

CLASSICAL-RELATIVISTIC VARIABILITY OF INERTIA OF PHOTON DISPLACEMENT MASS IN THE MATTER AND THE SPECIFIC VACUUM TEMPERATURE

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ABSTRACT

The effects of photon inertia on the determination of its trajectory were verified and the representation of a displacement mass characterized by the flow of the number of wavefronts and the decomposition of photon inertia into parts associated with translation and rotation motions was considered. It was found that with the relativistic increase of the photon's resistance to change its directional properties, it inhibits the relativistic trajectory of the second torque, of Minkowski, in an angular range of incidence. After synchronizations, in the OAM inversions, there are reductions of the inertia associated to the translational part that assumes classical predominance, where the relativistic trajectory is allowed while the photon offers less resistance to changes in its directional properties. The classical-relativistic variability of the photon inertia characterizes the classical or relativistic profile of the energy distribution in forms of motion, where adjustments of the rotational and translational parts can be performed as a function of the refractive index rate, temperature and angle of incidence. It was found that with increasing temperature of the refringent medium, the synchronizations displacement in the sense of the normal incidence. A specific vacuum temperature for the refringent medium was characterized, where the photon exhibits a classical-relativistic synchronization under all angles of incidence, characteristic of its immaterial state in vacuum.

Keywords: photon inertia, inertia momenta, photon inertia variability, photon inertia momenta variability, specific vacuum temperature, Abraham momenta, Minkowski momenta, relativistic energy wave, translational inertia, rotational inertia, photon displacement mass.

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1. INTRODUCTION

The growing demand of photonics in the development of new technologies, with special emphasis on the use for quantum computing and information [41], [42], [13], [25] conditions numerous studies to unveil the hidden secrets of different physical properties of light, that sometimes are to those that constitute the very essence of the physical entity, for example the nature of the photon mass that recently guides different studies ([34], [29], [21], [22], [36], [20], [2], [35], [28]), and even the wave-particle duality is discussed [31], [3], [20].

Naturally, in this scenario, the rest mass of the photon becomes a possibility discussed by some authors. Arbab [2], describes a model to treat experimental results that point to a rest mass in matter of the order of μm_e , while Grado-Caffaro and Grado-Caffaro [22] associates the rest mass to the wavelength.

Although this study does not deal with a possible rest mass, it is important to note that it may not be in a relativistic scenario when gravitationally deflected or while complex [34]. In this perspective, of the photon mass in different physical scenarios, we will consider that its mass may be composed of a classical part and a relativistic increment, partitioning the variability of inertia. Different studies partition physical quantities in different properties, as for example the total energy of light in wavelike and material parts [20]; the mass of a pulse in mechanical and electromagnetic parts [35]; the duality in wavelike and corpuscular parts [31]; the inertia in radiative, instantaneous and rotational parts [23] and the identities where one is associated with energy storage and the other material [26].

Recently, it has been verified the variability and control of physical properties, such as the speed as a function of the variability of the physical properties of the optical conductor. Sayrin et al. [8], doping glass fiber with cesium atoms obtained accentuated modulations of the refractive index by electromagnetically manipulating the transparency of the medium, which according to the authors reduces the pulse group velocity, resulting in slow light. Other studies also manipulate the properties of the medium to control the speed of light [40], [33], [24], [39]. The temperature of the refringent medium is a control variable of light speed [37], [24], where in different transmission analysis the refractive index is a function of temperature [18], [16], [1], [43], and pressure [30].

The induction of orbital angular momentum (OAM) also condition the photon to variability in its properties. Recent studies report that twisted beams in vacuum exhibit subluminal effects [19], [15], in turn Lyons et al. [4] discuss that the addition of OAM can accelerate the beam. The relationship of mass variation with velocity variation are analyzed for different particles [28].

The relativistic aspects of the photon-matter interaction in the refractive, conservative and doppler realm are recently treated by Cardoso [12], where the photon is subject to two distinct

torques, Abraham's and Minkowski's, introducing a relativistic trajectory that arises with delays, advances and synchronizations. The relativistic trajectory presents a smaller deflection in relation to the original trajectory when compared to the classical trajectory and the instants of synchronization of the trajectories arise with variations in the direction of the OAM.

In this analysis is presented the decomposition of the inertia of the photon in forms of motion, translational and rotational, verifying the effects of inertia in determining the trajectory while classical or relativistic [12]. The variation of the inertia of the OAM is verified as a function of the relative refractive index, temperature and angle of incidence.

2. METHODOLOGY

In this work, we considered translational energy conservation in OAM in the photon-matter interaction [12], in the transmission of a single photon in the transition between pairs of media with refractive and thermodynamic properties previously known in the literature.

In the treatments of inertia and moment of inertia, the relativistic energy wave (REW) models with the action of two non-additive torques in the refringent medium, Abraham's classical and Minkowski's relativistic were employed [12].

The variabilities of inertia and moment of inertia were verified as a function of the angle of incidence, refringence of the medium and its temperature. The estimates of the refractive index as a function of temperature considered the model presented by Djurišić and Stanić [18], for a wavelength of 589.3 nm, disregarding the margin of error³, according to the expression:

$$n(T) = 1,33455 - 0,0000553132 T - 0,00000112008 T^2 . \quad (1)$$

In treating the refractive index of air as a function of temperature, Walker's model [38] was adopted, given by:

$$n(T, p) = 1 + \frac{0,0002928}{1 + 0,0036 T} \frac{p}{76} , \quad (2)$$

disregarding the margin of error inherent in the model [38], assuming a constant atmospheric pressure of 76 cm Hg.

The inertia analysis considered that the photon has a displacement mass in vacuum and presents its massive characteristics in the photon-matter interaction, be it in the detection. It was considered the decomposition of the photon inertia in accordance with the energy decomposition in forms of motion.

³ For the analysis of the variability of the photon inertia as a function of temperature, with a diagnostic character of the system, the error margins of eq. (1) were disregarded, although it does not imply lower accuracy whereas the error margin is very small [18].

3. DEVELOPMENT AND DISCUSSION

In the conservation of the translation energy in OAM presented by Cardoso [12], there is conservation of the photon mass at transition interface between media pairs. Considering that after the Abraham torque an inversion of angular momentum is possible under the Minkowski torque, in the synchronization instants [12], illustrated in Figura 1, we proceeded to investigate the relativistic effects on the mass and consequently the responses of the photon inertia by means of the Abraham and Minkowski actions.

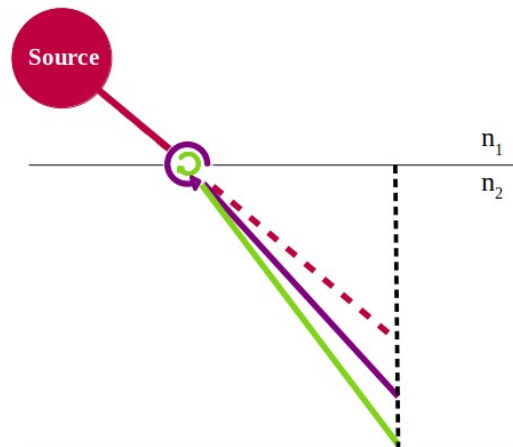


Figura 1- Characterization of the angular momentum variation in refraction due to Abraham and Minkowski torques. (Source: author's)

In agreement with Cardoso [12], the classical torque imposes the Abraham moment which acts as a relativistic ignition device for the photon, where the new state of motion is governed by the relativistic Minkowski moment, which is larger than Abraham's [12], [7], [5], [14].

Considering that the transition between two media occurs with constant frequency, we can know the variability of the moment of inertia of the photon from the variability of the natural OAM of photon-matter interaction, presented in Cardoso [12]. Assuming a photon of frequency equal to 60 PHz for simplicity, we can verify in Figura 2 that the signature of the variability are the same as those treated in the OAM, where it is observed that the synchronization is preceded with the increase of angular momentum, where the photon starts to assume the relativistic trajectory closer to the original trajectory, when compared to the classical one [12].

Clearly, the cause of this effect is associated with the increased ability of the photon to offer resistance to changes in its state of motion, attenuating the inclination in relation to the original trajectory. Considering that there are energy parcels employed to translational and OAM motions, as well as pondering that the directional properties of the trajectory are associated to the translational motion, we proceed to analyze the inertias in translational and rotational parts.

The variability of the moment of inertia is directly associated with the variations of the dimensions of the structure of the OAM, however the inertia of the photon is well characterized by the measure of its mass, which we will now address.

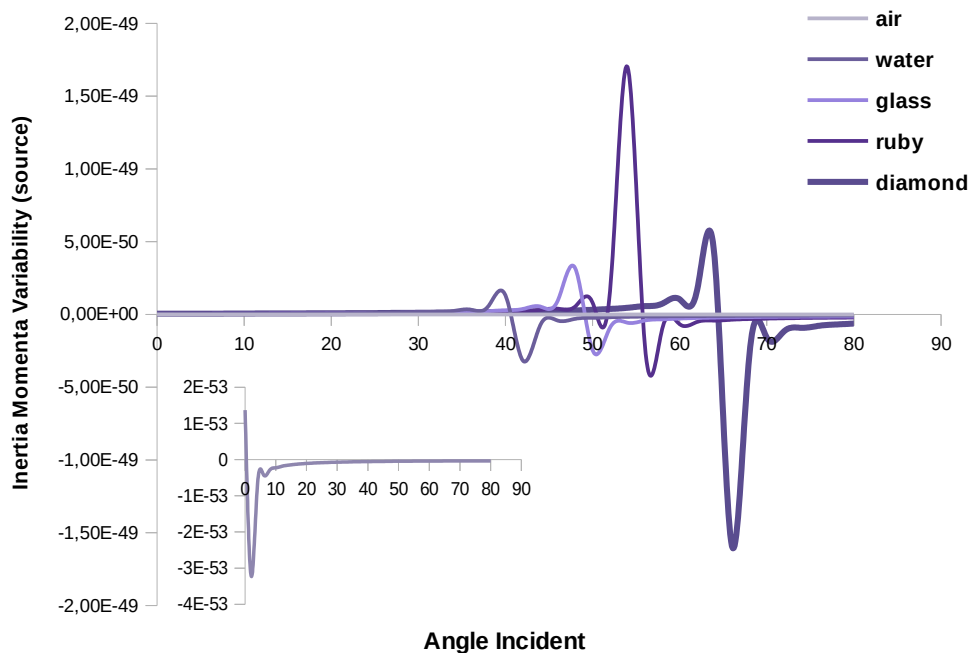


Figure 2- Variability of the photon moment of inertia as a function of the angle of incidence. (Source: author's)

Although the photon assumes a relativistic trajectory with certain delays, advances and synchronizations between source-observer [12], the relativistic ignition of the photon elapses under any incidence following the example of other studies where the estimates are consistent with the data, with normal incidence implied [9], [10]. In this sense, we will verify the behavior of the inertia of the photon when transiting between two media, with and without inclination with respect to the normal.

Let us consider a photon that in the material medium presents the behavior of a material energy wave of which the de Broglie wavelength is relativistically dilated [12], but that when passing through the vacuum, with velocity c , it becomes immaterial and the invisible visible spectrum becomes confused with the vacuum itself. In the material medium, the interaction with matter conditions it to materiality, demonstrating all its massiveness, as illustrated in Figura 3.

Considering the well-known mass-energy relation of the photon and the conservation of translational energy in OAM [12]:

$$\frac{1}{c^2} E = m_y \frac{v^2}{c^2} + \frac{1}{c^2} dE \quad , \quad (3)$$

where the energy variation is a function of the relative refractive index [12], such that conservation of mass is satisfied:

$$m_y = m_y n_{12}^2 + \frac{h\nu}{c^2}(1 - n_{12}^2) \quad , \quad (4)$$

the second term of eq. (3) is null in vacuum where $dE = 0$. In this sense, we can say that the photon presents a massive behavior exclusively when it conserves energy in another form of motion where dE is nonzero. In an analogy with Maxwell's displacement current, where there is no charge transport, we can treat through the wavefront number flux where the photon has a displacement mass:

$$\mu_0 \epsilon_0 \frac{\partial \phi}{\partial t} = [m_y n_{12}^2]_{\text{translational inertia}} + [m_y (1 - n_{12}^2)]_{\text{rotational inertia}} \quad , \quad (5)$$

where $\phi = hN(n_{12}, t)$, being $N(n_{12}, t)$ is the number of wavefronts as a function of relative refractive index and time, where flow will have greater representation of variability with increasing relative refractive index, according to Figura 4. The first term of eq. (7) is a measure of the inertia⁴ associated to the translation motion and the second term is the measure of inertia employed to the other form of motion, associated to OAM according to Cardoso [12], [10], [9],[9], [11].

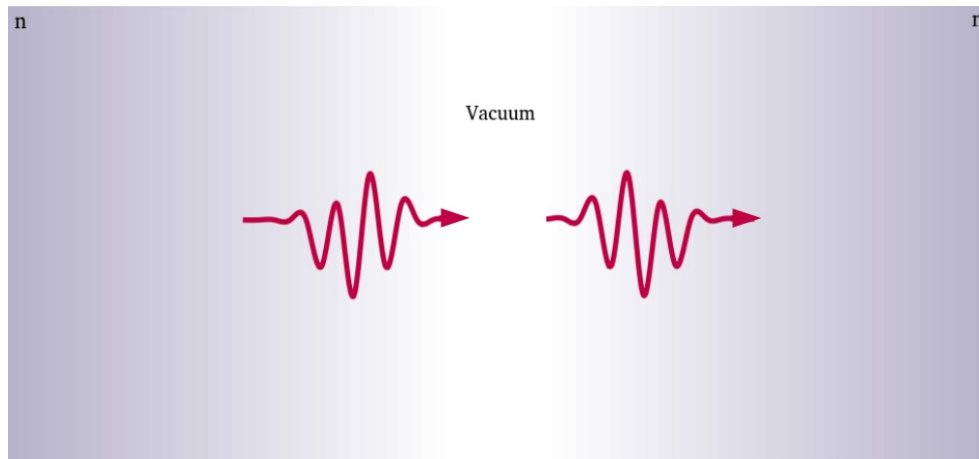


Figura 3- Illustration of a photon transiting through a material discontinuity.
(source: author's)

In Figura 4, it is observed in the normal incidence an increase of the inertia associated with the angular motion with the increase of the relative refractive index, which tends to correspond to the integrity of the representation of the photon inertia. In this sense, it is possible to say that with the increase of refractivity there is less resistance to change the directional properties of the photon, while there is greater resistance to changes in its state of angular motion.

The increase of temperature of the refringent medium indicates that for the same pair of media, under different temperatures, that the proportions between the parts of inertia of the photon in the refringent medium will be different where the relativistic effect decreases with the increase of

⁴ Although the inertia analysis was conducted through two contributions, translation and rotation, it is not about mass dissociation. The inertia measures consider the parts of energy used to translation and rotation movements in agreement with Cardoso [12].

temperature. Considering that the parts of inertia treated here represent proportions of the photon energy in forms of motion, naturally with the gradual increase or decrease one can adjust the energetic parts delivered to translation and OAM.

According to Cardoso [12], the relativistic increment can be written in terms of the classical increment, where the mass increment can dilate with angular symmetry:

$$[dm]_{\text{Relativistic}} = \gamma(\theta_1, n_{12})[dm]_{\text{Classic}} , \quad (6)$$

being:

$$\gamma(\theta_1, n_{12}) = \frac{\sin \theta_1}{\sqrt{1 - n_{12}^2}} . \quad (7)$$

It can be seen in Figura 5, that the balance of the translational and rotational inertia contributions is a function of the angle of incidence. The classical-relativistic variability of the photon inertia represents the difference between the parts of inertia given by eq. (5) and the parts considering the relativistic increment of eq.(6). It can be seen that the classically-relativistic variability of the translational is the negative of the variability of the rotational inertia:

$$[\Delta m]_{\text{translacional}} = -[\Delta m]_{\text{rotacional}} = -[1 - \gamma(\theta_1, n_{12})][dm]_{\text{Classic}} . \quad (8)$$

It can be seen that the inertia parts as a function of the refractive index for each material listed in Figura 4, are identified in the normal incidence through the rotational part in Figura 5. However, it is noted a predominance of the relativistic effect on the translational part in the normal incidence. As the relativistic effect decreases against the classical effect on the translational part, it becomes greater on the classical rotational part.

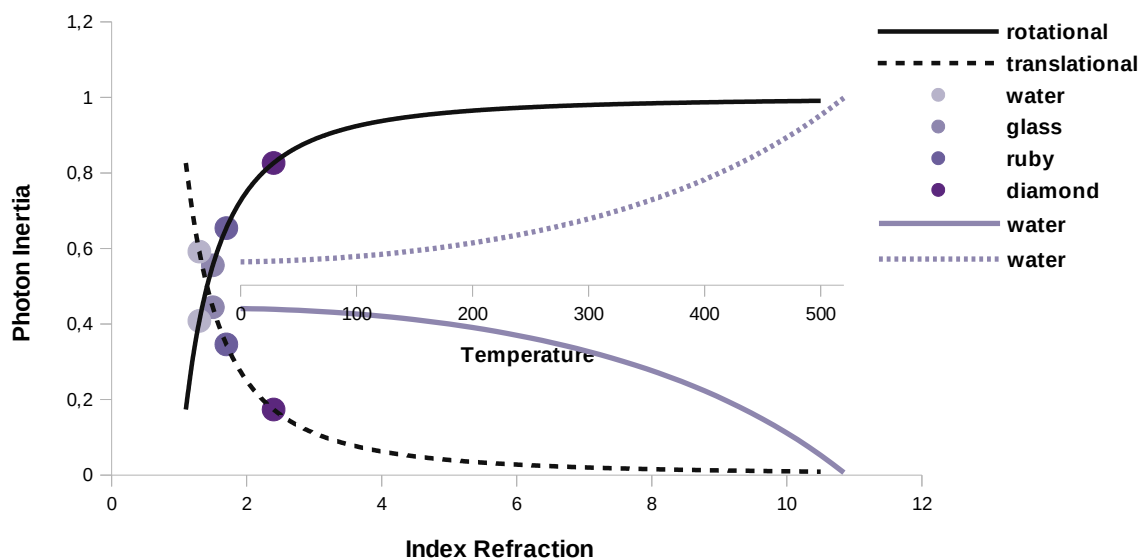


Figure 4- Behavior of the translational and rotational parts of the photon inertia, as a function of the refringence of the medium. (source: author's)

In the synchronizations presented in Cardoso [12], we find the Classical-Relativistic transition moment in the parts of inertia associated to the forms of movement Figura 5, where the translational part passes to the Classical and rotational domain with relativistic predominance.

This Classical-Relativistic variability of the photon inertia explains the delay of the relativistic trajectory [12]. When the photon transits between two media, it is received by the first Abraham torque, responsible for the energy variation eq.(3, 4) imposing the classical regime, which in turn acts as a relativistic ignition device of the photon triggering the second Minkowski torque. In this sense, the relativistic effect occurs even under normal incidence, but the trajectory is delayed due to the fact that the relativistic increase of the translation inertia after the first torque translates into greater resistance to changes in the photon's directional properties, where the second torque becomes relevant with the reduction of the relativistic effect on the translational part, precisely in the synchronizations where the classical translational part becomes predominant and as seen in Figura 4, in the classical region the photon offers less resistance to changes in its directional properties.

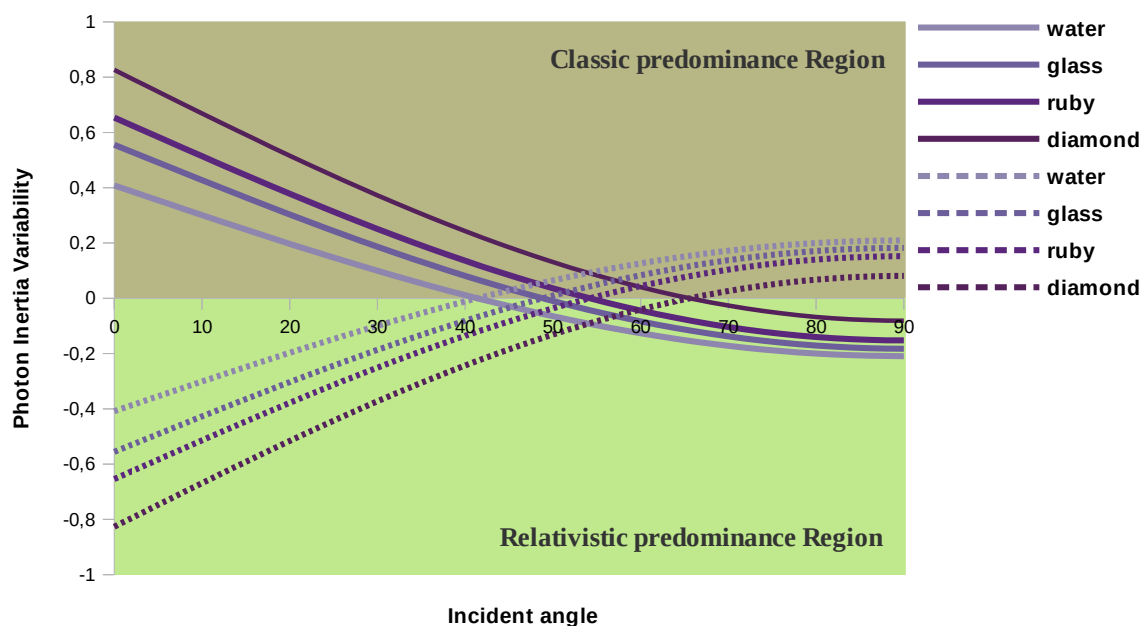


Figure 5 - Variability of photon inertia as a function of relative refractive index and angle of incidence. (source: author's)

The synchronization regions for a given group of materials may be advanced due to the thermodynamic state of the refringent medium, as for instance, the advancement of the incidence angle for the synchronization of the trajectories around 10 degrees, presented in the inversion of the variability of the moment of inertia presented in the main graph of Figura 6. Comparing with the variability of Figura 2, there is an advance⁵ of the synchronization points with increasing

⁵ The analysis of the variability of the photon moment of inertia as a function of temperature was conducted under the wavelength of 589.3 nm.

temperature and although Cardoso [12] has presented characteristic synchronizations for certain materials, it appears that for same material there will be different synchronization points as a function of temperature, as shown in the upper graph of Figura 6.

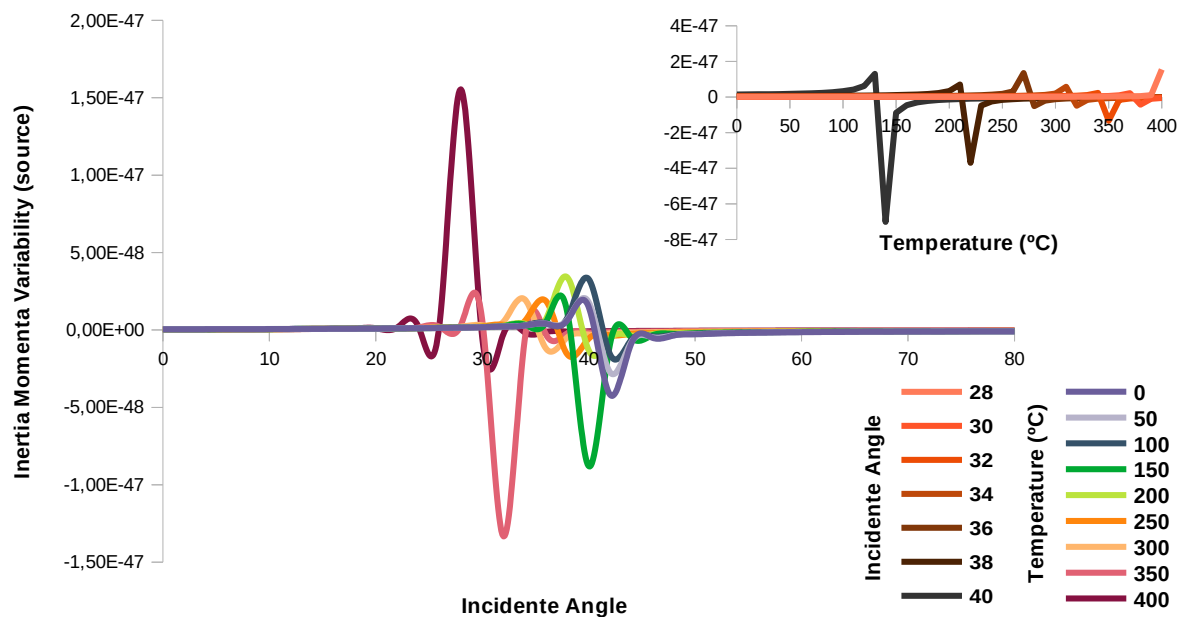


Figura 6- Variability of the photon moment of inertia as a function of temperature and angle of incidence. (source: author's)

In Figura 7, it is seen that for $n(T)$ according to eq. (1) there is a limited range of incidence angles with synchronization between classical and relativistic trajectories⁶ with increasing temperature, be it 0-40°. Considering that in water, regardless of temperature, synchronization would occur under an angle around 41°, naturally the scale adopted in Figura 7 suppresses a small range of angles that allow synchronization above 40°. For angles larger than 40°, the translational inertia variability is in the classical domain while the rotational inertia variability is in the relativistic domain, and there is no synchronization while the absence of intersections is verified.

Note that there is a limit temperature where $n(T)$ is the refractive index of vacuum. In these terms, we can say that each material has a critical temperature at which it begins to have vacuum characteristics, or simply a specific vacuum temperature. At this temperature matter merges to the vacuum for the photon, because the classical-relativistic variabilities of its inertia are null, where $dE = 0$, we will identify here the specific vacuum temperature as a region of classical-relativistic synchronization for any incidence. In the vicinity of the specific vacuum temperature, mostly the rotational inertia variability is in the relativistic domain while the translational inertia variability is in the classical domain.

⁶ The trajectory synchronizations in Figura 7 are found at the intersections of the translational and rotational inertial variabilities, at the central axis where the variability is zero.

The analysis conducted considered the function $n(T)$ according to eq. (1), indicating that the estimates of the refractive index as a function of thermodynamic parameters for other materials can present not only information of the control of inertia and consequently of the energy distribution of light in forms of motion, but also the specific vacuum temperature for different materials, including for metamaterials where the refractive index is negative [10].

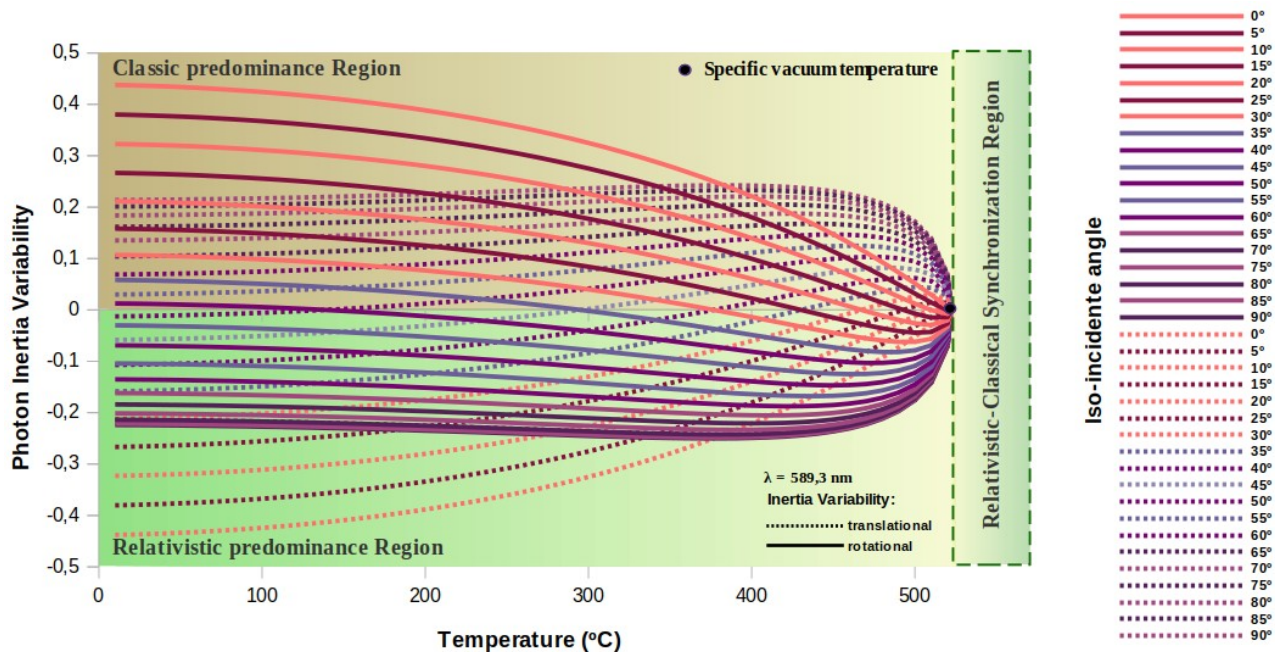


Figura 7- Variability of the photon inertia as a function of the angle of incidence and temperature, and the specific vacuum temperature. (Source: authors)

Recently experiments have been able to slow down photons by varying the properties of the refringent medium [8], [40], [33]. In the inertia domain, we verified the adjustment of translational and rotational inertias as a function of the relative refractive index and a second adjustment as a function of the angle of incidence, which in turn are directly associated with the energy employment to each form of motion. It is noted that the rates of refractive index and angle of incidence can act to modulate the energy distributions in the different forms of motion that can be found in a given pulse.

From the specific vacuum temperature perspective, at temperature extremes, where the photon interprets a material discontinuity, recent results demonstrate for temperatures near absolute zero, that matter can exhibit invisibility [27], [17], [32], where the number of scattered photons in elastic collisions decreases with temperature [27], with increased transmission in an ultra-cold Fermi degenerate gas [17] and suppressing not only scattering but also absorption with decreasing temperature [32].

In view of these results [27], [17], [32] the variability of the photon inertia at low temperatures was considered, as presented in Figura 8 for water and air, considering that they are different analyses, methodologies and systems, but if identify with respect to the photon state at low temperatures. This analysis shows that each material presents a specific temperature in which its optical properties are close to those of vacuum, where for low temperatures, considering eqs. (1, 2), it is found that the specific vacuum temperature is lower than absolute zero, being closer the lower the refractive index, where it is found that air presents a specific vacuum temperature around -277,818 °C.

In Figura 8, it is verified⁷ with materials of lower refractive index that with the decrease of temperature, the majority of the translational inertia variability of the photon is in the classical domain, while the rotational is in the relativistic domain, where in air, only two angles of incidence present synchronizations with the decrease of temperature, presenting cessation of classical and relativistic variability around the absolute zero. In water, with a greater number of synchronizations, indicates that the higher the refractive index, the lower the specific vacuum temperature at low temperatures ($T < 0$), without symmetry with the specific vacuum temperature at high temperatures ($T > 0$) treated in the Figura 7, for the same material.

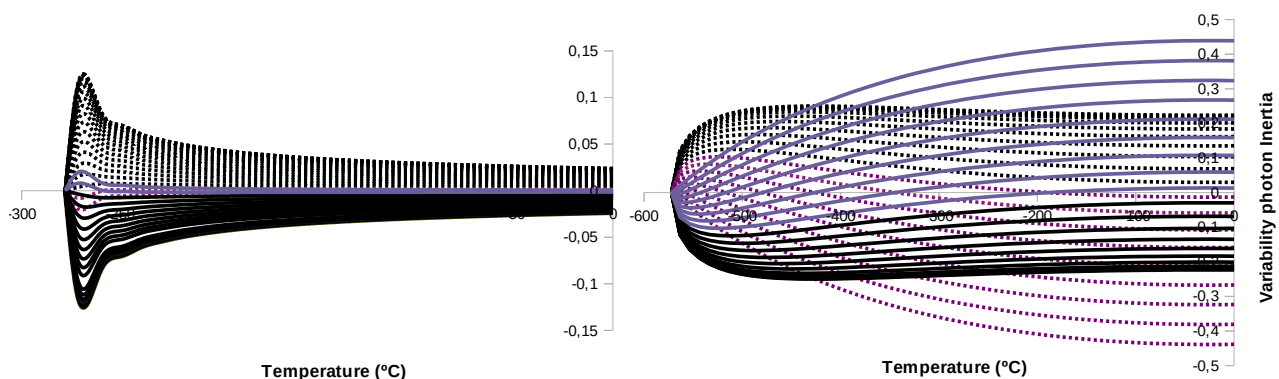


Figura 8 -Variability water (right) and air (left), for specific vacuum temperature in $\sim -277,818$ and $\sim -571,368$ °C, whit $\lambda = 589$ nm. (source: author's)

4. CONSIDERATIONS

The photon was treated as an immaterial quantity far from the interaction, where flux of the number of wavefronts characterizes a displacement mass of which the massive effect arises with the interaction as a consequence of the conservative properties of the photon-matter system, such that if there is no conservation of energy in some form of motion the total mass will correspond to the

⁷ The analysis conducted in the treatment of variability in Figura 8 considered two models for the refractive index as a function of temperature, $n(T)$, given by eqs. (1, 2), previously known in the literature. Other models for $n(T)$ can be adopted for the same analysis, which may attenuate or accentuate curves and/or vary in a narrow range the estimate of the specific vacuum temperature.

displacement mass. The massive effects were treated in the inertia and temperature domain of the refringent medium.

The analysis of the decomposition of the photon inertia into translational and rotational parts in the transition between two media allowed to verify an adjustment of the energy balance employed to the translational and OAM parts, in the perspective of the inertia, as a function of the refringence of the medium, the temperature and the angle of incidence.

The relativistic increase of the translational part of the inertia explains that the delays of the relativistic trajectories are due to the greater resistance of photons to changes in their directional properties, suppressing the trajectory imposed by the Minkowski torque in a range of incidence. The synchronizations of the classical and relativistic trajectories comprise the beginning of the classical predominance over the translational inertia where the photon offers less resistance to changes in its directional properties, allowing the Minkowski torque to impose its trajectory.

It was found that the relativistic trajectory is preceded by an increase in the moment of inertia as a function of the angle of incidence. The rotational part of the inertia presents a significant relativistic increase to the point of overlapping the classical part. During synchronization the predominance of the rotational part is relativistic but with a lower intensity of inertia compared to the one preceding synchronization, agreeing with the negative variations of the moment of inertia.

The analysis of the moment of inertia as a function of the refractive medium temperature verified that the synchronizations approach the normal incidence with increasing temperature and that each material has a wider range of angular incidence with synchronizations as a function of temperature. Synchronization over all angles of incidence, where classical and relativistic variabilities are zero, occurs at the specific vacuum temperature. The specific vacuum temperature is that of the refringent medium in which the photon is in a classical-relativistic synchronization under all angles of incidence, presenting no natural structuring of photon-matter interaction, becoming immaterial, as in vacuum.

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