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A Critical Analysis of the Quantum Nonlocality Problem: On the Polemic Assessment of what Bell Did

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Abstract: Despite their Nobel prize-winning empirical falsification, the interpretation of the Bell inequality remains controversial. An objective analysis of Bell's work on nonlocality shows that Bell's rationale calls for reconsidering a widespread argument on quantum nonlocality yielding a clearer formulation free from the usual obscurities that lead to misleading controversies. By dismissing unnecessary metaphysical tenets, it is possible to probe the core of the problem and determine under what rational assumptions locality or nonlocality become feasible alternatives. The approach renders a more balanced perspective on a long-standing polarized interpretative debate.

Keywords: Bell inequality; locality; nonlocality; local causality

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1. Introduction

Bell, of course, like Einstein, believed that quantum mechanics implies *"Spooky action at a distance."* However, contrary to widespread opinions, we sustain that the proof of quantum nonlocality should not involve the Bell inequality.

The Bell theorem is a straightforward mathematical result proving an uncontroversial fact: that quantum mechanics is not a non-conspiratorial local hidden variable theory[1].

Straining that clear mathematical theorem for including quantum nonlocality as its thesis leads to irreconcilable antagonistic interpretations. A characteristic exposition of these extreme positions is given through an exchange between Tim Maudlin, with his article "What Bell did" which suggested the title of this paper, and Reinhard Werner's comments, exemplifying the innumerable articles claiming quantum nonlocality based on the Bell inequality, on the one hand, and its rejection for the same reason, on the other [1–4].

Despite our agnostic attitude towards quantum nonlocality, we sustain that Bell's and Einstein's beliefs in the nonlocal character of quantum mechanics cannot be so easily dismissed by arguments concerning realism or "classicality" as many interpret.

The problem of nonlocality deals exclusively with the existence of superluminal influences. It is not about the dichotomy between "classicality" and "quantumness" which, owing to an incorrect approach, has become a widespread false paradigm, at least, concerning the nonlocality quandary. Indeed, claiming that quantum mechanics is local because it violates some obscure notion of realism [5] is at least as questionable as claiming it is nonlocal because it violates the Bell inequality.

We stress that to avoid incorrectly posed controversies, the arguments sustaining either quantum nonlocality or locality should be based exclusively on quantum mechanics' objective predictions, irrespective of eventual interpretations regarding the nature of the quantum state or the collapse of the wave function.

Section 2 presents a chronological review of Bell's main papers on nonlocality arguing that, in his most lucid writings, Bell distinctly formulated his inequality only after explicitly establishing the nonlocal character of quantum mechanics, either by previously assuming [6] or directly proving it [7,8].

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[1] Non-conspiratorial means the statistical independence hypothesis is assumed.

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So, John Bell is not responsible for the controversial approach of ascribing to quantum mechanics properties derived from a "classical inequality".

Section 3 analyzes Bell's actual quantum nonlocality argument which only appeared in 1975, more than ten years after the publication of his celebrated 1964 paper. We remark on a more formal approach to proving quantum mechanics is not locally causal and highlight the endemic inconsistency present in the usual approach but absent in Bell's lucid reasoning.

Since we deal with quantum mechanics interpretation, we also have "logically" valid counterarguments for quantum locality. If only for completeness, we briefly review some well-known counterarguments in section 4. Of course, none of such "correct" counterarguments rely on precepts such as "...the Bell inequality is based on classical physics..." or "...the Bell inequality assumes realism..."; not because they are necessarily incorrect but because they are irrelevant to the quantum nonlocality argument. Finally, we present our conclusions in sections 5 and 6.

2. Chronological Review Of Bell's Works On Nonlocality

We present a brief review of Bell's main papers on nonlocality. The key point is that nowhere in those papers can we find an explicit statement declaring that quantum mechanics' nonlocal character is a consequence of Bell's inequalities violations.

On the other hand, he explicitly proved quantum nonlocality without introducing hidden variables and before formulating his inequality at least on two occasions [7,8]. Therefore, the objective evidence favors that Bell did not consider his inequalities as a direct proof of quantum nonlocality but only of the impossibility of a non-conspiratorial local completion.

2.1. The 1964 Bell Theorem

A usual and widespread interpretation asserts that Bell formulated his inequality to prove that quantum mechanics is not a local theory, presenting the Bell theorem as a quantum nonlocality theorem. But an attentive reading of Bell's 1964 argument reveals two crucial facts that prove otherwise [6]:

1. Bell already considered quantum mechanics as nonlocal from the beginning, i.e., before formulating his inequality. Indeed, in the third line of the introduction, he wrote: *"These additional variables were to restore to the theory causality and locality."* That is, the inclusion of hidden variables into the theory was supposed to modify it and recover locality instead of proving its nonlocality.
2. In correspondence with the above-quoted sentence, Bell starts the conclusion section by saying: *"In a theory in which parameters are added to quantum mechanics...."*; so, clearly, he was not inferring properties of quantum mechanics, but only of a modified theory in which parameters are added.

So, Bell's stunning conclusion is not that quantum mechanics is nonlocal, which he took for granted, but that we cannot fix its nonlocality by completing it.

Bell resumed the argument where EPR has left it: if local, quantum mechanics must be incomplete. Since the orthodox interpretation asserts quantum mechanics is complete, then, according to EPR, it must be nonlocal.

The previous inference was implicit in his expression in a) asserting that additional variables were necessary to restore locality. Then, following an EPR-like reasoning, Bell derived a deterministic hidden variable model and proved, through his inequality, the untenability of a local completion. Again, the impossibility of an acceptable local completion only proves that we cannot modify orthodox quantum mechanics to make it local.

Thus, accepting the EPR reasoning, as Bell did, the inequality is unnecessary to prove quantum nonlocality, so claiming that Bell's inequality proves it (which Bell did not) would only be circular reasoning.

On the other hand, Bell's theorem is a mere mathematical theorem that should be free of any polemic if we strictly follow Bell's rationale, namely, that a local completion is untenable. Probably, that was what Richard Feynman meant when he said that Bell's theorem [9],

> "It is not an important theorem. It is simply a statement of something we know is true – a mathematical proof of it."

However, we disagree with Feynman on the unimportance of the Bell theorem since it was a significant advance in the Bohr-Einstein debate that, by 1964, remained stagnant for almost thirty years.

2.2. Bell'S Theorem After 1964

Bell's arguments evolved over the years. In later works, he abandoned the EPR polemic reasoning, which, to Einstein's dislike [10], was based on metaphysical prejudices about physical reality.

However, a persistent view exists that dispenses with Bell's later arguments. That view advocates a controversial reading of Bell's 1964 reasoning advertising the Bell theorem as a quantum nonlocality theorem. We call this view "radical non-localist".

According to the radical non-localist stance, the EPR argument is unassailable. They even consider it an "analytic concept", i.e., we cannot coherently deny it [1]. However, as we observed in the previous section, be it analytic or not, when we accept the EPR reasoning, we do not need the inequality to prove quantum nonlocality.

Except on the occasions where he left the issue ambiguous, Bell explicitly separated his arguments of quantum nonlocality from his inequality, which he used to prove the impossibility of a local completion. Above, ambiguous means he neither explicitly claimed his inequality proved quantum nonlocality nor said otherwise. To prove our previous assertion, next, we chronologically review Bell's main papers discussing nonlocality after 1964.

2.2.1. Introduction To The Hidden Variable Problem

In 1971 he wrote the paper "Introduction to the hidden-variable question" [11]. Here Bell did not explicitly mention quantum nonlocality but investigated the de Broglie-Bohm hidden variables theory and highlights its explicit nonlocal character as "the difficulty".

Bell concluded after formulating his inequality and proving that quantum mechanics violates it:

> "Thus the quantum-mechanical result cannot be reproduced by a hidden-variable theory which is local in the way described."

Note the adjective "local" refers to the hidden-variable theory so irrespective of whether he believed that quantum mechanics is nonlocal, he did not say his inequality proved it and any assertion in that direction would be purely speculative.

2.2.2. The Theory Of Local Beables

This article appeared in 1975. [2] Here Bell abandoned the EPR reasoning and introduced the concept of local causality. He argued that quantum mechanics violates this form of locality in section 3 without mentioning any inequality. He starts that section by asserting:

> "Ordinary quantum mechanics, even the relativistic quantum field theory, is not locally causal in the sense of (2)."

(2) above refers to local causality. Then he develops his quantum nonlocality argument. It is similar to the one given by Einstein in 1927.[3] In the same section, immediately after establishing the nonlocal character of quantum mechanics, Bell explored the problem of adding hidden variables. Then in section 4, "Locality inequality", he derived a stochastic Bell-CHSH inequality. Finally, in section 5, he established the impossibility of a local completion by proving that quantum mechanics violates his inequality, concluding:

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[2] Bell's work is reproduced in [7].
[3] Einstein's argument is reproduced by Laudisa [12] and also by Harrigan and Spekkens [13].

"So quantum mechanics is not embeddable in a locally causal theory as formulated above."

That is different from concluding, "So quantum mechanics is not a local theory" unless we force the interpretation. Otherwise, why would he bother to prove quantum mechanics violates local causality two sections before without using any inequality or hidden variables? We guess because he was well aware of the logical loophole of concluding quantum nonlocality directly from his inequality.

As Stapp once clearly explained [14]:

> "Thus whatever is proved is not a feature of quantum mechanics, but is a property of a theory that tries to combine quantum theory with quasi-classical features that go beyond what is entailed by quantum theory itself. One cannot logically prove properties of a system by establishing, instead, properties of a system modified by adding properties alien to the original system."

Above, "properties alien to the original system" rigorously mean variables that do not legitimately pertain to quantum mechanics. Although some have observed that the hidden variables can include the quantum state [15–17], the problem persists with the other "additional variables". As we observe in the section 3.1.3, the Bell inequality cannot be formulated without additional variables foreign to quantum mechanics, notwithstanding that one of those variables may include the quantum state as indicated by Bell himself [18].

2.2.3. Bertlmann'S Socks

In 1981 Bell wrote his celebrated paper "Bertlmann's socks and the nature of reality" [18]. On this occasion, Bell did not explicitly prove quantum nonlocality before formulating the inequality. He based his arguments on EPR. But he also uses common causes to explain local correlations.

This is one of the papers where he left ambiguous whether his inequality violation should be interpreted as proof of quantum nonlocality. Can we assume that Bell changed his mind about the meaning of his inequality? We do not think so because, in his last paper (cf. 2.2.4), he returned to his previous formulations, i.e., either assuming (1964) or proving (1975) quantum nonlocality without introducing hidden variables or mentioning any inequality.

In [18], Bell chose intuition and ease of interpretation over logical rigor. In Bell's own words, this paper was one of those that [19]:

> "...are nontechnical introductions to the subject. They are meant to be intelligible to nonphysicists."

That is why he spent great effort explaining the difference between quantum and classical entanglement through naive analogies, such as those of Mr. Bertlmann's socks.

2.2.4. La Nouvelle Cuisine

This is Bell's last paper which appeared in 1990 [8]. Here again, Bell's view of his inequality and quantum nonlocality is crystal clear. This time Bell mentions EPR in the two sections that concern us here. In section 8, when proving that "Ordinary quantum mechanics is not locally causal" without mentioning any inequality and without actually using an EPR argument. Then, in section 10, when explicitly introducing hidden variables as local common causes, for proving, through his inequality, that :

"Quantum mechanics cannot be embedded in a locally causal theory"

Although we could force the former statement to interpret that it means that quantum mechanics itself is nonlocal, the order in which he presents his argument does not favor that interpretation. First, he unambiguously established quantum nonlocality without any inequality and then, in a separate section, proved the impossibility of a local completion through his inequality, clearly separating the arguments.

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3. Bell'S Proof Of Quantum Nonlocality

In this section, we analyze Bell's explicit argument on the nonlocal character of quantum mechanics. In his 1975 paper, "The theory of local beables" [7], Bell gave an explicit argument for quantum nonlocality for the first time. This paper has four outstanding characteristics that were missing in 1964:

- A formal definition of locality that is directly applicable to quantum mechanics. He called it *local causality* (LC).
- An argument showing quantum mechanics violates LC and hence is nonlocal. Bell's argument, of course, does not involve his inequality.
- A justification for assuming *statistical independence* (SI) in his hidden variable model. In 1964, SI was an *ad hoc* implicit assumption.
- An absence of any reference to the EPR paper.

Next, we briefly address each of these characteristics.

3.1. Local Causality And Quantum Nonlocality

Firstly, we briefly review the LC concept. Since, as observed by Norsen [15], LC is a little known concept, appendix A contains a detailed explanation for those unfamiliar with it. Secondly, we prove like Bell, that quantum mechanics violates LC without the Bell inequality. Thirdly, we analyze the logical loophole in the usual quantum nonlocality argument.

3.1.1. Local Causality

The concept of LC was conceived to be directly applicable to not deterministic theories like quantum mechanics. Here it suffices to say that when distant experiments are independently performed, to exclude nonlocal effects, the existence of correlations must have a local common cause explanation.

More concretely, in the case of a Bell-type experiment, if $P(A, B \mid a, b)$ is the joint probability for Alice and Bob finding results A and B when their experimental settings are a and b respectively, we in general have that,

$$P(A, B \mid a, b) \neq P(A \mid a) P(B \mid b) \tag{1}$$

(1) means the results of the experiment are correlated. However if correlations are to be explained locally, after including all relevant factors represented by λ[4], we must have,

$$P(A, B \mid a, b, \lambda) = P(A \mid a, \lambda) P(B \mid b, \lambda) \tag{2}$$

(2) means that after all factors(known and unknown) are included, whatever Bob decides to do in his distant laboratory cannot influence Alice's local measurements; and vice versa.

3.1.2. Quantum Nonlocality

After defining local causality, Bell gave an argument explaining why, when considered complete, quantum mechanics violates it. He gave a qualitative explanation and concluded that in quantum mechanics "We simply do not have (2)", where (2) in his paper is LC.

We can recast Bell's and Einstein's arguments in more formal terms through the mathematical formulation of local causality. The crucial point is that (2) avoids a deterministic formulation or any "classical" prejudice and is directly applicable to quantum mechanics. If quantum mechanics is

[4] In case many common causes are required, λ represents a vector variable.

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complete and local, the locally causal explanation of its correlations must lie within the quantum state. In our case

$$\mid \psi\rangle = \frac{1}{\sqrt{2}}(\mid +\rangle \otimes \mid -\rangle - \mid -\rangle \otimes \mid +\rangle) \tag{3}$$

Thus, if locally causal, ordinary quantum mechanics must satisfy (2) when

$$\lambda = \mid \psi\rangle \tag{4}$$

However, choosing $a = b$, $A = 1$, and $B = -1$, an elementary quantum mechanical calculation gives

$$\begin{array}{rcl} P(1, -1 \mid a, a, \mid \psi\rangle) & = & P(1 \mid a, \mid \psi\rangle)P(-1 \mid a, \mid \psi\rangle) \\ \frac{1}{2} & \neq & \frac{1}{2}\frac{1}{2} = \frac{1}{4} \end{array} \tag{5}$$

Given that (5) is not widely known as a non-classical argument for quantum nonlocality, and some believe (4) is incorrect although Bell explained that λ could include the quantum state [18], the appendices C and B contain detailed explanations.

Since in (5) $1/2 \neq 1/4$, ordinary quantum mechanics lacks a locally causal explanation of its correlations, i.e., *the quantum state alone cannot screen-off events on one side from spacelike separated events on the other far away side. Hence, it conspicuously fails the LC locality criterion.*

Note (5) relies exclusively on quantum mechanical objective predictions. It depends only on the quantum formalism irrespective of any interpretation of the wave function. It is an argument in line with the Copenhagen approach, an operational definition that does not rely on metaphysical assumptions.

Formally, that is the counterargument against claims asserting the singlet correlations find a local common cause explanation in their preparation with the same generating event [20]. There is no doubt they find a common cause explanation in their preparation. Unfortunately, that common cause explanation is not a “local” common cause explanation because all we know from its preparation is its quantum state, and as (5) proves, it does not contain a “local common cause”. Nor does the magic of superposition justify those correlations, at least in a locally causal way [21].

3.1.3. The Endemic Logical Loophole and the Gist of the Controversy

Arguing that quantum mechanics is nonlocal because it violates the Bell inequality is unconvincing because proving Bell-type inequalities requires writing joint probabilities as

$$P(A, B \mid a, b) = \int P(A \mid a, \lambda)P(B \mid b, \lambda)P(\lambda)d\lambda \tag{6}$$

which is impossible without going beyond quantum mechanics because (5) proves that

$$P(A, B \mid a, b, \lambda = \mid \psi\rangle) \quad \neq \quad P(A \mid a, \lambda = \mid \psi\rangle)P(B \mid b, \lambda = \mid \psi\rangle) \tag{7}$$

Thus, we are confronted with the following facts:

1. The proof of any property based on (6) is not a property that can be unambiguously ascribed to quantum mechanics.
2. (7) is proved independently of (6), so the Bell inequality is not necessary to prove that good old orthodox operational quantum mechanics’ predictions indeed violate the local causality condition.

The violation of local causality by quantum mechanics through the singlet state is not widely known but was observed and discussed by some authors [15,17,22]. However, most cognoscenti commentators give it only a subsidiary importance. They prefer to turn to the Bell inequality as their main argument losing the opportunity to present a more solid proof that does not suffer from the usual weaknesses pointed out by the localists.

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A typical example of this approach is given, for instance, by Norsen [15] who ultimately presents the CHSH inequality as a quantum nonlocality proof when, after taking for granted statistical independence, he declares:

> "...the empirically violated Clauser-Horne-Shimony-Holt inequality can be derived from Bell's concept of local causality alone, without the need for further assumptions involving determinism, hidden variables, "realism," or anything of that sort."

That argument is unwarranted and justifies the opposite stance held by localists. As previously explained, the CHSH inequality cannot be formulated without hidden variables or common causes not present in quantum mechanics precisely because quantum mechanics violates (2), as proved by (5). The minute one writes (6) he goes beyond quantum theory and it becomes unclear how it concerns quantum mechanics.

Admittedly, the problem explained above can be considered a trivial logical loophole. However, an endemic loophole that is frequently exploited by localists to debunk even the most technical quantum nonlocality presentations like the one by Brunner et al. [23] where again (2) is correctly explained but finally (6) is highlighted as the "locality constraint", declaring,

> "This is the content of Bell's theorem, establishing the nonlocal character of quantum theory and of any model reproducing its predictions."

The localists contend that since the inequality based on (6) is not about quantum mechanics, it signals the nonlocality "of any model reproducing its predictions", except quantum mechanics, claiming that quantum "weirdness", for some unexplained reason, restores locality. Since, anyway, "nobody understands quantum mechanics", that seems to be an acceptable argument.

3.2. Statistical Independence

As we mentioned above, in Bell's 1964 paper, he implicitly assumed the hidden variables distribution function[5] $P(\lambda)$ was not conditional on the experimental settings a and b.

$$P(\lambda \mid a, b) = P(\lambda) \tag{8}$$

We can justify (8) by requiring the experimental settings to be independent of the same common factors λ affecting the results

$$P(a, b \mid \lambda) = P(a, b) \tag{9}$$

According to Bayes theorem we have

$$P(a, b \mid \lambda)P(\lambda) = P(\lambda \mid a, b)P(a, b) \tag{10}$$

Then from (9) and (10) we get (8). The ansatz (9) seems to be a reasonable assumption justifying (8).

Thus, (9) and (8) are equivalent and are known as *statistical independence*, *measurement independence*, *freedom*, or *no-conspiracy*. We shall come back to SI in sect. 4.3.

3.3. The EPR Paper

Although Bell conceived his 1964 paper as a continuation of the EPR argument, one of the virtues of his 1975 formulation is not referencing the EPR paper. Besides presumably being a classical-like argument, the EPR reasoning contains an unnecessary construction that has been the source of much superfluous metaphysical speculation, namely, the *elements of physical reality*.

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5 Note that the distribution function of the λ common causes is irrelevant for the definition of local causality. $P(\lambda)$ is necessary only to derive the Bell inequality.

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The reality criterion has a highly metaphysical burden because it assumes the existence of physical magnitudes from the mere possibility of predicting their values, notwithstanding that we do not indeed measure them. They are unnecessary because they are employed neither to prove quantum nonlocality (5) nor to derive the Bell inequality [24].

Bohr attacked the reality criterion [25]. Einstein did not write the EPR paper, and he did not like how it came out. In a letter to Schrödinger he wrote [10]:

> "But still it has not come out as well as I really wanted; on the contrary, the main point was, so to speak, buried by erudition."

Einstein based his argument for incompleteness on his separation principle and avoided reference to the reality criterion. Thus, it is worth noticing that Einstein and Bell distanced themselves from the excessive metaphysical baggage carried with the EPR elements of physical reality. Even in 1964, when Bell referenced the EPR article, he never mentioned the elements of physical reality.

4. Quantum Locality

We briefly mention three counterarguments that may justify considering quantum mechanics as a local theory. Only the third one contemplates the use of the Bell theorem and is related to the Bell inequality. We mention them only for completeness as possible logically admissible counterarguments.

These arguments are, of course, well-known and not new. There are also others we do not mention, such as the many-worlds interpretation and QBism, that also claim to preserve quantum locality.

4.1. Rejecting Local Causality

Bell's definition of LC can be rejected as the correct definition of locality. In that sense Jarrett [26] helped clarify the nature of local causality by decomposing it into the conjunction of two different conditions, which Shimony respectively called parameter independence (PI) and outcome independence (OI),[6]

$$LC \equiv PI \wedge OI \tag{11}$$

Shimony also proposed the more picturesque expressions controllable and uncontrollable nonlocality, respectively. We refer the non-specialist to Shimony [27] for a detailed explanation of these concepts.

Jarret proved that a theory complying with PI is no-signaling. He also showed quantum mechanics respects PI, hence is no-signaling. However, quantum mechanics violates OI, thus violating LC.

We can effectively block the argument in favor of quantum nonlocality by adopting parameter independence as the appropriate concept for locality and rejecting outcome independence as a necessary condition.

In summary, by accepting no-signaling (parameter independence) as a sufficient criterion for locality, we reject the more stringent condition of local causality, recovering quantum mechanics locality. Of course, those who claim quantum mechanics is not local will not accept the definition [28].

However, more rational discussions are possible by recognizing the correct arguments instead of superfluous discussions about metaphysical irrelevancies such as the preexistence prejudice of elements of physical reality.

It is fair to note that even some who can be considered radical nonlocalists accept that quantum nonlocality is not right out "action at a distance" [29].

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[6] Jarrett used the terms "locality" and "completeness", implying that PI alone is locality. Shimony terminology is better because it is more neutral.

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4.2. Rejecting Causation

Causation in physics was criticized by Bertrand Russell in 1912 [30] and was proposed to solve the quantum nonlocality problem by Van Fraassen [31]. There is no action at a distance simply because there is no need for a causal explanation.

According to Van Fraassen, *"In some cases, the methodological tactic of developing a causal theory will achieve this aim of empirical adequacy, in other cases it will not, and that is just the way the world is."* He claimed that a mythical picture of causal processes got grip on our imagination.

The need for a causal explanation is also called realism. However, this rational form of realism does not conflict with quantum superposition and has nothing to do with the pre-existence tenet usually implied with the expression "local realism".

Van Fraassen was also puzzled by the *"incredible metaphysical extravaganzas to which this subject has led"* [7].

4.3. Completing Quantum Mechanics

This approach is different from the former two because it implies going beyond orthodox quantum mechanics. If we are willing to accept local causality as the correct locality concept and recover a causal explanation, we must consider quantum mechanics as an emergent theory.

We can complete quantum mechanics with local hidden variables if we reject statistical independence. The 1975 version of the Bell theorem is

$$LC \wedge SI \rightarrow Bell \quad inequaqlity \tag{12}$$

Thus, it is possible to keep local causality in a hidden variables theory by rejecting statistical independence. Indeed, well-known local hidden variables models exist reproducing the singlet correlations violating statistical independence [33,34].

Whether statistical independence is a necessary physical condition is a contentious issue. According to some physicists, its rejection is a rational position [35–37]. Others, including John Bell [18], sustain its rejection as inadmissible since it purportedly compromises the experimental freedom implying unreasonable conspiracies.

5. Conclusions

The incorrect mixing of two distinct issues, the proofs of quantum nonlocality on the one hand and quantum completion on the other, has diverted the debate from the correct arguments and hindered it from advancing to more rational alternatives; giving a wrong perspective of the real interpretational difficulties.

We have argued for two key questions: a) the correct quantum nonlocality proof is not based on the Bell inequality which should be sensibly and unambiguously interpreted as a no-local-hidden variables theorem, and b) John Bell's quantum nonlocality argument was also not based on his inequality, which he used only to prove the impossibility of an acceptable local completion.

The first issue (a) is a consistency puzzle. It is formally sustained on Bell's concept of local causality, which orthodox operational quantum mechanics certainly violates without the need to introduce any elements extraneous to quantum theory or inequalities based on hidden variable models.

The second one (b) has a historical character. Although it is more subject to interpretation, we claim it follows from an unbiased and rigorous reading of Bell's writings.

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[7] In our opinion, the most baffling is the counterfactual definiteness assumption which should not be confused with the rational application of counterfactual reasoning [32].

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We conclude there are valid reasons to support both views, locality, and nonlocality. However, the correct arguments supporting or rejecting either of the two opposite positions are not as definite or trivial as the usual specious explanations uphold, exacerbating the different attitudes.

6. Epilogue

Although some may find it comforting to base the claim of quantum nonlocality on the Bell inequality to dismiss it as the consequence of different worldviews, classical vs. quantum [2,38,39], accepting the correct argument is not insurmountable, and may hint at deeper insights.

Distant simultaneity and nonlocality are closely related concepts. Both lack direct and objective physical determination. Admitting a certain degree of convention is necessary to maintain a coherent level of discourse. That is why the locality problem will remain controversial. However, it is essential to recognize its contentious nature for the correct motives instead of incorrect or dubious reasonings.

Relativity taught us to reject absolute simultaneity because of its lack of objective determination, likewise, quantum mechanics may be teaching us to reject action at a distance for the same reason, concretized through its nonsignaling property. Quantum mechanics may require a revision of our notion of causality, just as relativity prompted us to revise our concept of simultaneity.

The other possibility is that quantum mechanics is emergent and, because of Bell's theorem, that would require the acceptance of superdeterminism.[8] These options are still valid open questions, and pretending they are closed or inexistent is not the best scientific attitude.

Appendix

Appendix A Local Causality

Bell's definition of LC is a formalization of the idea that, according to relativity theory, interactions can happen only at a finite speed. It means that causes cannot have an instantaneous effect on distant events. *He formulated LC so that it can be applied to not deterministic theories like quantum mechanics. It is a locality argument that avoids a purportedly classical EPR-like reasoning. A concept directly applicable to orthodox quantum mechanics without distorting its nature.*

For the particular case that concerns us, i.e., the singlet state correlations in a Bell-type experiment, LC takes the following form. Let $P(A,B \mid a,b)$ be the probability of a joint measurement giving the results $A, B \in \{-1,+1\}$ conditional on the respective measurements directions a, b. The laws of probabilities require

$$P(A,B \mid a,b) = P(A \mid B,a,b)P(B \mid a,b) \tag{A1}$$

So far, it is just about probabilities. Let us now add some physics and assume that both observers, Alice and Bob, choose their measurement directions at the last moment so that both measurements are spacelike separated events. Then LC requires that neither the results A, B nor the measurement settings a, b made on one side can affect the state of affairs on the other side. However, we cannot exclude the existence of correlations. In the r.h.s of (A1), we can have that

$$P(A \mid B,a,b) \neq P(A \mid a) \tag{A2}$$
$$P(B \mid a,b) \neq P(B \mid b) \tag{A3}$$

notwithstanding that events A and a are spacelike separated from B and b. However, relativistic causality requires the correlations implied by (A2) and (A3) to be explained by local common causes λ. They are local because they are supposed to lie at the intersection of the backward light cones of the

[8] By superdeterminism, we mean violating the mathematical condition (8) without implying any particular interpretation regarding its admissibility.

measurement events. Once the common causes λ are specified, the inclusion of spacelike separated parameters in the l.h.s of (A2) and (A3) become redundant

$$P(A \mid B, a, b, \lambda) = P(A \mid a, \lambda) \tag{A4}$$
$$P(B \mid a, b, \lambda) = P(B \mid b, \lambda) \tag{A5}$$

Including λ in (A1)

$$P(A, B \mid a, b, \lambda) = P(A \mid B, a, b, \lambda)P(B \mid a, b, \lambda) \tag{A6}$$

Replacing (A4) and (A5) in (A6)

$$P(A, B \mid a, b, \lambda) = P(A \mid a, \lambda)P(B \mid b, \lambda) \tag{A7}$$

The last equation is also known as the screening-off condition. It is the formal expression of the intuitive idea behind relativistic locality and is Bell's definition of LC for the case at hand.

The common cause λ is usually called "hidden variables"; however, it is somewhat misleading to believe the λ variables are necessarily unknown parameters. The only condition they need to comply with is lying at the intersection of the backward light cones of the measuring events to constitute a local explanation of the correlations. It is also utterly misleading to think they are EPR elements of physical reality; on the contrary, their role is to eliminate any EPR-like argument. Furthermore, local causality is independent of the stochastic properties of the common causes. More concretely, they are independent of the statistical independence hypothesis.

Although Bell did not mention Reichenbach, his λ variables are according to Reichenbach's common cause principle [40]. The last point is relevant because one possibility to block the argument in favor of quantum nonlocality is to reject Reichenbach's principle of common causes [22].

Appendix B Common Causes Meaning

Some researchers find it perplexing that the quantum state $\mid \psi\rangle$ can be considered a common cause in the definition of local causality. That is owed to the incorrect metaphysical meaning usually attached to the λ variables as preexisting EPR elements of physical reality [41] or a necessarily classical concept.

Sustaining that λ is by necessity an element foreign to quantum mechanics amounts to forbidding the application of the local causality concept to quantum mechanics. It is particularly convenient for summarily dismissing its vexing nonlocal character decreeing it local by construction [2]. But the physical meaning of λ is not limited to classical or metaphysical concepts other than representing local common causes. Bell also explained that the hidden variables may include the quantum state [18]:

> "It is notable that in this argument nothing is said about the locality, or even localizability, of the variable λ. These variables could well include, for example, quantum mechanical state vectors, which have no particular localization in ordinary space-time."

Unfortunately, researchers often grossly overlook Bell's explanation of the meaning of the λ variables. They generally identify λ with metaphysical entities such as EPR preexisting values or believe they must necessarily be unknown parameters. We suspect such misunderstandings arise from the concrete example he used in his 1964 paper [6] where λ was a spin three-vector or from naive analogies he employed in his less technical expositions such as those in "Bertlmann's socks and the nature of reality" [18].

Appendix C Formal Proof Of Quantum Nonlocality

The particular case $\lambda = \mid \psi\rangle$ is necessary to formalize Bell's (and Einstein's) qualitative arguments of quantum nonlocality according to the rigorous definition of local causality. This step is necessary to

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test whether quantum mechanics can give a locally causal explanation of its correlations. *Thus, there is no valid argument against submitting quantum mechanics, within its own rules, to the local causality test.*

To see whether ordinary quantum mechanics complies with the local causality criterion, all we have to do is set $\lambda = \mid \psi\rangle$ in (A7) with $\mid \psi\rangle$ given by (3), where $\mid +\rangle$ and $\mid -\rangle$ denote the spin eigenstates in the z-direction. We assume that motion takes place in y direction with setting angles a and b lying the $x - z$ plane measured with respect to the z axis. If $\mid a, +\rangle$ and $\mid a, -\rangle$ are the spin eigenstates in the a direction

$$\mid a, +\rangle \quad = \quad +\cos\frac{a}{2} \mid +\rangle + \sin\frac{a}{2} \mid -\rangle \tag{A8}$$

$$\mid a, -\rangle \quad = \quad -\sin\frac{a}{2} \mid +\rangle + \cos\frac{a}{2} \mid -\rangle \tag{A9}$$

Analogously for the particle measured at the other laboratory, we have

$$\mid b, +\rangle \quad = \quad +\cos\frac{b}{2} \mid +\rangle + \sin\frac{b}{2} \mid -\rangle \tag{A10}$$

$$\mid b, -\rangle \quad = \quad -\sin\frac{b}{2} \mid +\rangle + \cos\frac{b}{2} \mid -\rangle \tag{A11}$$

The joint probability according the quantum formalism is

$$P(A, B \mid a, b, \mid \psi\rangle) = \langle\psi \mid (\mid a, A\rangle \otimes \mid b, B\rangle\langle a, A \mid \otimes \langle b, B \mid) \mid \psi\rangle \tag{A12}$$

Letting $A = +1$, $B = -1$ according to (3), (A8) and (A11)

$$\begin{aligned} P(+1, -1 \mid a, b, \mid \psi\rangle) &= \langle\psi \mid (\mid a, +\rangle \otimes \mid b, -\rangle\langle a, + \mid \otimes \langle b, - \mid) \mid \psi\rangle & \text{(A13)} \\ &= \frac{1}{\sqrt{2}}(\langle + \mid a, +\rangle\langle - \mid b, -\rangle - \langle - \mid a, +\rangle\langle + \mid b, -\rangle)\frac{1}{\sqrt{2}}()^* \\ &= \frac{1}{2}(\cos\frac{a}{2}\cos\frac{b}{2} + \sin\frac{a}{2}\sin\frac{b}{2})^2 & \text{(A14)} \\ &= \frac{1}{2}\cos^2(\frac{a - b}{2}) & \text{(A15)} \end{aligned}$$

Where $()^*$ represents the complex conjugate of the first factor in parenthesis. If we further assume $a = b$, (A15) gives

$$P(+1, -1 \mid a, a, \mid \psi\rangle) \quad = \quad \frac{1}{2} \tag{A16}$$

When we perform a measurement only in Alice's laboratory, the quantum formalism prescribes

$$\begin{aligned} P(+1, a, \mid \psi\rangle) &= \langle\psi \mid (\mid a, +\rangle\langle a, + \mid \otimes I) \mid \psi\rangle & \text{(A17)} \\ &= \langle\psi \mid [(\mid a, +\rangle\langle a, + \mid \otimes I) \mid \psi\rangle] & \text{(A18)} \\ &= \langle\psi \mid \left[\frac{1}{\sqrt{2}}(\mid a, +\rangle\langle a, + \mid +\rangle \otimes \mid -\rangle - \mid a, +\rangle\langle a, + \mid -\rangle \otimes \mid +\rangle)\right] \\ &= \langle\psi \mid \left[\frac{1}{\sqrt{2}}(\cos\frac{a}{2} \mid a, +\rangle \otimes \mid -\rangle - \sin\frac{a}{2} \mid a, +\rangle \otimes \mid +\rangle)\right] & \text{(A19)} \\ &= \frac{1}{2}\left[\cos\frac{a}{2}\langle + \mid a, +\rangle + \sin\frac{a}{2}\langle - \mid a, +\rangle\right] & \text{(A20)} \\ &= \frac{1}{2}\left[\cos^2\frac{a}{2} + \sin^2\frac{a}{2}\right] & \text{(A21)} \\ &= \frac{1}{2} & \text{(A22)} \end{aligned}$$

Where $I = \mid +\rangle\langle + \mid + \mid -\rangle\langle - \mid$ is the identity operator in the one particle two-dimensional Hilbert-space. In a similar way, performing a measurement only on Bob's laboratory we find

$$P(-1, b, \mid \psi\rangle) = \langle \psi \mid (I \otimes \mid b, -\rangle\langle b, - \mid) \mid \psi\rangle = \frac{1}{2} \tag{A23}$$

From (A16), (A22), and (A23), we obtain (5) formally proving that ordinary quantum mechanics lacks a local common cause explanation for its correlations. t

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