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Article

A Spatially Bounded Airspace Axiom

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Abstract: Free route airspace (FRA) is a new concept of European air transport. It has been designed to eliminate the negative impact of air traffic on the environment, minimize fuel consumption, simplify and expand flight planning. In our research, FRA is modelled by using graph theory, networks, and convex analysis. We also looked for answers to the question which are the basic mathematical properties of airspace? Basic mathematical properties are expressed using axioms. Therefore, in our work, we state the general, basic axiom of airspace based on our FRA research. The axiom is given using practical, significant entry points and geodetic coordinates.

Keywords: Earth-centered Earth-fixed; Free Route Airspace; Significant Points; Simple Airspace Polygon; Airspace Axiom;

1. Introduction - Coordinate Systems

We use *coordinate systems* to determine locations on and around the Earth. The airspace can be represented using *Cartesian coordinates* in an *Earth-centered-Earth-fixed* (ECEF) *coordinate frames* (or *Geocentric coordinate system*). Cartesian coordinate system use two or three numbers, a coordinate, to identify the location of a point. Each of these numbers indicates the distance between the point and a fixed reference point, called the origin. ECEF origin is at the center of Earth. The origin has coordinates $[0,0,0]$. The distance from the origin is defined for each point $[x,y,z]$. The unit that is used to measure the distance from the origin will be *meter (kilometer (km))* or *feet*. The points of airspace $[x,y,z]$ are located in a sphere bar, which can be expressed by inequality

$$r^2 \leq x^2 + y^2 + z^2 \leq R^2 \tag{1}$$

where r is the *radius of the globe* and R is the *highest flight level* (see next chapter). The value of R is approximately 100 km, for the purposes of our article. Beyond this distance ($R > 100$ km), the points already belong to the *cosmic space*.

The surface points of the Earth are not well described by inequality $r^2 \leq x^2 + y^2 + z^2$, so we use the projections on the Earth surface. The projection that is used to draw the Earth on a flat map will be expressed by *longitude* and *latitude*. We will use this projection of airspace

$$[longitude, latitude] \tag{2}$$

to state our axiom. It is a special kind of coordinate system, called a *geodetic coordinate system*, since they identify points on Earth's surface. A more detailed description of the coordinate systems can be found in the books [1], [5] and a brief explanation of the issue on the Internet [14]. Figure 1 contains the main elements of the system.

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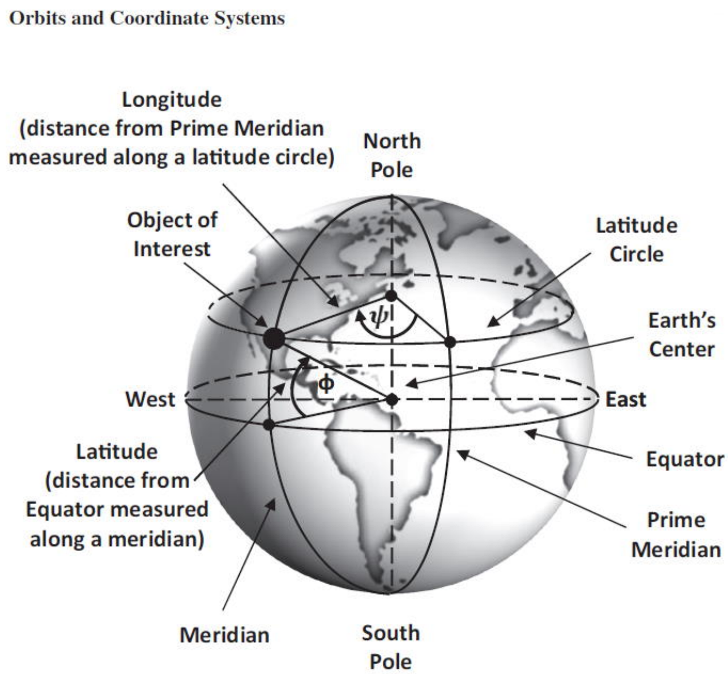

Figure 1. Geodetic (spherical) coordinate system (see [1])

When we also consider the flight *altitude*, we have a coordinate system with coordinates

$$[longitude, latitude, altitude] \tag{3}$$

There are conversion functions from coordinate system $[x, y, z]$ to *[longitude, latitude, altitude]* and vice versa. The coordinates *[longitude, latitude, altitude]* are given in degrees, minutes and seconds. A description of these coordinates and transfer functions can be found in [1],[13]. A specific application of the transfer function is located in program [12]. We assume that, each airspace, as a set of points $[x, y, z]$ or *[longitude, latitude, altitude]* is spatially bounded by relation (1).

We will not consider the *altitude* of the space for our purposes. The point *[longitude, latitude]* on the projection of the airspace means *Global Positioning System* (GPS) coordinates for geolocation of a point (for description of GPS system see [4]). Thus given airspace points with coordinates $[longitude, latitude]$ are called *significant points*. We examine what types of airspace areas are covered by these points and what mathematical properties these areas have.

2. Vertical Positions and Other Limitations

We only work in 2-dimensional spaces, which is given by *[longitude, latitude]*. The vertical position of an aircraft (or of obstacles) is not used when specifying axioms. The distance between two points is calculated based on a line, not a circular arc.

However, for completeness of the topic and possible further research, in this chapter, we will shortly describe different ways of indicating the vertical position in the airspace with which we could work in future.

In general, any 3-dimensional part of the atmosphere can be called airspace. However, from a legal perspective, airspace extends from the surface of the earth and through coastal waters to an unspecified height while it is bounded by the national border of the State.

In terms of airspace management and the range of navigation services provided, the airspace can be divided into two basic categories:

1. controlled airspace: this area extends to flight level 660, or 66 000 feet.

2. uncontrolled airspace: an area in which any flight activity is subject to basic rules and regulations without the issuance of air traffic control instructions. [15]

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Vertical positions are expressed in feet (ft); they can be expressed in meters (m) and the ways of how to indicate these positions are the following:

1. **Height / Absolute Altitude** is defined as a vertical distance of an aircraft above whatever surface the aircraft is flying over, e.g. building, crane, sea, etc. In other words, height refers to the distance between an aircraft and the ground. Height is expressed in feet AGL (*Above Ground Level*) or meters in some countries.

2. **Indicated Altitude (ALT)** is defined as a vertical distance of an aircraft above the *mean sea level* (MSL). Altitude is expressed in feet AMSL (*Above Mean Sea Level*) or meters in some countries.

3. **Flight Level (FL) / Pressure Altitude** is an indication of pressure, not of altitude. FL is used to ensure safe vertical separation between aircraft. It is defined as a vertical distance of an aircraft above the isobaric surface[1] of 1013.25hPa (hecto Pascal) or 29.92 in Hg (inches of Mercury) and is separated from other such surfaces by specific pressure intervals. It is also known as QNE. In aviation, 1013.25hPa (hector Pascal) / 29.92 in Hg (inches of Mercury) are referred to as the standard altimeter setting. The Flight Level is represented by FL, with the altitude in hundreds of feet (while being a multiple of 500ft) and so always ending in 0 or 5. FL is followed by three digits, for example, 10000 feet become Flight Level 100 = FL100; 3500 feet become Flight Level 035 = FL035. FL is an altitude flown on the standard QNH setting of 1013.25 hPa or 29.92 in Hg. By this, it is ensured that each aircraft is flying with reference to the same setting, which implies less chance of flying into each other. The airfields have different QNH (local altimeter settings) values because of differences in locations. Consequently, the 1013.25 hPa isobaric surface is a fictive curve that can be greater / lower than the mean sea level surface.

An important flight instrument necessary for a pilot to maintain the desired or assigned altitude during flight is called Altimeter.

The most common types of altimeters are barometric; they determine the vertical distance of an object (aircraft) above a fixed level by measuring air pressure differences (compare the pressure of outside static air to the standard pressure 1013hPa of air at sea level).

In relation to this, there are three references in use for barometric pressure:

- QFE (Field Elevation). This refers to a pressure set on the subscale of the altimeter to produce the height above the reference elevation being used. With the aerodrome QFE set in the subscale, the altimeter will read zero when an aircraft is on the runway and will give height above the runway when an aircraft is airborne.

- QNH (Height Above Sea Level). This refers to a pressure set on the subscale of the altimeter to produce the height above sea level. It reads runway elevation when the aircraft is on the runway.

- QNE. This refers to a pressure setting of 29.92 inches or 1013 hPa that will produce a standard atmosphere altitude and provides the basis for flight levels.[16]

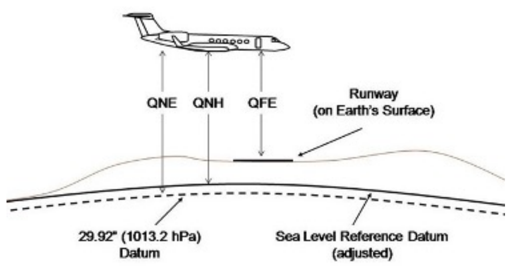

Figure 2. Flight Level

There are, of course, other types of altimeters that do not depend on the air pressure. For example, the Global Positioning System (GPS) receivers determine altitude by trilater-

[1] An isobaric surface is an invisible landscape that connects all points with the same atmospheric pressure.

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ation with four or more satellites. Radar and laser altimeters send a radio or laser signal toward the surface and measure the time it takes for the signal to bounce back. The time it takes for the signal to bounce back to the aircraft is translated to an elevation.

3. Mathematical Model of Free Route Airspace

3.1. Introduction to Free Route Airspace

To model Free Route Airspace (FRA) and to record all these created models and related algorithms, we are using the mathematical graph theory, networks, and technology Cloud Computing. These technologies and theories are essential to document the system, data structure of different objects, data analysis, and specific data of airspace and programs in which the models and algorithms are recorded (see [10] and [9]).

Free Route Airspace is one of the key Air Traffic Management (ATM[2]) functionalities. It is a new concept of operations created to modernize the airspace and contribute to the reduction of ATM costs, of air transport impact on the environment, fuel consumption, production of emissions and last but not least, it improves the efficiency of flights. FRA is defined as a specified volume of airspace in which the airspace users can freely plan a route by using the FRA Significant Points ([9], Documentation Section, Doc [3]). FRA Significant Points are specific points described (using the standard ICAO format) by geographical coordinates or by bearing and distance. They are published in the national States' Aeronautical Information Publications (AIP).

For an effective study of this system, in our view, the best way is to do it by mathematical modelling. In our research, the FRA system is a simplified representation, network. In this network, the graph vertices represent the individual airspace where FRA is implemented, and the edges represent connections between these airspace. Further operations performed in this network allow model the FRA graph and create the algorithms for calculating the most effective flight in this system, see ([9], Algorithms Section).

3.2. Graphs and Networks

In our previous work [8] we defined some basic concepts of graph theory. Now, we recall the relevant terms and notations from this work.

The complex concept FRA will be modelled by using graph theory, more precisely using the networks. We will use two-dimensional models for modelling. We define some basic concepts of graph theory.

Graph is a pair $G = (V, E)$, where V is a set of *vertices (nodes)*, and E is a set of *edges (arcs)*, connecting some pairs of vertices (nodes). Two vertices u and v of a graph are adjacent if they are connected by an edge $r = (u, v)$. For our purposes, we will consider no multiple edges (several edges are called multiple if they connect the same pair of vertices) and loops (edges beginning and ending at the same vertex). A graph is said to be *simple* if it contains no loops nor multiple edges. *Adjacency matrix* M of simple graph $G(V, E)$ is a 0-1 matrix (a matrix with elements 0 or 1) defined by $M(i, j) = 1 \iff (i, j) \in E$ and $M(i, j) = 0 \iff (i, j) \notin E$ for each $i, j \in V$. *Adjacency list* is a set of all edges of a simple graph (we can enter graphs in the computer program using the adjacency list).

Directed graph or *digraph* is a pair $D = (V, E)$, where V is a finite set of vertices, and E is a relation on V. Elements of the set E are called *directed edges* or *arcs*. A *simple digraph* contains no loops nor multiple arcs. *Path* $p = (v_0, v_1, ..., v_n)$ of length n in an digraph $D = (V, E)$ is a sequence if various vertices $v_0, v_1, ..., v_n \in V$, each pair $(v_{i-1}, v_i) \in E$ $(i = 1, ..., n)$ of which forms an arc.

[2] The dynamic, integrated management of air traffic and airspace including air traffic services, airspace management and air traffic flow management — safely, economically and efficiently — through the provision of facilities and seamless services in collaboration with all parties and involving airborne and ground-based functions.
Source: ICAO Doc 4444 PANS-ATM

A *weighted graph* is a graph in which a number (the weight) is assigned to each edge. Such weights might represent for example costs, lengths or capacities, depending on the problem.

Further definitions and characteristics of graphs can be found in publications [2].

In a basic definition, a network is defined as a collection of points joined together in pairs by lines. The points are referred to as vertices or nodes and the lines are referred to as edges or arcs. Any system that is composed of individual parts or components linked together in some way can be called Network. It can be, for example, the Internet, a collection of computers linked by data connections or human societies, collections of people linked by social interactions, or flights in air transport [3].

3.3. Encoding of the GFRA Graph Vertices and Edges

In this chapter, we define a graph for FRA. This graph is labeled as GFRA. By means of the GFRA, we can define a complex network ([11]). Let $GFRA = (V, E)$ be a graph, where individual FRA areas create the set of vertices V. We see this graph in figure 3. For better visualization, the vertex V_1 displays the SEE FRA (*South-East Europe Free Route Airspace* in 2021) area consisting of the airspace of Slovakia, Hungary, Romania, and Bulgaria. We can load the adjacency matrix of GFRA from the database [9] - Data and Data Structure Section, Data [2] - FRA_Matrix.csv.

Let E be the set of edges, and $(i, j) \in E$ if only if the vertices V_i and V_j are adjacent. The vertices are adjacent if there is a transition from V_i to V_j.

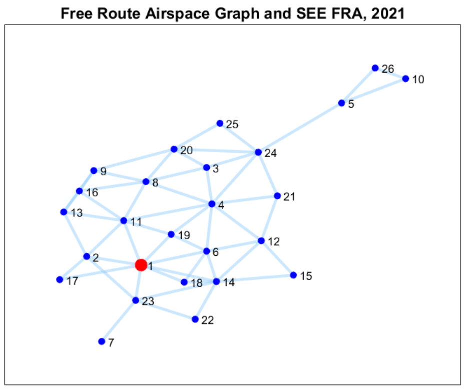

Figure 3. *Graph Free Route Airspace, 2021*

3.4. The FRA Significant Points

In the graph GFRA, each vertex is airspace. In this chapter, we will examine the properties of this airspace. We will examine how airspace is defined according to the FRA concept. The airspace is defined by *Significant Points.* The point is given by GPS coordinates, name, and other attributes. The FRA significant points, see ([9] Data Structure Section, and [7]), are divided on:

1. FRA Horizontal Entry Point (E) - It is a published significant point located on the horizontal boundary of FRA from which the FRA operations are allowed. It should be located exactly on the boundary of the relevant FRA area. *We define a Boolean function E(P). The function value E(P) is true if and only if P is an Entry Point.*

2. FRA Horizontal Exit Point (X) - It is a published significant point located on the horizontal boundary of FRA to which the FRA operations are allowed. It should be located exactly on the boundary of the relevant FRA area. *We define a Boolean function X(P). The function value X(P) is true if and only if P is an Exit Point.*

3. Combined FRA Horizontal Entry and Exit Point (EX) - It is a published significant point located on the horizontal boundary of FRA from which and to which the FRA operations are allowed. *We define a Boolean function EX(P). The function value EX(P) is true if and only if P is an Entry and Exit Point.*

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In some exceptional cases, the above-described points can be located inside or outside the relevant FRA (instead of being located at the horizontal border). This has to be within certain limits and after coordination with EUROCONTROL Network Manager (NM). Note: For more details, see EUROCONTROL FRA Design Guidelines ([9] Documentation Section).

4. FRA Arrival Connecting Point (A) - It is a published significant point to which FRA operations are allowed for arriving traffic to specific aerodromes. In a vertical dimension, this point could be also considered as an exit point (X) as any arriving traffic exits the FRA area through this point but it is not marked as (X).

5. FRA Departure Connecting Point (D) - It is a published significant point from which FRA operations are allowed for departing traffic from specific aerodromes. In a vertical dimension, this point could be also considered as an entry point (E) as any departing traffic enter the FRA area through this point but it is not marked as (E).

6. Combined FRA Arrival Connecting and FRA Departure Connecting Points (AD) - It is a published significant point to/from which the FRA operations are allowed for arriving/departing traffic to/from specific aerodromes. In a vertical dimension, this point could be considered as a combined FRA entry and exit (EX) point as any arriving/departing flight exits/enters vertically the FRA area but this point is not marked as such (EX).

The points in bullet 3-6 are referenced to the FRA area/State to which they are located but they can be also defined with reference to an aerodrome located in an adjacent FRA or non-FRA area.

7. FRA Intermediate Points (I) - It is a published or unpublished point defined by geographical coordinates or by bearing and distance via which FRA operations are allowed. This point is located inside the FRA area.

3.5. NFRA Network and Significant Points

By using the graph GFRA we define a complex network. In this network, each vertex of the Graph GFRA is defined by FRA significant points (see *Figure* 4 and [9] Algorithm Section). We define the FRA network as an object that contains the pairs $[\overline{V}, P_{\overline{V}}]$ where $\overline{V} \in V(GFRA)$, $P_{\overline{V}} = \{[LatP, LonP]; [LatP, LonP] \in \overline{V}\}$ is a set of the FRA significant points, and $[LatP, LonP]$ are *GPS coordinates of the given point*. We do not mention the edges of the network here. We assume that each vertex (significant point) can be connected to every vertex (significant point) within the area FRA. This network is called the *Free Route Airspace Network* and is denoted by $NFRA$, see ([9], see A[2] - The Vertex SEE FRA).

Note: $NFRA$ contains all significant points $P_{\overline{V}}$ (*GPS coordinates of the points*) in the FRA area $\overline{V}$. Each FRA significant point has assigned information to which FRA area it belongs.

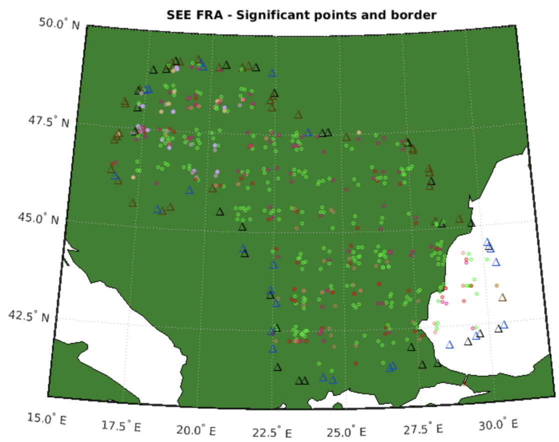

Figure 4. *SEE FRA significant points*

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4. Airspace Axiom Characteristics

FRA efficiency provides a rule that the flight can enter and exit the concerned airspace only once. With the aim of flight optimization, the idea arose to study the basic mathematical properties of airspace. The basic mathematical properties of airspace can best be expressed by specifying the basic axiom. These propositions are based on the FRA's analysis mentioned above. The pronounced axioms define the general characteristics of airspace.

4.1. A Simple Polygon

Now, we construct horizontal boundary $B(\overline{V})$ for each airspace $\overline{V} \in V(GFRA)$. The boundary $B(\overline{V})$ is an *FRA polygon* that is defined by special significant points (type 1. or 3. and country information). This polygon is a 2D (two-dimensional) projection of airspace into the plane and represents the FRA airspace. We use a 2D model because the significant points that create the airspace have two coordinates, latitude and longitude. We need to define terms polygon and simple polygon before pronouncing the axioms.

Definition 1. *The* polygon *is a two-dimensional closed curve consisting of a set of line segments (*sides *or* edges*) connected such that no two segments cross. The points where two edges meet are the polygon's* vertices*.*

Definition 2. *The* simple polygon *is a polygon that does not intersect itself and encloses only one region (polygon has no holes).*

Axiom. *(A Spatially Bounded Airspace Axiom)*

For each airspace $\overline{V}$ there is a simple polygon

$$B(\overline{V}) = \{[x_i, y_i]; [x_i, y_i] \in \overline{V}, i = 0, 1, \cdots, n, and\ x_0 = x_n, y_0 = y_n\} \quad (4)$$

where$[x_i, y_i]$ significant entry or entry and exit points of airspace are polygon's *vertices*, for $i = 0, 1, \cdots, n$.

Polygon $B(\overline{V})$ is called the $B(\overline{V})$ *airspace polygon*. We denote the number of polygon's vertices as $|B(\overline{V})|$. We can see in Figure 4 such a polygon for airspace SEE FRA. Now we will show that an airspace polygon is simple.

4.2. No Loop Property

Traffic overflying the concerned airspace is to be planned directly between FRA Entry, FRA Intermediate, and FRA Exit Points. This means that once the flight enters the airspace via the FRA Entry Point, then the next point that has to use is the FRA Intermediate Point $\rightarrow$ the flight cannot make the circle and use again the FRA Entry Point. The same applies for the FRA Intermediate Points (after using this type of point, the flight has to proceed to the next FRA Intermediate Point or the FRA Exit Point) and for the FRA Exit Points (after using this type of point, the flight has to exit the airspace; it cannot make the circle and to come back to the same point).

Proposition 1.

The airspace polygon $B(\overline{V})$ does not contain a loop. (No loop property).

4.3. No Hole Property

Wherever FRA is implemented, the whole area that belongs to the concerned state and where air traffic control of that country is provided has to be covered. According to the ERNIP Part 1, FRA forms an **integral part of the overall European ATM network** and flights remain subject to air traffic control. From that, we can assume the no hole property of airspace polygon.

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Proposition 2.

The airspace polygon $B(\overline{V})$ has one region (airspace does not contain a hole - no hole property).

We can express these properties mathematically as follows :

Lemma 1. *Let $B(\overline{V})$ be an airspace polygon and $|B(\overline{V})| = n$. If $[x_i, y_i], [x_j, y_j] \in B(\overline{V})$ then $[x_i, y_i] \neq [x_j, y_j]$ for all i,j : $0 < i \neq j < n$.*

Note that in fact the polygon $B(\overline{V})$ has only n vertices (not $n+1$), because the first and last vertices are identical. The notation from relation (4) makes it possible to express that a polygon is a closed polyline.

4.4. Airspace Polygon Creation

In this section airspace polygon creation will be discussed. We start the section with the definition of the *convex set*, *convex polygon* and *convex hull*.

Definition 3. *A set S is* convex *if for any two points $p, q \in S$ the line segment $\overline{pq} \subseteq S$.*

Definition 4. *A* convex polygon *is a polygon that is the boundary of a convex set.*

Definition 5. *The convex hull of set S is the smallest convex polygon containing points S. The convex hull of the set S is denoted by $conv(S)$.*

The method for creating an airspace polygon has several steps.

1) The *significant point selection* is the first step in our method .

Suppose that $P_{\overline{V}} = \{[LatP, LonP]; [LatP, LonP] \in \overline{V}\}$ is the set of all significant points of airspace $\overline{V}$. For the construction of the airspace polygon, it is necessary to select significant points from this set, which are Entry or Entry and Exit points.

Let's mark

$$SE_{\overline{V}} = \{[x, y] \in \overline{V}; (E(x, y) \vee EX(x, y) = 1)\}$$

set of all Entry or Entry and Exit points of airspace $\overline{V}$ and it *convex hull* $conv(SE_{\overline{V}})$.

2) The second step is the *construction of a convex hull* .

Suppose that $|SE_{\overline{V}}| = n$ and $|conv(SE_{\overline{V}})| = m$. We know more algorithms for finding a convex hull for a set of points. One such algorithm describes the article [6]. Convex hull of set $SE_{\overline{V}}$ is a convex polygon containing vertices from $SE_{\overline{V}}$. For simplicity, assume that the convex polygon is formed from points $[x_0, y_0], [x_1, y_1], \cdots, [x_m, y_m]$ and $x_0 = x_m, y_0 = y_m$.

3) The third step is the *polygon extension* - *defining a polyline* between the vertices $[x_i, y_i]$ and $[x_{i+1}, y_{i+1}]$ for all $i = 0, 1, \ldots, m-1$.

We can create without loops a sequence of points $[x_{ij}, y_{ij+1}] \in SE_{\overline{V}}$, for $j = 0, 1, \ldots, l-1$ that form the boundary of airspace for each section $\langle x_i, x_{i+1} \rangle$ (see 5).

Note, this method step is non-algorithmic. We define the neighborhood points according to geographical conditions.

4) The fourth step is the *renumbering polygon vertices*.

In addition, we need to determine the order of the points as they follow each other, the point indices are renumbered.

The output of our method is the airspace polygon

$$B(\overline{V}) = \{[x_i, y_i, (E(x_i, y_i) \vee EX(x_i, y_i) = 1), i]; [x_i, y_i] \in \overline{V}, i = 0, 1, \cdots, n, and\ x_0 = x_n, y_0 = y_n\} \quad (5)$$

The designation $[x_i, y_i, (E(x_i, y_i) \vee EX(x_i, y_i) = 1), i]$ means that $i-th$ significant point on airspace polygon $B(\overline{V})$ has latitude x_i and longitude y_i and this point is entry $(E(x_i, y_i) = 1)$ or entry and exit $(EX(x_i, y_i) = 1)$. In addition, the$[x_i, y_i]$ point is the $i-th$ vertex of the airspace polygon. Note, it is possible to construct such a polygon only for entry or only for (entry and exit) points.

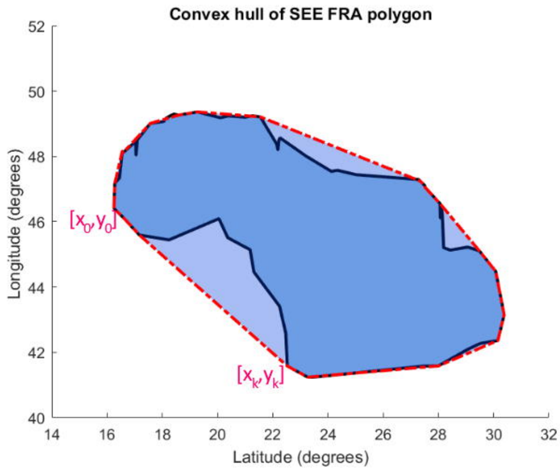

Figure 5. *SEE FRA airspace polygon and its convex hull*

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Let's note that, we can define the starting and ending point of polygon $[x_0, y_0] \in SE_{\overline{V}}$ as follows

$$x_0 = \min\{x; [x, y] \in \overline{V} \wedge (E(x, y) \vee EX(x, y) = 1)\} \quad (6)$$

$$y_0 = \min\{y; [x_0, y] \in \overline{V} \wedge (E(x_0, y) \vee EX(x_0, y) = 1)\} \quad (7)$$

We have shown that for any set of significant points of airspace it is possible to construct a simple airspace polygon, which means the horizontal boundary of airspace. The airspace polygon exists in the plane (on the 2D map) and on the sphere, on the surface of the earth.

5. Conclusion

The FRA airspace is defined by special, significant points. The significant point coordinates are expressed by longitude and latitude in a geodetic coordinate system. In our work, we have identified an axiom that defines airspace. Each airspace is given by a simple polygon of significant entry or (entry and exit) points. This polygon defines the horizontal boundary of airspace. The polygon does not contain holes and loops.

The publication was also created to support an Internet of Things project. Special sensors are to create a monitoring system in the project to protect isolated and high-risk populations against the spread of viral diseases. The common point between the free route airspace or airspace and the monitoring system are the significant points and a simple polygon on Earth's surface, which defines the investigated area.

5.1. Funding

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5.2. Code Availability

All algorithms that we used to write the article, are licensed and published in the repository *MATLAB File Exchange*. In addition, the algorithms are hosted in FRA Research Cloud ([9] - *Algorithms*).

5.3. Data Availability Statement

It should be noted that all algorithms work with the research data that are freely available online. Hence, all outputs are only estimations and are not to be used for operational purposes. Our data are stored in the clouds, see ([9] - *Data and Data Structures*). The main documentation of the FRA system can be found in references ([9] - *Documentation*).

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5.4. Authors' Contributions

Conceptualization, P.S. and M. F.; methodology, P.S. and M. F; software, P.S. and M. F.; formal analysis, M.B.; resources, M.F.; data curation, P.S. and M.F.; writing - original draft preparation, P.S. and M. F; writing - review and editing, P.S. and M. F; visualization, P.S. and M. F.; supervision, P.S. and M. F.; project administration M.B.

All authors have read and agreed to the published version of the manuscript.

5.5. Competing Interest

The authors declare no conflict of interest.

5.6. Disclaimer

The views expressed in this paper are the authors' own and do not represent a policy or position of EUROCONTROL.

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