953 (2.64)	0.15156 (0.5)	0.59112 (2.84)	-0.19141 (-1.02)	-0.88446 (-3.15)	-0.29579 (-1.07)
Construction house	0.31608 (0.63)	0.22079 (0.5)	0.80232 (1.75)	-0.53162 (-1.68)	0.33719 (1.18)	-0.16984 (-0.4)	-0.23379 (-0.56)

<sup>1</sup> The beta of regression is normalized coefficient. Adjusted coefficient of determination is 0.277. (T statistic value)

Multi-variate regression for mapping factors is final procedure to discover exposure level of Business-Index. The default index is influenced by real interest rate of South Korea and Inflation of the world factor as -0.92 and 0.78. Unsold home index is impacted by Growth rate of the world economy as much as 0.92. The Apartment price index is affected by Growth rate of the South Korea, Real interest rate of United States and Growth rate of the world economy as much as 0.76, 0.59 and -0.88. Any factors do not affect to Construction house factor under the 95 percent confidence level. Therefore, I consider default, unsold home and apartment price index to meaningful Business-Index for optimizing portfolio. Default and unsold home cause negative effect for institution's income, on the other hands, apartment price index is positive relation to income. Consequently, I should consider effect vector to income with benchmark of portfolio.

### 3. Constructing portfolio.

For constructing reference portfolio of factors, I separate asset classes as domestic stock, domestic bond, global stock, global bond, domestic real estate and global real estate. The benchmark data is from Bloomberg and KRX(Korea exchange) as 2002 to 2020. The information of data is shown in Table 9.

**Table 9.** Benchmark and asset class.

	Observation	Mean	Standard deviation	Benchmark
Domestic stock	19	7.48%	21.74%	KOSPI
Domestic bond	19	2.99%	3.24%	KRX All bond total index
Global stock	19	5.21%	19.61%	MSCI ACWI
Global bond	19	4.71%	5.04%	Barclays global aggregate bond index
Domestic real estate	19	1.27%	3.66%	Real Residential Property Prices for Republic of Korea
Global real estate	19	3.81%	13.89%	S&P Real Asset Index

I matched individual benchmark to macro factors as mapping procedures. For discovering coefficient of factors in asset, I use PCA reputedly to macro factors in benchmarks. The final coefficient computed by weighted average of explanation ability.

$$\beta_{i,j} = \sum PC_{i,j}^t * EA_{i,j}^t \quad (1)$$



The  $\beta_{i,j}$  means weighted coefficient score of  $j$  factor in  $i$  asset.  $PC_{i,j}^t$  is coefficient value and  $EA_{i,j}^t$  is explanation ability of  $t$  principal component. Coefficient score could be detected by principal components regression.

All of elbow point to principal component of benchmark is third PC. The condition is same with Business-Index analysis as 85% explanation ability. Domestic stock is affected negatively by real interest rate of South Korea and inflation of the world. Domestic bond is affected positively by all factors. Other scores of components can be affirmed as shown in Table 10.

**Table 10.** PCA score of asset classes.

	KRGDP	KRRATE	USRATE	WGDP	WCPI
Domestic stock	0.2002	-0.2855	0.3894	0.2710	-0.0991
Domestic bond	0.2043	0.2261	0.3799	0.2836	0.1681
Global stock			0.3871	0.2771	-0.1039
Global bond			-0.3975	0.2278	0.1531
Domestic real estate	0.1947	0.2664	0.3891	0.2844	0.1277
Global real estate			0.3877	0.2739	0.0895

<sup>1</sup> Since I should consider causality factor to benchmark, I eliminate domestic factor in global assets.

After basic analysis of macro factor and benchmark, I construct two portfolios. First, asset-only allocation approach is optimized as Mean-Variance process (H. Markowitz, 1952). Second, factor-based approach is optimized same procedure as Mean-Variance. That is, difference point of optimization procedure is only asset classes. First portfolio uses six asset classes to allocate portfolio, however second portfolio uses five factors to allocate. Furthermore, I must memorize that portfolio should support Business-Index. If, income decrease by negative economic condition, the return of portfolio would increase by positive correlation with negative macro factors.

$$\max_w [w_j \mu - \frac{1}{2} \left( (w^T \Sigma w)_j^2 + (w^T \Sigma w)_i^2 + \sum_{k=1}^N \left( \frac{1}{\rho_{p,v_k} + 1} \right)^2 \right)] \quad (\rho_{p,v_k} > 0) \quad (2)$$

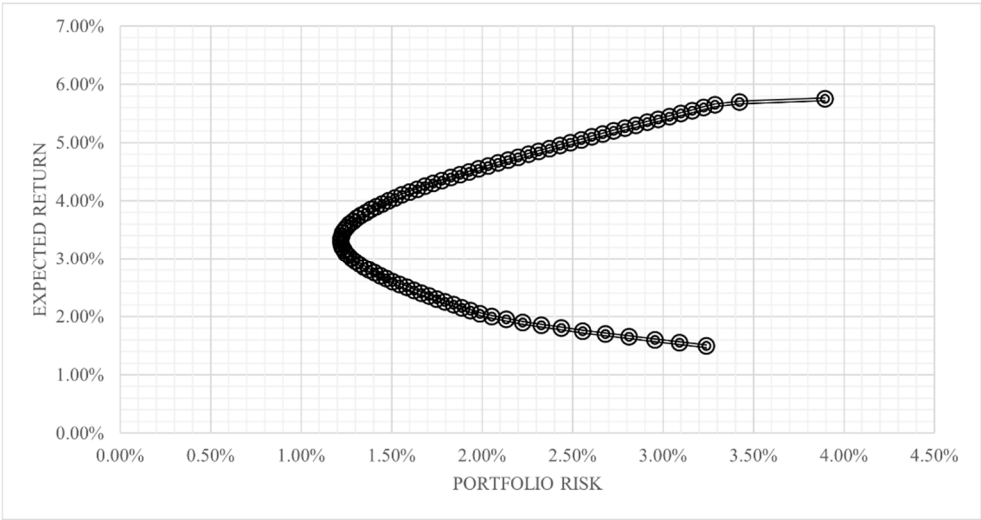
The objective function is similar with mean-variance approach, but I use covariance of factors and correlation of macro factors with Business-Index. Then, for searching 100% risky portfolio which usually called tangent portfolio, I adopted separate theory and risk aversion factor by Tobin and James[1984] and Sharpe[1964] as follows.

$$\max_{\lambda} \lambda(r, \sigma_j, \rho) = \frac{r - r_f}{\sqrt{\sigma_j^2 + \sigma_i^2 + \sum_{k=1}^N \left( \frac{1}{\rho_{p,v_k} + 1} \right)^2}} \quad (\rho_{p,v_k} > 0) \quad (3)$$

$$\mathcal{L}(w_i, w_j, \lambda) = \sum_{i=1}^N w_i - \frac{\sum_{j=1}^N \sum_{i=1}^N w_i w_j \sigma_i \sigma_j}{\sum_{i=1}^N w_i \sigma_i} \quad (4)$$

The  $\sigma_i$  is standard deviation of macro factors,  $w_j$  is weight to asset,  $v$  is rate of change Business-Index.  $p_n$  is factor portfolio  $n$  by historical change rate of macro-factor.  $\rho$  is correlation  $v$  with  $p_n$ . As I describe previous procedure, the final purpose of allocation is to seek weight of individual asset as portfolio. If I should obtain negative correlation Business-Index and macro factor,  $\rho_{p,v_k}$  would be reciprocal. So, to seek weight of asset from factors, I add restriction to optimize Lagrange function as follows. That is, I maximize utility function of change rate of Business-Index and factor-based portfolio with same trend of correlation.

For verifying factor-based approach is better than asset-only method for minimize risk under the stressed scenario, I establish strategy of two portfolio. The expected return is used from capital market assumption by Blackrock and Samsung Investment. The efficient frontier of asset-only approach portfolio is shown in Table 11. The market portfolio of asset-only is constructed by 47.03% as domestic bond, 6.73% as global stock, 4.49% as global bond, 41.57% as domestic real estate. The asset-only approach portfolio expected return is 3.60%, expected risk is 1.27%. However macro-factor risk is higher than factor-based portfolio as 1.87%.



**Figure 1.** Efficient frontier of Asset-only approach.

Since risk-return profile of asset-only portfolio imply optimizing solution as point of asset allocation, the efficient frontier of factor-based approach is not improved solution on the risk-return profile than asset-only portfolio. However, factor-based portfolio cause pareto improvement on the factor risk-return profile. Moreover, the correlation of negative scenario income with portfolio return. That is, when income decrease by negative circumstance, factor-based portfolio would increase. So, I compared three portfolios as asset-only, factor-based and factor-based portfolio with Business-Index’ factor correlation. The detail weight of asset and profile are shown in table 12.

**Table 11.** Cross section of three portfolio approach.

		Expected return	weight
Domestic stock	Asset-Only		0.00%
	Factor-based Correlation	6.29%	1.09%
	Factor-based		0.72%
Domestic bond	Asset-Only		47.03%
	Factor-based Correlation	1.49%	32.12%
	Factor-based		36.12%
Global stock	Asset-Only		6.73%
	Factor-based Correlation	6.3%	0.63%
	Factor-based		0.00%
Global bond	Asset-Only		4.49%
	Factor-based Correlation	3.1%	15.57%
	Factor-based		6.22%
Domestic real estate	Asset-Only		41.75%
	Factor-based Correlation	5.6%	27.63%
	Factor-based		31.05%
Global real estate	Asset-Only		0.00%
	Factor-based Correlation	6.2%	22.96%
	Factor-based		25.89%

	KRGDP	KRRATE	USRATE	WGDP	WCPI	Expected return	Factor risk
Asset-Only	57.69%	18.75%	2.25%	36.00%	0.00%	3.60%	1.82%
Factor-based Correlation	0.00%	0.00%	43.57%	45.88%	10.55%	4.96%	1.50%
Factor-based	0.00%	0.00%	55.12%	0.00%	44.88%	4.12%	0.73%

<sup>1</sup> Factor-based portfolio is computed by re-mapping macro factor process. In this process, I can use expected return of individual asset and factor covariance as portfolio factor risk. Factor-based with business correlation portfolio is computed by expected return of asset, covariance of factors and correlation portfolio and historical return of factors as mapping from Business-Index.

Asset-only approach portfolio is exposed by domestic growth of economic factor as 57.69%, Real interest rate of South Korea as 18.75%. Factor-based approach portfolio is exposed by Real interest rate of USA as 43.57% and world growth of economic and inflation factors. In other words, on the factor exposure risk-return profile, factor-based portfolio improved successfully risk adjustment return.

#### 4. Robustness

To add robustness of factor-based approach portfolio allocation, I should test risk-adjustment return draw down and macro-factor sensitivity through after COVID-19 scenario from 17-Dec-2021 to 17-Dec 2021. There is not rebalancing portfolio, the performance is measured by annual. The items for measuring performance are Sharpe ratio, max draw down, monthly correlation of return with change rate to income. Since, Real Residential Property Prices for Republic of Korea index is computed quarterly, I conduct smoothing of data as daily return.

There is risk free rate as 0.19% by US 3-year bond yield. I don't consider any trade cost for rebalancing is not executed. Although, asset-only approach portfolio had lower expected volatility and higher expected Sharpe ration than factor-based approach portfolio, former portfolio recorded lower Sharpe ratio than latter portfolio. Max draw-down

volatility of former portfolio is lower than latter. Table 14 reports cumulative return from 2020 to 2021 and performance measurement returns.

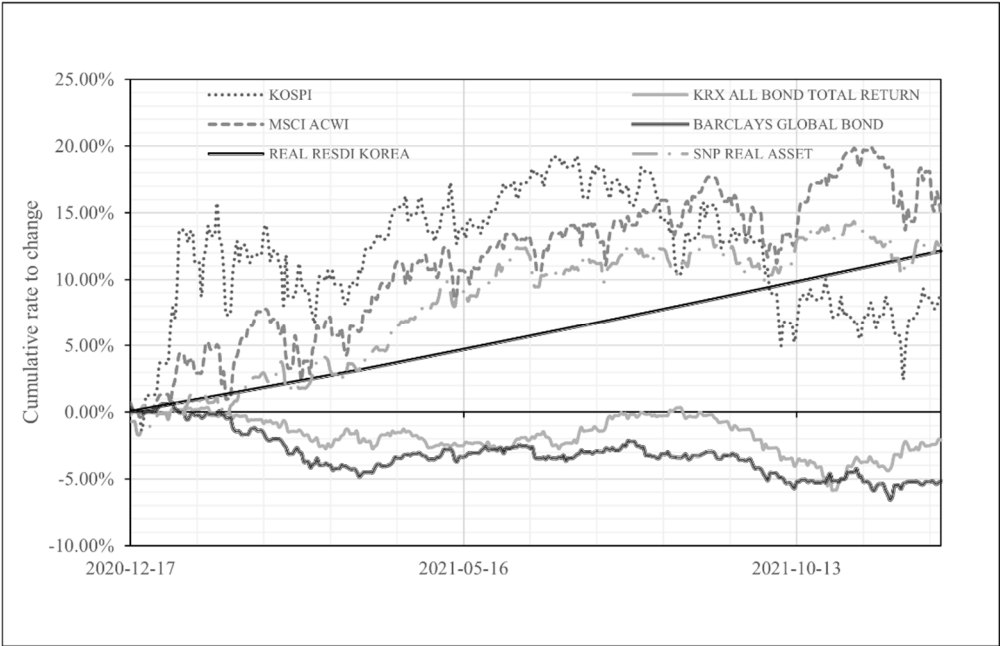


Figure 2. Cumulative rate of change index.

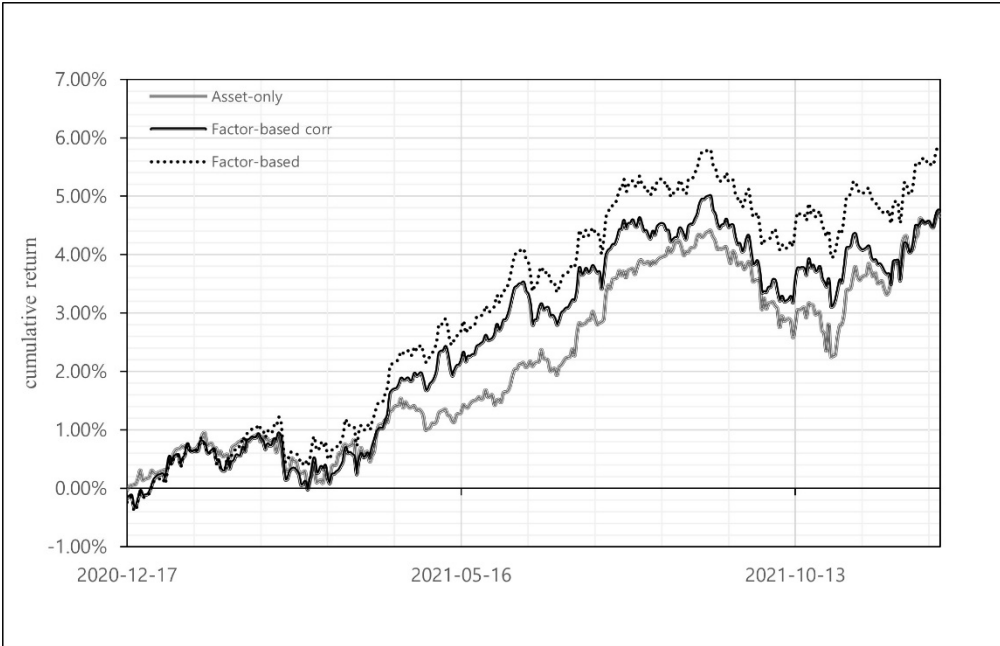
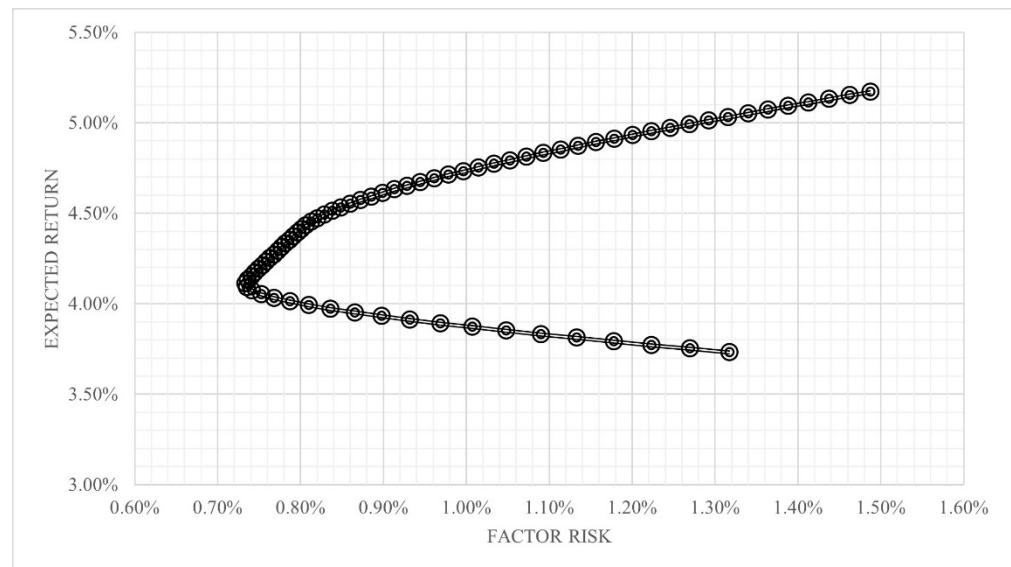


Figure 3. Performance comparison of strategy.

**Table 12.** Performance comparison of strategy

	return	risk	factor risk	Sharpe ratio	Factor Sharpe ratio	MDD
Asset-Only	4.65%	1.90%	2.06%	2.35	2.26	-0.55%
Factor-based Correlation	4.75%	1.92%	1.51%	2.38	3.14	-0.45%
Factor-based	5.83%	2.01%	0.73%	2.81	7.72	-0.44%
	Inflation Beta		Interest Beta			
Asset-Only	0.32%		-0.23%			
Factor-based Correlation	0.45%		-0.48%			
Factor-based	0.48%		-0.41%			

I report that factor-based portfolio has better risk-adjustment performance than asset-only portfolio. Especially, after COVID-19, inflation is excessively increased by recovery situation of economy. Therefore, I study to coefficient inflation index with each strategy. The inflation index is used by Horizon Kinetics Inflation Beneficiaries ETF (INFL) as same investing period. And I used US 3-year bond yield as interest index. Table 14 reports that asset-only strategy denotes lower inflation beta than factor-based, in which higher interest beta than factor-based also. That is, macro-factor volatility which influenced Business-Index could be controlled by factor-based approach strategy. Additionally, I report efficient frontier of factor profile as shown in Table 15.

**Figure 4.** Efficient Frontier of factor profile.

For improving robustness of this subject, from Table 14, I simulated three portfolio strategy performance. I should attend to comparison expected portfolio performance and posterior performance. Table 12, 14 show that optimizing portfolio in risk-return profile is not allocated as economic movement. Finally, I discovered portfolio which is optimized by macro factors has better risk-adjustment return than asset-only portfolio. However, factor-based with Business-Index portfolio denotes better performance of macro-factor sensitivity than others. From Table 8, default index related with change of income has -0.92 interest component and 0.77 inflation component. To block passing the business risk to investment, when default increase by change of inflation and interest, return of investment would increase together. At the same exhibit, situation when inflation increase would increase default, in which decreasing interest cause decreasing the rate of default and income. So, performance of the portfolio must have positive relationship with

increasing inflation and decreasing interest. Factor-based strategy has best correlation with inflation by 0.48%. However, factor-based with Business-Index strategy denotes best performance in interest crisis by -0.48%. In conclusion, factor-based with Business-Index correlation strategy is appropriate to block passing the business risk and decreasing macro factor risk in the crisis scenarios.

## 5. Conclusions

In this article, I suggest factor-based approach with controlled Business-Indexes. Many studies discovered that ALM(Asset Liability Management) method and LDI(Liability Driven Investment) strategy are useful about pension fund. However, not only pension fund but also financial institution should establish strategy with considering income and spending. That is, I suggest factor-based approach as framework. First, investigate proper Business-Indexes which can be considered as income and spending. Second, disassemble Business-Indexes to macro-factors, and selecting factors which influence to Business-Indexes. Third, compute mapping approach from asset classes to macro-factors. Forth, I simulate mean-variance optimization about factors, then, I should set objective function as weight to asset. In this process, simulation is worked by covariance of factors and calculation back to weight to asset.

Factor-based approach is also based modern portfolio theory by Markowitz[1952]. Asset-only approach is effective strategy in return and volatility. Especially, Mean-variance optimizing approach could be used to construct factor-based portfolio as useful method. In conclusion, the approach of this article denotes method for improving modern portfolio theory in line with management method evolution of financial institutions.

Although, I solve problem that assumption of expected macro-factor premium as using expected premium of individual asset and re-mapping process, the study how the performance is changed by currency hedged position is needed. I estimate that macro-factor of currency has multicollinearity problem with other factors. Since return of hedged position has excessive volatility and institutions can consider arbitrage asset as currency swap strategy, I expect following study about currency factors.

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