

Article

The Primordial Particle Accelerator of the Cosmos

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Abstract: In a previous paper we have shown that superluminal particles are allowed by the general relativistic theory of gravity provided that the metric is locally Euclidean. Here we calculate the probability density function of a canonical ensemble of superluminal particles as function of temperature. Although only the Lorentzian metric is stable for normal matter density, an Euclidian metric can be created under special gravitational circumstances and persist in a limited region of space-time consisting of the very early universe which is characterized by extremely high densities and temperatures. Superluminal particles also allow attaining thermodynamic equilibrium at a shorter duration and also suggest a rapid expansion of the matter density, thus making mechanism such as inflation (which demands invoking and ad-hoc scalar field) redundant. This is in accordance with Occam's razor.

Keywords: General Relativity, Euclidian Metric, Superluminality, Cosmological Inflation

1. Introduction

It is well known that our daily space-time is approximately of Lorentz (Minkowski) type with a metric $\eta_{\mu\nu} = \text{diag}(1, -1, -1, -1)$ (we adopt the standard notation in which Greek indices take the values 0, 1, 2, 3). The above statement is taken as one of the central assumptions of the theory of special relativity and has been supported by numerous experiments. But one should ask why should it be so?

Many textbooks [1] state that in the general theory of relativity any space-time is locally of the type $\eta_{\mu\nu} = \text{diag}(1, -1, -1, -1)$, although it can not be presented so globally due to the effect of matter. This is a part of the demands dictated by the well known equivalence principle. The above principle is taken to be one of the assumptions of general relativity other assumption such as diffeomorphism invariance, and the requirement that theory reduce to Newtonian gravity in the proper regime lead to the Einstein equations:

$$G_{\mu\nu} = -\frac{8\pi G}{c^4} T_{\mu\nu} \quad (1)$$

in which $G_{\mu\nu}$ is the Einstein tensor, $T_{\mu\nu}$ is the stress-energy tensor, G is the gravitational constant and c is the velocity of light.

The Principle of Equivalence rests on the equality of gravitational and inertial mass, demonstrated by Galileo, Huygens, Newton, Bessel, and Eötvös. Einstein reflected that, as a consequence, no external static homogeneous gravitational field could be detected in a freely falling elevator, for the observers, their test bodies, and the elevator itself would respond to the field with the same acceleration [1]. This means that the observer will experience himself as free, not feeling the effect of any force at all. Mathematically speaking for the observer space time is locally (but not globally) flat and Minkowskian.

The point is that one need not assume that space-time is locally Lorentz based on an empirical (unexplained) facts, rather one can derive this property from the field equations based on the stability of the Minkowskian solution. Other unstable flat solutions of non Minkowskian type, such as an Euclidian metric $\eta_{\mu\nu} = \text{diag}(1, 1, 1, 1)$ can exist in a limited region of space-time. In an Euclidian metric there are no speed limitations and thus the alleged particle can travel in faster than light speed [21]. The reader should notice that already Eddington [2, page 25] has considered the possibility that the universe contains



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different domains in which some domains are locally Lorentzian and others have some other local metric of the type $\eta_{\mu\nu} = \text{diag}(-1, -1, -1, -1)$ or the type $\eta_{\mu\nu} = \text{diag}(1, 1, -1, -1)$. The stability of those domains was not discussed by Eddington.

Many authors have suggested explanations to the locally Lorentzian nature of space-time [3–6]. What is common to all the above approaches is that additional theoretical structures & assumptions are needed. In previous works [7–10] it was shown that General relativistic equations and linear stability analysis suffice to obtain a unique choice of the Lorentzian metric being the only one which is stable. Other metrics are allowed but are unstable and thus can exist in only a limited region of space-time. The analysis will not be repeated here, the reader is referred to the original literature. It should be mentioned that the choice of coordinates in the Fisher approach to physics is also justified using the stability approach [11]. The nonlinear stability question of the Lorentzian metric was settled by D. Christodoulou & S. Klainerman [13]. As for the nonlinear instability of other spaces of constant metric this remains an open question at this time.

For non empty space-time the situation can be drastically different [12]. The existence of the intuitive partition of 4 dimensional space into "spatial" space and "temporal" time, is a feature of an almost empty space-time. This does not contradict the fact that such a partition can not be demonstrated in general solutions of Einstein's equations, such as the one discovered by Gödel [15]. But this problem is not a characteristic of exotic space-times rather it is a property of standard cosmological models.

Standard Cosmology has many fundamental problems those include the horizon problem, the flatness problem, the entropy problem and the monopole problem [16]. A possible solution to those problems were suggested by Alan Guth using his famous inflation theory [17]. Entropy problems which plagued the original inflation model has led to a new inflation model suggested by Linde [18] which solve the entropy problem but required fine tuning of parameters. The same criticism holds for chaotic inflation also suggest by Linde [19]. On 17 March 2014, astrophysicists of the BICEP2 collaboration announced the detection of inflationary gravitational waves in the B-mode power spectrum, which if confirmed, would provide clear experimental evidence for the theory of inflation [20]. However, on 19 June 2014, lowered confidence in confirming the findings was reported.

It is the opinion of the author of this paper that a basic flaw in common to all inflation models. All inflation models require to postulate one or more scalar fields which have no function, implication or purpose in nature except for their ad-hoc use in the inflation model. This is in sharp contradiction with the principle of Occam's razor which demand that a minimum number of assumptions will explain a maximum number of phenomena. Postulating a physical field for every phenomena does not serve the purpose of theoretical science. In the words of Einstein: "Everything should be made as simple as possible, but not simpler". Moreover, it will be shown that a perfectly good explanation within the frame-work of standard Cosmology does exist for the horizon problem if one looks closely at the metric changes of the Friedman-Lemaitre-Robertson-Walker metric.

The plan of this paper is as follows: in the first section we describe possible mechanisms of metric change. In the following section we describe a particle trajectory in a general flat space. Then we analyze particle trajectories in Lorentz space-time for the standard subluminal cases. The following section will discuss dynamics in the presence of an Euclidean metric. Next we analyze particle trajectories in a Lorentz space-time but now we assume that the particles are superluminal. The final section is devoted to statistical analysis of free particles of the three different types (Euclidian, subluminal Lorentzian and superluminal Lorentzian), in which we shall attempt to describe an equilibrium probability density function of a canonical ensemble of free particles. Then the possible physical implications of the current theory are described. Finally some concluding remarks are given.

2. Possible Mechanisms of Metric Change

It was shown in [7] that among the possible flat space metrics only the Lorentzian metric is stable and can persist for a considerable region of space-time. Nevertheless one may still inquire if a mechanism exists by which a metric change does occur (in the sense that the eigen-values of the metric change signs), can we create somehow a metric of the type $g_{\mu\nu} = \text{diag} (+1, +1, -1, -1)$ in some region of space-time? The answer obviously has to do with the only reason a metric should change according to equation (1) and this is $T_{\mu\nu}$. Looking at available solution of general relativity one finds that metric changes are quite common.

The Schwarzschild square interval (in terms of spherical coordinates t, r, θ, ϕ) is given by:

$$c^2 d\tau^2 = \left(1 - \frac{r_s}{r}\right) c^2 dt^2 - \frac{dr^2}{1 - \frac{r_s}{r}} - r^2 (d\theta^2 + \sin^2 \theta d\phi^2) \quad (2)$$

In which τ is the proper time, and r_s is the Schwarzschild radius (in meters) of the massive body, which is related to its mass M by $r_s = \frac{2GM}{c^2}$. It is obvious that while for $r > r_s$ the metric is locally (up to scaling) $g_{\mu\nu} = \text{diag} (+1, -1, -1, -1)$. For $r < r_s$ the metric is locally (up to scaling) $g_{\mu\nu} = \text{diag} (-1, +1, -1, -1)$. Hence the direction of temporal and (one) spatial axis is exchanged. Notice, however, that although the sign of the eigen-values did change we are still left with a Lorentzian metric.

Another example is the Friedman-Lemaitre-Robertson-Walker square interval which is well known in cosmological models:

$$c^2 d\tau^2 = c^2 dt^2 - a(t)^2 \left(\frac{dr^2}{1 - \kappa r^2} + r^2 (d\theta^2 + \sin^2 \theta d\phi^2) \right) \quad (3)$$

$a(t)$ is known as the "scale factor" and κ may be taken to have units of length^{-2} , in which case r has units of length and $a(t)$ is unitless. κ is then the Gaussian curvature of the space at the time when $a(t) = 1$. Hence for radial distances such that $r < \frac{1}{\sqrt{\kappa}}$ the metric is locally (up to scaling) $g_{\mu\nu} = \text{diag} (+1, -1, -1, -1)$ that is Lorentzian. However, for $r > \frac{1}{\sqrt{\kappa}}$ the metric is locally (up to scaling) $g_{\mu\nu} = \text{diag} (+1, +1, -1, -1)$. This means that a particle propagating in a radial direction will experience an Euclidean metric.

One should notice that in the above cases a signature change is accompanied by a metric singularity [14] while the signature changes considered by Eddington [2] involve zeros. However, metric singularities are not curvature singularities and can be removed by proper choice of coordinates.

It will be also interesting to find a metric which is completely Euclidean in some regime of space-time, while being Lorentzian in another such a transitory metric may take the form

$$g_{\mu\nu} = \text{diag} \left(+1, 2e^{-\frac{(x_\mu - x_{0\mu})^2}{\Delta^2}} - 1, 2e^{-\frac{(x_\mu - x_{0\mu})^2}{\Delta^2}} - 1, 2e^{-\frac{(x_\mu - x_{0\mu})^2}{\Delta^2}} - 1 \right) \quad (4)$$

which is necessary to create an Euclidean domain of a width Δ located at $x_{0\mu}$. More analytical effort is needed in order to describe accurately the conditions under which space-time will become locally completely Euclidean. Here we shall simply assume that a similar solution is attained at least in a limit region of space-time.

3. Particle Trajectories in Flat Space

Let us now look at a particle travelling in a space-time with a constant metric of arbitrary form. Such a particle can be described by the Action \mathcal{A} :

$$\mathcal{A} = -mc \int d\tau - e \int A^\alpha dx_\alpha \quad (5)$$

In the above τ is the trajectory interval:

$$d\tau^2 = \left| \eta^{\alpha\beta} dx_\alpha dx_\beta \right| = |dx_\alpha dx^\alpha| \quad (6)$$

x_α are the particle coordinates (raising and lowering indices is done using the metric as is customary), m is the particle mass, e is the particle charge and A^α are some functions of the particle coordinates (that transform as a four dimensional vector). Basic variational analysis leads to the following equations of motion:

$$m \frac{du^\alpha}{d\tau} = -\frac{e}{c} u^\beta (\partial_\beta A^\alpha - \partial^\alpha A_\beta), \quad u^\alpha \equiv \frac{dx^\alpha}{d\tau} \quad (7)$$

in which the metric $\eta_{\alpha\beta}$ can be of any flat type: Lorentzian, Euclidean etc.

3.1. Lorentz Space-Time

Let us assume a Lorentz Space-Time with a metric $\eta_{\mu\nu} = \text{diag}(1, -1, -1, -1)$. Hence space-time is dissected into spatial and temporal coordinates. The spatial coordinates are $\vec{x} = (x_1, x_2, x_3)$ and the temporal coordinate is x_0 . Since it is customary to measure time in different units (seconds) than space (meters) we write $x_0 = ct$, in which c serves as a units conversion factor. We now define the velocity: $\vec{v} \equiv \frac{d\vec{x}}{dt}$, $v = |\vec{v}|$. In a similar way we dissect A_α into temporal and spatial parts:

$$A_\alpha = (A_0, A_1, A_2, A_3) \equiv (A_0, \vec{A}) \equiv \left(\frac{\phi}{c}, \vec{A}\right) \quad (8)$$

the factor $\frac{1}{c}$ in the last term allows us to obtain the equations in MKS units, it is not needed in other types of unit systems. Using equation (8), we can define a magnetic field:

$$\vec{B} = \vec{\nabla} \times \vec{A} \quad (9)$$

($\vec{\nabla}$ has the standard definition of vector analysis) and an electric field:

$$\vec{E} = -\frac{\partial \vec{A}}{\partial t} - \vec{\nabla} \phi \quad (10)$$

For subluminal particles $v < c$ we can then write $d\tau^2$ as:

$$d\tau^2 = c^2 dt^2 \left(1 - \frac{v^2}{c^2}\right), \quad d\tau = c dt \sqrt{1 - \frac{v^2}{c^2}} \quad (11)$$

And using the above equations one can write the spatial part of equation (7) as:

$$\frac{d}{dt} \left(m \frac{\vec{v}}{\sqrt{1 - \frac{v^2}{c^2}}} \right) = e \left(\vec{E} + \vec{v} \times \vec{B} \right) \quad (12)$$

The above equation shows clearly that a subluminal particle in a Lorentz space must remain subluminal. Since as the particle is accelerated to c its "effective mass" $m_{eff} \equiv \frac{m}{\sqrt{1 - \frac{v^2}{c^2}}}$ becomes infinite. On the other hand for superluminal particles (which are $v > c$ at $\tau = 0$) we can write $d\tau^2$ as:

$$d\tau^2 = c^2 dt^2 \left(\frac{v^2}{c^2} - 1\right), \quad d\tau = c dt \sqrt{\frac{v^2}{c^2} - 1} \quad (13)$$

And using the above equations one can write the spatial part of equation (7) as:

$$\frac{d}{dt} \left(m \frac{\vec{v}}{\sqrt{\frac{v^2}{c^2} - 1}} \right) = e (\vec{E} + \vec{v} \times \vec{B}) \quad (14)$$

Here the difficulty would be near the velocity c , in which its "effective mass" $m_{eff} \equiv \frac{m}{\sqrt{\frac{v^2}{c^2} - 1}}$ becomes infinite. In the absence of forces the velocity of the above particle remains constant and superluminal. We conclude that in a Lorentz space time there is a difficulty to pass the velocity c from below as is well known. In any case luminal particle with $v = c$ have $d\tau = 0$ which make this parameter unsuitable to describe the trajectory for those type of particles.

3.2. Euclidean Space-Time

Let us assume an Euclidean space-time with a metric $\eta_{\mu\nu} = \text{diag} (+1, +1, +1, +1)$. Here space-time is dissected (arbitrarily) into spatial and temporal coordinates as in the Lorentz space which are measured in the customary units. Again we dissect A_α into temporal and spatial parts as in equation (8). Using equation (8), we can define the magnetic field as in equation (9) but the electric field is defined now as:

$$\vec{E} = -\frac{\partial \vec{A}}{\partial t} + \vec{\nabla} \phi \quad (15)$$

notice that this definition for the electric field is different than in the Lorentz space but is necessary in order to maintain Faraday's law. For all particles either (subluminal or superluminal) we can than write $d\tau^2$ as:

$$d\tau^2 = c^2 dt^2 \left(1 + \frac{v^2}{c^2} \right), \quad d\tau = c dt \sqrt{1 + \frac{v^2}{c^2}} \quad (16)$$

And using the above equations one can write the spatial part of equation (7) as:

$$\frac{d}{dt} \left(m \frac{\vec{v}}{\sqrt{1 + \frac{v^2}{c^2}}} \right) = e (\vec{E} - \vec{v} \times \vec{B}) \quad (17)$$

The above equation shows clearly that particles in an Euclidean space are quite indifferent to passing the velocity c .

4. Statistical Physics

The definition of a probability density function is intimately connected to the notion of phase space. This in turn arises naturally when time is the independent variational variable. The path to the Hamiltonian formalism goes through defining a Lagrangian at the action per unit time and through the Lagrangian one can define the canonical momenta and finally the Hamiltonian. We shall follow this route.

4.1. Lagrangian and Canonical Momenta

Let us write the action given in equation (5) as:

$$\mathcal{A} = -mc \int d\tau - e \int A^\alpha dx_\alpha = - \int dt \left[mc \frac{d\tau}{dt} + e A^\alpha \frac{dx_\alpha}{dt} \right] \quad (18)$$

Introducing the notation:

$$v_\alpha \equiv \frac{dx_\alpha}{dt} = (c, \vec{v}) \quad (19)$$

and using the definition of equation (6) it follows that:

$$\frac{d\tau}{dt} = \sqrt{\left| \frac{dx_\alpha}{dt} \frac{dx^\alpha}{dt} \right|} = \sqrt{|v_\alpha v^\alpha|} = \sqrt{|c^2 + v_i v^i|} \quad (20)$$

in which the Latin indices are $i \in \{1, 2, 3\}$. The upper index v^i has the following meaning:

$$v^i = \begin{cases} +v_i & \text{Euclidean metric} \\ -v_i & \text{Lorentz metric} \end{cases} \quad (21)$$

Thus:

$$\begin{aligned} \frac{d\tau}{dt} &= \begin{cases} \sqrt{|c^2 + \vec{v}^2|} & \text{Euclidean metric} \\ \sqrt{|c^2 - \vec{v}^2|} & \text{Lorentz metric} \end{cases} \\ &= \begin{cases} \sqrt{c^2 + v^2} & \text{Euclidean metric} \\ \sqrt{c^2 - v^2} & \text{Lorentz metric subluminal case } v < c \\ \sqrt{v^2 - c^2} & \text{Lorentz metric superluminal case } v > c \end{cases}, \end{aligned} \quad (22)$$

It follows from equation (18) that one can define a Lagrangian:

$$\begin{aligned} \mathcal{A} = \int dt L, \quad L &\equiv - \left[mc \frac{d\tau}{dt} + e A^\alpha \frac{dx_\alpha}{dt} \right] = - \left[mc \sqrt{|v_\alpha v^\alpha|} + e A^\alpha v_\alpha \right] \\ &= - \left[mc \sqrt{|c^2 + v_i v^i|} + e c A^0 + e A^i v_i \right]. \end{aligned} \quad (23)$$

This leads to a canonical momentum of the form:

$$p^i \equiv \frac{\partial L}{\partial v_i} = -mc \frac{\pm v^i}{\sqrt{|c^2 + v_i v^i|}} - e A^i \quad (24)$$

the sign is decided according whether the absolute value changes or does not change the sign of $c^2 + v_i v^i$. It does not change sign of course in the standard subluminal Lorentzian case but also the Euclidean case. On the other the superluminal Lorentzian case involves a sign change. Hence:

$$p^i = \begin{cases} -mc \frac{v^i}{\sqrt{|c^2 + v_i v^i|}} - e A^i & \text{Lorentzian subluminal or Euclidean} \\ +mc \frac{v^i}{\sqrt{|c^2 + v_i v^i|}} - e A^i & \text{Lorentzian superluminal} \end{cases} \quad (25)$$

Introducing the standard notation:

$$\beta_i \equiv \frac{v_i}{c}, \quad \beta^i \equiv \frac{v^i}{c}, \quad \beta \equiv \frac{v}{c}, \quad \gamma \equiv \frac{1}{\sqrt{|1 + \frac{v_i v^i}{c^2}|}} = \frac{1}{\sqrt{|1 + \beta_i \beta^i|}} \quad (26)$$

in which:

$$\gamma = \begin{cases} \frac{1}{\sqrt{1 + \frac{v^2}{c^2}}} & \text{Euclidean metric} \\ \frac{1}{\sqrt{1 - \frac{v^2}{c^2}}} & \text{Lorentz metric subluminal case } v < c \\ \frac{1}{\sqrt{\frac{v^2}{c^2} - 1}} & \text{Lorentz metric superluminal case } v > c \end{cases}. \quad (27)$$

We may write:

$$p^i = \begin{cases} -\gamma m v^i - e A^i & \text{Lorentzian subluminal or Euclidean} \\ +\gamma m v^i - e A^i & \text{Lorentzian superluminal} \end{cases} \quad (28)$$

We can define a "free" momentum as:

$$p_f^i = p^i + e A^i = \begin{cases} -\gamma m v^i & \text{Lorentzian subluminal or Euclidean} \\ +\gamma m v^i & \text{Lorentzian superluminal} \end{cases} \quad (29)$$

this is a misnomer, as the case $A^i = 0$ includes the free particle case but applies also to the case in which the particle moves under the influence of a scalar electric potential. Defining a canonical momentum vector we have:

$$\vec{p} \equiv (p^1, p^2, p^3) = \begin{cases} \gamma m \vec{v} + e \vec{A} & \text{Lorentzian subluminal} \\ -\gamma m \vec{v} - e \vec{A} & \text{Euclidean} \\ -\gamma m \vec{v} + e \vec{A} & \text{Lorentzian superluminal} \end{cases} \quad (30)$$

in which we have used equation (21) and also:

$$A^i = \begin{cases} +A_i & \text{Euclidean metric} \\ -A_i & \text{Lorentz metric.} \end{cases} \quad (31)$$

And similarly:

$$\vec{p}_f \equiv (p_f^1, p_f^2, p_f^3) = \begin{cases} \gamma m \vec{v} & \text{Lorentzian subluminal} \\ -\gamma m \vec{v} & \text{Euclidean} \\ -\gamma m \vec{v} & \text{Lorentzian superluminal.} \end{cases} \quad (32)$$

Interestingly the "free" momentum has the same direction as the velocity only in the (standard) Lorentzian subluminal case, in all other cases the momentum direction is the opposite. The Lorentzian subluminal case reduces to the classical result for low velocities:

$$v \ll c \Rightarrow \gamma \simeq 1 \Rightarrow \vec{p} \simeq m \vec{v} + e \vec{A} \quad (33)$$

In all cases the magnitude of the "free" momentum is:

$$p_f = |\vec{p}_f| = \gamma m v. \quad (34)$$

Thus a subluminal Lorentzian "free" particle will have zero momentum for $v = 0$ and infinite momentum for $v = c$ in which the momentum is an increasing function of v (see figure 1), hence the phase space is non compact. In the Euclidean case the small velocity momentum is similar to the Lorentzian case, however, the momentum space in this case is limited inside a "momentum sphere" hence it is compact. Thus a subluminal Lorentzian "free" particle will have zero momentum for $v = 0$ and a momentum of $p_{fE} = mc$ for $v = \infty$. The momentum is an increasing function of velocity which is bounded. (see figures 2 and 3). Finally for a superluminal Lorentzian particle we have $p_{fLsup} = \infty$ for the case $v = c$ and $p_{fLsup} = mc$ for the case $v = \infty$. The momentum is a decreasing function of velocity (see figure 4) which differs considerably from our habits and physical intuition. The momentum space is not compact, however, it has spherical hole in its middle of radius mc . One could say that the set union of the momentum space in the Euclidean case and the momentum space in the superluminal Lorentzian case is equal to the momentum space of the subluminal Lorentzian case, this situation is depicted in figure 5. In terms of the γ notation we can write the Lagrangian as:

$$L = - \left[\frac{mc^2}{\gamma} + ecA^0 + eA^i v_i \right]. \quad (35)$$

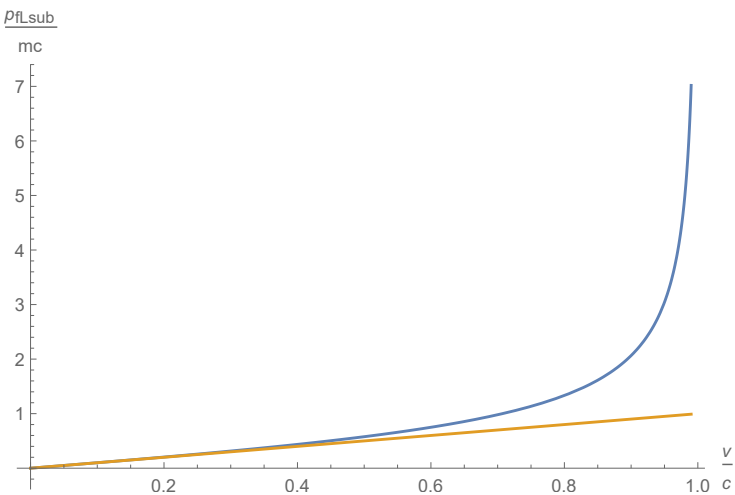


Figure 1. Free momentum for Lorentzian subluminal particles, the correct expression is compared with the classical one. The blue line is the correct expression while the orange line is the classical approximation.

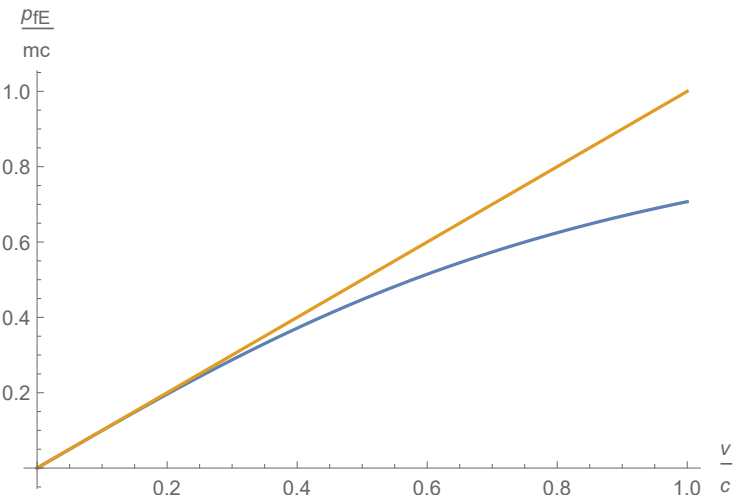


Figure 2. Free momentum for Euclidean subluminal particles, the correct expression is compared with the classical one. The blue line is the correct expression while the orange line is the classical approximation.

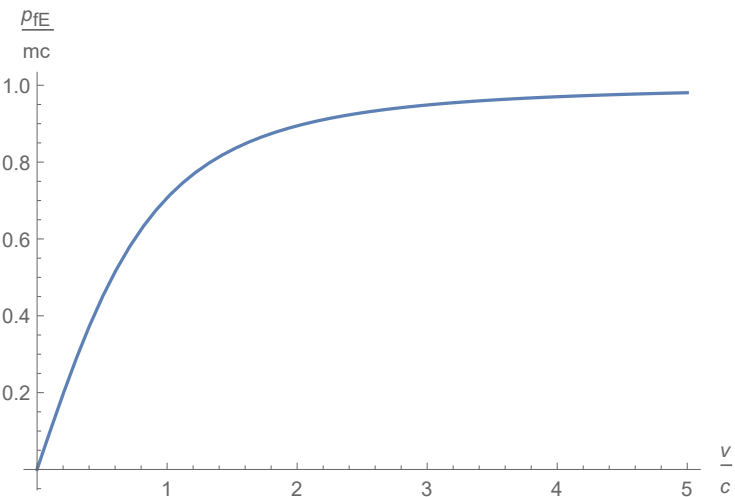


Figure 3. Free momentum for Euclidean particles.

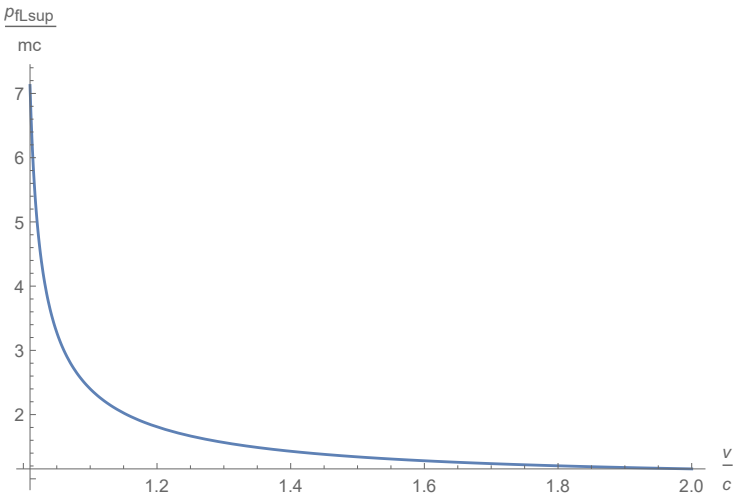


Figure 4. Free momentum for Lorentzian superluminal particles.

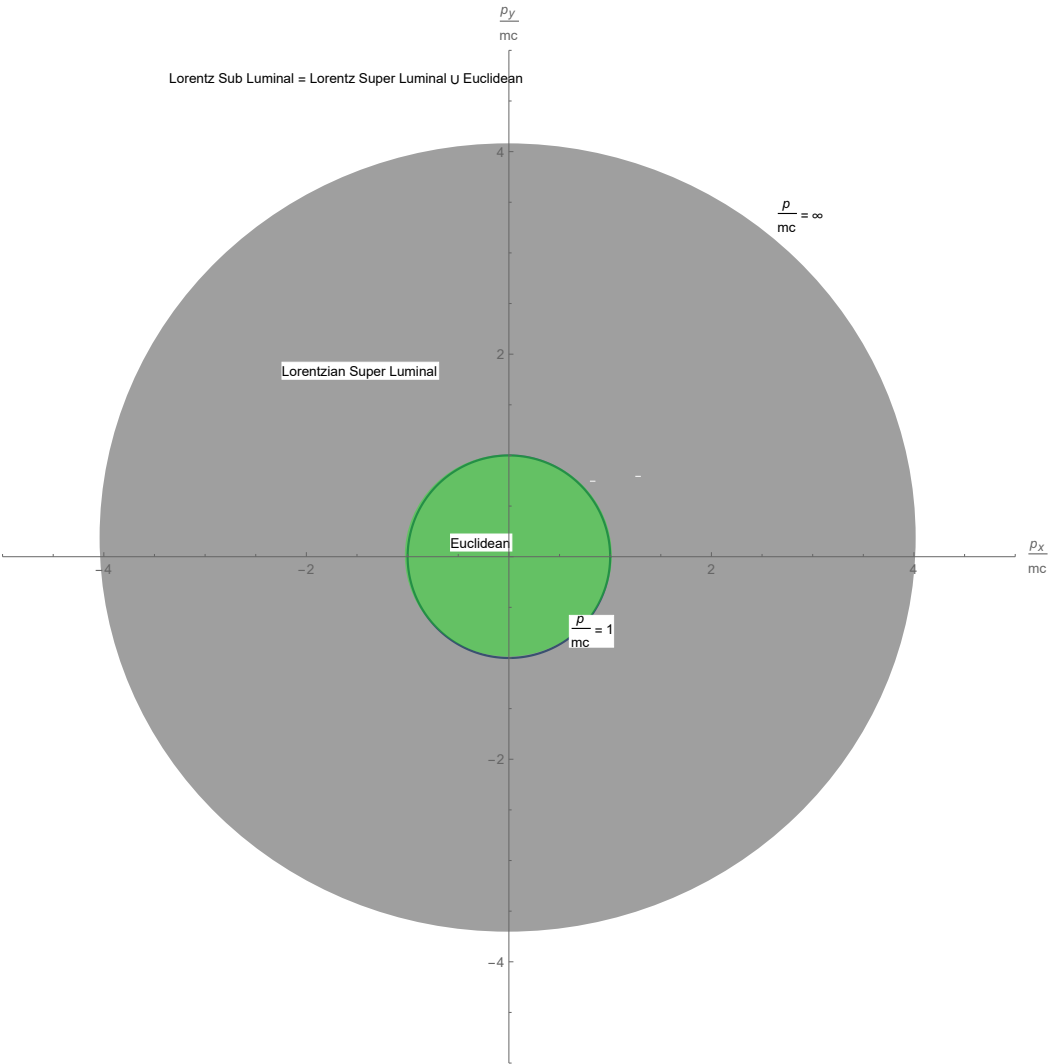


Figure 5. A depiction of the cross section of the momentum space for the cases considered: Euclidean (a compact sphere) Lorentzian superluminal (an infinite domain with a spherical hall) and Lorentzian subluminal (the entire momentum plane). The later is a union of the former.

For small velocities in a Lorentzian space time we have:

$$\gamma \simeq 1 + \frac{1}{2} \frac{v^2}{c^2}, \quad \gamma^{-1} \simeq 1 - \frac{1}{2} \frac{v^2}{c^2}. \quad (36)$$

Hence the classical Lagrangian would be:

$$L_c = \frac{1}{2}mv^2 - (mc^2 + e\phi - e\vec{A} \cdot \vec{v}) = \frac{1}{2}mv^2 + e\vec{A} \cdot \vec{v} - e\phi - mc^2. \quad (37)$$

Finally we will be interested in the dependence of v on p_f , first notice that:

$$m\gamma = \frac{p_f}{v} \Rightarrow m^2v^2 = p_f^2\gamma^{-2} = p_f^2 \left| 1 + \frac{v_i v^i}{c^2} \right|. \quad (38)$$

Thus we obtain:

$$v = \begin{cases} \frac{p_f}{\sqrt{m^2 + \frac{p_f^2}{c^2}}} & \text{Lorentzian subluminal} \\ \frac{p_f}{\sqrt{m^2 - \frac{p_f^2}{c^2}}} & \text{Euclidean} \\ \frac{p_f}{\sqrt{\frac{p_f^2}{c^2} - m^2}} & \text{Lorentzian superluminal} \end{cases} \quad (39)$$

4.2. Hamiltonian and Energy

Once we have the canonical momenta and Lagrangian it is straight forward to calculate the Hamiltonian:

$$\begin{aligned} H &= \vec{v} \cdot \vec{p} - L = p^i v_i - L = p^i v_i + \frac{mc^2}{\gamma} + ecA^0 + eA^i v_i \\ &= (\mp \gamma m v^i - eA^i) v_i + \frac{mc^2}{\gamma} + ecA^0 + eA^i v_i = \mp \gamma m v^i v_i + \frac{mc^2}{\gamma} + ecA^0. \end{aligned} \quad (40)$$

in the above the minus sign is for the Euclidean and subluminal Lorentzian cases while the plus sign is for the superluminal Lorentzian case. This can be simplified as follows:

$$\begin{aligned} H &= \gamma m \left(\mp v^i v_i + \frac{c^2}{\gamma^2} \right) + e\phi = \gamma m \left(\mp v^i v_i + c^2 \left| 1 + \beta_i \beta^i \right| \right) + e\phi \\ &= \gamma m \left(\mp v^i v_i + \left| c^2 + v_i v^i \right| \right) + e\phi = \pm \gamma m c^2 + e\phi. \end{aligned} \quad (41)$$

The plus sign in the last terms belongs to the Euclidean and subluminal Lorentzian cases while the minus sign is for the superluminal Lorentzian case. Explicitly:

$$H = \begin{cases} \gamma m c^2 + e\phi & \text{Euclidean and subluminal Lorentzian} \\ -\gamma m c^2 + e\phi & \text{superluminal Lorentzian} \end{cases} \quad (42)$$

The energy of the particle is $E_n = H$ and will remain constant as long as ϕ does not explicitly depend on time. The structure of the Hamiltonian is:

$$H = E_k + e\phi \quad (43)$$

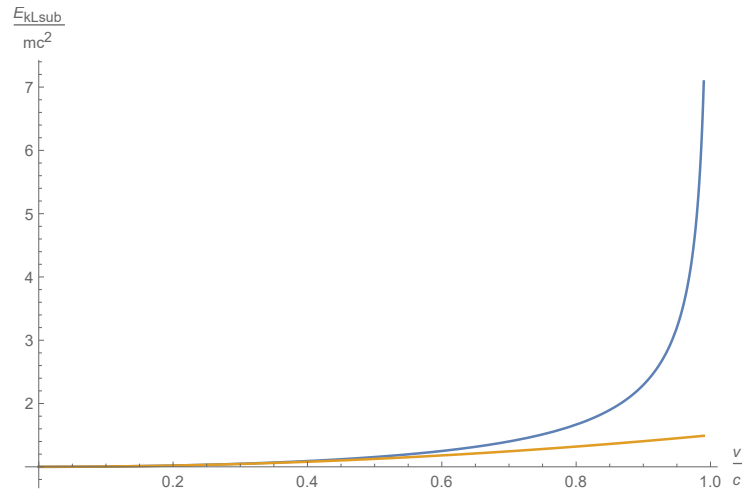


Figure 6. Kinetic energy for Lorentzian subluminal particles, the correct expression is compared with the classical one. The blue line is the correct expression while the orange line is the classical approximation

in which:

$$E_k \equiv \begin{cases} \frac{mc^2}{\sqrt{1+\frac{v^2}{c^2}}} & \text{Euclidean metric} \\ \frac{mc^2}{\sqrt{1-\frac{v^2}{c^2}}} & \text{Lorentz metric subluminal case } v < c \\ -\frac{mc^2}{\sqrt{\frac{v^2}{c^2}-1}} & \text{Lorentz metric superluminal case } v > c \end{cases} \quad (44)$$

The kinetic energy has the following attributes. For the Lorentz subluminal case (which is the standard case) the minimal value for the kinetic energy is the rest energy $E_{kLsub \min} = mc^2$ obtained for $v = 0$. However, it can reach an infinite value for velocities approaching the speed of light in vacuum c :

$$E_{kLsub \max} = \lim_{v \rightarrow c} E_{kLsub} = +\infty. \quad (45)$$

It is always positive for all values of v . In the classical case in which $v \ll c$ we can partition the energy into a "classical kinetic energy" and a rest energy:

$$E_{kLsub} \simeq mc^2 + E_{kLsubc}, \quad E_{kLsubc} \equiv \frac{1}{2}mv^2, \quad v \ll c. \quad (46)$$

The expression E_{kLsub} is depicted in figure 6. In the Euclidean case the kinetic energy is always positive, it has a maximal value for a particle in rest: $E_{kE \max} = mc^2$ and a minimal value of zero for a particle travelling at an infinite speed, we recall that there are no speed limitation in an Euclidean space-time.

$$E_{kE \min} = \lim_{v \rightarrow +\infty} E_{kE} = 0. \quad (47)$$

Curiously one can define a "classical kinetic energy" also in the Euclidean case, but it will be negative:

$$E_{kE} \simeq mc^2 + E_{kEc}, \quad E_{kEc} \equiv -\frac{1}{2}mv^2, \quad v \ll c. \quad (48)$$

The expression E_{kE} is depicted in figure 7 for subluminal velocities and in figure 8 for superluminal velocities. Finally we consider the superluminal Lorentzian kinetic energy, which differs from the previous cases in the attribute that it is always non positive. Its

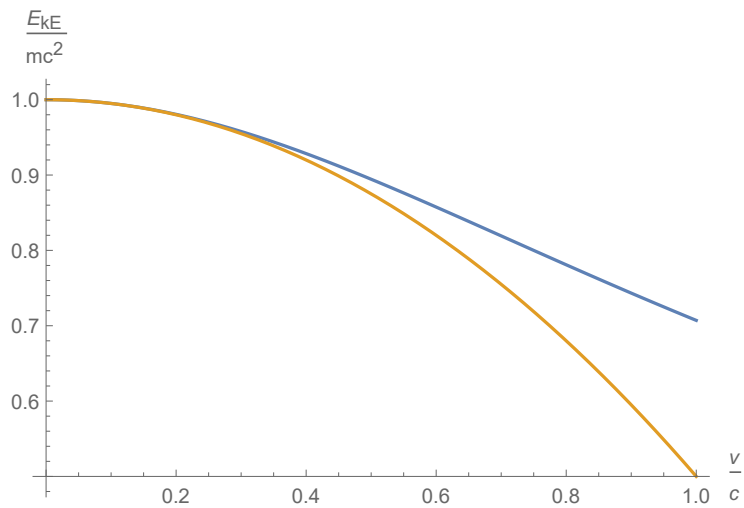


Figure 7. Kinetic energy for Euclidean particles, the correct expression (blue) is compared with the classical approximation (orange).

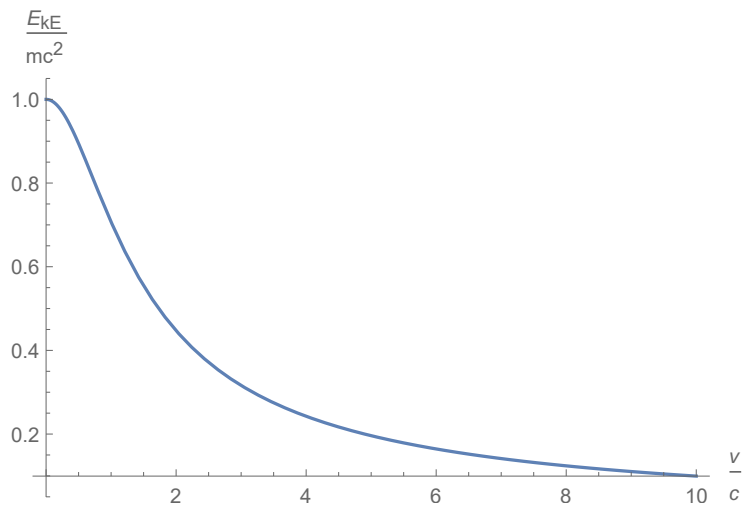


Figure 8. Kinetic energy for Euclidean particles of subluminal and superluminal velocities.

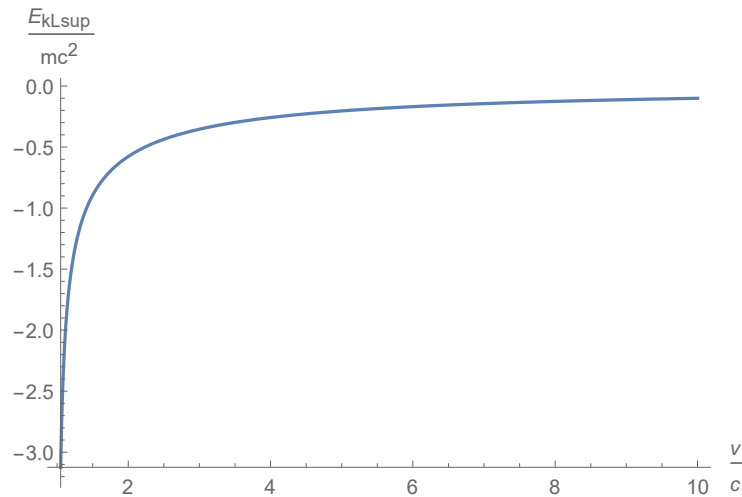


Figure 9. Kinetic energy for Lorentzian particles of superluminal velocities.

maximal value of zero is attained for infinite velocities and it can reach minus infinity when the velocity of the particle is reduced down to the speed of light c , a limit it cannot reach. Thus:

$$E_{kLsup \max} = \lim_{v \rightarrow +\infty} E_{kLsup} = 0, \quad E_{kLsup \min} = \lim_{v \rightarrow c} E_{kLsup} = -\infty. \quad (49)$$

of course there is no sense of discussing the classical limit in this case, as by definition superluminality requires $v > c$. The expression E_{kLsup} is depicted in figure 9. Let us suppose that the potentials are time independent and the energy is conserved, it follows that the energy E_n is conserved and for any two points \vec{x}_1 and \vec{x}_2 on the trajectory:

$$E_n = E_{k1} + e\phi_1 = E_{k2} + e\phi_2 \Rightarrow E_{k2} = E_{k1} + e(\phi_1 - \phi_2) \quad (50)$$

Thus the kinetic energy can be increased or decreased using a potential difference, this is of course well known and is used for accelerating and decelerating charged particles in electrostatic accelerators like the tandem accelerator located in Ariel university [24]. For a subluminal particle an increase in the kinetic energy means an increase in velocity and thus using a potential difference a charged particle can be accelerated. Similarly by using a potential difference with an opposite sign the particle becomes slower as its kinetic energy is reduced. For Euclidean particles the situation is opposite, particles with higher kinetic energy are slower and with low kinetic energy are faster, nevertheless, potential differences can be still used to achieve acceleration and deceleration. Superluminal Lorentzian particles are similar to the subluminal Lorentzian particles in the sense that the (negative) kinetic energy would be lower for slower particles and higher for faster particles. Another important difference between Euclidean and Lorentzian particles is lack of velocity limits in the former. In fact it is easy to see that using a finite energy equal to its rest mass (mc^2) a particle can be accelerated from zero velocity to infinite velocity in an Euclidean space-time. This is of course impossible for a Lorentzian particle. In the sub luminal case we will need an infinite amount of energy to accelerate the particle to the speed of light, while for a superluminal particle an infinite amount of energy will be needed to reduce its speed to the speed of light. This entails an infinite potential difference and thus is physically impossible. We remark that even if the electromagnetic potentials are time dependent (as in the popular accelerator scheme of RF Linacs [25]) this limitations cannot be avoided because that although the total energy of the particle can be increased in this scenario and is not necessarily constant, it cannot increase to infinite values (which should be supplied from an infinite reservoir). Thus in a Lorentzian universe subluminal and superluminal particles must be separated by their velocities forever. We will discuss the cosmological and other implications of those facts later in this paper.

To conclude this subsection we would like to express the Hamiltonian as a function of the coordinates and the momenta. Using equation (42) and equations (38) and (39) this can be written as follows:

$$H = \begin{cases} c^2 \sqrt{m^2 + \frac{p_f^2}{c^2}} + e\phi & \text{Lorentzian subluminal} \\ c^2 \sqrt{m^2 - \frac{p_f^2}{c^2}} + e\phi & \text{Euclidean} \\ -c^2 \sqrt{\frac{p_f^2}{c^2} - m^2} + e\phi & \text{Lorentzian superluminal} \end{cases} \quad (51)$$

For Lorentzian classical particles equation (33) holds hence $p_f \simeq mv$ and thus:

$$H_c \simeq mc^2 + \frac{p_f^2}{2m} + e\phi \quad (52)$$

Using equation (29) we arrive at:

$$H(\vec{x}, \vec{p}) = \begin{cases} c^2 \sqrt{m^2 + \frac{(\vec{p} - e\vec{A}(\vec{x}))^2}{c^2}} + e\phi(\vec{x}) & \text{Lorentzian subluminal} \\ c^2 \sqrt{m^2 - \frac{(\vec{p} + e\vec{A}(\vec{x}))^2}{c^2}} + e\phi(\vec{x}) & \text{Euclidean} \\ -c^2 \sqrt{\frac{(\vec{p} - e\vec{A}(\vec{x}))^2}{c^2} - m^2} + e\phi(\vec{x}) & \text{Lorentzian superluminal} \end{cases} \quad (53)$$

the first line in the above is Jackson's [23] formula (12.17). In the classical case we have:

$$H_c(\vec{x}, \vec{p}) \simeq mc^2 + \frac{(\vec{p} - e\vec{A}(\vec{x}))^2}{2m} + e\phi(\vec{x}) \quad (54)$$

4.3. Statistical Physics of "Classical" Particles

Once we have a phase space we may discuss what form of the probability density function for a particle to be in a specific part of this space. Elementary considerations show [26] that in thermal equilibrium this function must depend on the constants of motion of the system, in particular its energy.

$$f_{\text{system}} = f_{\text{system}}(H) = f_{\text{system}}(E_{\text{system}}) \quad (55)$$

Further if the system can be partitioned into two sub systems A and B of which the interaction is negligible (as in the case of free particles) it follows that:

$$f_{\text{system}}(E_{\text{system}}) = f_{\text{system}}(E_A + E_B) = f_A(E_A)f_B(E_B) \quad (56)$$

This leads after a few trivial steps to the result that:

$$f = \frac{e^{-\beta_T H}}{Z} \quad (57)$$

Z the normalization constant also known as the partition function. $\beta_T = \frac{1}{k_B T}$, in which k_B is the Boltzmann constant:

$$k_B \equiv 1.380649 \cdot 10^{-23} \text{ m}^2 \text{ kg s}^{-2} \text{ K}^{-1} \quad (58)$$

and T is the temperature measured in degrees Kelvin. In what follows we will consider only free particles, more over we will consider only a single free particle. In this case:

$$Z = \int e^{-\beta_T H} d^3 p \quad (59)$$

In all the relevant cases H depends only the absolute value $p = |\vec{p}|$, hence the integral simplifies to:

$$Z = 4\pi \int e^{-\beta_T H} p^2 dp \quad (60)$$

We will start with the more familiar case of a subluminal Lorentzian particle and consider the more exotic cases later.

4.3.1. A Low speed Lorentzian particle

For a small velocity free Lorentzian particle equation (54) takes the form:

$$H_c = mc^2 + \frac{p^2}{2m} \quad (61)$$

hence:

$$f(\vec{p}) = \frac{e^{-\beta_T mc^2} e^{-\beta_T \frac{p^2}{2m}}}{Z}. \quad (62)$$

The partition function can be calculated to be:

$$Z = 4\pi \int e^{-\beta_T H_c} p^2 dp = (2\pi mk_B T)^{\frac{3}{2}} e^{-\beta_T mc^2} \quad (63)$$

Thus we obtain the well known Maxwell-Boltzmann probability density function:

$$f(\vec{p}) = \frac{e^{-\frac{p^2}{2mk_B T}}}{(2\pi mk_B T)^{\frac{3}{2}}}. \quad (64)$$

This is a typical Gaussian distribution with a null average and a variance which is linear in the temperature and a standard deviation which is the square root of the same:

$$E[p^i] = 0, \quad E[p^{i2}] = mk_B T, \quad \sigma_{p^i} = \sqrt{mk_B T}. \quad (65)$$

Using equation (46) we obtain the well known result for the average of the classical kinetic energy:

$$E[E_{kLsub}] = E[\frac{1}{2}mv^2] = E[\frac{p^2}{2m}] = \frac{3mk_B T}{2m} = \frac{3}{2}k_B T \quad (66)$$

Thus the average of the total kinetic energy which includes a rest energy term:

$$E[E_{kLsub}] \simeq mc^2 + \frac{3}{2}k_B T. \quad (67)$$

In what follows expressions will appear simpler using a normalized momenta and β_T defined as follows:

$$\vec{p}' \equiv \frac{\vec{p}}{mc}, \quad \lambda \equiv \beta_T mc^2 = \frac{mc^2}{k_B T}. \quad (68)$$

Thus low λ means high temperature, and high λ means low temperature. In terms of those we may write the classical distribution as:

$$f(\vec{p}') = \left(\frac{\lambda}{2\pi}\right)^{\frac{3}{2}} e^{-\frac{\lambda p'^2}{2}}, \quad Z' = \left(\frac{2\pi}{\lambda}\right)^{\frac{3}{2}} e^{-\lambda}. \quad (69)$$

Thus:

$$\lim_{\lambda \rightarrow 0} Z' = \infty, \quad \lim_{\lambda \rightarrow \infty} Z' = 0. \quad (70)$$

In term of λ the average energy becomes:

$$\bar{E}'_{kLsub} = \frac{E[E_{kLsub}]}{mc^2} \simeq 1 + \frac{3}{2\lambda} \quad (71)$$

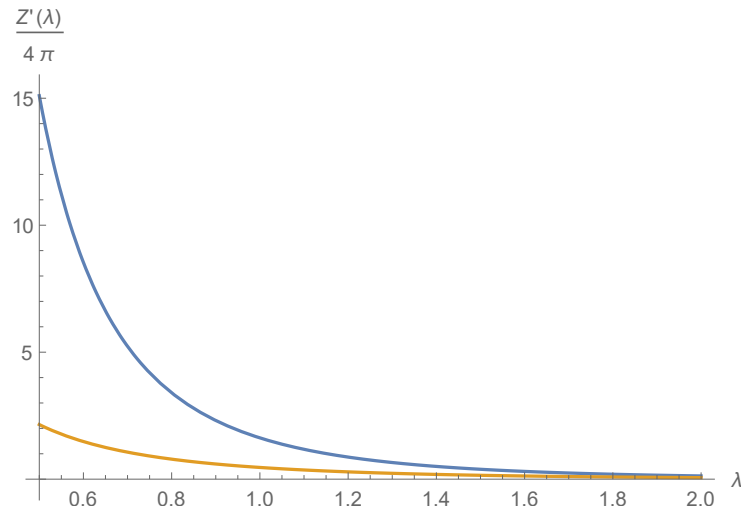


Figure 10. Partition function for a free subluminal Lorentzian particle, the blue line is the correct value while the orange represents the low velocity approximation.

for high λ (low velocities).

4.3.2. A Lorentzian particle

Generally speaking the Maxwell-Boltzmann probability density function does not describe the momentum distribution function for a Lorentzian particle unless the velocities are much smaller than the speed of light. Taking into account equation (53) for a free particle, and equation (57) we arrive at:

$$f(\vec{p}') = \frac{e^{-\beta_T H}}{Z'} = \frac{e^{-\lambda \sqrt{1+p'^2}}}{Z'} \quad (72)$$

in which:

$$Z'(\lambda) = 4\pi \int_0^\infty e^{-\lambda \sqrt{1+p'^2}} p'^2 dp' \quad (73)$$

The above expression cannot be evaluated analytically, but can be easily evaluated numerically (see figure 10) in which we compare the results to the classical case. As can be clearly seen the results converge for high λ (low temperature) but differ considerably for small λ (high temperature). For a Lorentzian subluminal particle we have:

$$\lim_{\lambda \rightarrow 0} Z' = \infty, \quad \lim_{\lambda \rightarrow \infty} Z' = 0, \quad (74)$$

the above result is similar to the case of a classical particle. Having calculated the partition function we are now in a position to calculate the probability density function. We present a two dimensional plot in figure 11, and two cross section for low and high λ in figures 12 and 13. It is clear that the Maxwell-Boltzmann approximation is only appropriate for low temperatures (high λ) but fails completely at high temperatures. Finally we calculate the average energy:

$$\begin{aligned} \bar{E}'_{kLsub}(\lambda) &= \frac{E[E_{kLsub}]}{mc^2} = E[\sqrt{1+p'^2}] = 4\pi \int_0^\infty \sqrt{1+p'^2} f(\vec{p}') p'^2 dp' \\ &= \frac{4\pi}{Z'(\lambda)} \int_0^\infty \sqrt{1+p'^2} e^{-\lambda \sqrt{1+p'^2}} p'^2 dp' = -\frac{1}{Z'(\lambda)} \frac{dZ'(\lambda)}{d\lambda} = -\frac{d \ln Z'(\lambda)}{d\lambda} \end{aligned} \quad (75)$$

this expression can be evaluated numerically and is depicted in figure 14 for high λ and figure 15 for low λ . Again we notice that the classical approximation is only valid for high λ (low temperature). In any case (exact or approximated) the average energy is a decreasing

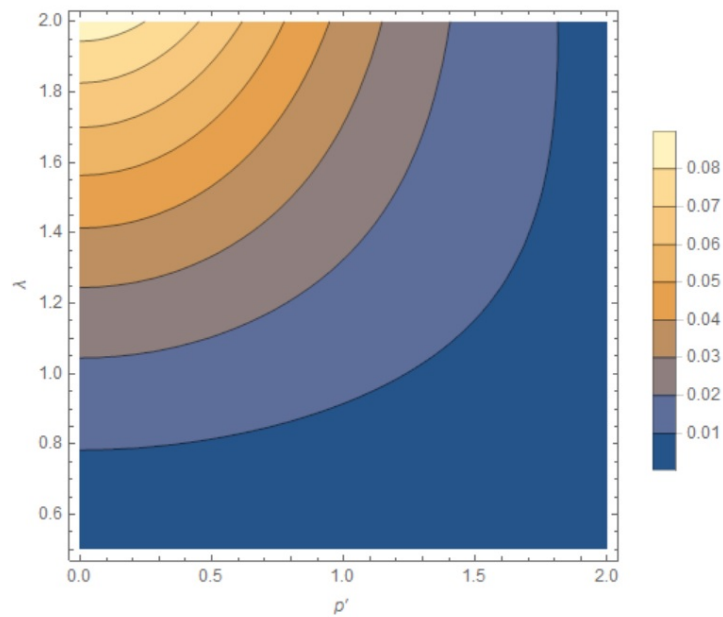


Figure 11. Probability density function for a free subluminal Lorentzian particle as function of p' and λ .

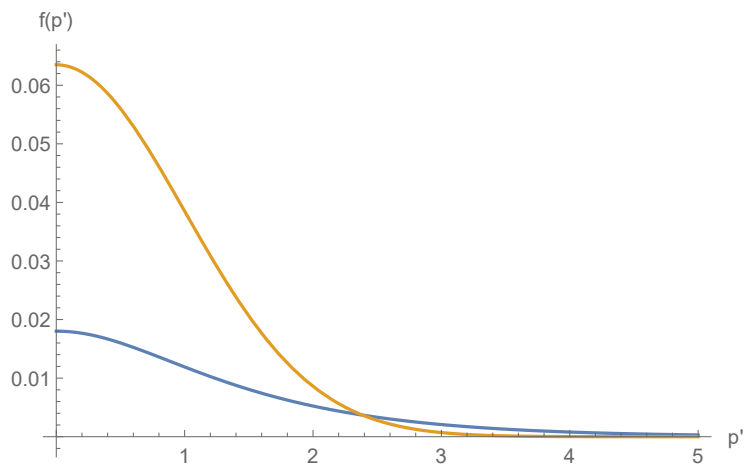


Figure 12. Probability density function for a free subluminal Lorentzian particle as function of p' for $\lambda = 1$. The blue line depicts the correct value, while the orange line depicts the Maxwell-Boltzmann approximation.

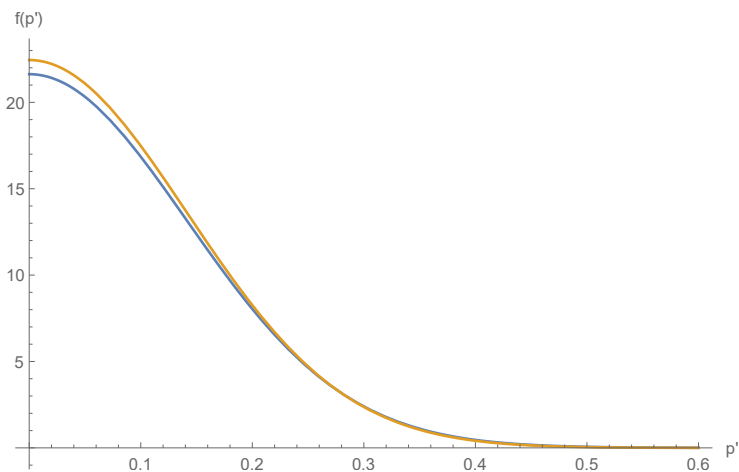


Figure 13. Probability density function for a free subluminal Lorentzian particle as function of p' for $\lambda = 50$. The blue line depicts the correct value, while the orange line depicts the Maxwell-Boltzmann approximation.

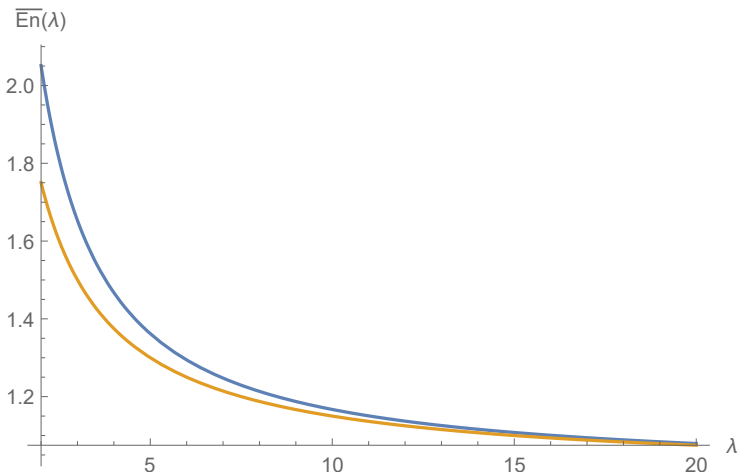


Figure 14. Average energy for free Lorentzian subluminal particles with high λ . The blue line depicts the correct value, while the orange line depicts the Maxwell-Boltzmann approximation.

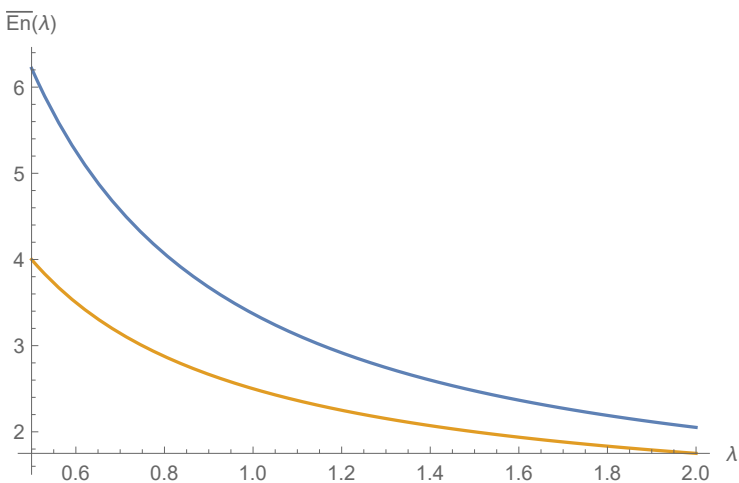


Figure 15. Average energy for free Lorentzian subluminal particles with high λ . The blue line depicts the correct value, while the orange line depicts the Maxwell-Boltzmann approximation.

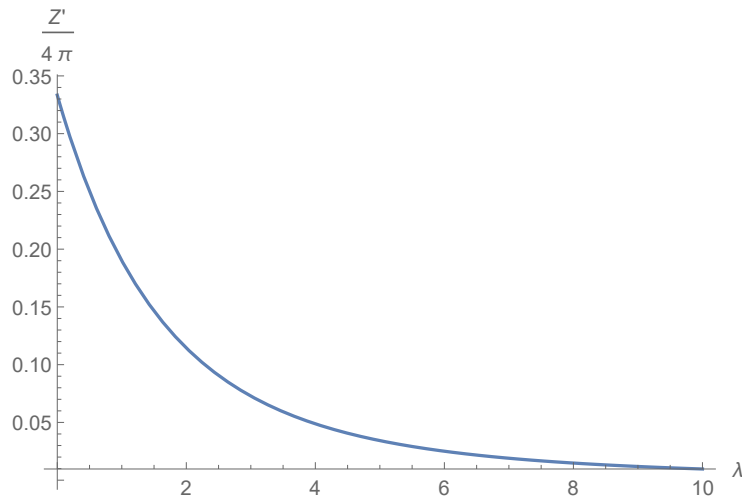


Figure 16. Partition function for a free Euclidean particle.

function of λ or an increasing function of temperature as might be expected. However, for Euclidean particles the results are less intuitive as will be shown below.

4.3.3. An Euclidean particle

As we saw in previous sections, Euclidean particles share with Lorentzian particle the energy positivity property. However, they differ in the structure of their phase space considerably thus taking into account equation (53) for a free particle, and equation (57) we arrive at:

$$f(\vec{p}') = \frac{e^{-\beta_T H}}{Z'} = \frac{e^{-\lambda \sqrt{1-p'^2}}}{Z'} \quad (76)$$

in which:

$$Z'(\lambda) = 4\pi \int_0^1 e^{-\lambda \sqrt{1-p'^2}} p'^2 dp' \quad (77)$$

in which we recall that a free Euclidean particle phase space is compact and that $0 \leq p' \leq 1$. The above expression cannot be evaluated analytically, but can be easily evaluated numerically (see figure 16). As can be clearly seen the partition function is a decreasing function of λ or an increasing function of temperature. For an Euclidean particle we have:

$$\lim_{\lambda \rightarrow 0} Z' = \frac{4\pi}{3}, \quad \lim_{\lambda \rightarrow \infty} Z' = 0, \quad (78)$$

Having calculated the partition function we are now in a position to calculate the probability density function. We present a two dimensional plot in figure 17, and a cross section in figure 76. It is remarkable that it is more probable to find an Euclidean particle with high momentum than in low momentum, this is in sharp contradiction to the situation for Lorentzian subluminal particles that prefer to stay in lower momenta. Of course in both cases high momenta means high velocity. However, this fact correlates well with the energy being a decreasing function of velocity in the Euclidean case. Finally we calculate the average energy:

$$\begin{aligned} \bar{E}'_{kE}(\lambda) &= \frac{E[E_{kE}]}{mc^2} = E[\sqrt{1-p'^2}] = 4\pi \int_0^1 \sqrt{1-p'^2} f(\vec{p}') p'^2 dp' \\ &= \frac{4\pi}{Z'(\lambda)} \int_0^1 \sqrt{1-p'^2} e^{-\lambda \sqrt{1-p'^2}} p'^2 dp' = -\frac{1}{Z'(\lambda)} \frac{dZ'(\lambda)}{d\lambda} = -\frac{d \ln Z'(\lambda)}{d\lambda} \end{aligned} \quad (79)$$

this expression can be evaluated numerically and is depicted in figure 19. Thus the average energy in the Euclidean case is a decreasing function of λ as in the Lorentzian subluminal case. Thus it is an increasing function of temperature as might be expected. However, for

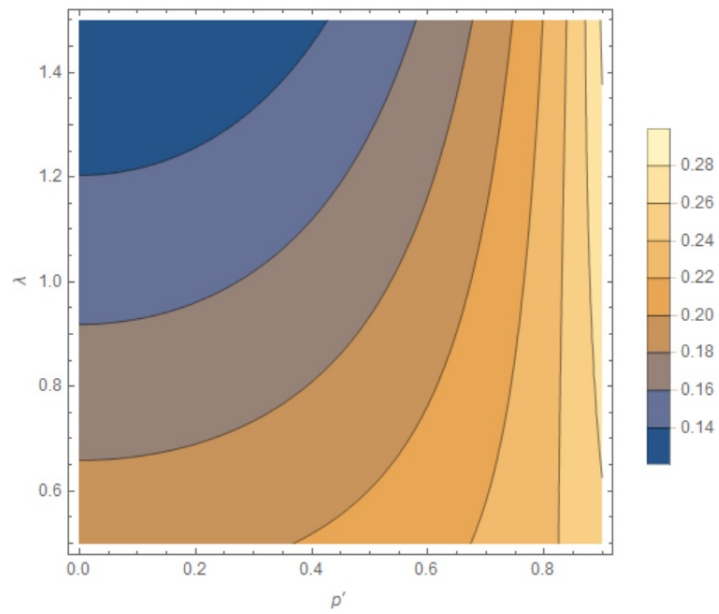


Figure 17. Probability density function for a free Euclidean particle as function of p' and λ .

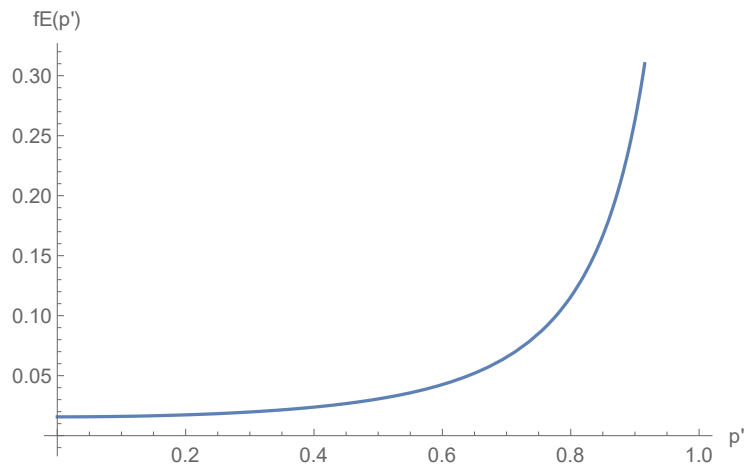


Figure 18. Probability density function for a free Euclidean particle as function of p' for $\lambda = 5$.

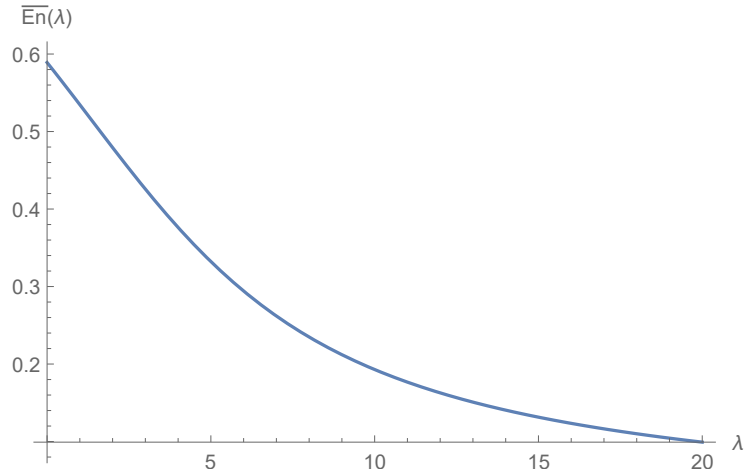


Figure 19. Average energy for free Euclidean particles with high λ .

Euclidean particles high temperature and high average energies entail low velocities. And low temperatures in the scale of mc^2 imply high velocities. A cooling down Euclidean universe will have a larger proportion of extremely fast moving particles.

4.3.4. A Lorentzian superluminal particle

As we saw in previous sections, the phase space of a free Lorentzian superluminal particle is complementary to the phase space of an Euclidean particle. Moreover, this case is unique with respect to the two previous cases as its free energy is negative. Taking into account equation (53) for a free particle, and equation (57) we arrive at:

$$f(\vec{p}') = \frac{e^{-\beta_T H}}{Z'} = \frac{e^{\lambda \sqrt{p'^2 - 1}}}{Z'} \quad (80)$$

in which:

$$Z'(\lambda) = 4\pi \int_1^\infty e^{\lambda \sqrt{p'^2 - 1}} p'^2 dp' \quad (81)$$

in which we recall that for a superluminal Lorentzian free particle phase space the unit sphere is excluded thus $1 < p'$. It easy to see that area below this probability density function is infinite for every $\lambda > 0$ and thus Z' diverges. The proof is as a follows: choose any finite $p'_L \gg 1$ thus we may write:

$$Z'(\lambda) = 4\pi \int_1^{p'_L} e^{\lambda \sqrt{p'^2 - 1}} p'^2 dp' + 4\pi \int_{p'_L}^\infty e^{\lambda \sqrt{p'^2 - 1}} p'^2 dp' \quad (82)$$

now:

$$4\pi \int_{p'_L}^\infty e^{\lambda \sqrt{p'^2 - 1}} p'^2 dp' \simeq 4\pi \int_{p'_L}^\infty e^{\lambda p'} p'^2 dp' \quad (83)$$

the right hand expression can be calculated analytically:

$$4\pi \int_{p'_L}^\infty e^{\lambda p'} p'^2 dp' = e^{\lambda p'} \left[\frac{p'^2}{\lambda} - \frac{2p'}{\lambda^2} + \frac{2}{\lambda^3} \right] \Big|_{p'_L}^\infty = \infty \quad (84)$$

hence $Z'(\lambda) = \infty$ for $\lambda > 0$. There are two possible conclusions at this stage, either that a thermal equilibrium distribution is impossible for the superluminal Lorentzian free particles, or that a thermal equilibrium does exist but with a negative λ which entails a negative temperature. Admittedly this is a strange concept, however, if we are to accept superluminal Lorentzian particles in thermal equilibrium there is no way around it. Hence:

$$f(\vec{p}') = \frac{e^{-|\lambda| \sqrt{p'^2 - 1}}}{Z'} \quad (85)$$

in which:

$$Z'(\lambda) = 4\pi \int_1^\infty e^{-|\lambda| \sqrt{p'^2 - 1}} p'^2 dp' \quad (86)$$

The above expression cannot be evaluated analytically, but can be easily evaluated numerically (see figure 20). As can be clearly seen the partition function is a decreasing function of $|\lambda|$ or an increasing function of temperature. For a free superluminal Lorentzian particle we have:

$$\lim_{|\lambda| \rightarrow 0} Z' = \infty, \quad \lim_{|\lambda| \rightarrow \infty} Z' = 0, \quad (87)$$

Having calculated the partition function we are now in a position to calculate the probability density function. We present a two dimensional plot in figure 21, and a cross section in figure 22. Thus superluminal particles have a higher probability to be at lower momenta which is the situation for Lorentzian subluminal particles. However, for superluminal

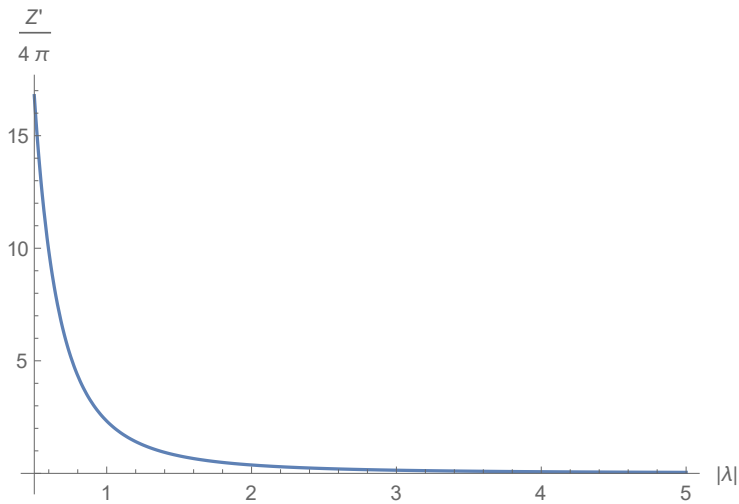


Figure 20. Partition function for a free superluminal Lorentzian particle.

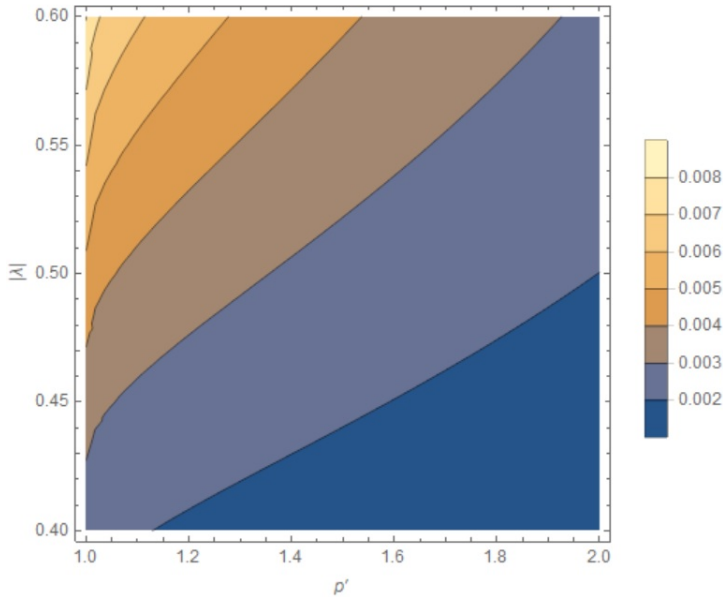


Figure 21. Probability density function for free superluminal Lorentzian particle as function of p' and $|\lambda|$.

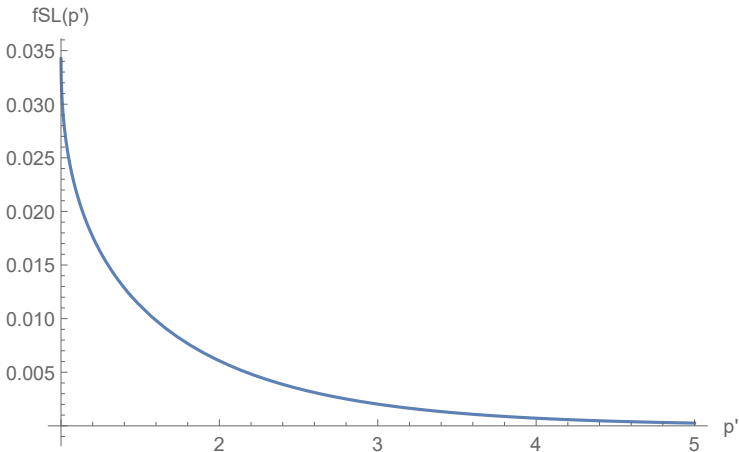


Figure 22. Probability density function for a free superluminal Lorentzian particle as function of p' for $|\lambda| = 1$.

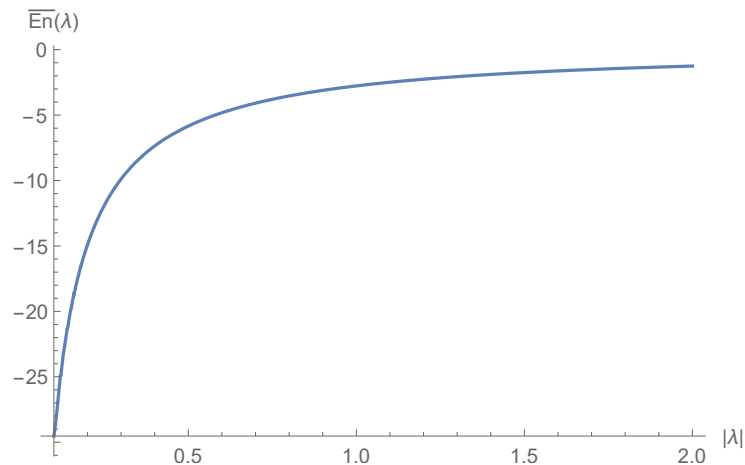


Figure 23. Average energy for free Lorentzian subluminal particles with high $|\lambda|$.

particles low momenta means high velocity (and not low velocity). Hence super luminal particles will tend to have $v \gg c$. Finally we calculate the average energy:

$$\begin{aligned} \bar{E}'_{kE}(\lambda) &= \frac{E[E_{kE}]}{mc^2} = -E[\sqrt{p'^2 - 1}] = -4\pi \int_1^\infty \sqrt{p'^2 - 1} f(\vec{p}') p'^2 dp' \\ &= -\frac{4\pi}{Z'(\lambda)} \int_1^\infty \sqrt{p'^2 - 1} e^{-|\lambda| \sqrt{p'^2 - 1}} p'^2 dp' = \frac{1}{Z'(\lambda)} \frac{dZ'(\lambda)}{d|\lambda|} = \frac{d \ln Z'(\lambda)}{d|\lambda|} \end{aligned} \quad (88)$$

this expression can be evaluated numerically and is depicted in figure 23. Thus the average energy or free Lorentzian superluminal particles is a increasing function of $|\lambda|$ contrary to the Lorentzian subluminal case. Thus it is an decreasing function of the absolute temperature but an increasing function of the true temperature as might be expected. We notice that for moderate temperatures the average energy flattens near zero energy level.

5. Some Possible Cosmological and Physical Implications

Suppose that the universe is Euclidean at $t = 0$, once its starts to expand the temperature drops and the Euclidean particles become faster thus increasing the rate of the universe expansion and thermalization, obviously there is no horizon (homogeneity) problem for Euclidean particles. As the universe increases further the temperature continues to drop making the particles even faster, thus creating a positive feedback loop. This increased expansion is cosmological inflation, but without an ad hoc inflationary field [22]. This is the primordial particle accelerator of the cosmos. We notice that a Higgs type fields do not give the correct density perturbation spectrum [22], hence one is forced to postulate a new field which is not a part of any particle model and thus is a possible but inelegant solution of the homogeneity problem. Alternatively one can speculate that homogeneity is achieved by ordinary matter which can become superluminal as the current analysis shows.

However, as the universe expands to a certain limit the density drops and the Euclidean metric becomes unstable [7] and a Lorentzian metric develops instead. In a Lorentzian space-time we have two distinct particle species that cannot mix, the subluminal particles that we are familiar with, and the superluminal particles which tend to reach higher and higher velocities and are thus moving to the further reaches of the universe, quite beyond our reach. Those particles may be what is perceived as dark energy [27] which affect the velocities of very distant supernovae and the CMB spectrum. Λ CDM cosmology predicts that $0.76 \pm 0.02\%$ of the universe are made of an unexplained "dark energy" component, obviously the Occam razor principle will vindicate a model in which such a ad-hoc component is not needed.

Another obvious physical implication of the previous analysis involve a far fetched technological scenario, is which a particle is accelerated to a velocity close to the velocity c in

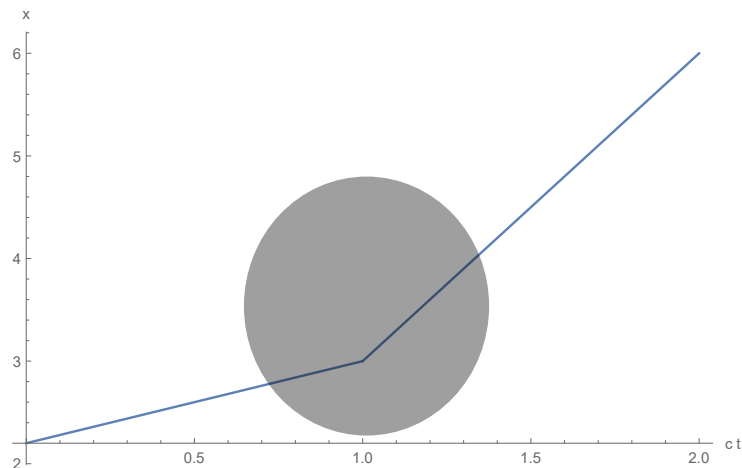


Figure 24. A schematic acceleration scheme of a particle in an Euclidean portion of space-time.

a Lorentz space-time, enter into an artificially created Euclidean space-time and accelerated further in this region to velocities above the speed c and emerge in a Lorentz space in which it will remain above the speed c for ever unless it is decelerated in an Euclidean space again (see figure 24). This may happen to a particle which travels radially in a Friedman-Lemaitre-Robertson-Walker metric passing outwards the critical radius of $r_c = \frac{1}{\sqrt{k}}$ and then coming back at superluminal velocities. However, this will be very difficult to do artificially. Obviously a metric change will require a significant $T_{\mu\nu}$ according to equation (1). Taking into account that the largest metric deviation from the Lorentzian metric is the solar system on the surface of the sun in $h_{00} \sim 10^{-6}$ [28], it does not seem conceivable that such a metric change can be indeed implemented.

Last but not least one should remember that although classical physics is assumed to take place in a Lorentzian background, quantum field theory calculations are done in an Euclidean background using the Wick rotation. This is usually justified on the basis that it is an analytic continuation. But an analytic continuation is a mathematical technique which has no physical justification in Lorentzian space-time but makes perfect sense if part of space-time, in particular the part which is very close to the particle is Euclidean. Hence one may speculate that each elementary particle may carry with it a "bubble" of a microscopic Euclidean space-time.

6. Conclusions

We have shown that general relativity allows for non-Lorentzian space-times in particular this is allowed in part of the Friedman-Lemaitre-Robertson-Walker universe. The result of which is that superluminal particles can exist in such a cosmology. Some of the cosmological implications of superluminal particles regarding the homogeneity problem, and dark energy problems are underlined. Some other possible implications of non Lorentzian metrics which are not connected to superluminality but may be a consequence of non-Euclidean metrics are also suggested. Of course much more detailed analysis is needed to reach a definite conclusion regarding any of the above physical problems but the existence of non-Lorentzian space-times and superluminal particles suggests a plausible solution.

In the scope of the current paper we have only considered canonical ensembles in the number of particles is fixed, however, at high energies pair creation from the vacuum is possible, hence a grand canonical ensemble should be studied. Quantum mechanical effects were also out of the scope of the current paper which concentrated on classical effects only.

Finally, an Euclidean metric will effect the energy momentum tensor thus effecting the allowed solutions of the Friedman-Lemaitre-Robertson-Walker universe. An exact mathematical model describing the transition from the Euclidean to the Lorentzian universe in which we leave in, is left for future studies.

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