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A Critical Analysis of the Quantum Nonlocality Problem: On the Polemic Assessment of What Bell Did

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Abstract: The alleged nonlocal character of quantum mechanics is inextricably related to the formulation of the Bell theorem. That relation, however, is commonly incorrectly assessed. The departure from the clear line of reasoning that John Bell tried to convey has led to a polarization of part of the scientific community into radical irreconcilable positions. We show how the correct appreciation of Bell's work calls for reinterpreting the usual significance given to the Bell theorem yielding a more rational perspective of the problem. We formalize the correct argument sustaining the nonlocal character of quantum mechanics and also comment on alternatives that may justify considering quantum mechanics as a local theory.

Keywords: Bell inequality; locality; nonlocality; local causality

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1. Introduction

Nonlocality can be justified in different ways. For instance, it is well-known that contextuality already implies nonlocality [1–4].

Here, however, we pursue the analysis of quantum nonlocality instead of just nonlocality, which may refer only to hidden variables theories.

We point out that, according to Bell's approach, the usual arguments upholding the nonlocal character of quantum mechanics based on the Bell theorem are usually inappropriately posed.

That situation promotes the extreme opposite position of the localists, sometimes leading to heated debates [5–8]. The localists' stance reflects in expressions such as:

> The terms "non-locality" or "quantum non-locality" are buzzwords in foundations of quantum mechanics and quantum information. Most of scientists treat these terms as a more handy expression equivalent to the clumsy "violation of Bell's inequalities". Unfortunately, some treat them seriously. [9]

Contrary to the prevailing opinion, we remark that an unbiased reading of Bell's work shows that he did not claim his inequality proves quantum nonlocality.

We begin by following the historical development of Bell's ideas revealing the nature of misunderstandings that constitute the source of the existing conflicts. In Section 2, we review Bell's 1964 formulation. Then in Section 3, we analyze Bell's later works where he gave arguments for quantum nonlocality. We propose a slight argumentation change that puts on a more formal base the reasons sustaining quantum nonlocality.

Of course, since we are dealing with quantum mechanics interpretation, we also have counterarguments in favor of quantum locality. We briefly analyze those arguments in Section 4. Finally, we present our conclusions in Sections 5 and 6.

2. The 1964 Bell Theorem

We propose a different reading of Bell's 1964 theorem [10] from the traditional one. We claim our proposal is more according to what Bell intended in 1964 and clarifies the origin of the conflicts between localists and non-localists. The difference is subtle but crucial.

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The usual reading asserts that Bell formulated his inequality to prove that quantum mechanics is not a local theory, presenting the Bell theorem as a quantum nonlocality theorem. But a careful reading reveals two relevant facts:

a) Bell already considered quantum mechanics as nonlocal from the beginning, i.e., before formulating his inequality. Indeed, in the third line of the introduction, he wrote: *"These additional variables were to restore to the theory causality and locality."* That is, without additional variables, quantum mechanics remains an acausal nonlocal theory.
b) Bell starts the conclusion section by saying: *"In a theory in which parameters are added to quantum mechanics...."*; so, clearly, he was not inferring properties of quantum mechanics, but only of a theory in which parameters are added.

Bell took the argument where EPR [11] has left it, namely, if local, quantum mechanics must be incomplete. Then proved through his inequality that a local completion was untenable. Therefore, the impossibility of a local completion only proves that we cannot fix orthodox acausal nonlocal quantum theory.

The last conclusion is correct only if we assume, as Bell did, the not local character of orthodox complete quantum mechanics. Thus, if we follow Bell's argument, claiming that the Bell theorem proves quantum nonlocality (which Bell did not) would only be circular reasoning.

The EPR reasoning is polemic, and famously Bohr rejected it. Rejecting the EPR argument [1] implies rejecting quantum nonlocality from the begining. On the other hand, Bell's theorem is a mere mathematical theorem that should be free of any polemic if we follow Bell's conclusion, namely, that a local completion is untenable. Richard Feynman put that clearly when he said that Bell's theorem

> It is not an important theorem. It is simply a statement of something we know is true – a mathematical proof of it. [13]

Bell's arguments evolved over the years. In later works, he abandoned the EPR reasoning dissipating the fog around it. But most commentators ignore Bell's later arguments for some reason insisting on a distorted polemic reading of Bell's 1964 reasoning advertising the Bell theorem as a quantum nonlocality theorem.

3. Bell's Proof of Quantum Nonlocality

In 1975 Bell [2] wrote an improved version of his seminal 1964 paper. The new version has four outstanding characteristics that were missing in 1964:

- A rigorous definition of locality he called *local causality* (LC).
- A proof that quantum mechanics violates LC, therefore, is nonlocal.
- A physical justification for assuming what later became known as the *statistical independence* (SI) hypothesis. In 1964, SI was an *ad hoc* implicit assumption.
- An absence of any reference to the EPR paper.

Next, we briefly address each of these characteristics. Now the reasons for quantum nonlocality are more obviously separated from his inequality formulation than in 1964. So their mixing resulting in controversial and obscure arguments can not be attributed to John Bell.

3.1. Local Causality

Bell's definition of LC is a formalization of the idea that, according to relativity theory, interactions can happen only at a finite speed. It means that causes cannot have an instantaneous effect on distant

1 Not surprisingly, Bohr's rejection [12] is at least as polemic as EPR.
2 Bell's work is reproduced in [14].

events. *He formulated LC so that it can be applied to not deterministic theories like quantum mechanics. It is a locality argument that avoids a purportedly classical EPR-like reasoning. A concept directly applicable to orthodox quantum mechanics without distorting its nature.*

For the particular case that concerns us, i.e., the singlet state correlations in a Bell-type experiment, LC takes the following form. Let $P(A, B \mid a, b)$ be the probability of a joint measurement giving the results $A, B \in \{-1, +1\}$ conditional on the respective measurements directions a, b. The laws of probabilities require

$$P(A, B \mid a, b) = P(A \mid B, a, b) P(B \mid a, b) \tag{1}$$

So far, it is just about probabilities. Let us now add some physics and assume that both observers, Alice and Bob, choose their measurement directions at the last moment so that both measurements are spacelike separated events. Then LC requires that neither the results A, B nor the measurement settings a, b made on one side can affect the state of affairs on the other side. However, we cannot exclude the existence of correlations. In the r.h.s of (1), we can have that

$$P(A \mid B, a, b) \neq P(A \mid a) \tag{2}$$
$$P(B \mid a, b) \neq P(B \mid b) \tag{3}$$

notwithstanding that events A and a are spacelike separated from B and b. However, relativistic causality requires the correlations implied by (2) and (3) to be explained by local common causes λ. They are local because they are supposed to lie at the intersection of the backward light cones of the measurement events. Once the common causes λ are specified, the inclusion of spacelike separated parameters in the l.h.s of (2) and (3) become redundant

$$P(A \mid B, a, b, \lambda) = P(A \mid a, \lambda) \tag{4}$$
$$P(B \mid a, b, \lambda) = P(B \mid b, \lambda) \tag{5}$$

Including λ in (1)

$$P(A, B \mid a, b, \lambda) = P(A \mid B, a, b, \lambda) P(B \mid a, b, \lambda) \tag{6}$$

Replacing (4) and (5) in (6)

$$P(A, B \mid a, b, \lambda) = P(A \mid a, \lambda) P(B \mid b, \lambda) \tag{7}$$

The last equation is also known as the screening-off condition. It is the formal expression of the intuitive idea behind relativistic locality and is Bell's definition of LC for the case at hand.

The common cause λ is usually called "hidden variables"; however, it is somewhat misleading to believe the λ variables are necessarily unknown parameters. The only condition they need to comply with is lying at the intersection of the backward light cones of the measuring events to constitute a local explanation of the correlations. It is also utterly misleading to think they are EPR elements of physical reality; on the contrary, their role is to eliminate any EPR-like argument. Furthermore, local causality is independent of the stochastic properties of the common causes. More concretely, they are independent of the statistical independence hypothesis.

Although Bell did not mention Reichenbach, his λ variables are according to Reichenbach's common cause principle [15]. The last point is relevant because, as we shall see later, one possibility to block the argument in favor of quantum nonlocality is to reject Reichenbach's principle of common causes [16].

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3.2. Quantum Nonlocality

After defining local causality, Bell gave an argument explaining why, when considered complete, quantum mechanics violates it. Bell's argument is similar to the one given by Einstein in 1927.[3]

We can recast Bell's and Einstein's arguments in more formal terms through the mathematical formulation of local causality. The crucial point is that (7) avoids the polemic around an EPR-like classical argument. If quantum mechanics is complete and local, the locally causal explanation of its correlations must lie within the quantum state. In our case

$$\mid\psi\rangle = \frac{1}{\sqrt{2}}(\mid +\rangle\otimes\mid -\rangle - \mid -\rangle\otimes\mid +\rangle) \tag{8}$$

Thus, if locally causal, ordinary quantum mechanics must satisfy (7) when

$$\lambda = \mid\psi\rangle \tag{9}$$

However, choosing $a = b$, $A = 1$, and $B = -1$, an elementary quantum mechanical calculation gives

$$\begin{array}{ccc} P(1,-1 \mid a, a, \mid\psi\rangle) & = & P(1 \mid a, \mid\psi\rangle)P(-1 \mid a, \mid\psi\rangle) \\ \frac{1}{2} & \neq & \frac{1}{2}\frac{1}{2} = \frac{1}{4} \end{array} \tag{10}$$

Given that (10) is not widely known as a non-classical argument for quantum nonlocality, and some find (9) inappropriate, appendix 7 contains a detailed explanation.

Since in (10) $1/2 \neq 1/4$, ordinary quantum mechanics lacks a locally causal explanation of its correlations, i.e., *the quantum state alone cannot screen-off events on one side from spacelike separated events on the other far away side. Hence, it conspicuously fails the LC locality criterion.*

Note that (10) is not an EPR-like argument. It relies exclusively on quantum mechanical objective predictions. It is independent of the wave function interpretation and wave function collapse. In particular, it is independent of the ontic or epistemic nature of the quantum state, depending only on the quantum formalism irrespective of any interpretation. It is an argument in line with the Copenhagen approach, an operational definition that does not rely on metaphysical assumptions.

Formally, that is the counterargument against claims asserting the singlet correlations find a local common cause explanation in their preparation with the same generating event [19]. There is no doubt they find an explanation in their preparation. Unfortunately, that explanation is not locally causal because all we know from its preparation is its quantum state, and as (10) proves, it does not contain a common cause explanation. Nor does the magic of superposition justify those correlations, at least in a locally causal way [20].

Note that Bell never invoked his inequality for proving quantum nonlocality. Either in 1964, or 1975, Bell introduced his inequality only after taking for granted that quantum mechanics is not a local theory. The logically dubious approach of using a "classical inequality" to derive properties of quantum theory is not ascribable to John Bell. Unfortunately, often the same does not apply either to localists or non-localists [5,6,9,20–23].

3.3. Statistical Independence

As we mentioned above, in Bell's 1964 paper, he implicitly assumed the hidden variables distribution function[4] $P(\lambda)$ was not conditional on the experimental settings a and b.

$$P(\lambda \mid a, b) = P(\lambda) \tag{11}$$

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3 Einstein's argument is reproduced by Laudisa in [17] and also by Harrigan and Spekkens in [18].
4 Note that the distribution function of the λ common causes is irrelevant for the definition of local causality. $P(\lambda)$ is necessary only to derive the Bell inequality.

We can justify (11) by requiring the experimental settings to be independent of the same common factors λ affecting the results

$$P(a,b \mid \lambda) = P(a,b) \tag{12}$$

According to Bayes theorem we have

$$P(a,b \mid \lambda)P(\lambda) = P(\lambda \mid a,b)P(a,b) \tag{13}$$

Then from (12) and (13) we get (11). The ansatz (12) seems to be a reasonable assumption justifying (11).

Thus, (12) and (11) are equivalent and are known as *statistical independence*, *measurement independence*, *freedom*, or *no-conspiracy*. We shall come back to SI in sect. 4.3.

3.4. The EPR Paper

Although Bell conceived his 1964 paper as a continuation of the EPR argument, one of the virtues of his 1975 formulation is not referencing the EPR paper. Besides presumably being a classical-like argument, the EPR reasoning contains an unnecessary construction that has been the source of much superfluous metaphysical speculation, namely, the *elements of physical reality*.

The reality criterion has a highly metaphysical burden because it assumes the existence of physical magnitudes from the mere possibility of predicting their values, notwithstanding that we do not indeed measure them. They are unnecessary because they are employed neither to prove quantum nonlocality (10) nor to derive the Bell inequality [24].

Bohr attacked the reality criterion [12]. Einstein did not write the EPR paper, and he did not like how it came out. In a letter to Schrödinger he wrote

> But still it has not come out as well as I really wanted; on the contrary, the main point was, so to speak, buried by erudition [25]

Einstein based his argument for incompleteness in his separation principle and avoided any reference to the reality criterion. Thus, it is worth noticing that Einstein and Bell distanced themselves from the original EPR elements of physical reality criterion. Even in 1964, when Bell referenced the EPR article, he never mentioned the elements of physical reality.

4. Quantum Locality

We shall review three counterarguments that justify considering quantum mechanics as a local theory. Only one of them, the third one, contemplates the use of the Bell inequality and is related to Bell's theorem.

We do not advocate for any of those positions but only mention them as possible logically correct counterarguments.

4.1. Rejecting Local Causality

Jarrett [26] helped clarify the nature of local causality by decomposing it into the conjunction of two different conditions, which Shimony respectively called parameter independence (PI) and outcome independence (OI)

$$LC \equiv PI \wedge OI \tag{14}$$

Shimony also proposed the more picturesque expressions controllable and uncontrollable nonlocality, respectively. We refer the non-specialist to Ref. [27] for a detailed explanation of these concepts.

Jarret proved that a theory complying with PI is no-signaling. He also showed quantum mechanics respects PI, hence is no-signaling. However, quantum mechanics violates OI, thus violating LC.

We can effectively block the argument in favor of quantum nonlocality by adopting parameter independence as the appropriate concept for locality and rejecting outcome independence as a necessary condition. Of course, those who claim quantum mechanics is not local will not accept the definition. However, more rational discussions are possible by explicitly acknowledging the different criteria.

In summary, by accepting no-signaling (parameter independence) as a sufficient criterion for locality, we reject the more stringent condition of local causality, recovering quantum mechanics locality.

4.2. Rejecting Realism

Usually, realism is ascribed obscure meanings as a "classicality" hypothesis and has been justly criticized as a necessary hypothesis to derive the Bell inequality [7,28–30].

Notwithstanding that Bell-type inequalities violations are not proof of quantum nonlocality, when we interpret realism as the need for a causal explanation, its rejection allows for recovering quantum mechanics' locality by discarding local causality.

Causation in physics has been criticized by Bertrand Russell in 1912 [31] and was again proposed to solve the quantum nonlocality problem by Van Fraassen in 1982 [32]. There is no action at a distance simply because there is no need for a causal explanation.

Whether we like it or not, when we interpret realism as a causal explanation, its rejection constitutes a logically correct option to solve the quantum nonlocality problem.

4.3. Completing Quantum Mechanics

This approach is different from the former two because it implies going beyond orthodox quantum mechanics. If we are willing to accept local causality as the correct locality concept and recover a causal explanation, we must consider quantum mechanics as an emergent theory.

We can complete quantum mechanics with local hidden variables if we reject statistical independence. The 1975 version of the Bell theorem is

$$LC \wedge SI \rightarrow Bell \quad inequaqlity \tag{15}$$

Thus, we can retain local causality in a hidden variables theory by rejecting statistical independence. Indeed, well-known local hidden variables models exist reproducing the singlet correlations violating statistical independence [33,34].

Whether statistical independence is a necessary physical condition is a contentious issue. According to some physicists, its rejection is a rational position [35–37]. Others, including John Bell [38], sustain its rejection as inadmissible since it purportedly compromises the experimental freedom implying unreasonable conspiracies.

5. Conclusions

Rationalizing the current controversy around the not local character of quantum mechanics requires a radical interpretational change.

A stance we dub as radical non-localist [5,22,23,39] interprets the Bell inequality violations as definitive proofs of quantum nonlocality. That interpretation, however, is controversial, poorly supported, and does not reflect Bell's clear line of reasoning. It spawns a radical localist stance [6,9,19,20] asserting there exist no solid arguments sustaining the nonlocal character of quantum mechanics. Radical localists base their assertions on plain statements such as:

> Thus the usual derivations of CHSH and other Bell inequalities employ classical physics to discuss quantum systems, so it is not surprising when these inequalities fail to agree with quantum predictions, or the experiments that confirm these predictions. [19]

In fact, since all derivations (either deterministic or stochastic) of Bell-type inequalities are not based on quantum formalism, the former claim is arguably well justified, notwithstanding radical non-localists' opposition [5,17,40].

The current impasse arises from the incorrect mixing of two different issues, the arguments for quantum nonlocality on the one hand and quantum completion on the other. We propose a clear separation of those arguments that imply changing the usual interpretation of the Bell theorem as a quantum nonlocality theorem. Our proposed reinterpretation, however, is far from being original. It is already there, explicit and well-documented, in the writings that John Bell has left us.

In 1964, Bell assumed nonlocality from the beginning, and only afterward he derived his inequality to prove that a rational local completion is untenable. Then, in 1975 [14] and again in 1990 [41], he proved that quantum mechanics is not locally causal before setting out his inequality.[5] In all cases, the conclusion he drew from his line of reasoning, i.e., the thesis of his theorem, was:

> Quantum mechanics cannot be embedded in a locally causal theory. [41]

Because the alleged locally causal theory is a hidden variables theory, it is questionable to argue that quantum nonlocality should follow from the Bell theorem. John Bell never made such a claim.[6]

On the other hand, his non-localist followers reverse Bell's reasoning claiming that quantum nonlocality follows from the Bell inequality violations [5,22,23,39,42] rendering a convenient motivation to localists for readily dismissing the awkward nonlocal character of quantum mechanics [6,9,20,21].

We call for the scientific community to vindicate John Bell and not to distort his clear line of reasoning as he laid it out from 1975 onwards. The allegedly not local character of orthodox quantum mechanics arises because it violates local causality (equivalently the common cause principle), not because it violates Bell-type inequalities.

It could be confusing that since local causality is a hypothesis of the Bell inequality in (15), wouldn't the inequality violation by quantum mechanics implies that it violates local causality? The answer is negative because the target of the inequality is "classical theories", i.e., theories containing parameters that are not present in orthodox quantum mechanics (see discussion after (32)).

Thus, quantum mechanics' violation of local causality requires independent proof as given by (10). Hence, the Bell inequality violations were never an issue for quantum nonlocality. They signal the impossibility of a local completion when statistical independence is assumed. Despite most non-localists' contention, *the Bell theorem is a no-local-hidden-variables theorem, not a quantum nonlocality theorem. John Bell never claimed otherwise.*

6. Final Remarks

Simultaneity and nonlocality are closely related concepts. Both lack direct and clear-cut physical determination. Admitting a certain degree of convention is necessary if we want to maintain a coherent level of discourse. Although the locality problem will remain controversial, it is essential to recognize its contentious nature for the correct motives instead of incorrect or obscure reasonings.

The quantum nonlocality problem cannot be summarily dismissed by looking for trivial conceptual loopholes within the Bell-type inequalities and Bell's arguments or by discarding superfluous metaphysical ideas [43]. Quantum mechanics may require a revision of our notion of causality, just as relativity prompted us to revise our concept of simultaneity. The other possibility is that quantum mechanics is emergent and, because of Bell's theorem, that would require the acceptance of superdeterminism [35–37]. These options are still valid open questions, and pretending they are closed or inexistent is not the best scientific attitude.

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[5] Unfortunately, Bell did not make this point explicitly clear in one of his most celebrated papers, "Bertlmann's socks and the nature of reality" [38].

[6] We avoid speculating on what Bell might have thought or believed. We base our assertion on what he has written in his published papers.

APPENDIX

7. Common Causes and the Quantum State

Some researchers find it perplexing that the quantum state $|\psi\rangle$ can be considered a common cause in the definition of local causality. That is so owed to the metaphysical meaning usually attached to the λ variables as preexisting EPR elements of physical reality [44] or a necessarily classical concept.

Sustaining that λ is by necessity an element foreign to quantum mechanics amounts to forbidding the application of the local causality concept to quantum mechanics. It is particularly convenient for summarily dismissing its annoying nonlocal character decreeing it local by construction [6]. But the physical meaning of λ is not limited to classical or metaphysical concepts other than representing local common causes. Bell also explained that the hidden variables may include the quantum state:

> It is notable that in this argument nothing is said about the locality, or even localizability, of the variable λ. These variables could well include, for example, quantum mechanical state vectors, which have no particular localization in ordinary space-time. [38]

Unfortunately, researchers often grossly overlook Bell's explanation of the meaning of the λ variables. They generally identify λ with metaphysical entities such as preexisting values [43] or believe they must necessarily be unknown parameters.

The particular case $\lambda = |\psi\rangle$ is necessary to formalize Bell's (and Einstein's) qualitative arguments of quantum nonlocality according to the rigorous definition of local causality. This step is in complete agreement with viewing the quantum state as already containing the local common cause capable of explaining the quantum correlations. At least if quantum mechanics, as some people claim, is locally causal. *Thus, there is no valid argument against submitting quantum mechanics, within its own rules, to the local causality test.*

To see whether ordinary quantum mechanics complies with the local causality criterion, all we have to do is set $\lambda = |\psi\rangle$ in (7) with $|\psi\rangle$ given by (8), where $|+\rangle$ and $|-\rangle$ denote the spin eigenstates in the z-direction. We assume that motion takes place in y direction with setting angles a and b lying the $x - z$ plane measured with respect to the z axis. If $|a,+\rangle$ and $|a,-\rangle$ are the spin eigenstates in the a direction

$$|a,+\rangle = +\cos\frac{a}{2}|+\rangle + \sin\frac{a}{2}|-\rangle \quad (16)$$

$$|a,-\rangle = -\sin\frac{a}{2}|+\rangle + \cos\frac{a}{2}|-\rangle \quad (17)$$

Analogously for the particle measured at the other laboratory, we have

$$|b,+\rangle = +\cos\frac{b}{2}|+\rangle + \sin\frac{b}{2}|-\rangle \quad (18)$$

$$|b,-\rangle = -\sin\frac{b}{2}|+\rangle + \cos\frac{b}{2}|-\rangle \quad (19)$$

The joint probability according the quantum formalism is

$$P(A,B\mid a,b,|\psi\rangle) = \langle\psi\mid(|a,A\rangle\otimes|b,B\rangle\langle a,A|\otimes\langle b,B|)\mid\psi\rangle \quad (20)$$

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Letting $A = +1$, $B = -1$ according to (8), (16) and (19)

$$\begin{aligned} P(+1,-1 \mid a,b,\mid\psi\rangle) &= \langle\psi \mid (\mid a,+\rangle\otimes \mid b,-\rangle\langle a,+\mid\otimes\langle b,-\mid) \mid \psi\rangle & (21)\\ &= \frac{1}{\sqrt{2}}(\langle + \mid a,+\rangle\langle - \mid b,-\rangle - \langle - \mid a,+\rangle\langle + \mid b,-\rangle)\frac{1}{\sqrt{2}}()^* \\ &= \frac{1}{2}(\cos\frac{a}{2}\cos\frac{b}{2} + \sin\frac{a}{2}\sin\frac{b}{2})^2 & (22)\\ &= \frac{1}{2}\cos^2(\frac{a-b}{2}) & (23)\end{aligned}$$

Where $()^*$ represents the complex conjugate of the first factor in parenthesis. If we further assume $a = b$, (23) gives

$$P(+1,-1 \mid a,a,\mid\psi\rangle) = \frac{1}{2} \quad (24)$$

When we perform a measurement only in Alice's laboratory, the quantum formalism prescribes

$$\begin{aligned} P(+1,a,\mid\psi\rangle) &= \langle\psi \mid (\mid a,+\rangle\langle a,+\mid\otimes I) \mid \psi\rangle & (25)\\ &= \langle\psi \mid [(\mid a,+\rangle\langle a,+\mid\otimes I) \mid \psi\rangle] & (26)\\ &= \langle\psi \mid \left[\frac{1}{\sqrt{2}}(\mid a,+\rangle\langle a,+\mid +\rangle\otimes\mid -\rangle - \mid a,+\rangle\langle a,+\mid -\rangle\otimes\mid +\rangle)\right] \\ &= \langle\psi \mid \left[\frac{1}{\sqrt{2}}(\cos\frac{a}{2}\mid a,+\rangle\otimes\mid -\rangle - \sin\frac{a}{2}\mid a,+\rangle\otimes\mid +\rangle)\right] & (27)\\ &= \frac{1}{2}\left[\cos\frac{a}{2}\langle + \mid a,+\rangle + \sin\frac{a}{2}\langle - \mid a,+\rangle\right] & (28)\\ &= \frac{1}{2}\left[\cos^2\frac{a}{2} + \sin^2\frac{a}{2}\right] & (29)\\ &= \frac{1}{2} & (30)\end{aligned}$$

Where $I = \mid +\rangle\langle + \mid + \mid -\rangle\langle - \mid$ is the identity operator in the one particle two-dimensional Hilbert-space. In a similar way, performing a measurement only on Bob's laboratory we find

$$\begin{aligned} P(-1,b,\mid\psi\rangle) &= \langle\psi \mid (I\otimes \mid b,-\rangle\langle b,-\mid) \mid \psi\rangle & (31)\\ &= \frac{1}{2} & (32)\end{aligned}$$

From (24), (30), and (32), we obtain (10) formally proving that ordinary quantum mechanics lacks a local common cause explanation for its correlations.

As we explain in the main text and contrary to widespread beliefs, Bell inequality violation does not tell us that quantum mechanics is not local. Nonlocality follows from quantum formalism itself. John Bell (and Einstein) never claimed otherwise. What is puzzling about the Bell theorem is that we cannot complete quantum mechanics with additional parameters under reasonable assumptions, proving that the experimentally tested quantum predictions seem to be hopelessly nonlocal. Probably that would have disappointed Einstein.

A similar argument for quantum nonlocality using the concept of local causality alone was also made, for instance, by Norsen. However, ambiguity arises when he ultimately presents the CHSH inequality as a quantum nonlocality proof when, after taking for granted statistical independence, he declares:

> ...the empirically violated Clauser-Horne-Shimony-Holt inequality can be derived from Bell's concept of local causality alone, without the need for further assumptions involving determinism, hidden variables, "realism," or anything of that sort. [40]

In our opinion, that move is unwarranted and justifies the opposite stance held by localists. Indeed, the CHSH inequality cannot be formulated without hidden variables or common causes not present in quantum mechanics precisely because quantum mechanics violates (7), as proved by (10). Certainly, proving Bell-type inequalities require writing joint probabilities as

$$P(A,B \mid a,b) = \int P(A \mid a,\lambda)P(B \mid b,\lambda)P(\lambda)d\lambda \tag{33}$$

which is impossible without going beyond quantum mechanics. Admittedly, it is a trivial logical loophole. However, an endemic loophole that is frequently exploited by localists to debunk even the most lucid quantum nonlocality presentations like the one by Brunner et al. [39] where again (7) is correctly explained but finally (33) is highlighted as the "locality constraint", declaring

> This is the content of Bell's theorem, establishing the nonlocal character of quantum theory and of any model reproducing its predictions.

Quantum localists respond by saying that since the inequality based on (33) is not about quantum mechanics, it signals the nonlocality of something else. That is why the Bell theorem concerns the impossibility of a local completion of quantum mechanics. It does not prove orthodox quantum mechanics' nonlocal character.

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