Article

Identify the mathematical models generated by the Table Curve 3D program

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Abstract: This article describes the methodology used to identify the mathematical model that describes the correlations between the input parameters of an experiment and the parameters being followed. As a technological process, the aerodynamic separation was chosen, respectively, the behavior of a solid particle within an ascending vertical airflow. The experimental data obtained were used to identify two parameters, the average linear velocity, and the angular velocity, and through the Table Curve 3D program was developed a mathematical model which describes the dependence between the input parameters (the shape and size of the solid particle and the velocity of the airflow) and the monitored parameters. In order to determine a single mathematical equation that describes as accurately as possible the correlation between the input variables and those obtained, a pyramid-type analysis was designed. The determination of the mathematical equation started from the number of equations generated by the Table Curve 3D program, then the equations with a correlation coefficient greater than 0.85 were chosen, and finally, the common equations were identified. Respecting the working methodology was identified one equation which has for the average linear velocity a correlation coefficient $r^2$ between 0.88-0.99 and 0.86-0.99 for the angular velocity.

Keywords: mathematical models; Table Curve 3D; correlation coefficient.

1. Introduction

Within the process industry there are several operations which involve the use of certain physical-mechanical characteristics of solid particles [1-6]. One of these is the aerodynamic behavior of the solid particle, respectively its aerodynamic separation [7-17].

Aerodynamics aims to study the dynamic interaction between fluid (respectively the air) and various categories of solid particles, generically called aeromechanical structures. Depending on the categories in which the bodies can be grouped, at present one can talk about the following distinct branches of aerodynamics, which also constitute the main applications of this science [18-26]:

- The aerodynamics of aircraft, which study load-bearing wings, tail assemblies, fuselages, or other components of an aircraft (nacelles, pillars, landing gear, etc.) both as isolated structures, but also interdependently;
- The aerodynamics of cars, which mainly study the flow around the bodies, but which also addresses problems related to air flow in the engine compartment or in the passenger compartment;
- Aerodynamics of paddle rotors, as in the case of propeller propulsive or wind turbines;
• Industrial aerodynamics deals with the study of other categories of bodies, such as buildings, bridges, antennas, various infrastructure elements subject to the action of the wind or air currents.

Following a set of experimental determinations, it is advisable to design a mathematical model, thus generating equations with which they can be tracking, analysed and optimized different industrial processes [27-41].

In order to create a mathematical model corresponding for study, many works have been made and are presented in the specialized literature [28, 29, 31, 32, 34, 38, 41-44], but a rather difficult problem arises when it is desired to identify a mathematical model that corresponds to several studies:

- Studies that have been performed at different time intervals or in different locations but at the same time;
- Studies conducted by different authors, but which have the same research field.

In this paper the aim was to identify common mathematical models for a real case, using a pyramid system as the method of identification. In order to present this method, a study was used which aimed to monitor the behavior of a solid polystyrene particle (of different sizes) in an industrial process, respectively the study of the behavior of a solid particle in a vertical airflow [10]. Once again, I would like to mention that the article aims to identify the mathematical model that describes as accurately as possible the correlations between the input parameters used in the study and the parameters followed in this study, an equation that is valid for several parameters analyzed.

2. Materials and Methods

The study of the behavior of a solid particle in an ascending vertical airflow was performed using a laboratory stand which is provided with an ascending vertical air channel and three chambers of soothing (Figure 1). To carry out this study, the laboratory stand was made of transparent material in order to allow visualization of the behavior of the solid particle in the air channel [11, 14, 45-47].

The passage of the solid particle through the tracking area was very fast and that is why it was chosen to film its behavior. For this purpose, a camera has been used that can film a number of processes that are carried out quickly, respectively a high-speed camera. The film thus obtained is subjected to analysis and by means of video and image processing software, a series of parameters of the behavior of the solid particle in the air stream ascending have been determined [10, 45, 46].

Figure 1. Image of laboratory stand [10].

The area in which the behavior of the solid particle was analyzed had a circular section, respectively the beginning area of the laboratory stand, an area in which the rooms of soothing are not positioned.
As solid particles used in this set of experimental determinations, particles of different sizes of polystyrene (Φ27, Φ35, Φ47 and Φ56) were used which were deformed to obtain different sphericity of them [10].

Following this article, the experimental values obtained for the following parameters are summarized:
- The average value of the instantaneous speed;
- Average angular velocity.

In order to study these parameters, the speed of the airflow generated by the fan of the installation used was also varied, obtained the following values thereof: 4.519 m/s, 4.896 m/s, 5.775 m/s, 6.277 m/s and 7.784 m/s [10].

3. Results

Following the analysis of the experimental results it was possible to identify the dependence between the input parameters, respectively between [10]:
- the value of the velocity of the airflow that passes through the vertical tubing, a parameter that depends on the operating mode of the installation used;
- the value of the solid particle sphericity parameter which depends on the type of particle used in the experimental determinations;

and the parameters followed, respectively:
- the average value of the instantaneous speed (Figure 2);
- average angular velocity (Figure 3).

From the analysis of the graphical representations in Figure 2 it is observed that the average values of the instantaneous velocity vary directly in proportion to the variation of the airflow that crosses the working installation and inversely proportional to the variation of the sphericity of the particles used. The lowest obtained value of this parameter is 1.61 m/s corresponding to a velocity of airflow of 4.89 m/s for the solid particle with Φ27 and a sphericity of 1, and the highest value is 4.97 m/s for the solid particle with Φ35 and a sphericity of 0.82 at an airflow velocity of 7.78 m/s [10].

In the case of the representations in Figure 3, it is found that the values of the average angular velocity vary inversely proportional to the variation of the sphericity of the solid particles and directly proportional to the variation of the airflow passing through the lab stand. The lowest obtained value of the average angular velocity is 108.6 rad/s corresponding to a velocity of airflow of 4.51 m/s for the solid particle with Φ47 and a sphericity of 1, and the highest value is 424.6 rad/s in the case of an airflow velocity of 6.27 m/s for a particle with sphericity Φ27 [10].

![Figure 2](image_url)

Figure 2. The variation of the linear average velocity as a function of the variation of the airflow velocity and the variation of the solid particle shape for particles of different sizes [10, 48].
4. Identifying the equation

The generation of mathematical models, which correspond to the results, was achieved using TableCurve 3D software, a program that can generate different types of response surfaces, surfaces that correspond to the 450 million equations existing in the program database [49].

In order to generate the mathematical model a series of working steps must be observed, these being described in Figure 4.

The work steps for identifying the mathematical model corresponding to the study carried out are described below:

1. The values obtained experimentally are entered in an excel file corresponding to Excel 97-2003;
2. Table Curve 3D program allows inserting excel file data [49];
3. The parameters corresponding to the three axes are selected, the input parameters are entered on the OX and OY axes and on the OZ axis the tracked parameter;
4. Table Curve 3D program [49] generates equations corresponding to the values introduced, and in Figures 5 and 6 their number is presented according to the value of the correlation coefficient $r^2$ for all sets of experiments;

![Figure 4. Steps of generating mathematical models.](image)
Figure 5. Representation of the number of equations generated by the TableCurve 3D program according to the value of the coefficient $r^2$ for the average value of the instantaneous speed [48].

Figure 6. Representation of the number of equations generated by the TableCurve 3D program according to the value of the coefficient $r^2$ for the variation of the average angular velocity [48].

The number of equations generated by the Table Curve 3D program was:
• for the average value of the instantaneous velocity the largest number of equations, 338, were generated for the particle with the diameter of Φ27, and the smallest number of equations, 178, was generated for the particle with Φ56;
• the same type of analysis was also performed in the case of the average angular velocity parameter obtained for the largest number of equations, 444, for the particle with the diameter of Φ36, and the smallest number of equations, also of 178, was generated for the same particle as in the case of the average value of the instantaneous velocity, that is for Φ56.

5. The obtained equations are grouped according to the value of the correlation coefficient $r^2$ (Figure 7);

![Figure 7](image)

**Figure 7.** Grouping the equations obtained by the value of the coefficient $r^2$ [48]: (a) the average value of the instantaneous speed; (b) average angular velocity.

The analysis of this type of representation shows that:
• In the case of the parameter: average value of the instantaneous speed, the values of the coefficient $r^2$ are distributed as follows:
  o In the case of the particle with Φ27 the maximum weighting, of 74.8 %, corresponds to the variation of $r^2$ in the range 0.9 - 1, and the smallest weighting, of 0 %, for the range of variation $r^2$ of 0.3 - 0.4;
  o For the particle with Φ36 the largest number of equations generated are those which correspond to the interval 0.9-1, are 196, and the smallest number of equations correspond to the interval $r^2$ equal to 0.3 - 0.4;
  o For the particle with Φ47, the maximum weighting of this characteristic is also for the interval 0.9 - 1 and is 84.09 %, and in the case of the intervals 0.5 - 0.6; 0.1 - 0.2 and 0 - 0.1 no equations were generated;
  o For the last dimension of the studied particle, Φ56, it is found that the largest number of equations were generated for $r^2$ in the range 0.8-0.9, respectively of 96 equations, and in the case of the range 0.2 - 0.3 no equation was generated;
• For the parameter: the value of the average angular velocity the values of the coefficient $r^2$ are distributed as follows:
  o For the particle with Φ27 was generated a maximum number of 173 equations corresponding to the range 0.8-0.9 and a minimum of 2 equations for the range 0.2-0.3;
  o In the case of the particle with Φ36 the maximum weighting, of 82.2 %, corresponds to the variation of $r^2$ in the range 0.9-1, and the smallest weighting, of 0.45 %, for the variation range of $r^2$ of 0.3 to 0.4;
  o For the particle with Φ47 a maximum number of 205 equations corresponding to the variation of $r^2$ in the range 1-0.9 and a minimum number of 0 equations corresponding to the variation of $r^2$ within the range 0.3-0.4 was identified;
  o For the particle whose diameter is Φ56, a number of 81 equations were generated for an $r^2$ in the range 0.8-0.9 and represents the interval with most equations. The intervals where no equations are generated are for $r^2$ between 0.6-0.7 and 0.5-0.6.
6. The next step is to identify common equations. In this stage, the group corresponding to an experimental set whose number of equations is the largest is identified, respectively:
   a. For the average value of the instantaneous velocity, the equations corresponding to the set of experiments for the particle with diameter Φ27 there are 338 equations;
   b. For the value of the average angular velocity, the equations corresponding to the set of experiments for the particle with diameter Φ36 there are 444 equations.
Each equation corresponds to a number, which helps to identify common equations. Therefore, an analysis is carried out in relation to the set of equations chosen as reference. The results of the analysis obtained are presented in Figure 8. Within this representation it should be specified that:
   • No 3 means that the equation in the reference set is found in the other three sets of equations;
   • No 2 represents that the equation in the reference set is found only in two sets of equations;
   • No 1 represents that the equation in the reference set is found only in a set of equations;
   • No 0 means that the equation in the reference set is not found in any set of equation.

![Figure 8](image_url)

**Figure 8.** Identifying common equations [48]: (a) the average value of the instantaneous speed; (b) average angular velocity.

In this way, for the variation of the average value of the instantaneous velocity, a number of 48 common equations from a number of 338 equations could be identified, and for the variation of the average angular velocity a number of 45 common equations from a number of 444 equations.

7. From the common equations an analysis is carried out in order to choose the equations whose correlation coefficient r2 is as close to the value of 0.99 (Figure 9);
Figure 9. Analysis of common equations according to the value of the correlation coefficient \( r^2 \) [48]: (a) the average value of the instantaneous speed; (b) average angular velocity.

5. Choosing the best equation

For this set of experiments, a common equation was chosen, both for the variation of the average value of the instantaneous speed and for the variation of the average angular velocity (equation no. 1). The graphical representations of the equation were generated by the Table Curve 3d program [49] and are presented within the Figure 10.

Corresponding to equation 1, for the two parameters studied, the values of the correlation coefficients \( r^2 \) are presented in Table 1 and the values of the coefficients describing the equation are presented in Table 2.

\[
z = a + b \cdot \ln x + c \cdot (\ln x)^2 + d \cdot y + e \cdot y^2,
\]

(1)

![Figure 10](image)

(a) Representation of the response surfaces corresponding to the chosen equation [49]: (a) for the average value of the instantaneous velocity for the particle with dimensions of \( \Phi 47 \); (b) for the average angular velocity corresponding to the particle with \( \Phi 27 \) diameter.

<table>
<thead>
<tr>
<th>The solid particle diameter</th>
<th>Source of sampling</th>
<th>The value of the correlation coefficients, ( r^2 )</th>
</tr>
</thead>
<tbody>
<tr>
<td>27</td>
<td>the average value of the instantaneous speed data</td>
<td>0.98404352</td>
</tr>
<tr>
<td>35</td>
<td></td>
<td>0.954536506</td>
</tr>
<tr>
<td>47</td>
<td></td>
<td>0.991563069</td>
</tr>
<tr>
<td>56</td>
<td></td>
<td>0.8806499</td>
</tr>
<tr>
<td>27</td>
<td>the average angular velocity</td>
<td>0.883216778</td>
</tr>
<tr>
<td>35</td>
<td></td>
<td>0.918608489</td>
</tr>
<tr>
<td>47</td>
<td></td>
<td>0.992395157</td>
</tr>
<tr>
<td>56</td>
<td></td>
<td>0.868026975</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Diameter of solid particle</th>
<th>a</th>
<th>b</th>
<th>c</th>
<th>d</th>
<th>e</th>
</tr>
</thead>
<tbody>
<tr>
<td>The average value of the instantaneous speed</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>27</td>
<td>-50.23232724</td>
<td>-11.95975744</td>
<td>-34.97830761</td>
<td>17.47111956</td>
<td>-1.415462687</td>
</tr>
<tr>
<td>35</td>
<td>-49.20607684</td>
<td>-0.92288797</td>
<td>16.90007259</td>
<td>15.17783217</td>
<td>-1.067222164</td>
</tr>
<tr>
<td>47</td>
<td>-1.820559781</td>
<td>0.557052644</td>
<td>52.79511238</td>
<td>0.978623048</td>
<td>0.001785313</td>
</tr>
</tbody>
</table>

Table 1. The correlation coefficients value for the both speeds [49].

Table 2. The coefficients value in the obtained equations [49].
The average angular velocity

<table>
<thead>
<tr>
<th>Diameter</th>
<th>Average Value</th>
<th>Maximum Value</th>
<th>Minimum Value</th>
<th>Standard Deviation</th>
<th>Real Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>27</td>
<td>1559.120656</td>
<td>-519.420112</td>
<td>561.116060</td>
<td>-41.90513018</td>
<td></td>
</tr>
<tr>
<td>35</td>
<td>645.2588717</td>
<td>574.907661</td>
<td>198.608978</td>
<td>-11.88403605</td>
<td></td>
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<tr>
<td>47</td>
<td>3171.846461</td>
<td>436.8999213</td>
<td>1245.268354</td>
<td>-114.7711104</td>
<td></td>
</tr>
<tr>
<td>56</td>
<td>999.2614963</td>
<td>-297.4052482</td>
<td>297.6328447</td>
<td>-18.55319149</td>
<td></td>
</tr>
</tbody>
</table>

From the analysis of the mathematical models, generated by the TableCurve 3D program, it is observed that the value of the correlation coefficient is between 0.86 and 0.99, so that the values obtained through the mathematical models coincide with the real values used for the models elaboration.

In order to highlight the difference between the two values, the real values and the values obtained through mathematical models, the program allows making a graphical representation that bears the name of the residual graph. Within this type of graph, the difference between the surface generated by the mathematical models is realized, the surface that in this case coincides with the XOY plane, and the experimental values. These residual graphs [49] are shown in Figures 11 and 12.

Figure 11. The residual graphs corresponding to the variation of the average value of the instantaneous speed [49]: (a) the particle with the diameter of \( \Phi 27 \); (b) the particle with a diameter of \( \Phi 35 \); (c) the particle with the diameter of \( \Phi 47 \); (d) the particle with a diameter of \( \Phi 56 \).
Figure 11. The residual graphs corresponding to the variation of the average angular velocity [49]: (a) the particle with the diameter of Φ 27; (b) the particle with a diameter of Φ 35; (c) the particle with the diameter of Φ 47; (d) the particle with a diameter of Φ 56.

Comparing the two data sets, those obtained from the experimental determinations and used for the creation of mathematical models, and those generated by replacing the values of the input parameters within the chosen equations, it is observed that:

- For the residual graphs corresponding to the variation of the average value of the instantaneous speed, the following results were obtained:
  - For the particle with Φ27 the minimum difference between the two values is 0.01 and the maximum value of the difference is 0.25;
  - For the particle with Φ35 the minimum difference between the two values is 0.01 and the maximum difference value is 0.28;
  - For the particle with Φ47 the minimum difference between the two values is 0.003 and the maximum value of the difference is 0.08;
  - For the particle with Φ56 the minimum difference between the two values is 0.04 and the maximum value of the difference is 0.28;

- For the residual graphs corresponding to the variation of the average angular velocity were obtained:
  - For the particle with Φ27 the minimum difference between the two values is 4.5 and the maximum value of the difference is 56.3;
  - For the particle with Φ35 the minimum difference between the two values is 2.2 and the maximum value is 28.2;
  - For the particle with Φ47 the minimum difference between the two values is 1.1 and the maximum value of the difference is 5.8;
  - For the particle with Φ56 the minimum difference between the two values is 2.1 and the maximum value of the difference is 12.8.

6. Discussion
The working method presented in this article aims to help identify in a simple way the mathematical model, generated from the data obtained experimentally, regardless of the experimental field.

7. Conclusions

The process industry involves performing different operations in order to obtain a series of products. To optimize these processes, a series of experiential studies are carried out, after that which mathematical models can be realized.

In this article, a series of mathematical models were made using a series of experimental data. These were obtained from the studies carried out in order to analyze the influence exerted by the size of the solid particle, its shape, and the value of the velocity of the air current on the behavior of a solid particle in an upward vertical air flow.

The mathematical models were created using TableCurve 3D software and generated general equations valid for the two parameters studied.

The values of the correlation coefficients $r^2$, corresponding to the equations generated through the TableCurve 3D software, have values ranging from 0.86 to 0.99.

Following the analysis performed on the common equations, a general equation was identified that describes the dependence between the input parameters and the parameters followed.

The verification of the mathematical models was realized by plotting the graphs of the difference between the surface obtained with the help of the mathematical models and the surface obtained by means of the real data thus obtaining a residual graphical representation.

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References

2. Panainte, M.; Mosnegutu, E.; Savin, C.; Nedeff, V. Mărunțirea produselor agroalimentare; Publisher: Meron, Romania, 2005.


