Motor of Mutual Retention

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Abstract

This work proposes a new theoretical approach for the next generation of extremely efficient motors, whose impact will be substantial for future sustainable technological development. Building on well-known physical concepts, this approach introduces the concept of equilibrium of mutual retention, upon which a device remains in an equilibrium state between movement and attraction-repulsion. This new theoretical approach showed promising results in computer simulation experiments, indicating that the equilibrium of mutual retention can allow for a new way of modeling an extremely efficient motor. Finally, a theoretical efficiency analysis showed the need to expand the limits of some physical concepts already established.

Keywords: Equilibrium of mutual retention, Self-contained energy of mutual retention, Motor of mutual retention

1. Introduction

One of the strategies to mitigate the impacts of the greenhouse effect, the principal cause of climate change, is to optimize and improve the de-

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sign of electric motors, which represent 40-50% of the total global electricity consumption, to make them much more efficient [1].

Consequently, many countries are adopting mandatory regulations to boost the electric motor industry to the efficiency classes established by the International Electrotechnical Commission (IEC) [2], namely: IE1 - Standard Efficiency; IE2 - High Efficiency; IE3 - Premium Efficiency; IE4 - Super Premium Efficiency; and IE5 - Ultra Premium Efficiency.

However, a common point in all the efficiency classes is that they aim to considerably reduce the losses inherent to the various motor components.

Although the strategy of improving efficiency without reducing losses seems like a nonsensical hypothesis, the challenge is worth exploring and reporting because it suggests new possibilities. Therefore, the objective of this work is to present a disruptive proposal that introduces a new theoretical approach for the sustainable technological development of the next generation of motors.

In this context, starting from a foundation of well-known physical concepts, the presented methodology leads to a new theoretical approach that proposes the concept of equilibrium of mutual retention between movement and attraction-repulsion, on which the motor remains in this equilibrium state.

Finally, it is worth noting that the subsequent sections are structured to achieve the proposed objective and represent the main contribution of this work, that is, they present the basic principles on which the new theoretical approach rests.

Therefore, the methods section gradually brings the mathematical and physical developments that lead to the equilibrium of mutual retention. Then, the results section presents evidence of some of the basic principles of the new theoretical approach. In the end, the discussion section analyzes the simulated results and discusses the evidence of the other basic principles. Furthermore, other consequences of this new approach are highlighted, such as motor efficiency.

2. Method

2.1. Halbach Cylinder

The Halbach cylinder [3, 4] is a hollow cylinder made of a ferromagnetic material, such that its magnetization distribution $\vec{M}(\rho, \varphi)$ produces an intense magnetic flux density $\vec{B}(\rho, \varphi)$, which in cylindrical coordinates, are given, respectively, by

$$\vec{M}(\rho,\varphi) = M_r\left[cos\left(k\varphi\right)\hat{\rho} + sin\left(k\varphi\right)\hat{\varphi}\right] \tag{1}$$

$$\vec{B}\left(\rho,\varphi\right) = B_k\rho^{(k-1)}\left[cos\left(k\varphi\right)\hat{\rho} - sin\left(k\varphi\right)\hat{\varphi}\right] \tag{2}$$

such that

$$B_k = \begin{cases} B_r\frac{k}{k-1}\left[1-\left(\frac{r_{in}}{r_{ou}}\right)^{k-1}\right]\left(\frac{1}{r_{in}}\right)^{k-1} & \text{if } k>1 \\ B_r ln\left(\frac{r_{ou}}{r_{in}}\right)^{k-1} & \text{if } k=1 \\ B_r\frac{k}{k-1}\left[1-\left(\frac{r_{ou}}{r_{in}}\right)^{k-1}\right]\left(\frac{1}{r_{ou}}\right)^{k-1} & \text{if } k\leq -1 \end{cases} \tag{3}$$

where M_r is the ferromagnetic remanence, B_r is the magnitude of the remanent magnetic flux density, k is a integer that defines the order of the Halbach cylinder, r_{in} is the inner radius of the Halbach cylinder and r_{ou} is the outer radius of the Halbach cylinder.

2.2. Magnetic Force

Consider in the vicinity of the Halbach cylinder a magnetic dipole moment $\vec{\mu}$ given by

$$\vec{\mu} = \mu_\rho\hat{\rho} + \mu_\varphi\hat{\varphi} \tag{4}$$

where μ_ρ is the radial component and μ_φ is the tangential component.

The magnetic force $\vec{F}$ acting on the dipole is given by [5]

$$\vec{F} = \nabla(\vec{\mu}\cdot\vec{B}) \tag{5}$$

As a result of Eqs. from 2 to 5, the components of the magnetic force ($\vec{F} = F_\rho\hat{\rho} + F_\varphi\hat{\varphi}$) can be written as [6]

$$F_\rho = -B_k\rho^{(k-2)}\left(k-1\right)\left[\mu_\varphi sin\left(k\varphi\right) - \mu_\rho cos\left(k\varphi\right)\right] \tag{6}$$

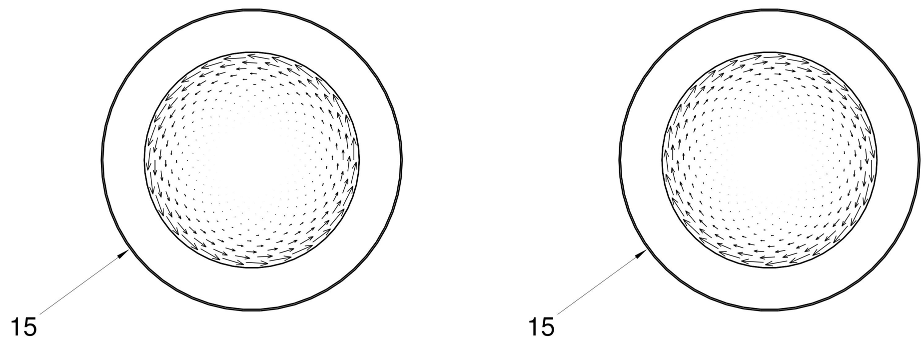
$$F_\varphi = -B_k\rho^{(k-2)}k\left[\mu_\varphi cos\left(k\varphi\right) + \mu_\rho sin\left(k\varphi\right)\right] \tag{7}$$

where F_ρ is the radial component and F_φ is the tangential component.

2.3. Proof Dipole

Now consider the magnetic dipole $\vec{\mu}$ as a proof dipole. This proof dipole will be used to analyze the direction of the magnetic force $\vec{F}$, such that the angle θ between the proof dipole $\vec{\mu}$ and the magnetic flux density $\vec{B}$ will be constant at all points in space inside the Halbach cylinder.

In this condition, for example, for $k = 9$ and for $\theta = \pm 90°$, the Figs. 1a and 1b show the magnetic force $\vec{F}$ (represented by black arrows) acting on the proof dipole $\vec{\mu}$ located at various points within the Halbach cylinder (identified by the reference number 15 according to [6]).



(a) $\theta = -90°$ (b) $\theta = +90°$

Figure 1: Direction of magnetic force $\vec{F}$ acting on the proof dipole $\vec{\mu}$ when $\theta = \pm 90°$ [6].

It can be seen in Figs. 1a and 1b that the magnetic force $\vec{F}$ is always tangent to the circumferences described by the proof dipole $\vec{\mu}$, that is, $\vec{F}$ only has a tangential component $\vec{F}_\varphi$ and the radial component $\vec{F}_\rho$ is always null.

Therefore, the tangential component $\vec{F}_\varphi$ is able to move the magnetic dipole $\vec{\mu}$ along a circumference of radius r_o, where r_o is distance between the center of the Halbach cylinder and the center of mass of the dipole, provided that the angle θ is favorable for this.

2.4. Velocity Ratio

One way to keep $\vec{F}_\varphi \neq \vec{0}$ is to impose a compound motion on the magnetic dipole $\vec{\mu}$, as illustrated in Fig. 2, where the reference number 37 identifies the dipole $\vec{\mu}$, such that the ratio of intrinsic angular velocity $\vec{\omega}_i(t)$ to orbital angular velocity $\vec{\omega}_o(t)$, named the velocity ratio, is expressed by [6]

$$\frac{\vec{\omega}_i(t)}{\vec{\omega}_o(t)} = -k \tag{8}$$

where k is the order of the Halbach cylinder.

Since the angular velocity is the derivative of the angular displacement with respect to time t, it can be shown that [6]

$$\theta_i(t) = -\left[k\varphi_o(t) - k\varphi_o(t_0) - \theta_i(t_0)\right] \tag{9}$$

where $\theta_i(t) = |\vec{\theta}_i(t)|$ is the magnitude of the intrinsic angular displacement of the magnetic dipole $\vec{\mu}$ with respect to its center of mass, $\theta_i(t_0)$ is the magnitude of the intrinsic angular displacement at the initial instant $t_0 = 0s$, named the misalignment angle and denoted by θ_d, such that $\theta_i(t_0) = \theta_d$, $\varphi_o(t) = |\vec{\varphi}_o(t)|$ is the magnitude of the orbital angular displacement of the magnetic dipole $\vec{\mu}$ with respect to the center of the Halbach cylinder and $\varphi_o(t_0) = |\vec{\varphi}_o(t_0)|$ is the magnitude of the orbital angular displacement at the initial instant $t_0 = 0s$.

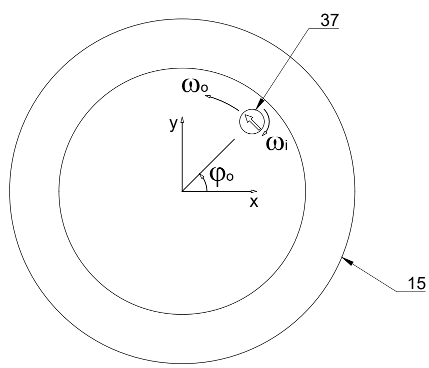


Figure 2: Compound motion of the magnetic dipole $\vec{\mu}$ (identified by the reference number 37) inside the Halbach cylinder (identified by the reference number 15) [6].

In this way, the radial component μ_ρ and the tangential component μ_φ are given, respectively, by [6]

$$\mu_\rho = +\mu_r cos[\theta_i(t)] = +\mu_r cos\left[k\varphi_o(t) - k\varphi_o(t_0) - \theta_d\right] \tag{10}$$

$$\mu_\varphi = +\mu_r sin[\theta_i(t)] = -\mu_r sin\left[k\varphi_o(t) - k\varphi_o(t_0) - \theta_d\right] \tag{11}$$

where μ_r is the magnitude of the magnetic dipole $\vec{\mu}$.

Therefore, when the magnetic dipole $\vec{\mu}$ is located in the ordered pair (ρ_o, φ_o), i.e. $\rho \equiv \rho_o(t)$ and $\varphi \equiv \varphi_o(t)$, then the components of the magnetic force $\vec{F}$ in (ρ_o, φ_o), given by the Eqs. 6 and 7, can be rewritten as [6]

$$\vec{F}_\rho = (k-1)F_k cos[k\varphi_o(t_0) + \theta_d]\hat{\rho} \tag{12}$$

$$\vec{F}_\varphi = -kF_k sin[k\varphi_o(t_0) + \theta_d]\hat{\varphi} \quad (13)$$

where $F_k = \mu_r B_k \rho_o^{(k-2)}$.

The Fig. 3 shows in the same graph the variations of the radial component $\vec{F}_\rho$ and the tangential component $\vec{F}_\varphi$ as a function of the misalignment angle θ_d, where $F_k = \mu_r B_k \rho_o^{(k-2)} = 1N$, $k = 10$ and $\varphi_o(t_0) = 0°$.

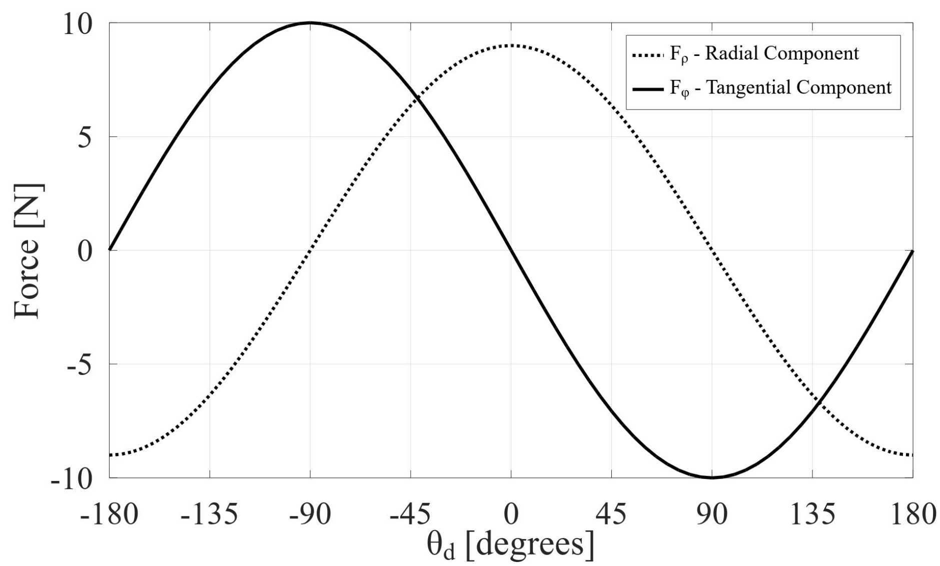
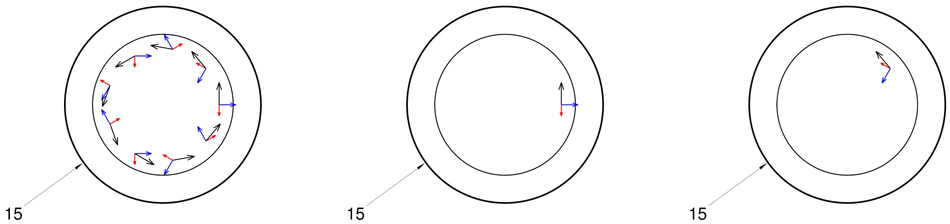


Figure 3: The radial component $\vec{F}_\rho$ and the tangential component $\vec{F}_\varphi$ as a function of the misalignment angle θ_d.

It can be seen in the Fig. 3 that there is only a radial component $\vec{F}_\rho$ when $\theta_d = 0°$ or $\theta_d = \pm 180°$, such that $\vec{F} = \vec{F}_\rho$. On the other hand, there is only a tangential component $\vec{F}_\varphi$ when $\theta_d = \pm 90°$, such that $\vec{F} = \vec{F}_\varphi$. Finally, at other angles, like $\theta = \pm 45°$, there is both a radial component $\vec{F}_\rho$ and a tangential component $\vec{F}_\varphi$, such that $\vec{F} = \vec{F}_\rho + \vec{F}_\varphi$.

As an example, the Figs. from 4a to 4c illustrate the magnetic force $\vec{F}$ acting on a magnetic dipole $\vec{\mu}$ inside the Halbach cylinder of order $k = 13$ for nine instants of time corresponding to $\varphi_o(t)$ increasing in intervals of 40°. The misalignment angle θ_d between the magnetic dipole $\vec{\mu}$ and the magnetic flux density $\vec{B}$ at each of these instants is constant and always equals $-90°$.

Initially, the Fig. 4b shows that the magnetic dipole $\vec{\mu}$ (red arrow) is at rest at the instant $t = 0s$, forming a misalignment angle θ_d equals $-90°$ with



(a) All time overlay (b) $\varphi_o(t) = 0°$ (c) $\varphi_o(t) = 40°$

Figure 4: Magnetic force $\vec{F}$ (black arrow) acting on a magnetic dipole $\vec{\mu}$ (red arrow) inside a Halbach cylinder of order $k = 13$, responsible for creating the magnetic flux density $\vec{B}$ (blue arrow). Magnetic dipole $\vec{\mu}$ has $\vec{\omega}_i/\vec{\omega}_o = -13$, $\theta_d = -90°$ e $\rho_o = r_o$.

magnetic flux density $\vec{B}$ (blue arrow). In this condition, the magnetic force $\vec{F}$ (black arrow) acting on the dipole is tangent to the circumference r_o.

Assuming that the velocity ratio given by Eq. 8 is satisfied by some mechanism, then, due to the action of the magnetic force $\vec{F}$, the magnetic dipole $\vec{\mu}$ starts to move along a circumference of radius r_o with orbital angular velocity $\vec{\omega}_o$, while, simultaneously, performing a circular motion around its own center of mass with intrinsic angular velocity $\vec{\omega}_i$.

This compound motion (orbital $\vec{\omega}_o$ and intrinsic $\vec{\omega}_i$) is responsible for keeping the misalignment angle θ_d constant and equal to $-90°$. This, in turn, keeps the tangential component $\vec{F}_\varphi$ not null, as seen in Fig. 4c.

2.5. Planetary Gear System

A planetary gear system performs the velocity ratio given by Eq. 8, where the planetary gears have a rotary motion transmitted to the carriers, such that the carriers have orbital angular velocity $\vec{\omega}_o$ and planetary gears have intrinsic angular velocities $\vec{\omega}_i$.

To satisfy the velocity ratio $\vec{\omega}_i/\vec{\omega}_o = -k$ when the planetary gear is inside the ring gear, the ratio of the number of teeth of the ring gear Z_r by the number of teeth of each planetary gear Z_p is given by [6]

$$\frac{Z_r}{Z_p} = |k| + 1 \tag{14}$$

Consequently, the magnetic dipoles $\vec{\mu}$ are located on a circumference of radius r_o given by [6]

$$r_o = kr_p \tag{15}$$

where r_p is the pitch radius of the planetary gear.

Before continuing, the list below identifies the reference numbers of some elements comprising the motor of mutual retention [6], while Figs. from 5a to 5f show the arrangement of these elements:

- the stator (2) comprises: Halbach cylinder (15); and ring gear (13);
- the Halbach cylinder (15) comprises: segments (41);
- the rotor (28) comprises: carriers (30 and 34); and satellites (35);
- the satellite (35) comprises: satellite axis (36); magnetic dipole $\vec{\mu}$ (37); and planetary gears (38).

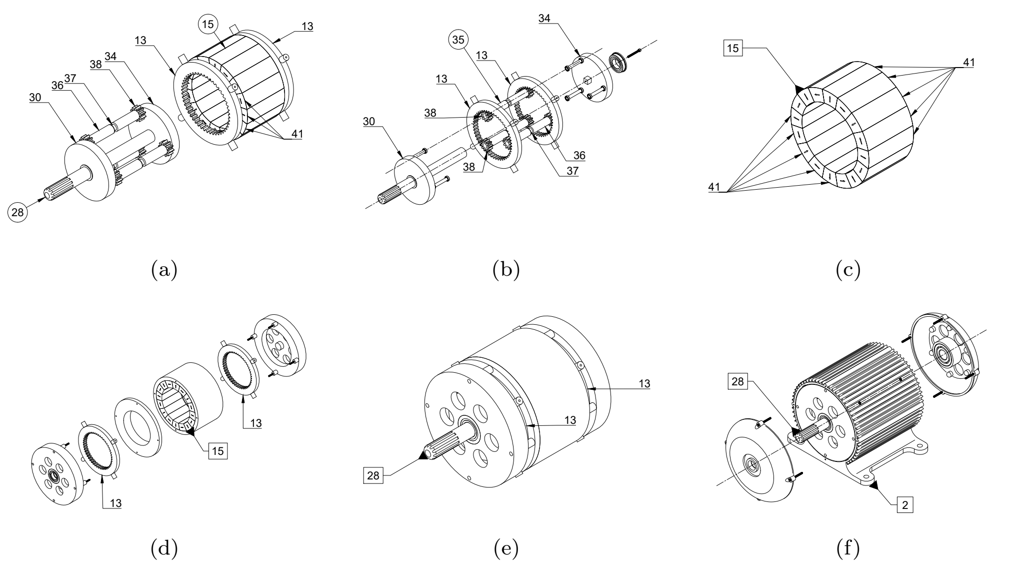


Figure 5: Arrangement of some elements that comprise the motor of mutual retention [6].

After establishing this nomenclature, note that the resultant force $\vec{R}$ acting on each satellite has a radial component $\vec{R}_\rho$, responsible for the centripetal force, and a component tangential $\vec{R}_\varphi$, responsible for the orbital torque $\vec{\tau}_o$ with respect to the origin of the inertial coordinate system xyz, such that

$$\vec{R} = \vec{R}_\rho + \vec{R}_\varphi \tag{16}$$

Also note that the velocity ratio given by Eq. 8 ensures that the misalignment angle θ_d remains constant along the entire trajectory traversed by the magnetic dipole $\vec{\mu}$, such that the work W_F done by the magnetic force $\vec{F}$ on the closed path C is zero, because $\vec{F}$ is a conservative force, but the work W_R of the resultant force $\vec{R}$ on each satellite is non-zero, that is, by the work-energy theorem [7, 8], we have

$$W_R = \oint_C \vec{R} \cdot d\vec{r} = T_B - T_A = \Delta T \neq 0 \tag{17}$$

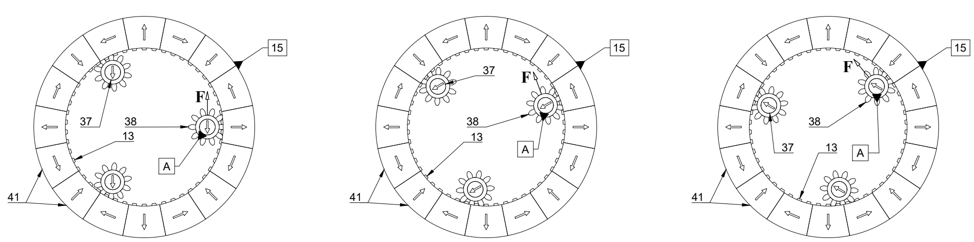
where T_A and T_B are the kinetic energies in an initial A and final B configuration, respectively.

Furthermore, the resultant force $\vec{R}$ has two main parts of an active nature, that is, they are capable of impelling movement: the contribution of the magnetic force $\vec{F}$; and the equivalent contribution due to magnetic torque $\vec{\tau}$, given by [7]

$$\vec{\tau} = \vec{\mu} \times \vec{B} \tag{18}$$

Due to the restrictions imposed on the movement of each satellite (orbital $\vec{\omega}_o$ and intrinsic $\vec{\omega}_i$), restrictions inherent to the gear system, the tangential component $\vec{F}_\varphi$ influences the orbital torque $\vec{\tau}_o$ and the magnetic torque $\vec{\tau}$ influences the intrinsic torque $\vec{\tau}_i$. On the other hand, the other contributions that make up the resultant force $\vec{R}$ are constraint forces due to the contact between the other elements of the gear system.

As an example, consider the Figs. from 6a to 6c that illustrate the evolution of the motion of three magnetic dipoles $\vec{\mu}$ (37), where $\theta_d = -90°$ and the Halbach cylinder (15) of order $k = 4$ is implemented with 16 segments (41). The arrows inside the dipoles (37) and the segments (41) indicate the vector orientation of the magnetization of the permanent magnets.



(a) $\varphi_o = 0°$ e $\theta_i = -90°$ (b) $\varphi_o = 20°$ e $\theta_i = -170°$ (c) $\varphi_o = 40°$ e $\theta_i = -250°$

Figure 6: Evolution of the motion of three magnetic dipoles $\vec{\mu}$ (37) mounted on a gear system at three time instants corresponding to φ_o increasing in intervals of 20° [6].

The Figs. from 6a to 6c illustrate the magnetic force $\vec{F}$ acting on the magnetic dipole $\vec{\mu}$ indicated with the letter A. Note that the magnetic force $\vec{F}$ always has a component tangent to the trajectory described by the $\vec{\mu}$.

2.6. Equilibrium of Mutual Retention

As previously described, the resultant force $\vec{R}$ on each satellite (35) performs a non-zero work W_R due to the actions of magnetic force $\vec{F}$, magnetic torque $\vec{\tau}$, and constraint forces, that guarantee the velocity ratio $\vec{\omega}_i/\vec{\omega}_o = -k$.

Thus, when $\theta_d \neq 0°$ or $\theta_d \neq \pm 180°$, the tangential component $\vec{F}_\varphi$ is responsible for impelling tangential motion to the magnetic dipole $\vec{\mu}$ (37), which is fixed to the planetary gear (38) through the satellite axis (36).

As each magnetic dipole $\vec{\mu}$ begins its orbital angular displacement $\vec{\varphi}_o$ with orbital angular velocity $\vec{\omega}_o$, due to the action of the magnetic force $\vec{F}$, the velocity ratio $\vec{\omega}_i/\vec{\omega}_o = -k$, due to the constraints of movement imposed by the gear system, guarantees that $\vec{\mu}$ has intrinsic angular velocity $\vec{\omega}_i$.

In turn, this compound motion of each magnetic dipole $\vec{\mu}$ ensures that the misalignment angle θ_d remains constant and hence there is always a tangential component $\vec{F}_\varphi$ responsible for impelling tangential motion to dipole $\vec{\mu}$.

Therefore, it can be deduced that there is an equilibrium between movement (orbital $\vec{\omega}_o$ and intrinsic $\vec{\omega}_i$), whose velocity ratio is $\vec{\omega}_i/\vec{\omega}_o = -k$, and opposing forces (force of attraction and force of repulsion), whose resultants are the magnetic force $\vec{F}$ and the magnetic torque $\vec{\tau}$.

This equilibrium represents an interaction of mutual retention, in which the magnetic force $\vec{F}$ and the magnetic torque $\vec{\tau}$, of an active nature, are responsible for maintain and sustain the velocity ratio $\vec{\omega}_i/\vec{\omega}_o = -k$ and similarly the velocity ratio is responsible for maintain and sustain $\vec{F}$ and $\vec{\tau}$.

Before proceeding, note that the other constraint forces, together with the magnetic force $\vec{F}$ and the magnetic torque $\vec{\tau}$, are responsible for ensure the velocity ratio $\vec{\omega}_i/\vec{\omega}_o = -k$. However, these constraint forces are not, by themselves, the propelling spring capable of impelling movement to the set because they are not active forces.

In continuity, this equilibrium of mutual retention means that the "movemen" ($\vec{\omega}_i$ and $\vec{\omega}_o$) is always working in conjunction with "attraction - repulsion" ($\vec{F}$ and $\vec{\tau}$), that is, they are self-contained [9].

That way, the "movement" ($\vec{\omega}_i$ and $\vec{\omega}_o$) is contained and controlled by the "attraction-repulsion" ($\vec{F}$ and $\vec{\tau}$), or, in other words, the "attraction -

repulsion" ($\vec{F}$ and $\vec{\tau}$) sustains and maintains the "movement" ($\vec{\omega}_i$ and $\vec{\omega}_o$) [9].

In particular, the "attraction" provides the 'bonding' to retain the momentum of the "movement" and put it under control, the "repulsion" provides the 'rejection' to ensure continuity of "movement" and the "movement" provides the 'impetus' to ensure that the "attraction-repulsion" remains active, where there is a reciprocal exchange between "movement" and "attraction-repulsion" permanently on an equal footing [9].

Finally, it is possible to infer from the postulates described in [9] that *the equilibrium - the mutual retention between "movement" ($\vec{\omega}_i$ and $\vec{\omega}_o$) and "attraction - repulsion" ($\vec{F}$ and $\vec{\tau}$) - "to move about" and "to remain bonded" - creates an infinite spiral of self-contained energy of mutual retention U_{mr}.*

2.7. Motor of Mutual Retention

Based on this theoretical approach, the motor of mutual retention is a device configured to remain in an equilibrium of mutual retention and convert self-contained energy of mutual retention U_{mr} into mechanical energy E [6].

In summary, a physical realization of the motor of mutual retention comprises at least one magnetic dipole $\vec{\mu}$ immersed in a magnetic flux density $\vec{B}$, whose main contribution comes from the Halbach cylinder of order k. Furthermore, the velocity ratio $\vec{\omega}_i/\vec{\omega}_o = -k$, which describes the compound motion of the magnetic dipole $\vec{\mu}$, is conveniently chosen to suit the variation of magnetic flux density $\vec{B}$, such that the magnetic force $\vec{F}$ and the magnetic torque $\vec{\tau}$ are the active nature propulsive action to move magnetic dipole $\vec{\mu}$. Thus, with a simple adjustment of the angular position θ_d between the magnetic dipole $\vec{\mu}$ and the magnetic flux density $\vec{B}$, it is possible to control the velocity, the acceleration and the torque applied to the motor load [6].

Finally, it is possible to correlate the inventive concept - *the equilibrium of mutual retention between "movement – attraction – repulsion" creates an infinite spiral of self-contained energy of mutual retention U_{mr} and this energy is converted into mechanical energy E* - with the well-known physical concepts [6].

Therefore, the "equilibrium" is correlated with the condition of the motor, in which the "movement" and the "attraction-repulsion" compensate each other to maintain the state of the motor preserved. The "mutual retention" emphasizes that the "movement" and the "attraction-repulsion" happen reciprocally in a permanent state, such that the "movement" maintains and sustains the "attraction-repulsion" and the "attraction-repulsion" maintains and sustains the “movement”. The “movement” is correlated with the physical quantity whose measurable property is associated with velocity. The “attraction-repulsion” is correlated with physical quantities whose measurable properties are associated with force and torque. The “infinite spiral” emphasizes a progressive process tending to infinity as long as equilibrium persists. The “self-contained energy of mutual retention U_{mr}” is correlated with the physical quantity whose measurable property is associated with energy. The expression “self-contained” emphasizes that the “movement” and the “attraction-repulsion” have everything needed to work together eternally. The expression “this energy is converted into mechanical energy E” is correlated with the First Law of Thermodynamics which deals with the conservation of energy and its transformations [6].

3. Results

This section presents the results of the validation of the basic principles related to the behavior of the magnetic force $\vec{F}$ in a realistic situation.

For this, the ideal magnetic dipoles were implemented by permanent magnets. Likewise, the ideal Halbach cylinder was implemented by a segmented Halbach cylinder, where each segment comprises a permanent magnet.

The dipoles were placed inside the Halbach cylinder and subjected to the velocity ratio $\vec{\omega_i}/\vec{\omega_o} = -k$, where the misalignment angle θ_d was made equal to $-90°$.

The Finite Element Method Magnetics (FEMM) software [10] was used to simulate the behavior of the magnetic force $\vec{F}$. This software uses the finite element method to solve two-dimensional electromagnetic problems, despite accepting depth specifications in planar problems. Furthermore, Lua Programming Language was used to automate the simulations.

3.1. Simulations

The Table 1 presents the parameters and specifications of the simulations, in addition to indicating some constructive parameters, such as r_p and r_r.

The Fig. 7a shows the simulation result referring to Table 1 with the indication of some elements that comprise the motor [6].

The Fig. 7b shows the graph of the components of the magnetic force $\vec{F}$ acting on each permanent magnet as a function of the misalignment angle θ_d, where $\varphi_o(t_o) = \beta_i$ is the orbital angular displacement of each dipole at the initial instant $t = 0s$, such that $\beta_i \in [0°, 120°, 240°]$.

Parameters	Specifications	Description
k	4	Order of Halbach cylinder
N_i	3	Number of magnetic dipoles
N_s	32	Halbach implemented by N_s magnets
$\vec{\omega_i}/\vec{\omega_o}$	-4	Velocity ratio (Lua script)
θ_d	$-90°$	Misalignment angle
β_i	$120°$	Angular sector between the dipoles
r_d	$5\,mm$	Radius of magnets
r_p	$10\,mm$	Pitch radius of the planetary gear
r_o	$40\,mm$	Magnets over circumference r_o
r_r	$50\,mm$	Pitch radius of the ring gear
r_{in}	$50\,mm$	Inner radius of Halbach
r_{ou}	$70\,mm$	Outer radius of Halbach
r_b	$100\,mm$	Radius of boundary condition
Boundary	Prescrited A	Software parameters
Depth	$30\,mm$	Software parameters
Dipoles	$NdFeB\ N55$	Dipoles are permanent magnets
Halbach	$NdFeB\ N55$	Segments are permanent magnets
Medium	Air	Space is filled with air
Problem	Magnetics	Software parameters
Units	Millimeters	Software parameters

Table 1: Parameters and specifications for simulations.

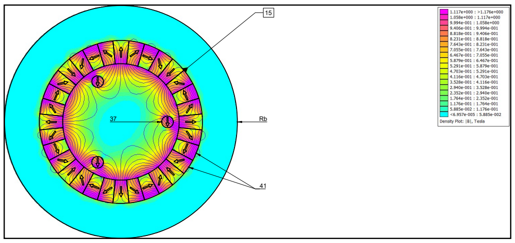
The Fig. 7c shows the components of the magnetic force $\vec{F}$ acting on each magnetic dipole $\vec{\mu}$ as a function of the orbital angular displacement φ_o, when the velocity ratio is $\vec{\omega_i}/\vec{\omega_o} = -4$ and the misalignment angle $\theta_d = -90°$.

The Fig. 7d shows the orbital torque $\vec{\tau_o}$ acting on the magnetic dipoles $\vec{\mu}$ as a function of the orbital angular displacement φ_o, when the velocity ratio is $\vec{\omega_i}/\vec{\omega_o} = -4$ and the misalignment angle $\theta_d = -90°$.

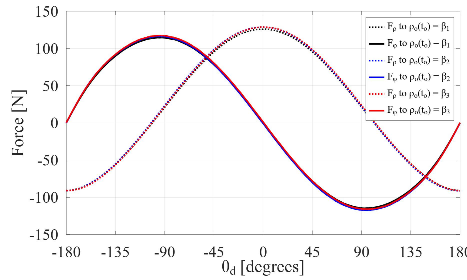
4. Discussion

4.1. Analysis of Results

Firstly, the analysis of the equation $\vec{F} = \nabla(\vec{\mu} \cdot \vec{B})$ says that, unlike the gravitational force, it is possible to create a region of space in which the vector orientation of the magnetic force $\vec{F}$ is easily manipulated, because this force is the gradient of the dot product between two vectors independent,



(a)



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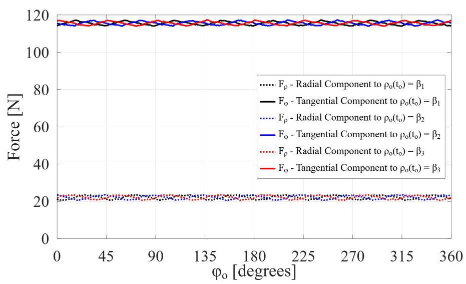
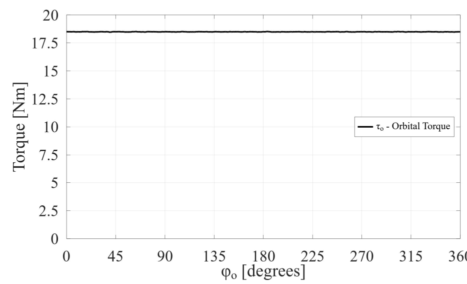


(c)



(d)

Figure 7: Simulations: (a) FEMM result. (b) $\vec{F}$ as a function of the θ_d. (c) $\vec{F}$ as a function of the φ_o. (d) $\vec{\tau_o}$ as a function of the φ_o [6].

$\vec{\mu}$ and $\vec{B}$. Therefore, as the magnetic flux density $\vec{B}$ is a function of the coordinates, a variation of the misalignment angle θ_d is able to direct the magnetic force $\vec{F}$ in any desired direction. According to this observation, by imposing a compound motion on the magnetic dipole $\vec{\mu}$, through a velocity ratio conveniently chosen to suit the variation of the vector field $\vec{B}$, the magnetic force $\vec{F}$ can have a component tangential to the trajectory described by the dipole, provided that θ_d is favorable to this.

This manipulation of the magnetic force $\vec{F}$ in any desired direction is one of the basic principles observed in the Fig. 7b. The reason why the radial component $\vec{F_\rho}$ is shifted upwards is due to the implementation by permanent magnets, whose cylindrical shape (radius $r_d = 5mm$ and height $h_d = 30mm$) is composed of several domains of elementary dipoles related mainly to the electron spin quantum number.

Again, the Fig. 7c shows that the behavior of the magnetic force $\vec{F}$ is in accordance with another basic principle related to the velocity ratio $\vec{\omega_i}/\vec{\omega_o} = -k$ and to the misalignment angle θ_d. This means that compound motion of the dipoles keeps the tangential component $\vec{F_\varphi}$ always present, provided that θ_d has an appropriate value for this.

Similarly, the Fig. 7d shows that the orbital torque $\vec{\tau}_o$, like the magnetic force $\vec{F}$, keeps the rotation counterclockwise and the magnitude approximately constant.

The undulations seen in the Figs. 7c and 7d result from the segmented implementation of the Halbach cylinder and the interaction between the dipoles. Note that, by the superposition principle, all magnetic elements contribute to the magnetic flux density $\vec{B}$. However, the main contribution of $\vec{B}$ comes from the Halbach cylinder.

Therefore, the analysis of the results is consistent with the basic principles about the behavior of the magnetic force $\vec{F}$ presented in the methodology.

Furthermore, the Fig. 7c shows that, although the dimensions are on the order of millimeters, the tangential component $\vec{F}_\varphi$ is on the order of hundreds of Newtons. This means that a force of 100 N applied to a mass of $1kg$ produces an acceleration of $100\, m/s^2$ or a force of $100\, N$ applied to a mass of $100kg$ produces an acceleration of $1\, m/s^2$.

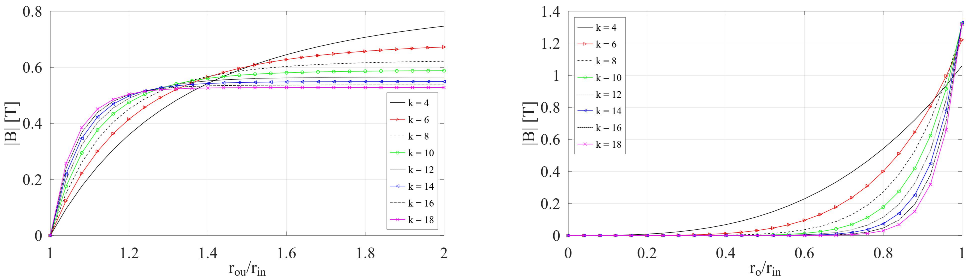
Consequently, the analysis from the point of view of the magnetic force $\vec{F}$ shows that this force of an active nature is capable of altering the state of rest or motion of the magnetic dipole $\vec{\mu}$. It means that, in the motor of mutual retention, the friction force to keep the satellite (35) stationary must have a magnitude equal to the magnetic force $\vec{F}$. However, since the moving parts of the gear system have reduced friction, the tangential component $\vec{F}_\varphi$ governs the movement of the rotor (28).

Before finishing, look at the Figs. 8a and 8b, where $B_r = 1.25T$ and $r_{in} = (k+1)r_p$. The Fig. 8a shows that the higher the value of k, the faster $|\vec{B}|$ reaches its maximum value, such that the ratio r_{ou}/r_{in}, which represents the wall thickness of the Halbach cylinder, is a relevant design parameter. The Fig. 8b shows that the closer the magnetic dipole $\vec{\mu}$ is to the inner wall of the Halbach cylinder, the larger $|\vec{B}|$, such that the ratio r_o/r_{in} is another design parameter that must also be taken into account.

Finally, despite the validation of the behavior of the magnetic force $\vec{F}$, further discussions presented below should be taken into account to bring consistency to the new theoretical approach.

4.2. Laws of Conservation

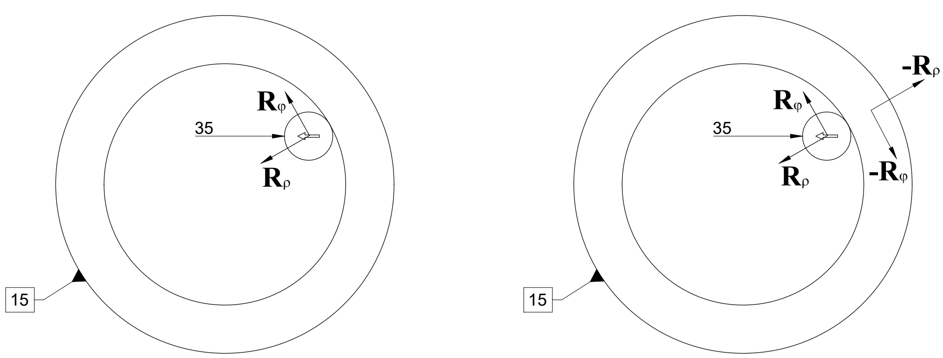
The Fig. 9a shows an idealized model of the motor, in which the satellite (35) comprises an elementary magnetic dipole $\vec{\mu}$ coupled to the center of a cylinder of radius r_p, infinitesimal height, mass m_p and material with zero



(a) (b)

Figure 8: (a) $|\vec{B}|$ as a function of the ratio r_{ou}/r_{in}, when $r_o = kr_p$. (b) $|\vec{B}|$ as a function of the ratio r_o/r_{in}, when $r_{ou} = 1.4r_{in}$ [6].

magnetic susceptibility (motor rotor). The dipole is located on a circumference of radius $r_o = kr_p$, where r_p represents the pitch radius of the planetary gear. The ideal Halbach cylinder (15) has an inner radius $r_{in} = r_o + r_p$, where r_{in} represents the pitch radius of the ring gear (motor stator). Furthermore, consider that the cylinders are in contact and that the movement ($\vec{\omega_i} = -k\vec{\omega_o}$) occurs without slipping and without losses.



(a) (b)

Figure 9: Idealized model of the motor of mutual retention.

Finally, consider that the magnetic dipole $\vec{\mu}$ forms a misalignment angle θ_d with the magnetic flux density $\vec{B}$, such that $\vec{F_\varphi} \neq \vec{0}$ and the magnetic potential energy U_m is given by [7]

$$U_m = -\vec{\mu} \cdot \vec{B} = -|\vec{\mu}|\left|\vec{B}\right| cos(\theta_d) \tag{19}$$

According to the Hamiltonian formalism [11], the Hamiltonian $\mathcal{H}$ of the idealized model is given by [6]

$$\mathcal{H}\left(\varphi, p_{\varphi}, t\right) = \frac{p_{\varphi}^{2}}{2\left(I_{o} - kI_{i}\right)} - |\vec{\mu}| \left|\vec{B}\right| cos\left(\theta_{d}\right) + U_{mr} \tag{20}$$

where the first term is the kinetic energy T, the second term is the magnetic potential energy U_m, and the third term is the self-contained energy of mutual retention U_{mr}, which represents the energy latent to the system due to equilibrium of mutual retention, p_{φ} is the canonical moment conjugated to the generalized coordinate $\varphi \equiv \varphi_o$, I_o is the orbital moment of inertia of the satellite, which represents the satellite's moment of inertia seen as a point mass about the center of the Halbach cylinder, and I_i is the satellite's intrinsic moment of inertia, which represents the satellite's moment of inertia about its center of mass.

From Hamilton's equations [11], we have [6]

$$\dot{\varphi} = \frac{\partial \mathcal{H}}{\partial p_{\varphi}} = \frac{p_{\varphi}}{\left(I_{o} - kI_{i}\right)} \tag{21}$$

$$\dot{p}_{\varphi} = -\frac{\partial \mathcal{H}}{\partial \varphi} = 0 \tag{22}$$

$$\frac{d\mathcal{H}}{dt} = \frac{\partial \mathcal{H}}{\partial t} = 0 \tag{23}$$

The Eq. 23 means that if the Hamiltonian $\mathcal{H}$ does not explicitly depend on the time t, then $\mathcal{H}$ is a constant of motion [11], that is [6],

$$T + U_m + U_{mr} = constant\, of\, motion \tag{24}$$

such that [6]

$$\Delta T + \Delta U_m + \Delta U_{mr} = 0 \tag{25}$$

where $\Delta U_m = 0$ for the case under analysis, that is, there is no variation in magnetic potential energy, since $U_m = -|\vec{\mu}||\vec{B}|cos(\theta_d)$ remains constant because θ_d, $|\vec{\mu}|$ and $|\vec{B}|$ do not change (although the direction of vector $\vec{B}$ varies, its magnitude remains constant along the circumference of radius r_o, see Eq. 2), evidencing the fact that there is no conversion of magnetic potential energy U_m in kinetic energy T, and the conservation of the total energy is satisfied by introducing the self-contained energy of mutual retention U_{mr} due to the equilibrium of mutual retention, since $\vec{R}_{\varphi} \neq \vec{0}$ because $\vec{F}_{\varphi} \neq \vec{0}$.

The Eq. 22 means that if the Hamiltonian $\mathcal{H}$ does not explicitly depend on the generalized coordinate φ, then the canonical moment p_φ conjugate to φ is a constant of motion [11], that is [6],

$$p_\varphi = (I_o - kI_i)\,\dot{\varphi} = constant\,of\,motion \tag{26}$$

such that [6]

$$\vec{L}_r + \vec{L}_s = \vec{0} \tag{27}$$

where p_φ represents the angular momentum of the rotor $\vec{L}_r$ and $\vec{L}_s$ is the angular momentum of the stator, and, consequently, the conservation of the total angular momentum is satisfied. Note that even though the stator remains stationary, $\vec{L}_s = -\vec{L}_r$ due to Newton's Third Law [7, 8].

At this point, note that when Hamiltonian $\mathcal{H}$ does not explicitly depend on a coordinate q_i, this coordinate is said to be cyclic or ignorable and, consequently, the canonical moment p_i conjugated to q_i is constant of motion [11].

Therefore, as the Hamiltonian of Eq. 20 does not depend on the other cylindrical coordinates ρ and z, then the conservation of total linear momentum is also satisfied, as illustrated by Fig. 9b, as a result of Newton's Third Law [7, 8], such that it is possible to demonstrate [6]

$$\vec{R}_\varphi^{(r)} = -\vec{R}_\varphi^{(s)} = \frac{2}{3}\vec{F}_\varphi \tag{28}$$

$$\vec{R}_\rho^{(r)} = -\vec{R}_\rho^{(s)} = -m_p r_p \dot{\varphi}^2 \hat{\rho} \tag{29}$$

$$\vec{R}_z^{(r)} = -\vec{R}_z^{(s)} \tag{30}$$

where the superscript (r) denotes force applied to the rotor, the superscript (s) denotes force applied to the stator, and $\vec{R}_\rho^{(r)}$ is the centripetal force.

In particular, the intrinsic torque $\vec{\tau}_i$ is given by

$$\vec{\tau}_i = \vec{\tau}_s - \vec{\tau} \tag{31}$$

where $\vec{\tau}_s$ is the torque due to constraint forces acting on the satellite and $\vec{\tau}$ is the magnetic torque, whose magnitude is $|\vec{\tau}| = |\vec{\mu}|\left|\vec{B}\right| sin\,(\theta_d)$ [7], such that

$$|\vec{\tau}_i| = r_p\left|\vec{F}_{\theta_i}\right| = \frac{r_p}{3}\left|\vec{F}_\varphi\right| = \frac{1}{3}|\vec{\mu}|\left|\vec{B}\right| sin\,(\theta_d) = \frac{1}{3}|\vec{\tau}| \tag{32}$$

thus

$$|\vec{\tau}_s| = |\vec{\tau}_i| + |\vec{\tau}| = \frac{4}{3}\,|\vec{\tau}| \tag{33}$$

where $\vec{F}_{\theta_i} = \frac{1}{3}\vec{F}_\varphi$ is the resultant tangential force that the stator applies to the satellite.

The Eq. 31 makes it clear that the magnetic torque $\vec{\tau}$ also influences the movement, despite being in opposite direction of the intrinsic torque $\vec{\tau}_i$, such that the magnetic torque $\vec{\tau}$ acts as a natural "brake". However, $|\vec{\tau}_s| > |\vec{\tau}|$ and, consequently, $\vec{\omega}_i/\vec{\omega}_o = -k$ remains valid, i.e. the intrinsic angular velocity direction $\vec{\omega}_i$ is given by $\vec{\tau}_i$ due to the action of the force $\vec{F}_{\theta_i}$.

Finally, note that the above deductions assumed that $U_{mr} \neq f(t, \rho, \varphi, z)$, this is, U_{mr} is not a function of the variables t, ρ, φ, and z. Therefore, the description of the dynamics of the idealized model, due to the action of the magnetic force $\vec{F}$, the magnetic torque $\vec{\tau}$ and the constraint forces, that guarantee the velocity ratio $\vec{\omega}_i/\vec{\omega}_o = -k$, where $U_{mr} \neq f(t, \rho, \varphi, z)$, satisfies the Laws of Conservation.

4.3. Theoretical Efficiency

The efficiency of a common motor is given by

$$\eta = \frac{E_u}{E_s} \tag{34}$$

where E_s is the input supplied energy and E_u is the output useful energy E_u.

For the case of the motor of mutual retention, the supplied energy E_s is that necessary to misalign each magnetic dipole $\vec{\mu}$ about the magnetic flux density $\vec{B}$, for example, when the Halbach cylinder is rotated, such that [6]

$$E_s = N_i|\vec{\mu}||\vec{B}||cos(\theta_d)| \tag{35}$$

As for energy E_u, remember that the motor of mutual retention converts self-contained energy of mutual retention U_{mr} into mechanical energy E, such that [6]

$$E = E_r + E_l \tag{36}$$

where E_r is the useful energy available at the rotor shaft, i.e. $E_u = E_r$, and E_l is the energy associated with losses, such as energy dissipated in the form of heat, raising the temperature of the system.

While the motor of mutual retention remains in the equilibrium of mutual retention, the efficiency is given by [6]

$$\eta = \frac{E_u}{E_s} \begin{cases} \leq 1 & \text{if } E_u \leq E_s \\ > 1 & \text{if } E_u > E_s \end{cases} \tag{37}$$

Therefore, the theoretical efficiency of the motor of mutual retention is higher than any efficiency class established by the International Electrotechnical Commission [2] because the efficiency of any electric motor is always less than 1.

4.4. Laws of Thermodynamics

Again, it is possible to infer from the postulates described in [9] that

- the Source of the energy that sustains the Universe is in two states:
  - in equilibrium in the invisible dimension, where the energy is imperceptible, infinite, and eternal, corresponding to the self-contained energy of mutual retention U_{mr}; or
  - in activity in the visible dimension, where the energy is perceptible and finite, corresponding to the kinetic energy T and the potential energy U, where $E = T + U$;
- when there is no equilibrium of mutual retention in the visible dimension of matter, the equation $\Delta T + \Delta U = 0$ represents the First Law of Thermodynamics, where initial energy equals final energy;
- when there is equilibrium of mutual retention in the visible dimension of matter, the equation $\Delta T + \Delta U = -\Delta U_{mr}$ continues to be consistent with the First Law of Thermodynamics, where initial energy equals final energy.

Therefore, expanding the limits of the First Law of Thermodynamics corresponds to [6]

$$\underbrace{\Delta T + \Delta U}_{perceptible} = \underbrace{-\Delta U_{mr}}_{imperceptible} \tag{38}$$

Thus, when there is an equilibrium of mutual retention in the material dimension, there is the conversion of self-contained energy of mutual retention U_{mr}, of imperceptible nature, in kinetic energy T and/or in energy potential

U, of perceptible nature. The fact that U_{mr} is of imperceptible nature means that U_{mr} does not depend on material dimensions, i.e. $U_{mr} \neq f(t, \rho, \varphi, z)$, whatever the function f.

Note that the variation in kinetic energy ΔT comprises any energy that has a nature associated with the motion of the elements of the system, such as thermal energy and sound energy. Likewise, the variation in potential energy ΔU comprises any energy that has a nature associated with the relative position of the elements of the system, such as gravitational potential energy and elastic potential energy.

Therefore, the First Law of Thermodynamics remains valid with the introduction of the self-contained energy of mutual retention U_{mr} due to the equilibrium of mutual retention. The Second Law of Thermodynamics also remains valid because the entropy analysis of the system remains the same. Finally, the Zeroth Law of Thermodynamics explains how heat exchanges between bodies occur.

4.5. Limitations

Although the analyzes (simulated and theoretical) suggest that the motor in the equilibrium of mutual retention can work, multiphysics simulations were not performed. Therefore, the author suggests the MFEM [12] and Elmer FEM [13] softwares for multiphysics simulations that consider constraint and dissipative forces.

Finally, the lack of understanding about how to perform these multiphysics simulations was the main reason for not using these softwares. However, the author's limitation may be the inspiration of other researchers.

5. Conclusion

In light of the new theoretical approach presented, the motor of mutual retention is a device capable of extracting energy directly from the Source that sustains the entire Universe [9]. For this, the motor must remain in an equilibrium of mutual retention between "movement" and "attraction-repulsion" [6]. This equilibrium state creates an infinite spiral of self-contained energy of mutual retention U_{mr} [9] which is converted into mechanical energy E [6].

Contrary to what is stated in [14], the analysis of the magnetic force $\vec{F}$ allows us to conclude that the motor remains in motion while there is the equilibrium of mutual retention. On the other hand, energy analysis leads to the introduction of the self-contained energy of mutual retention U_{mr}.

Furthermore, the analysis of conservation laws through the Hamiltonian formalism for an idealized model of the motor shows that the conservation of total energy, conservation of linear momentum, and conservation of angular momentum are all satisfied.

Consequently, the new theoretical approach shows that a motor in the equilibrium of mutual retention can have an efficiency greater than 100% because the self-contained energy of mutual retention U_{mr} is available in abundance. Thus, this motor is a good candidate for the next generation of motors with extreme efficiency.

Therefore, this new theoretical approach brings a conceptual advance to the First Law of Thermodynamics. Thus, with the introduction of the energy U_{mr} because of the equilibrium of mutual retention, the energy transfer takes place from the invisible dimension to the visible dimension [9].

In conclusion, the motor of mutual retention meets the energy demand of the entire modern society with a relevant economic, social, and environmental impact, significantly reducing the emission of greenhouse gases and the degradation of nature.

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