

Article

Not peer-reviewed version

Global Offensive Alliance in Strong Neutrosophic Graphs

[Mohammadesmail Nikfar](#)*

Posted Date: 28 January 2022

doi: 10.20944/preprints202201.0429.v1

Keywords: Modified Neutrosophic Number; Global Offensive Alliance; Complete Neutrosophic Graph



Preprints.org is a free multidisciplinary platform providing preprint service that is dedicated to making early versions of research outputs permanently available and citable. Preprints posted at Preprints.org appear in Web of Science, Crossref, Google Scholar, Scilit, Europe PMC.

Copyright: This open access article is published under a Creative Commons CC BY 4.0 license, which permit the free download, distribution, and reuse, provided that the author and preprint are cited in any reuse.

Disclaimer/Publisher's Note: The statements, opinions, and data contained in all publications are solely those of the individual author(s) and contributor(s) and not of MDPI and/or the editor(s). MDPI and/or the editor(s) disclaim responsibility for any injury to people or property resulting from any ideas, methods, instructions, or products referred to in the content.

Global Offensive Alliance in Strong Neutrosophic Graphs

Mohammadesmail Nikfar

Independent Researcher

DrHenryGarrett@gmail.com

Twitter's ID: @DrHenryGarrett | ©DrHenryGarrett.wordpress.com

Abstract

New setting is introduced to study the global offensive alliance. Global offensive alliance is about a set of vertices which are applied into the setting of neutrosophic graphs. Neighborhood has the key role to define this notion. Also, neighborhood is defined based on strong edges. Strong edge gets a framework as neighborhood and after that, too close vertices have key role to define global offensive alliance based on strong edges. The structure of set is studied and general results are obtained. Also, some classes of neutrosophic graphs containing complete, empty, path, cycle, star, and wheel are investigated in the terms of set, minimal set, number, and neutrosophic number. Neutrosophic number is defined in new way. It's first time to define this type of neutrosophic number in the way that, three values of a vertex are used and they've same share to construct this number. It's called "modified neutrosophic number". Summation of three values of vertex makes one number and applying it to a set makes neutrosophic number of set. This approach facilitates identifying minimal set and optimal set which forms minimal-global-offensive-alliance number and minimal-global-offensive-alliance-neutrosophic number. Two different types of sets namely global-offensive alliance and minimal-global-offensive alliance are defined. Global-offensive alliance identifies the sets in general vision but minimal-global-offensive alliance takes focus on the sets which deleting a vertex is impossible. Minimal-global-offensive-alliance number is about minimum cardinality amid the cardinalities of all minimal-global-offensive alliances in a given neutrosophic graph. New notions are applied in the settings both individual and family. Family of neutrosophic graphs is studied in the way that, the family only contains same classes of neutrosophic graphs. Three types of family of neutrosophic graphs including m-family of neutrosophic stars with common neutrosophic vertex set, m-family of odd complete graphs with common neutrosophic vertex set, and m-family of odd complete graphs with common neutrosophic vertex set are studied. The results are about minimal-global-offensive alliance, minimal-global-offensive-alliance number and its corresponded sets, minimal-global-offensive-alliance-neutrosophic number and its corresponded sets, and characterizing all minimal-global-offensive alliances. The connection of global-offensive-alliances with dominating set and chromatic number are obtained. The number of connected components has some relations with this new concept and it gets some results. Some classes of neutrosophic graphs behave differently when the parity of vertices are different and in this case, path, cycle, and complete illustrate these behaviors. Two applications concerning complete model as individual and family, under the titles of time table and scheduling conclude the results and they

give more clarifications. In this study, there's an open way to extend these results into the family of these classes of neutrosophic graphs. The family of neutrosophic graphs aren't study deeply and with more results but it seems that analogous results are determined. Slight progress is obtained in the family of these models but there are open avenues to study family of other models as same models and different models. There's a question. How can be related to each other, two sets partitioning the vertex set of a graph? The ideas of neighborhood and neighbors based on strong edges illustrate open way to get results. A set is global offensive alliance when two sets partitioning vertex set have uniform structure. All members of set have more amount of neighbors in the set than out of set. It leads us to the notion of global offensive alliance. Different edges make different neighborhoods but it's used one style edge titled strong edge. These notions are applied into neutrosophic graphs as individuals and family of them. Independent set as an alliance is a special set which has no neighbor inside and it implies some drawbacks for these notions. Finding special sets which are well-known, is an open way to purse this study. Special set which its members have only one neighbor inside, characterize the connected components where the cardinality of its complement is the number of connected components. Some problems are proposed to pursue this study. Basic familiarities with graph theory and neutrosophic graph theory are proposed for this article.

Keywords: Modified Neutrosophic Number, Global Offensive Alliance, Complete Neutrosophic Graph
AMS Subject Classification: 05C17, 05C22, 05E45

1 Background

Fuzzy set in Ref. [16], neutrosophic set in Ref. [2], related definitions of other sets in Refs. [2, 14, 15], graphs and new notions on them in Refs. [5–12], neutrosophic graphs in Ref. [3], studies on neutrosophic graphs in Ref. [1], relevant definitions of other graphs based on fuzzy graphs in Ref. [13], related definitions of other graphs based on neutrosophic graphs in Ref. [4], are proposed.

In this section, I use two subsections to illustrate a perspective about the background of this study.

1.1 Motivation and Contributions

In this study, there's an idea which could be considered as a motivation.

Question 1.1. *Is it possible to use mixed versions of ideas concerning “Global Offensive Alliance”, “Modified Neutrosophic Number” and “Complete Neutrosophic Graph” to define some notions which are applied to neutrosophic graphs?*

It's motivation to find notions to use in any classes of neutrosophic graphs. Real-world applications about time table and scheduling are another thoughts which lead to be considered as motivation. Connections amid two vertices have key roles to assign global-offensive alliance, minimal-global-offensive alliance, minimal-global-offensive-alliance number, and minimal-global-offensive-alliance-neutrosophic number. Thus they're used to define new ideas which conclude to the structure global offensive alliance. The concept of having strong edge inspires me to study the behavior of strong edges in the way that, two types of numbers and set, e.g., global-offensive alliance, minimal-global-offensive alliance, minimal-global-offensive-alliance number, and minimal-global-offensive-alliance-neutrosophic number are the cases of study in the

settings of individuals and in settings of families. Also, there are some avenues to extend these notions.

The framework of this study is as follows. In the beginning, I introduce basic definitions to clarify about preliminaries. In subsection “Preliminaries”, new notions of global- offensive alliance, minimal-global-offensive alliance, minimal-global-offensive-alliance number, and minimal-global-offensive-alliance-neutrosophic number are introduced and are clarified as individuals. In section “General Results For Neutrosophic Graphs”, general sets have the key role in this way. General results are obtained and also, the results about the connections between dominating set and chromatic number with the notion of global-offensive alliance are elicited. Classes of neutrosophic graphs are studied in the terms of global-offensive alliance, minimal-global-offensive alliance, minimal-global-offensive-alliance number, and minimal-global-offensive-alliance-neutrosophic number in section “Classes of Neutrosophic Graphs” as individuals. In section “Classes of Neutrosophic Graphs”, both numbers have applied into individuals. As a concluding result, there are three statements about the family of neutrosophic graphs as m-family of neutrosophic stars with common neutrosophic vertex set, m-family of odd complete graphs with common neutrosophic vertex set, and m-family of even complete graphs with common neutrosophic vertex set in section “Family of Neutrosophic Graphs.” The clarifications are also presented in section “Family of Neutrosophic Graphs” for introduced results. In section “Applications in Time Table and Scheduling”, two applications are posed for global-offensive alliance concerning time table and scheduling when the suspicions are about choosing some subjects and the mentioned models are complete as individual and uniform family. In section “Open Problems”, some problems and questions for further studies are proposed. In section “Conclusion and Closing Remarks”, gentle discussion about results and applications is featured. In section “Conclusion and Closing Remarks”, a brief overview concerning advantages and limitations of this study alongside conclusions is formed.

1.2 Preliminaries

In this subsection, basic material which is used in this article, is presented. Also, new ideas and their clarifications are elicited.

Basic idea is about the model which is used. First definition introduces basic model.

Definition 1.2. (Graph).

$G = (V, E)$ is called a **graph** if V is a set of objects and E is a subset of $V \times V$ (E is a set of 2-subsets of V) where V is called **vertex set** and E is called **edge set**. Every two vertices have been corresponded to at most one edge.

Neutrosophic graph is the foundation of results in this paper which is defined as follows. Also, some related notions are demonstrated.

Definition 1.3. (Neutrosophic Graph).

$NTG = (V, E, \sigma = (\sigma_1, \sigma_2, \sigma_3), \mu = (\mu_1, \mu_2, \mu_3))$ is called a **neutrosophic graph** if it's graph, $\sigma_i : V \rightarrow [0, 1]$, $\mu_i : E \rightarrow [0, 1]$, and for every $v_i v_j \in E$,

$$\mu(v_i v_j) \leq \sigma(v_i) \wedge \sigma(v_j).$$

(i) : σ is called **neutrosophic vertex set**.

(ii) : μ is called **neutrosophic edge set**.

(iii) : $|V|$ is called **order** of NTG and it's denoted by $\mathcal{O}(NTG)$.

(iv) : $\Sigma_{v \in V} \sigma(v)$ is called **neutrosophic order** of NTG and it's denoted by $\mathcal{O}_n(NTG)$. 68

(v) : $|E|$ is called **size** of NTG and it's denoted by $\mathcal{S}(NTG)$. 69

(vi) : $\Sigma_{e \in E} \Sigma_{i=1}^3 \mu_i(e)$ is called **neutrosophic size** of NTG and it's denoted by $\mathcal{S}_n(NTG)$. 70
71

Some classes of well-known neutrosophic graphs are defined. These classes of neutrosophic graphs are used to form this study and the most results are about them. 72
73

Definition 1.4. Let $NTG : (V, E, \sigma, \mu)$ be a neutrosophic graph. Then 74

(i) : a sequence of vertices $P : x_0, x_1, \dots, x_n$ is called **path** where $x_i x_{i+1} \in E, i = 0, 1, \dots, n-1$; 75
76

(ii) : **strength** of path $P : x_0, x_1, \dots, x_n$ is $\bigwedge_{i=0, \dots, n-1} \mu(x_i x_{i+1})$; 77

(iii) : **connectedness** amid vertices x_0 and x_n is

$$\mu^\infty(x, y) = \bigwedge_{P: x_0, x_1, \dots, x_n} \bigwedge_{i=0, \dots, n-1} \mu(x_i x_{i+1});$$

(iv) : a sequence of vertices $P : x_0, x_1, \dots, x_n$ is called **cycle** where $x_i x_{i+1} \in E, i = 0, 1, \dots, n-1$ and there are two edges xy and uv such that $\mu(xy) = \mu(uv) = \bigwedge_{i=0, 1, \dots, n-1} \mu(v_i v_{i+1})$; 78
79
80

(v) : it's **t-partite** where V is partitioned to t parts, V_1, V_2, \dots, V_t and the edge xy implies $x \in V_i$ and $y \in V_j$ where $i \neq j$. If it's complete, then it's denoted by $K_{\sigma_1, \sigma_2, \dots, \sigma_t}$ where σ_i is σ on V_i instead V which mean $x \notin V_i$ induces $\sigma_i(x) = 0$; 81
82
83

(vi) : t-partite is **complete bipartite** if $t = 2$, and it's denoted by K_{σ_1, σ_2} ; 84

(vii) : complete bipartite is **star** if $|V_1| = 1$, and it's denoted by S_{1, σ_2} ; 85

(viii) : a vertex in V is **center** if the vertex joins to all vertices of a cycle. Then it's **wheel** and it's denoted by W_{1, σ_2} ; 86
87

(ix) : it's **complete** where $\forall uv \in V, \mu(uv) = \sigma(u) \wedge \sigma(v)$; 88

(x) : it's **strong** where $\forall uv \in E, \mu(uv) = \sigma(u) \wedge \sigma(v)$. 89

The notions of neighbor and neighborhood are about some vertices which have one edge with a fixed vertex. These notions presents vertices which are close to a fixed vertex as possible. Based on strong edge, it's possible to define different neighborhood as follows. 90
91
92
93

Definition 1.5. (Strong Neighborhood). 94

Let $NTG : (V, E, \sigma, \mu)$ be a neutrosophic graph. Suppose $x \in V$. Then

$$N_s(x) = \{y \in N(x) \mid \mu(xy) = \sigma(x) \wedge \sigma(y)\}.$$

New notion is defined between two types of neighborhoods for a fixed vertex. A minimal set and some numbers are introduced in this way. The next definition has main role in every results which are given in this essay. 95
96
97

Definition 1.6. Let $NTG : (V, E, \sigma, \mu)$ be a neutrosophic graph. Then 98

(i) a set S is called **global-offensive alliance** if

$$\forall a \in V \setminus S, |N_s(a) \cap S| > |N_s(a) \cap (V \setminus S)|;$$

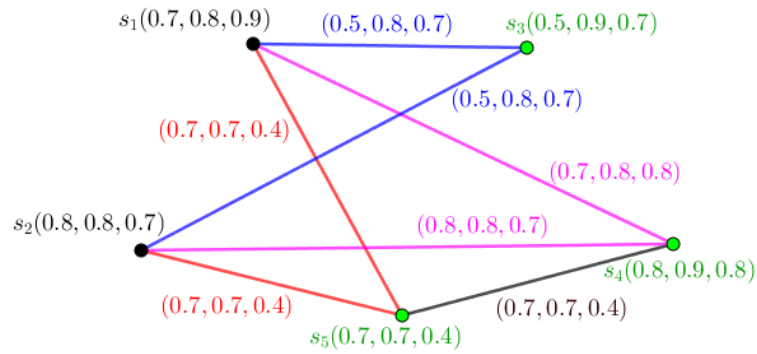


Figure 1. The set of black circles is minimal-global-offensive alliance.

(ii) $\forall S' \subseteq S$, S is global offensive alliance but S' isn't global offensive alliance. Then S is called **minimal-global-offensive alliance**;

(iii) **minimal-global-offensive-alliance number** of NTG is

$$\bigwedge_{S \text{ is a minimal-global-offensive alliance.}} |S|$$

and it's denoted by Γ ;

(iv) **minimal-global-offensive-alliance-neutrosophic number** of NTG is

$$\bigwedge_{S \text{ is a minimal-global-offensive alliance.}} \sum_{s \in S} \sum_{i=1}^3 \sigma_i(s)$$

and it's denoted by Γ_s .

Some clarifications are given for new definition which is presented in the paper as first time. Using new notions to make familiarity with main part of this article.

Example 1.7. Consider Figure (1).

(i) $S_1 = \{s_1, s_2\}$, $S_2 = \{s_3, s_5\}$, $S_3 = \{s_3, s_4\}$, $S_4 = \{s_4, s_5\}$ are only minimal-global-offensive alliances but only $S_3 = \{s_3, s_4\}$ is optimal such that forms minimal-global-offensive-alliance-neutrosophic number and minimal-global-offensive-alliance number;

(ii) $N = \{s_2, s_5\}$ isn't global-offensive alliance. Since

$$\begin{aligned} \exists s_1 \in V \setminus N, |N_s(s_1) \cap N| &= 1 < 2 = |N_s(s_1) \cap (V \setminus N)| \\ \exists s_1 \in V \setminus N, |N_s(s_1) \cap N| &= 1 \not\geq 2 = |N_s(s_1) \cap (V \setminus N)| \\ \exists s_1 \in V \setminus N, |N_s(s_1) \cap N| &\not\geq |N_s(s_1) \cap (V \setminus N)|; \end{aligned}$$

(iii) $\Gamma_s = 4.6$;

(iv) $\Gamma = 2$.

2 General Results For Neutrosophic Graphs

In this section, general results are given based on new definition. Some relations between new definition with dominating set and chromatic number are provided. The relation amid these two types of new numbers with fundamental numbers of neutrosophic graphs as order and neutrosophic order are clarified in the terms of vertices.

Proposition 2.1. Let $NTG : (V, E, \sigma, \mu)$ be a strong neutrosophic graph. If S is global-offensive alliance, then $\forall v \in V \setminus S, \exists x \in S$ such that

$$(i) \quad v \in N_s(x);$$

$$(ii) \quad vx \in E.$$

Proof. (i). Suppose $NTG : (V, E, \sigma, \mu)$ is a strong neutrosophic graph. Consider $v \in V \setminus S$. Since S is global-offensive alliance,

$$\forall z \in V \setminus S, |N_s(z) \cap S| > |N_s(z) \cap (V \setminus S)|$$

$$v \in V \setminus S, |N_s(v) \cap S| > |N_s(v) \cap (V \setminus S)|$$

$$v \in V \setminus S, \exists x \in S, v \in N_s(x).$$

(ii). Suppose $NTG : (V, E, \sigma, \mu)$ is a strong neutrosophic graph. Consider $v \in V \setminus S$. Since S is global-offensive alliance,

$$\forall z \in V \setminus S, |N_s(z) \cap S| > |N_s(z) \cap (V \setminus S)|$$

$$v \in V \setminus S, |N_s(v) \cap S| > |N_s(v) \cap (V \setminus S)|$$

$$v \in V \setminus S, \exists x \in S : v \in N_s(x)$$

$$v \in V \setminus S, \exists x \in S : vx \in E, \mu(vx) = \sigma(v) \wedge \sigma(x).$$

$$v \in V \setminus S, \exists x \in S : vx \in E.$$

□

Definition 2.2. Let $NTG : (V, E, \sigma, \mu)$ be a strong neutrosophic graph. Suppose S is a set of vertices. Then

(i) S is called **dominating set** if $\forall v \in V \setminus S, \exists s \in S$ such that either $v \in N_s(s)$ or $vs \in E$;

(ii) $|S|$ is called **chromatic number** if $\forall v \in V, \exists s \in S$ such that either $v \in N_s(s)$ or $vs \in E$ implies s and v have different colors.

Example 2.3. Consider Figure (1).

(i) $S = \{s_3, s_4\}$ is minimal dominating set;

(ii) $S = \{s_3, s_4\}$ is minimal-global-offensive alliance;

(iii) chromatic number is three.

Proposition 2.4. Let $NTG : (V, E, \sigma, \mu)$ be a strong neutrosophic graph. If S is global-offensive alliance, then

(i) S is dominating set;

(ii) there's $S \subseteq S'$ such that $|S'|$ is chromatic number.

Proof. (i). Suppose $NTG : (V, E, \sigma, \mu)$ is a strong neutrosophic graph. Consider $v \in V \setminus S$. Since S is global-offensive alliance, either

$$\forall z \in V \setminus S, |N_s(z) \cap S| > |N_s(z) \cap (V \setminus S)|$$

$$v \in V \setminus S, |N_s(v) \cap S| > |N_s(v) \cap (V \setminus S)|$$

$$v \in V \setminus S, \exists x \in S, v \in N_s(x)$$

or

$$\begin{aligned}
& \forall z \in V \setminus S, |N_s(z) \cap S| > |N_s(z) \cap (V \setminus S)| \\
& v \in V \setminus S, |N_s(v) \cap S| > |N_s(v) \cap (V \setminus S)| \\
& v \in V \setminus S, \exists x \in S : v \in N_s(x) \\
& v \in V \setminus S, \exists x \in S : vx \in E, \mu(vx) = \sigma(v) \wedge \sigma(x) \\
& v \in V \setminus S, \exists x \in S : vx \in E.
\end{aligned}$$

It implies S is dominating set.

(ii). Suppose $NTG : (V, E, \sigma, \mu)$ is a strong neutrosophic graph. Consider $v \in V \setminus S$. Since S is global-offensive alliance, either

$$\begin{aligned}
& \forall z \in V \setminus S, |N_s(z) \cap S| > |N_s(z) \cap (V \setminus S)| \\
& v \in V \setminus S, |N_s(v) \cap S| > |N_s(v) \cap (V \setminus S)| \\
& v \in V \setminus S, \exists x \in S, v \in N_s(x)
\end{aligned}$$

or

$$\begin{aligned}
& \forall z \in V \setminus S, |N_s(z) \cap S| > |N_s(z) \cap (V \setminus S)| \\
& v \in V \setminus S, |N_s(v) \cap S| > |N_s(v) \cap (V \setminus S)| \\
& v \in V \setminus S, \exists x \in S : v \in N_s(x) \\
& v \in V \setminus S, \exists x \in S : vx \in E, \mu(vx) = \sigma(v) \wedge \sigma(x) \\
& v \in V \setminus S, \exists x \in S : vx \in E.
\end{aligned}$$

Thus every vertex $v \in V \setminus S$, has at least one neighbor in S . The only case is about the relation amid vertices in S in the terms of neighbors. It implies there's $S \subseteq S'$ such that $|S'|$ is chromatic number. \square

Proposition 2.5. Let $NTG : (V, E, \sigma, \mu)$ be a strong neutrosophic graph. Then

$$(i) \quad \Gamma \leq \mathcal{O};$$

$$(ii) \quad \Gamma_s \leq \mathcal{O}_n.$$

Proof. (i). Suppose $NTG : (V, E, \sigma, \mu)$ is a strong neutrosophic graph. Let $S = V$.

$$\begin{aligned}
& \forall z \in V \setminus S, |N_s(z) \cap S| > |N_s(z) \cap (V \setminus S)| \\
& v \in V \setminus V, |N_s(v) \cap V| > |N_s(v) \cap (V \setminus V)| \\
& v \in \emptyset, |N_s(v) \cap V| > |N_s(v) \cap \emptyset| \\
& v \in \emptyset, |N_s(v) \cap V| > |\emptyset| \\
& v \in \emptyset, |N_s(v) \cap V| > 0
\end{aligned}$$

It implies V is global-offensive alliance. For all set of vertices S , $S \subseteq V$. Thus for all set of vertices S , $|S| \leq |V|$. It implies for all set of vertices S , $|S| \leq \mathcal{O}$. So for all set of vertices S , $\Gamma \leq \mathcal{O}$.

(ii). Suppose $NTG : (V, E, \sigma, \mu)$ is a strong neutrosophic graph. Let $S = V$.

$$\begin{aligned}
& \forall z \in V \setminus S, |N_s(z) \cap S| > |N_s(z) \cap (V \setminus S)| \\
& v \in V \setminus V, |N_s(v) \cap V| > |N_s(v) \cap (V \setminus V)| \\
& v \in \emptyset, |N_s(v) \cap V| > |N_s(v) \cap \emptyset| \\
& v \in \emptyset, |N_s(v) \cap V| > |\emptyset| \\
& v \in \emptyset, |N_s(v) \cap V| > 0
\end{aligned}$$

It implies V is global-offensive alliance. For all set of neutrosophic vertices S , $S \subseteq V$. Thus for all set of neutrosophic vertices S , $\sum_{s \in S} \sum_{i=1}^3 \sigma_i(s) \leq \sum_{v \in V} \sum_{i=1}^3 \sigma_i(v)$. It implies for all set of neutrosophic vertices S , $\sum_{s \in S} \sum_{i=1}^3 \sigma_i(s) \leq \mathcal{O}_n$. So for all set of neutrosophic vertices S , $\Gamma_s \leq \mathcal{O}_n$. \square

Proposition 2.6. Let $NTG : (V, E, \sigma, \mu)$ be a strong neutrosophic graph which is connected. Then

$$(i) \quad \Gamma \leq \mathcal{O} - 1;$$

$$(ii) \quad \Gamma_s \leq \mathcal{O}_n - \sum_{i=1}^3 \sigma_i(x).$$

Proof. (i). Suppose $NTG : (V, E, \sigma, \mu)$ is a strong neutrosophic graph. Let $S = V - \{x\}$ where x is arbitrary and $x \in V$.

$$\begin{aligned} \forall z \in V \setminus S, |N_s(z) \cap S| &> |N_s(z) \cap (V \setminus S)| \\ v \in V \setminus V - \{x\}, |N_s(v) \cap (V - \{x\})| &> |N_s(v) \cap (V \setminus (V - \{x\}))| \\ |N_s(x) \cap (V - \{x\})| &> |N_s(x) \cap \{x\}| \\ |N_s(x) \cap (V - \{x\})| &> |\emptyset| \\ |N_s(x) \cap (V - \{x\})| &> 0 \end{aligned}$$

It implies $V - \{x\}$ is global-offensive alliance. For all set of vertices $S \neq V$, $S \subseteq V - \{x\}$. Thus for all set of vertices $S \neq V$, $|S| \leq |V - \{x\}|$. It implies for all set of vertices $S \neq V$, $|S| \leq \mathcal{O} - 1$. So for all set of vertices S , $\Gamma \leq \mathcal{O} - 1$.

(ii). Suppose $NTG : (V, E, \sigma, \mu)$ is a strong neutrosophic graph. Let $S = V - \{x\}$ where x is arbitrary and $x \in V$.

$$\begin{aligned} \forall z \in V \setminus S, |N_s(z) \cap S| &> |N_s(z) \cap (V \setminus S)| \\ v \in V \setminus V - \{x\}, |N_s(v) \cap (V - \{x\})| &> |N_s(v) \cap (V \setminus (V - \{x\}))| \\ |N_s(x) \cap (V - \{x\})| &> |N_s(x) \cap \{x\}| \\ |N_s(x) \cap (V - \{x\})| &> |\emptyset| \\ |N_s(x) \cap (V - \{x\})| &> 0 \end{aligned}$$

It implies $V - \{x\}$ is global-offensive alliance. For all set of neutrosophic vertices $S \neq V$, $S \subseteq V - \{x\}$. Thus for all set of neutrosophic vertices $S \neq V$, $\sum_{s \in S} \sum_{i=1}^3 \sigma_i(s) \leq \sum_{v \in V - \{x\}} \sum_{i=1}^3 \sigma_i(v)$. It implies for all set of neutrosophic vertices $S \neq V$, $\sum_{s \in S} \sum_{i=1}^3 \sigma_i(s) \leq \mathcal{O}_n - \sum_{i=1}^3 \sigma_i(x)$. So for all set of neutrosophic vertices S , $\Gamma_s \leq \mathcal{O}_n - \sum_{i=1}^3 \sigma_i(x)$. \square

3 Classes of Neutrosophic Graphs

In this section, behaviors of some classes of neutrosophic graphs are analyzed when new definition is applied. In this way, the parity of number of vertices differentiate the results about some classes of neutrosophic graphs. Paths, cycles and complete are some classes of neutrosophic graphs which the parity of number of vertices get different results.

Proposition 3.1. Let $NTG : (V, E, \sigma, \mu)$ be an odd path. Then

- (i) the set $S = \{v_2, v_4, \dots, v_{n-1}\}$ is minimal-global-offensive alliance;
- (ii) $\Gamma = \lfloor \frac{n}{2} \rfloor + 1$ and corresponded set is $S = \{v_2, v_4, \dots, v_{n-1}\}$;
- (iii) $\Gamma_s = \min\{\sum_{s \in S = \{v_2, v_4, \dots, v_{n-1}\}} \sum_{i=1}^3 \sigma_i(s), \sum_{s \in S = \{v_1, v_3, \dots, v_{n-1}\}} \sum_{i=1}^3 \sigma_i(s)\}$;
- (iv) the sets $S_1 = \{v_2, v_4, \dots, v_{n-1}\}$ and $S_2 = \{v_1, v_3, \dots, v_{n-1}\}$ are only minimal-global-offensive alliances.

Proof. (i). Suppose $NTG : (V, E, \sigma, \mu)$ is an odd path. Let $S = \{v_2, v_4, \dots, v_{n-1}\}$ where for all $v_i, v_j \in \{v_2, v_4, \dots, v_{n-1}\}$, $v_i v_j \notin E$ and $v_i, v_j \in V$.

$$\begin{aligned} v \in \{v_1, v_3, \dots, v_n\}, |N_s(v) \cap \{v_2, v_4, \dots, v_{n-1}\}| &= 2 > 0 = |N_s(v) \cap \{v_1, v_3, \dots, v_n\}| \\ \forall z \in V \setminus S, |N_s(z) \cap S| &= 2 > 0 = |N_s(z) \cap (V \setminus S)| \\ \forall z \in V \setminus S, |N_s(z) \cap S| &> |N_s(z) \cap (V \setminus S)| \\ v \in V \setminus \{v_2, v_4, \dots, v_{n-1}\}, |N_s(v) \cap \{v_2, v_4, \dots, v_{n-1}\}| &> |N_s(v) \cap (V \setminus \{v_2, v_4, \dots, v_{n-1}\})| \end{aligned}$$

It implies $S = \{v_2, v_4, \dots, v_{n-1}\}$ is global-offensive alliance. If $S = \{v_2, v_4, \dots, v_{n-1}\} - \{v_i\}$ where $v_i \in \{v_2, v_4, \dots, v_{n-1}\}$, then

$$\exists v_{i+1} \in V \setminus S, |N_s(z) \cap S| = 1 = 1 = |N_s(z) \cap (V \setminus S)|$$

$$\exists v_{i+1} \in V \setminus S, |N_s(z) \cap S| = 1 \neq 1 = |N_s(z) \cap (V \setminus S)|$$

$$\exists v_{i+1} \in V \setminus S, |N_s(z) \cap S| \neq |N_s(z) \cap (V \setminus S)|.$$

So $\{v_2, v_4, \dots, v_{n-1}\} - \{v_i\}$ where $v_i \in \{v_2, v_4, \dots, v_{n-1}\}$ isn't global-offensive alliance. It induces $S = \{v_2, v_4, \dots, v_{n-1}\}$ is minimal-global-offensive alliance.

(ii) and (iii) are trivial.

(iv). By (i), $S_1 = \{v_2, v_4, \dots, v_{n-1}\}$ is minimal-global-offensive alliance. Thus it's enough to show that $S_2 = \{v_1, v_3, \dots, v_{n-1}\}$ is minimal-global-offensive alliance.

Suppose $NTG : (V, E, \sigma, \mu)$ is an odd path. Let $S = \{v_1, v_3, \dots, v_{n-1}\}$ where for all $v_i, v_j \in \{v_1, v_3, \dots, v_{n-1}\}$, $v_i v_j \notin E$ and $v_i, v_j \in V$.

$$v \in \{v_2, v_4, \dots, v_n\}, |N_s(v) \cap \{v_1, v_3, \dots, v_{n-1}\}| = 2 > 0 = |N_s(v) \cap \{v_2, v_4, \dots, v_n\}|$$

$$\forall z \in V \setminus S, |N_s(z) \cap S| = 2 > 0 = |N_s(z) \cap (V \setminus S)|$$

$$\forall z \in V \setminus S, |N_s(z) \cap S| > |N_s(z) \cap (V \setminus S)|$$

$$v \in V \setminus \{v_1, v_3, \dots, v_{n-1}\}, |N_s(v) \cap \{v_1, v_3, \dots, v_{n-1}\}| > |N_s(v) \cap (V \setminus \{v_1, v_3, \dots, v_{n-1}\})|$$

It implies $S = \{v_1, v_3, \dots, v_{n-1}\}$ is global-offensive alliance. If

$S = \{v_1, v_3, \dots, v_{n-1}\} - \{v_i\}$ where $v_i \in \{v_1, v_3, \dots, v_{n-1}\}$, then

$$\exists v_{i+1} \in V \setminus S, |N_s(z) \cap S| = 1 = 1 = |N_s(z) \cap (V \setminus S)|$$

$$\exists v_{i+1} \in V \setminus S, |N_s(z) \cap S| = 1 \neq 1 = |N_s(z) \cap (V \setminus S)|$$

$$\exists v_{i+1} \in V \setminus S, |N_s(z) \cap S| \neq |N_s(z) \cap (V \setminus S)|.$$

So $\{v_1, v_3, \dots, v_{n-1}\} - \{v_i\}$ where $v_i \in \{v_1, v_3, \dots, v_{n-1}\}$ isn't global-offensive alliance. It induces $S = \{v_1, v_3, \dots, v_{n-1}\}$ is minimal-global-offensive alliance. \square

Example 3.2. Consider Figure (2).

(i) $S_1 = \{s_1, s_3, s_4\}$ and $S_2 = \{s_2, s_4\}$ are only minimal-global-offensive alliances;

(ii) $S_1 = \{s_1, s_3, s_4\}$ is optimal such that only forms minimal-global-offensive-alliance-neutrosophic number but not minimal-global-offensive-alliance number;

(iii) $S_2 = \{s_2, s_4\}$ is optimal such that only forms minimal-global-offensive-alliance number but not minimal-global-offensive-alliance-neutrosophic number;

(iv) $N = \{s_1, s_3\}$ isn't global-offensive alliance. Since there are two instances but only one of them is enough;

(a) First counterexample for the statement " $N = \{s_1, s_3\}$ is global-offensive alliance.";

$$\exists s_4 \in V \setminus N, |N_s(s_4) \cap N| = 1 = 1 = |N_s(s_4) \cap (V \setminus N)|$$

$$\exists s_4 \in V \setminus N, |N_s(s_4) \cap N| = 1 \neq 1 = |N_s(s_4) \cap (V \setminus N)|$$

$$\exists s_4 \in V \setminus N, |N_s(s_4) \cap N| \neq |N_s(s_4) \cap (V \setminus N)|;$$

(b) second counterexample for the statement " $N = \{s_1, s_3\}$ is global-offensive alliance.".

$$\exists s_5 \in V \setminus N, |N_s(s_5) \cap N| = 0 < 1 = |N_s(s_5) \cap (V \setminus N)|$$

$$\exists s_5 \in V \setminus N, |N_s(s_5) \cap N| = 0 \neq 1 = |N_s(s_5) \cap (V \setminus N)|$$

$$\exists s_5 \in V \setminus N, |N_s(s_5) \cap N| \neq |N_s(s_5) \cap (V \setminus N)|.$$

(v) $\Gamma_s = 3.1$ and corresponded set is $S_1 = \{s_1, s_3, s_4\}$;

(vi) $\Gamma = 2$ and corresponded set is $S_2 = \{s_2, s_4\}$.

Proposition 3.3. Let $NTG : (V, E, \sigma, \mu)$ be an even path. Then

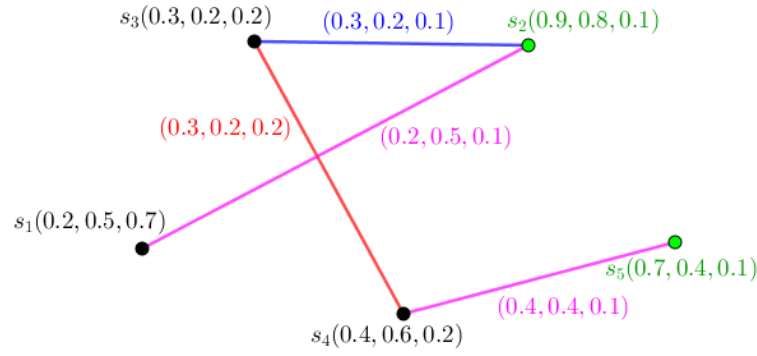


Figure 2. The set of black circles is minimal-global-offensive alliance.

- (i) the set $S = \{v_2, v_4, \dots, v_n\}$ is minimal-global-offensive alliance;
- (ii) $\Gamma = \lfloor \frac{n}{2} \rfloor$ and corresponded sets are $\{v_2, v_4, \dots, v_n\}$ and $\{v_1, v_3, \dots, v_{n-1}\}$;
- (iii) $\Gamma_s = \min\{\sum_{s \in S=\{v_2, v_4, \dots, v_n\}} \sum_{i=1}^3 \sigma_i(s), \sum_{s \in S=\{v_1, v_3, \dots, v_{n-1}\}} \sum_{i=1}^3 \sigma_i(s)\}$;
- (iv) the sets $S_1 = \{v_2, v_4, \dots, v_n\}$ and $S_2 = \{v_1, v_3, \dots, v_{n-1}\}$ are only minimal-global-offensive alliances.

Proof. (i). Suppose $NTG : (V, E, \sigma, \mu)$ is an even path. Let $S = \{v_2, v_4, \dots, v_n\}$ where for all $v_i, v_j \in \{v_2, v_4, \dots, v_n\}$, $v_i v_j \notin E$ and $v_i, v_j \in V$.

$$\begin{aligned} v \in \{v_1, v_3, \dots, v_{n-1}\}, & |N_s(v) \cap \{v_2, v_4, \dots, v_n\}| = 2 > 0 = |N_s(v) \cap \{v_1, v_3, \dots, v_{n-1}\}| \\ \forall z \in V \setminus S, & |N_s(z) \cap S| = 2 > 0 = |N_s(z) \cap (V \setminus S)| \\ \forall z \in V \setminus S, & |N_s(z) \cap S| > |N_s(z) \cap (V \setminus S)| \\ v \in V \setminus \{v_2, v_4, \dots, v_n\}, & |N_s(v) \cap \{v_2, v_4, \dots, v_n\}| > |N_s(v) \cap (V \setminus \{v_2, v_4, \dots, v_n\})| \end{aligned}$$

It implies $S = \{v_2, v_4, \dots, v_n\}$ is global-offensive alliance. If $S = \{v_2, v_4, \dots, v_n\} - \{v_i\}$ where $v_i \in \{v_2, v_4, \dots, v_n\}$, then

$$\begin{aligned} \exists v_{i+1} \in V \setminus S, & |N_s(z) \cap S| = 1 = 1 = |N_s(z) \cap (V \setminus S)| \\ \exists v_{i+1} \in V \setminus S, & |N_s(z) \cap S| = 1 \not> 1 = |N_s(z) \cap (V \setminus S)| \\ \exists v_{i+1} \in V \setminus S, & |N_s(z) \cap S| \not> |N_s(z) \cap (V \setminus S)|. \end{aligned}$$

So $\{v_2, v_4, \dots, v_n\} - \{v_i\}$ where $v_i \in \{v_2, v_4, \dots, v_n\}$ isn't global-offensive alliance. It induces $S = \{v_2, v_4, \dots, v_n\}$ is minimal-global-offensive alliance.

(ii) and (iii) are trivial.

(iv). By (i), $S_1 = \{v_2, v_4, \dots, v_n\}$ is minimal-global-offensive alliance. Thus it's enough to show that $S_2 = \{v_1, v_3, \dots, v_{n-1}\}$ is minimal-global-offensive alliance. Suppose $NTG : (V, E, \sigma, \mu)$ is an even path. Let $S = \{v_1, v_3, \dots, v_{n-1}\}$ where for all $v_i, v_j \in \{v_1, v_3, \dots, v_{n-1}\}$, $v_i v_j \notin E$ and $v_i, v_j \in V$.

$$\begin{aligned} v \in \{v_2, v_4, \dots, v_n\}, & |N_s(v) \cap \{v_1, v_3, \dots, v_{n-1}\}| = 2 > 0 = |N_s(v) \cap \{v_2, v_4, \dots, v_n\}| \\ \forall z \in V \setminus S, & |N_s(z) \cap S| = 2 > 0 = |N_s(z) \cap (V \setminus S)| \\ \forall z \in V \setminus S, & |N_s(z) \cap S| > |N_s(z) \cap (V \setminus S)| \\ v \in V \setminus \{v_1, v_3, \dots, v_{n-1}\}, & |N_s(v) \cap \{v_1, v_3, \dots, v_{n-1}\}| > |N_s(v) \cap (V \setminus \{v_1, v_3, \dots, v_{n-1}\})| \end{aligned}$$

It implies $S = \{v_1, v_3, \dots, v_{n-1}\}$ is global-offensive alliance. If $S = \{v_1, v_3, \dots, v_{n-1}\} - \{v_i\}$ where $v_i \in \{v_1, v_3, \dots, v_{n-1}\}$, then

$$\begin{aligned} \exists v_{i+1} \in V \setminus S, & |N_s(z) \cap S| = 1 = 1 = |N_s(z) \cap (V \setminus S)| \\ \exists v_{i+1} \in V \setminus S, & |N_s(z) \cap S| = 1 \not> 1 = |N_s(z) \cap (V \setminus S)| \\ \exists v_{i+1} \in V \setminus S, & |N_s(z) \cap S| \not> |N_s(z) \cap (V \setminus S)|. \end{aligned}$$

So $\{v_1, v_3, \dots, v_{n-1}\} - \{v_i\}$ where $v_i \in \{v_1, v_3, \dots, v_{n-1}\}$ isn't global-offensive alliance. It induces $S = \{v_1, v_3, \dots, v_{n-1}\}$ is minimal-global-offensive alliance. \square

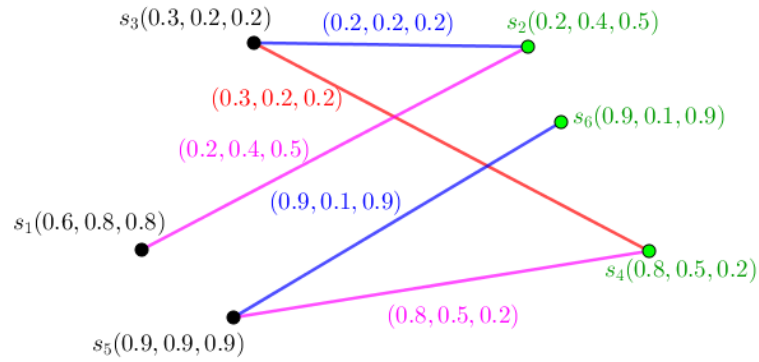


Figure 3. The set of black circles is minimal-global-offensive alliance.

Example 3.4. Consider Figure (3).

- (i) $S_1 = \{s_1, s_3, s_5\}$ and $S_2 = \{s_2, s_4, s_6\}$ are only minimal-global-offensive alliances;
- (ii) $S_2 = \{s_2, s_4, s_6\}$ is optimal such that forms both minimal-global-offensive-alliance-neutrosophic number and minimal-global-offensive-alliance number;
- (iii) $S_1 = \{s_1, s_3, s_5\}$ is optimal such that only forms minimal-global-offensive-alliance number but not minimal-global-offensive-alliance-neutrosophic number;
- (iv) $N = \{s_1, s_3\}$ isn't global-offensive alliance. Since there are three instances but only one of them is enough;

- (a) First counterexample for the statement " $N = \{s_1, s_3\}$ is global-offensive alliance.";

$$\begin{aligned} \exists s_4 \in V \setminus N, |N_s(s_4) \cap N| = 1 &= 1 = |N_s(s_4) \cap (V \setminus N)| \\ \exists s_4 \in V \setminus N, |N_s(s_4) \cap N| = 1 &\neq 1 = |N_s(s_4) \cap (V \setminus N)| \\ \exists s_4 \in V \setminus N, |N_s(s_4) \cap N| &\neq |N_s(s_4) \cap (V \setminus N)|. \end{aligned}$$

- (b) second counterexample for the statement " $N = \{s_1, s_3\}$ is global-offensive alliance.";

$$\begin{aligned} \exists s_5 \in V \setminus N, |N_s(s_5) \cap N| = 0 < 1 &= |N_s(s_5) \cap (V \setminus N)| \\ \exists s_5 \in V \setminus N, |N_s(s_5) \cap N| = 0 &\neq 1 = |N_s(s_5) \cap (V \setminus N)| \\ \exists s_5 \in V \setminus N, |N_s(s_5) \cap N| &\neq |N_s(s_5) \cap (V \setminus N)|; \end{aligned}$$

- (c) third counterexample for the statement " $N = \{s_1, s_3\}$ is global-offensive alliance.";

$$\begin{aligned} \exists s_6 \in V \setminus N, |N_s(s_6) \cap N| = 0 < 1 &= |N_s(s_6) \cap (V \setminus N)| \\ \exists s_6 \in V \setminus N, |N_s(s_6) \cap N| = 0 &\neq 1 = |N_s(s_6) \cap (V \setminus N)| \\ \exists s_6 \in V \setminus N, |N_s(s_6) \cap N| &\neq |N_s(s_6) \cap (V \setminus N)|. \end{aligned}$$

- (v) $\Gamma_s = 4.5$ and corresponded set is $S_2 = \{s_2, s_4, s_6\}$;
- (vi) $\Gamma = 3$ and corresponded sets are $S_1 = \{s_1, s_3, s_5\}$ and $S_2 = \{s_2, s_4, s_6\}$.

Proposition 3.5. Let $NTG : (V, E, \sigma, \mu)$ be an even cycle. Then

- (i) the set $S = \{v_2, v_4, \dots, v_n\}$ is minimal-global-offensive alliance;
- (ii) $\Gamma = \lfloor \frac{n}{2} \rfloor$ and corresponded sets are $\{v_2, v_4, \dots, v_n\}$ and $\{v_1, v_3, \dots, v_{n-1}\}$;

(iii) $\Gamma_s = \min\{\Sigma_{s \in S=\{v_2, v_4, \dots, v_n\}} \sigma(s), \Sigma_{s \in S=\{v_1, v_3, \dots, v_{n-1}\}} \sigma(s)\};$ 352

(iv) the sets $S_1 = \{v_2, v_4, \dots, v_n\}$ and $S_2 = \{v_1, v_3, \dots, v_{n-1}\}$ are only 353
minimal-global-offensive alliances. 354

Proof. (i). Suppose $NTG: (V, E, \sigma, \mu)$ is an even cycle. Let $S = \{v_2, v_4, \dots, v_n\}$ where 355
for all $v_i, v_j \in \{v_2, v_4, \dots, v_n\}$, $v_i v_j \notin E$ and $v_i, v_j \in V$. 356

$v \in \{v_1, v_3, \dots, v_{n-1}\}$, $|N_s(v) \cap \{v_2, v_4, \dots, v_n\}| = 2 > 0 = |N_s(v) \cap \{v_1, v_3, \dots, v_{n-1}\}|$ 357

$\forall z \in V \setminus S$, $|N_s(z) \cap S| = 2 > 0 = |N_s(z) \cap (V \setminus S)|$ 358

$\forall z \in V \setminus S$, $|N_s(z) \cap S| > |N_s(z) \cap (V \setminus S)|$ 359

$v \in V \setminus \{v_2, v_4, \dots, v_n\}$, $|N_s(v) \cap \{v_2, v_4, \dots, v_n\}| > |N_s(v) \cap (V \setminus \{v_2, v_4, \dots, v_n\})|$ 360

It implies $S = \{v_2, v_4, \dots, v_n\}$ is global-offensive alliance. If 361

$S = \{v_2, v_4, \dots, v_n\} - \{v_i\}$ where $v_i \in \{v_2, v_4, \dots, v_n\}$, then 362

$\exists v_{i+1} \in V \setminus S$, $|N_s(z) \cap S| = 1 = 1 = |N_s(z) \cap (V \setminus S)|$ 363

$\exists v_{i+1} \in V \setminus S$, $|N_s(z) \cap S| = 1 \neq 1 = |N_s(z) \cap (V \setminus S)|$ 364

$\exists v_{i+1} \in V \setminus S$, $|N_s(z) \cap S| \not> |N_s(z) \cap (V \setminus S)|$. 365

So $\{v_2, v_4, \dots, v_n\} - \{v_i\}$ where $v_i \in \{v_2, v_4, \dots, v_n\}$ isn't global-offensive alliance. 366
It induces $S = \{v_2, v_4, \dots, v_n\}$ is minimal-global-offensive alliance. 367

(ii) and (iii) are trivial. 368

(iv). By (i), $S_1 = \{v_2, v_4, \dots, v_n\}$ is minimal-global-offensive alliance. Thus it's 369
enough to show that $S_2 = \{v_1, v_3, \dots, v_{n-1}\}$ is minimal-global-offensive alliance. 370

Suppose $NTG: (V, E, \sigma, \mu)$ is an odd path. Let $S = \{v_1, v_3, \dots, v_{n-1}\}$ where for all 371
 $v_i, v_j \in \{v_1, v_3, \dots, v_{n-1}\}$, $v_i v_j \notin E$ and $v_i, v_j \in V$. 372

$v \in \{v_2, v_4, \dots, v_n\}$, $|N_s(v) \cap \{v_1, v_3, \dots, v_{n-1}\}| = 2 > 0 = |N_s(v) \cap \{v_2, v_4, \dots, v_n\}|$ 373

$\forall z \in V \setminus S$, $|N_s(z) \cap S| = 2 > 0 = |N_s(z) \cap (V \setminus S)|$ 374

$\forall z \in V \setminus S$, $|N_s(z) \cap S| > |N_s(z) \cap (V \setminus S)|$ 375

$v \in V \setminus \{v_1, v_3, \dots, v_{n-1}\}$, $|N_s(v) \cap \{v_1, v_3, \dots, v_{n-1}\}| > |N_s(v) \cap (V \setminus \{v_1, v_3, \dots, v_{n-1}\})|$ 376

It implies $S = \{v_1, v_3, \dots, v_{n-1}\}$ is global-offensive alliance. If 377

$S = \{v_1, v_3, \dots, v_{n-1}\} - \{v_i\}$ where $v_i \in \{v_1, v_3, \dots, v_{n-1}\}$, then 378

$\exists v_{i+1} \in V \setminus S$, $|N_s(z) \cap S| = 1 = 1 = |N_s(z) \cap (V \setminus S)|$ 379

$\exists v_{i+1} \in V \setminus S$, $|N_s(z) \cap S| = 1 \neq 1 = |N_s(z) \cap (V \setminus S)|$ 380

$\exists v_{i+1} \in V \setminus S$, $|N_s(z) \cap S| \not> |N_s(z) \cap (V \setminus S)|$. 381

So $\{v_1, v_3, \dots, v_{n-1}\} - \{v_i\}$ where $v_i \in \{v_1, v_3, \dots, v_{n-1}\}$ isn't global-offensive 382
alliance. It induces $S = \{v_1, v_3, \dots, v_{n-1}\}$ is minimal-global-offensive alliance. \square 383

Example 3.6. Consider Figure (4). 384

(i) $S_1 = \{s_1, s_3, s_5\}$ and $S_2 = \{s_2, s_4, s_6\}$ are only minimal-global-offensive alliances; 385

(ii) $S_2 = \{s_2, s_4, s_6\}$ is optimal such that forms both 386
minimal-global-offensive-alliance-neutrosophic number and 387
minimal-global-offensive-alliance number; 388

(iii) $S_1 = \{s_1, s_3, s_5\}$ is optimal such that only forms minimal-global-offensive-alliance 389
number but not minimal-global-offensive-alliance-neutrosophic number; 390

(iv) $N = \{s_1, s_3\}$ isn't global-offensive alliance. Since there are three instances but 391
only one of them is enough; 392

(a) First counterexample for the statement " $N = \{s_1, s_3\}$ is global-offensive 393
alliance."; 394

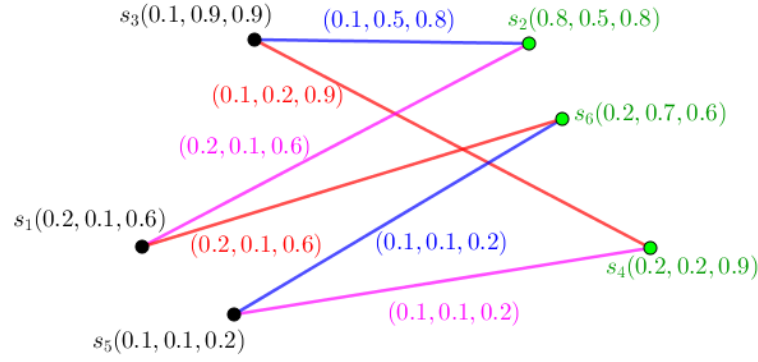


Figure 4. The set of black circles is minimal-global-offensive alliance.

$$\exists s_4 \in V \setminus N, |N_s(s_4) \cap N| = 1 = 1 = |N_s(s_4) \cap (V \setminus N)| \quad 395$$

$$\exists s_4 \in V \setminus N, |N_s(s_4) \cap N| = 1 \neq 1 = |N_s(s_4) \cap (V \setminus N)| \quad 396$$

$$\exists s_4 \in V \setminus N, |N_s(s_4) \cap N| \neq |N_s(s_4) \cap (V \setminus N)|. \quad 397$$

(b) second counterexample for the statement “ $N = \{s_1, s_3\}$ is global-offensive alliance.”; 398
399

$$\exists s_5 \in V \setminus N, |N_s(s_5) \cap N| = 0 < 1 = |N_s(s_5) \cap (V \setminus N)| \quad 400$$

$$\exists s_5 \in V \setminus N, |N_s(s_5) \cap N| = 0 \neq 1 = |N_s(s_5) \cap (V \setminus N)| \quad 401$$

$$\exists s_5 \in V \setminus N, |N_s(s_5) \cap N| \neq |N_s(s_5) \cap (V \setminus N)|; \quad 402$$

(c) third counterexample for the statement “ $N = \{s_1, s_3\}$ is global-offensive alliance.”; 403
404

$$\exists s_6 \in V \setminus N, |N_s(s_6) \cap N| = 0 < 1 = |N_s(s_6) \cap (V \setminus N)| \quad 405$$

$$\exists s_6 \in V \setminus N, |N_s(s_6) \cap N| = 0 \neq 1 = |N_s(s_6) \cap (V \setminus N)| \quad 406$$

$$\exists s_6 \in V \setminus N, |N_s(s_6) \cap N| \neq |N_s(s_6) \cap (V \setminus N)|. \quad 407$$

(v) $\Gamma_s = 3.2$ and corresponded set is $S_2 = \{s_2, s_4, s_6\}$; 408

(vi) $\Gamma = 3$ and corresponded sets are $S_1 = \{s_1, s_3, s_5\}$ and $S_2 = \{s_2, s_4, s_6\}$. 409

Proposition 3.7. Let $NTG : (V, E, \sigma, \mu)$ be an odd cycle. Then 410

(i) the set $S = \{v_2, v_4, \dots, v_{n-1}\}$ is minimal-global-offensive alliance; 411

(ii) $\Gamma = \lfloor \frac{n}{2} \rfloor + 1$ and corresponded set is $S = \{v_2, v_4, \dots, v_{n-1}\}$; 412

(iii) $\Gamma_s = \min\{\sum_{s \in S=\{v_2, v_4, \dots, v_{n-1}\}} \sum_{i=1}^3 \sigma_i(s), \sum_{s \in S=\{v_1, v_3, \dots, v_{n-1}\}} \sum_{i=1}^3 \sigma_i(s)\}$; 413

(iv) the sets $S_1 = \{v_2, v_4, \dots, v_{n-1}\}$ and $S_2 = \{v_1, v_3, \dots, v_{n-1}\}$ are only minimal-global-offensive alliances. 414
415

Proof. (i). Suppose $NTG : (V, E, \sigma, \mu)$ is an odd cycle. Let $S = \{v_2, v_4, \dots, v_{n-1}\}$ where for all $v_i, v_j \in \{v_2, v_4, \dots, v_{n-1}\}$, $v_i v_j \notin E$ and $v_i, v_j \in V$. 416
417

$$v \in \{v_1, v_3, \dots, v_n\}, |N_s(v) \cap \{v_2, v_4, \dots, v_{n-1}\}| = 2 > 0 = |N_s(v) \cap \{v_1, v_3, \dots, v_n\}| \quad 418$$

$$\forall z \in V \setminus S, |N_s(z) \cap S| = 2 > 0 = |N_s(z) \cap (V \setminus S)| \quad 419$$

$$\forall z \in V \setminus S, |N_s(z) \cap S| > |N_s(z) \cap (V \setminus S)| \quad 420$$

$$v \in V \setminus \{v_2, v_4, \dots, v_{n-1}\}, |N_s(v) \cap \{v_2, v_4, \dots, v_{n-1}\}| > |N_s(v) \cap (V \setminus \{v_2, v_4, \dots, v_{n-1}\})| \quad 421$$

It implies $S = \{v_2, v_4, \dots, v_{n-1}\}$ is global-offensive alliance. If 422

$S = \{v_2, v_4, \dots, v_{n-1}\} - \{v_i\}$ where $v_i \in \{v_2, v_4, \dots, v_{n-1}\}$, then 423

$$\exists v_{i+1} \in V \setminus S, |N_s(z) \cap S| = 1 = 1 = |N_s(z) \cap (V \setminus S)|$$

$$\exists v_{i+1} \in V \setminus S, |N_s(z) \cap S| = 1 \neq 1 = |N_s(z) \cap (V \setminus S)|$$

$$\exists v_{i+1} \in V \setminus S, |N_s(z) \cap S| \neq |N_s(z) \cap (V \setminus S)|.$$

So $\{v_2, v_4, \dots, v_{n-1}\} - \{v_i\}$ where $v_i \in \{v_2, v_4, \dots, v_{n-1}\}$ isn't global-offensive alliance. It induces $S = \{v_2, v_4, \dots, v_{n-1}\}$ is minimal-global-offensive alliance.

(ii) and (iii) are trivial.

(iv). By (i), $S_1 = \{v_2, v_4, \dots, v_{n-1}\}$ is minimal-global-offensive alliance. Thus it's enough to show that $S_2 = \{v_1, v_3, \dots, v_{n-1}\}$ is minimal-global-offensive alliance.

Suppose $NTG: (V, E, \sigma, \mu)$ is an odd cycle. Let $S = \{v_1, v_3, \dots, v_{n-1}\}$ where for all $v_i, v_j \in \{v_1, v_3, \dots, v_{n-1}\}$, $v_i v_j \notin E$ and $v_i, v_j \in V$.

$$v \in \{v_2, v_4, \dots, v_n\}, |N_s(v) \cap \{v_1, v_3, \dots, v_{n-1}\}| = 2 > 0 = |N_s(v) \cap \{v_2, v_4, \dots, v_n\}|$$

$$\forall z \in V \setminus S, |N_s(z) \cap S| = 2 > 0 = |N_s(z) \cap (V \setminus S)|$$

$$\forall z \in V \setminus S, |N_s(z) \cap S| > |N_s(z) \cap (V \setminus S)|$$

$$v \in V \setminus \{v_1, v_3, \dots, v_{n-1}\}, |N_s(v) \cap \{v_1, v_3, \dots, v_{n-1}\}| > |N_s(v) \cap (V \setminus \{v_1, v_3, \dots, v_{n-1}\})|$$

It implies $S = \{v_1, v_3, \dots, v_{n-1}\}$ is global-offensive alliance. If

$S = \{v_1, v_3, \dots, v_{n-1}\} - \{v_i\}$ where $v_i \in \{v_1, v_3, \dots, v_{n-1}\}$, then

$$\exists v_{i+1} \in V \setminus S, |N_s(z) \cap S| = 1 = 1 = |N_s(z) \cap (V \setminus S)|$$

$$\exists v_{i+1} \in V \setminus S, |N_s(z) \cap S| = 1 \neq 1 = |N_s(z) \cap (V \setminus S)|$$

$$\exists v_{i+1} \in V \setminus S, |N_s(z) \cap S| \neq |N_s(z) \cap (V \setminus S)|.$$

So $\{v_1, v_3, \dots, v_{n-1}\} - \{v_i\}$ where $v_i \in \{v_1, v_3, \dots, v_{n-1}\}$ isn't global-offensive alliance. It induces $S = \{v_1, v_3, \dots, v_{n-1}\}$ is minimal-global-offensive alliance. \square

Example 3.8. Consider Figure (5).

(i) $S_1 = \{s_1, s_3, s_4\}$ and $S_2 = \{s_2, s_4\}$ are only minimal-global-offensive alliances;

(ii) $S_2 = \{s_2, s_4\}$ is optimal such that forms both minimal-global-offensive-alliance-neutrosophic number and minimal-global-offensive-alliance number;

(iii) $S_1 = \{s_1, s_3, s_5\}$ is optimal such that not only doesn't form minimal-global-offensive-alliance number but also doesn't form minimal-global-offensive-alliance-neutrosophic number;

(iv) $N = \{s_1, s_3\}$ isn't global-offensive alliance. Since there are two instances but only one of them is enough;

(a) First counterexample for the statement " $N = \{s_1, s_3\}$ is global-offensive alliance.";

$$\exists s_4 \in V \setminus N, |N_s(s_4) \cap N| = 1 = 1 = |N_s(s_4) \cap (V \setminus N)|$$

$$\exists s_4 \in V \setminus N, |N_s(s_4) \cap N| = 1 \neq 1 = |N_s(s_4) \cap (V \setminus N)|$$

$$\exists s_4 \in V \setminus N, |N_s(s_4) \cap N| \neq |N_s(s_4) \cap (V \setminus N)|;$$

(b) second counterexample for the statement " $N = \{s_1, s_3\}$ is global-offensive alliance.";

$$\exists s_5 \in V \setminus N, |N_s(s_5) \cap N| = 0 < 1 = |N_s(s_5) \cap (V \setminus N)|$$

$$\exists s_5 \in V \setminus N, |N_s(s_5) \cap N| = 0 \neq 1 = |N_s(s_5) \cap (V \setminus N)|$$

$$\exists s_5 \in V \setminus N, |N_s(s_5) \cap N| \neq |N_s(s_5) \cap (V \setminus N)|.$$

(v) $\Gamma_s = 3.5$ and corresponded set is $S_2 = \{s_2, s_4\}$;

(vi) $\Gamma = 2$ and corresponded set is $S_2 = \{s_2, s_4\}$.

Proposition 3.9. Let $NTG: (V, E, \sigma, \mu)$ be star. Then

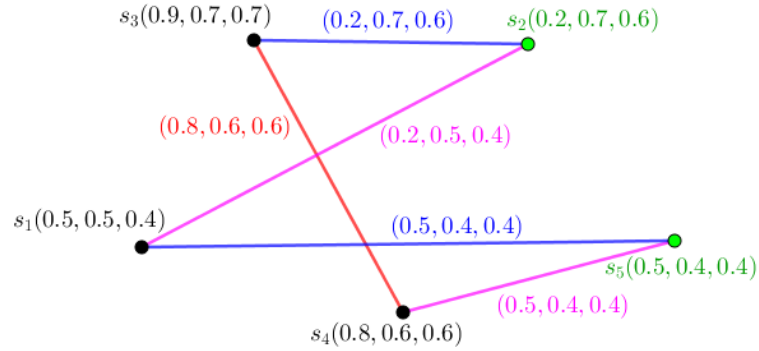


Figure 5. The set of black circles is minimal-global-offensive alliance.

(i) the set $S = \{c\}$ is minimal-global-offensive alliance;

(ii) $\Gamma = 1$;

(iii) $\Gamma_s = \sum_{i=1}^3 \sigma_i(c)$;

(iv) the sets $S = \{c\}$ and $S \subset S'$ are only global-offensive alliances.

Proof. (i). Suppose $NTG : (V, E, \sigma, \mu)$ is a star.

$$\forall v \in V \setminus \{c\}, |N_s(v) \cap \{c\}| = 1 > 0 = |N_s(v) \cap (V \setminus \{c\})|$$

$$\forall z \in V \setminus S, |N_s(z) \cap S| = 1 > 0 = |N_s(z) \cap (V \setminus S)|$$

$$\forall z \in V \setminus S, |N_s(z) \cap S| > |N_s(z) \cap (V \setminus S)|$$

$$v \in V \setminus \{c\}, |N_s(v) \cap \{c\}| > |N_s(v) \cap (V \setminus \{c\})|$$

It implies $S = \{c\}$ is global-offensive alliance. If $S = \{c\} - \{c\} = \emptyset$, then

$$\exists v \in V \setminus S, |N_s(z) \cap S| = 0 = 0 = |N_s(z) \cap (V \setminus S)|$$

$$\exists v \in V \setminus S, |N_s(z) \cap S| = 0 \not> 0 = |N_s(z) \cap (V \setminus S)|$$

$$\exists v \in V \setminus S, |N_s(z) \cap S| \not> |N_s(z) \cap (V \setminus S)|.$$

So $S = \{c\} - \{c\} = \emptyset$ isn't global-offensive alliance. It induces $S = \{c\}$ is minimal-global-offensive alliance.

(ii) and (iii) are trivial.

(iv). By (i), $S = \{c\}$ is minimal-global-offensive alliance. Thus it's enough to show that $S \subseteq S'$ is minimal-global-offensive alliance. Suppose $NTG : (V, E, \sigma, \mu)$ is a star. Let $S \subseteq S'$.

$$\forall v \in V \setminus \{c\}, |N_s(v) \cap \{c\}| = 1 > 0 = |N_s(v) \cap (V \setminus \{c\})|$$

$$\forall z \in V \setminus S', |N_s(z) \cap S'| = 1 > 0 = |N_s(z) \cap (V \setminus S')|$$

$$\forall z \in V \setminus S', |N_s(z) \cap S'| > |N_s(z) \cap (V \setminus S')|$$

It implies $S' \subseteq S$ is global-offensive alliance. \square

Example 3.10. Consider Figure (6).

(i) $S = \{s_1\}$ is only minimal-global-offensive alliance;

(ii) $S = \{s_1\}$ is optimal such that forms both minimal-global-offensive-alliance-neutrosophic number and minimal-global-offensive-alliance number;

(iii) S' including $S = \{s_1\}$ only forms global-offensive-alliance but not minimal-global-offensive-alliance;

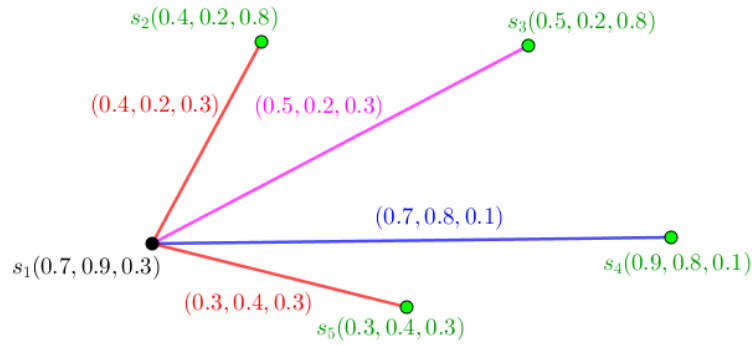


Figure 6. The set of black circles is minimal-global-offensive alliance.

(iv) $N = \{s_3, s_4\}$ isn't global-offensive alliance. Since there are three instances but only one of them is enough;

(a) First counterexample for the statement " $N = \{s_3, s_4\}$ is global-offensive alliance.";

$$\exists s_1 \in V \setminus N, |N_s(s_1) \cap N| = 2 = 2 = |N_s(s_1) \cap (V \setminus N)|$$

$$\exists s_1 \in V \setminus N, |N_s(s_1) \cap N| = 2 \neq 2 = |N_s(s_1) \cap (V \setminus N)|$$

$$\exists s_1 \in V \setminus N, |N_s(s_1) \cap N| \neq |N_s(s_1) \cap (V \setminus N)|;$$

(b) second counterexample for the statement " $N = \{s_3, s_4\}$ is global-offensive alliance.";

$$\exists s_2 \in V \setminus N, |N_s(s_2) \cap N| = 0 < 1 = |N_s(s_2) \cap (V \setminus N)|$$

$$\exists s_2 \in V \setminus N, |N_s(s_2) \cap N| = 0 \neq 1 = |N_s(s_2) \cap (V \setminus N)|$$

$$\exists s_2 \in V \setminus N, |N_s(s_2) \cap N| \neq |N_s(s_2) \cap (V \setminus N)|;$$

(c) third counterexample for the statement " $N = \{s_3, s_4\}$ is global-offensive alliance.";

$$\exists s_5 \in V \setminus N, |N_s(s_5) \cap N| = 0 < 1 = |N_s(s_5) \cap (V \setminus N)|$$

$$\exists s_5 \in V \setminus N, |N_s(s_5) \cap N| = 0 \neq 1 = |N_s(s_5) \cap (V \setminus N)|$$

$$\exists s_5 \in V \setminus N, |N_s(s_5) \cap N| \neq |N_s(s_5) \cap (V \setminus N)|.$$

(v) $\Gamma_s = 1.9$ and corresponded set is $S = \{s_1\}$;

(vi) $\Gamma = 1$ and corresponded set is $S = \{s_1\}$.

Proposition 3.11. Let $NTG : (V, E, \sigma, \mu)$ be wheel. Then

(i) the set $S = \{v_1, v_3\} \cup \{v_6, v_9 \dots, v_{i+6}, \dots, v_n\}_{i=1}^{6+3(i-1) \leq n}$ is minimal-global-offensive alliance;

(ii) $\Gamma = |\{v_1, v_3\} \cup \{v_6, v_9 \dots, v_{i+6}, \dots, v_n\}_{i=1}^{6+3(i-1) \leq n}|$;

(iii) $\Gamma_s = \Sigma_{\{v_1, v_3\} \cup \{v_6, v_9 \dots, v_{i+6}, \dots, v_n\}_{i=1}^{6+3(i-1) \leq n}} \Sigma_{i=1}^3 \sigma_i(s)$;

(iv) the set $\{v_1, v_3\} \cup \{v_6, v_9 \dots, v_{i+6}, \dots, v_n\}_{i=1}^{6+3(i-1) \leq n}$ is only minimal-global-offensive alliance.

Proof. (i). Suppose $NTG : (V, E, \sigma, \mu)$ is a wheel. Let

$S = \{v_1, v_3\} \cup \{v_6, v_9 \dots, v_{i+6}, \dots, v_n\}_{i=1}^{6+3(i-1) \leq n}$. There are either

$$\begin{aligned} \forall z \in V \setminus S, |N_s(z) \cap S| = 2 > 1 = |N_s(z) \cap (V \setminus S)| \\ \forall z \in V \setminus S, |N_s(z) \cap S| > |N_s(z) \cap (V \setminus S)| \end{aligned}$$

or

$$\begin{aligned} \forall z \in V \setminus S, |N_s(z) \cap S| = 3 > 0 = |N_s(z) \cap (V \setminus S)| \\ \forall z \in V \setminus S, |N_s(z) \cap S| > |N_s(z) \cap (V \setminus S)| \end{aligned}$$

It implies $S = \{v_1, v_3\} \cup \{v_6, v_9 \dots, v_{i+6}, \dots, v_n\}_{i=1}^{6+3(i-1) \leq n}$ is global-offensive alliance. If $S' = \{v_1, v_3\} \cup \{v_6, v_9 \dots, v_{i+6}, \dots, v_n\}_{i=1}^{6+3(i-1) \leq n} - \{z\}$ where $z \in S = \{v_1, v_3\} \cup \{v_6, v_9 \dots, v_{i+6}, \dots, v_n\}_{i=1}^{6+3(i-1) \leq n}$, then There are either

$$\begin{aligned} \forall z \in V \setminus S', |N_s(z) \cap S'| = 1 < 2 = |N_s(z) \cap (V \setminus S')| \\ \forall z \in V \setminus S', |N_s(z) \cap S'| < |N_s(z) \cap (V \setminus S')| \\ \forall z \in V \setminus S', |N_s(z) \cap S'| \not\geq |N_s(z) \cap (V \setminus S')| \end{aligned}$$

or

$$\begin{aligned} \forall z \in V \setminus S', |N_s(z) \cap S'| = 1 = 1 = |N_s(z) \cap (V \setminus S')| \\ \forall z \in V \setminus S', |N_s(z) \cap S'| = |N_s(z) \cap (V \setminus S')| \\ \forall z \in V \setminus S', |N_s(z) \cap S'| \not\geq |N_s(z) \cap (V \setminus S')| \end{aligned}$$

So $S' = \{v_1, v_3\} \cup \{v_6, v_9 \dots, v_{i+6}, \dots, v_n\}_{i=1}^{6+3(i-1) \leq n} - \{z\}$ where $z \in S = \{v_1, v_3\} \cup \{v_6, v_9 \dots, v_{i+6}, \dots, v_n\}_{i=1}^{6+3(i-1) \leq n}$ isn't global-offensive alliance. It induces $S = \{v_1, v_3\} \cup \{v_6, v_9 \dots, v_{i+6}, \dots, v_n\}_{i=1}^{6+3(i-1) \leq n}$ is minimal-global-offensive alliance.

(ii), (iii) and (iv) are obvious. □

Example 3.12. Consider Figure (7).

- (i) $S = \{s_1, s_3, s_5\}$ is only minimal-global-offensive alliance;
- (ii) $S = \{s_1, s_3, s_5\}$ is optimal such that forms both minimal-global-offensive-alliance-neutrosophic number and minimal-global-offensive-alliance number;
- (iii) S' including $S = \{s_2, s_4, s_5\}$ only forms global-offensive-alliance but not minimal-global-offensive-alliance;
- (iv) $N = \{s_1, s_3\}$ isn't global-offensive alliance. Since there is one instance and only one instance is enough;
 - (a) First counterexample for the statement " $N = \{s_1, s_3\}$ is global-offensive alliance.";

$$\begin{aligned} \exists s_5 \in V \setminus N, |N_s(s_5) \cap N| = 1 = 1 = |N_s(s_5) \cap (V \setminus N)| \\ \exists s_5 \in V \setminus N, |N_s(s_5) \cap N| = 1 \not\geq 1 = |N_s(s_5) \cap (V \setminus N)| \\ \exists s_5 \in V \setminus N, |N_s(s_5) \cap N| \not\geq |N_s(s_5) \cap (V \setminus N)|; \end{aligned}$$
- (v) $\Gamma_s = 4.9$ and corresponded set is $S = \{s_1, s_3, s_5\}$;
- (vi) $\Gamma = 3$ and corresponded set is $S = \{s_1, s_3, s_5\}$.

Proposition 3.13. Let $NTG : (V, E, \sigma, \mu)$ be an odd complete. Then

- (i) the set $S = \{v_i\}_{i=1}^{\lfloor \frac{n}{2} \rfloor + 1}$ is minimal-global-offensive alliance;
- (ii) $\Gamma = \lfloor \frac{n}{2} \rfloor + 1$;
- (iii) $\Gamma_s = \min\{\sum_{s \in S} \sum_{i=1}^3 \sigma_i(s)\}_{S=\{v_i\}_{i=1}^{\lfloor \frac{n}{2} \rfloor + 1}}$;

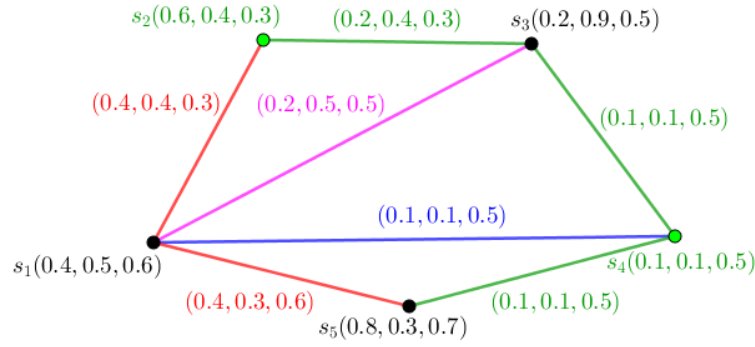


Figure 7. The set of black circles is minimal-global-offensive alliance.

(iv) the set $S = \{v_i\}_{i=1}^{\lfloor \frac{n}{2} \rfloor + 1}$ is only minimal-global-offensive alliances.

Proof. (i). Suppose $NTG : (V, E, \sigma, \mu)$ is odd complete. Let $S = \{v_i\}_{i=1}^{\lfloor \frac{n}{2} \rfloor + 1}$. Thus

$$\forall z \in V \setminus S, |N_s(z) \cap S| = \lfloor \frac{n}{2} \rfloor + 1 > \lfloor \frac{n}{2} \rfloor - 1 = |N_s(z) \cap (V \setminus S)|$$

$$\forall z \in V \setminus S, |N_s(z) \cap S| > |N_s(z) \cap (V \setminus S)|$$

It implies $S = \{v_i\}_{i=1}^{\lfloor \frac{n}{2} \rfloor + 1}$ is global-offensive alliance. If $S' = \{v_i\}_{i=1}^{\lfloor \frac{n}{2} \rfloor + 1} - \{z\}$ where $z \in S = \{v_i\}_{i=1}^{\lfloor \frac{n}{2} \rfloor + 1}$, then

$$\forall z \in V \setminus S, |N_s(z) \cap S'| = \lfloor \frac{n}{2} \rfloor = \lfloor \frac{n}{2} \rfloor = |N_s(z) \cap (V \setminus S)|$$

$$\forall z \in V \setminus S, |N_s(z) \cap S'| \not> |N_s(z) \cap (V \setminus S)|$$

So $S' = \{v_i\}_{i=1}^{\lfloor \frac{n}{2} \rfloor + 1} - \{z\}$ where $z \in S = \{v_i\}_{i=1}^{\lfloor \frac{n}{2} \rfloor + 1}$ isn't global-offensive alliance. It induces $S = \{v_i\}_{i=1}^{\lfloor \frac{n}{2} \rfloor + 1}$ is minimal-global-offensive alliance.

(ii), (iii) and (iv) are obvious. \square

Example 3.14. Consider Figure (8).

(i) $S_1 = \{s_1, s_2, s_3\}, S_2 = \{s_1, s_2, s_4\}, S_3 = \{s_1, s_2, s_5\}, S_4 = \{s_1, s_3, s_4\}, S_5 = \{s_1, s_3, s_5\}, S_6 = \{s_2, s_3, s_4\}, S_7 = \{s_2, s_3, s_5\}, S_8 = \{s_3, s_4, s_5\}$ are only minimal-global-offensive alliances;

(ii) $S_6 = \{s_2, s_3, s_4\}$ is optimal such that forms both minimal-global-offensive-alliance-neutrosophic number and minimal-global-offensive-alliance number;

(iii) $S = \{s_3, s_4, s_5\}$ only forms minimal-global-offensive-alliance number but not minimal-global-offensive-alliance-neutrosophic;

(iv) $N = \{s_3, s_4\}$ isn't global-offensive alliance. Since there is three instances and only one instance is enough;

(a) First counterexample for the statement " $N = \{s_3, s_4\}$ is global-offensive alliance.";

$$\exists s_1 \in V \setminus N, |N_s(s_1) \cap N| = 2 = 2 = |N_s(s_1) \cap (V \setminus N)|$$

$$\exists s_1 \in V \setminus N, |N_s(s_1) \cap N| = 2 \not> 2 = |N_s(s_1) \cap (V \setminus N)|$$

$$\exists s_1 \in V \setminus N, |N_s(s_1) \cap N| \not> |N_s(s_1) \cap (V \setminus N)|;$$

(b) second counterexample for the statement " $N = \{s_3, s_4\}$ is global-offensive alliance.";

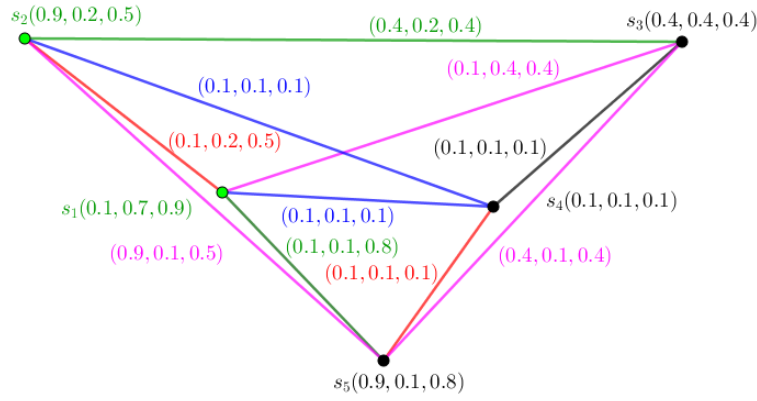


Figure 8. The set of black circles is minimal-global-offensive alliance.

$$\exists s_2 \in V \setminus N, |N_s(s_2) \cap N| = 2 = 2 = |N_s(s_2) \cap (V \setminus N)|$$

$$\exists s_2 \in V \setminus N, |N_s(s_2) \cap N| = 2 \neq 2 = |N_s(s_2) \cap (V \setminus N)|$$

$$\exists s_2 \in V \setminus N, |N_s(s_2) \cap N| \neq |N_s(s_2) \cap (V \setminus N)|;$$

(c) third counterexample for the statement “ $N = \{s_3, s_4\}$ is global-offensive alliance.”.

$$\exists s_5 \in V \setminus N, |N_s(s_5) \cap N| = 2 = 2 = |N_s(s_5) \cap (V \setminus N)|$$

$$\exists s_5 \in V \setminus N, |N_s(s_5) \cap N| = 2 \neq 2 = |N_s(s_5) \cap (V \setminus N)|$$

$$\exists s_5 \in V \setminus N, |N_s(s_5) \cap N| \neq |N_s(s_5) \cap (V \setminus N)|;$$

(v) $\Gamma_s = 3.3$ and corresponded set is $S_6 = \{s_2, s_3, s_4\}$;

(vi) $\Gamma = 3$ and corresponded sets are

$S_1 = \{s_1, s_2, s_3\}, S_2 = \{s_1, s_2, s_4\}, S_3 = \{s_1, s_2, s_5\}, S_4 = \{s_1, s_3, s_4\}, S_5 = \{s_1, s_3, s_5\}, S_6 = \{s_2, s_3, s_4\}, S_7 = \{s_2, s_3, s_5\}, S_8 = \{s_3, s_4, s_5\}$ which are only minimal-global-offensive alliances.

Proposition 3.15. Let $NTG : (V, E, \sigma, \mu)$ be an even complete. Then

(i) the set $S = \{v_i\}_{i=1}^{\lfloor \frac{n}{2} \rfloor}$ is minimal-global-offensive alliance;

(ii) $\Gamma = \lfloor \frac{n}{2} \rfloor$;

(iii) $\Gamma_s = \min\{\sum_{s \in S} \sum_{i=1}^3 \sigma_i(s)\}_{S=\{v_i\}_{i=1}^{\lfloor \frac{n}{2} \rfloor}};$

(iv) the set $S = \{v_i\}_{i=1}^{\lfloor \frac{n}{2} \rfloor}$ is only minimal-global-offensive alliances.

Proof. (i). Suppose $NTG : (V, E, \sigma, \mu)$ is even complete. Let $S = \{v_i\}_{i=1}^{\lfloor \frac{n}{2} \rfloor}$. Thus

$$\forall z \in V \setminus S, |N_s(z) \cap S| = \lfloor \frac{n}{2} \rfloor > \lfloor \frac{n}{2} \rfloor - 1 = |N_s(z) \cap (V \setminus S)|$$

$$\forall z \in V \setminus S, |N_s(z) \cap S| > |N_s(z) \cap (V \setminus S)|$$

It implies $S = \{v_i\}_{i=1}^{\lfloor \frac{n}{2} \rfloor}$ is global-offensive alliance. If $S' = \{v_i\}_{i=1}^{\lfloor \frac{n}{2} \rfloor} - \{z\}$ where $z \in S = \{v_i\}_{i=1}^{\lfloor \frac{n}{2} \rfloor}$, then

$$\forall z \in V \setminus S, |N_s(z) \cap S| = \lfloor \frac{n}{2} \rfloor - 1 < \lfloor \frac{n}{2} \rfloor + 1 = |N_s(z) \cap (V \setminus S)|$$

$$\forall z \in V \setminus S, |N_s(z) \cap S| \neq |N_s(z) \cap (V \setminus S)|$$

So $S' = \{v_i\}_{i=1}^{\lfloor \frac{n}{2} \rfloor} - \{z\}$ where $z \in S = \{v_i\}_{i=1}^{\lfloor \frac{n}{2} \rfloor}$ isn't global-offensive alliance. It induces $S = \{v_i\}_{i=1}^{\lfloor \frac{n}{2} \rfloor}$ is minimal-global-offensive alliance.

(ii), (iii) and (iv) are obvious. \square

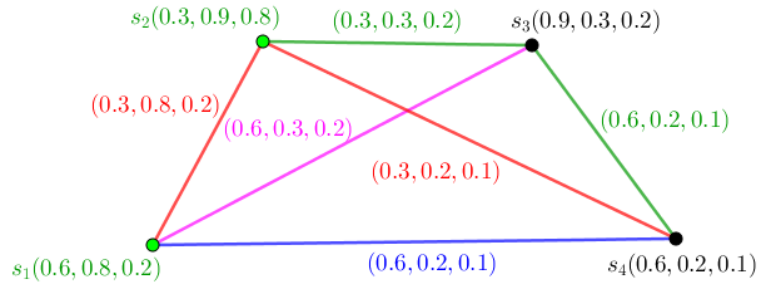


Figure 9. The set of black circles is minimal-global-offensive alliance.

Example 3.16. Consider Figure (13).

- (i) $S_1 = \{s_1, s_2\}, S_2 = \{s_1, s_3\}, S_3 = \{s_1, s_4\}, S_4 = \{s_2, s_3\}, S_5 = \{s_2, s_4\}, S_6 = \{s_3, s_4\}$ are only minimal-global-offensive alliances;
- (ii) $S_6 = \{s_3, s_4\}$ is optimal such that forms both minimal-global-offensive-alliance-neutrosophic number and minimal-global-offensive-alliance number;
- (iii) $S = \{s_1, s_3\}$ only forms minimal-global-offensive-alliance number but not minimal-global-offensive-alliance-neutrosophic;
- (iv) $N = \{s_1\}$ isn't global-offensive alliance. Since there is three instances and only one instance is enough;
 - (a) First counterexample for the statement " $N = \{s_1\}$ is global-offensive alliance.";

$$\begin{aligned} \exists s_2 \in V \setminus N, |N_s(s_2) \cap N| &= 1 < 2 = |N_s(s_2) \cap (V \setminus N)| \\ \exists s_2 \in V \setminus N, |N_s(s_2) \cap N| &= 1 \not\geq 2 = |N_s(s_2) \cap (V \setminus N)| \\ \exists s_2 \in V \setminus N, |N_s(s_2) \cap N| &\not\geq |N_s(s_2) \cap (V \setminus N)|; \end{aligned}$$
 - (b) second counterexample for the statement " $N = \{s_1\}$ is global-offensive alliance.";

$$\begin{aligned} \exists s_3 \in V \setminus N, |N_s(s_3) \cap N| &= 1 < 2 = |N_s(s_3) \cap (V \setminus N)| \\ \exists s_3 \in V \setminus N, |N_s(s_3) \cap N| &= 1 \not\geq 2 = |N_s(s_3) \cap (V \setminus N)| \\ \exists s_3 \in V \setminus N, |N_s(s_3) \cap N| &\not\geq |N_s(s_3) \cap (V \setminus N)|; \end{aligned}$$
 - (c) third counterexample for the statement " $N = \{s_1\}$ is global-offensive alliance.".

$$\begin{aligned} \exists s_4 \in V \setminus N, |N_s(s_4) \cap N| &= 1 < 2 = |N_s(s_4) \cap (V \setminus N)| \\ \exists s_4 \in V \setminus N, |N_s(s_4) \cap N| &= 1 \not\geq 2 = |N_s(s_4) \cap (V \setminus N)| \\ \exists s_4 \in V \setminus N, |N_s(s_4) \cap N| &\not\geq |N_s(s_4) \cap (V \setminus N)|; \end{aligned}$$
- (v) $\Gamma_s = 2.3$ and corresponded set is $S_6 = \{s_3, s_4\}$;
- (vi) $\Gamma = 2$ and corresponded set is $S_6 = \{s_3, s_4\}$.

4 Family of Neutrosophic Graphs

In this section, new definition is applied into family of some classes of neutrosophic graphs which in this family, all neutrosophic graphs have common neutrosophic vertex set. In the case of complete model, the parity of number of vertices concludes to have different results. Clarifications and demonstrations are given for every result as usual.

Proposition 4.1. Let \mathcal{G} be a m -family of neutrosophic stars with common neutrosophic vertex set. Then

- (i) the set $S = \{c_1, c_2, \dots, c_m\}$ is minimal-global-offensive alliance for \mathcal{G} ;
- (ii) $\Gamma = m$ for \mathcal{G} ;
- (iii) $\Gamma_s = \sum_{i=1}^m \sum_{j=1}^3 \sigma_j(c_i)$ for \mathcal{G} ;
- (iv) the sets $S = \{c_1, c_2, \dots, c_m\}$ and $S \subset S'$ are only minimal-global-offensive alliances for \mathcal{G} .

Proof. (i). Suppose $NTG : (V, E, \sigma, \mu)$ is a star.

$$\begin{aligned} \forall v \in V \setminus \{c\}, |N_s(v) \cap \{c\}| &= 1 > 0 = |N_s(v) \cap (V \setminus \{c\})| \\ \forall z \in V \setminus S, |N_s(z) \cap S| &= 1 > 0 = |N_s(z) \cap (V \setminus S)| \\ \forall z \in V \setminus S, |N_s(z) \cap S| &> |N_s(z) \cap (V \setminus S)| \\ v \in V \setminus \{c\}, |N_s(v) \cap \{c\}| &> |N_s(v) \cap (V \setminus \{c\})| \end{aligned}$$

It implies $S = \{c_1, c_2, \dots, c_m\}$ is global-offensive alliance or \mathcal{G} . If $S = \{c\} - \{c\} = \emptyset$, then

$$\begin{aligned} \exists v \in V \setminus S, |N_s(z) \cap S| &= 0 = 0 = |N_s(z) \cap (V \setminus S)| \\ \exists v \in V \setminus S, |N_s(z) \cap S| &= 0 \neq 0 = |N_s(z) \cap (V \setminus S)| \\ \exists v \in V \setminus S, |N_s(z) \cap S| &\not> |N_s(z) \cap (V \setminus S)|. \end{aligned}$$

So $S = \{c\} - \{c\} = \emptyset$ isn't global-offensive alliance for \mathcal{G} . It induces $S = \{c_1, c_2, \dots, c_m\}$ is minimal-global-offensive alliance for \mathcal{G} .

(ii) and (iii) are trivial.

(iv). By (i), $S = \{c_1, c_2, \dots, c_m\}$ is minimal-global-offensive alliance for \mathcal{G} . Thus it's enough to show that $S \subseteq S'$ is minimal-global-offensive alliance for \mathcal{G} . Suppose $NTG : (V, E, \sigma, \mu)$ is a star. Let $S \subseteq S'$.

$$\begin{aligned} \forall v \in V \setminus \{c\}, |N_s(v) \cap \{c\}| &= 1 > 0 = |N_s(v) \cap (V \setminus \{c\})| \\ \forall z \in V \setminus S', |N_s(z) \cap S'| &= 1 > 0 = |N_s(z) \cap (V \setminus S')| \\ \forall z \in V \setminus S', |N_s(z) \cap S'| &> |N_s(z) \cap (V \setminus S')| \end{aligned}$$

It implies $S' \subseteq S$ is global-offensive alliance for \mathcal{G} . □

Example 4.2. Consider Figure (10).

- (i) $S = \{s_1\}$ is only minimal-global-offensive alliance for \mathcal{G} ;
- (ii) $S = \{s_1\}$ is optimal such that forms both minimal-global-offensive-alliance-neutrosophic number and minimal-global-offensive-alliance number for \mathcal{G} ;
- (iii) S' including $S = \{s_1\}$ only forms global-offensive-alliance but not minimal-global-offensive-alliance for \mathcal{G} ;
- (iv) $N = \{s_3, s_4\}$ isn't global-offensive alliance for \mathcal{G} . Since there are two instances for every member of \mathcal{G} but only one of them is enough; for every member of \mathcal{G} , we have same following instances;
 - (a) First counterexample for the statement “ $N = \{s_3, s_4\}$ is global-offensive alliance for \mathcal{G} .”;

$$\begin{aligned} \exists s_1 \in V \setminus N, |N_s(s_1) \cap N| &= 2 = 2 = |N_s(s_1) \cap (V \setminus N)| \\ \exists s_1 \in V \setminus N, |N_s(s_1) \cap N| &= 2 \neq 2 = |N_s(s_1) \cap (V \setminus N)| \\ \exists s_1 \in V \setminus N, |N_s(s_1) \cap N| &\not> |N_s(s_1) \cap (V \setminus N)|; \end{aligned}$$

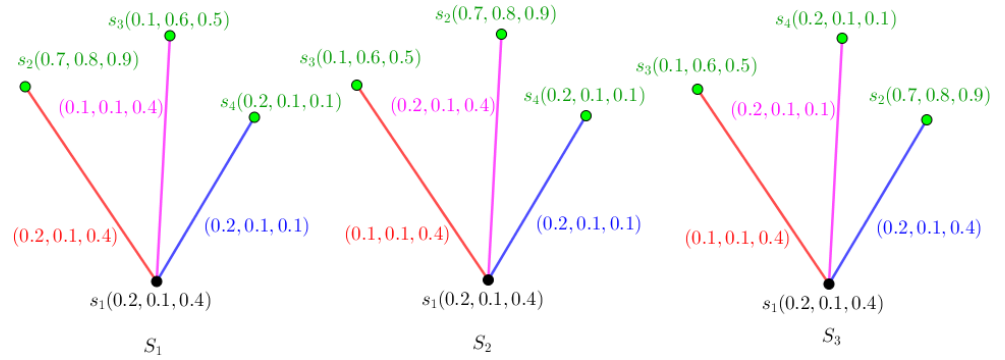


Figure 10. The set of black circles is minimal-global-offensive alliance.

(b) second counterexample for the statement “ $N = \{s_3, s_4\}$ is global-offensive alliance for \mathcal{G} .”;

$$\begin{aligned} \exists s_2 \in V \setminus N, |N_s(s_2) \cap N| = 0 < 1 = |N_s(s_2) \cap (V \setminus N)| \\ \exists s_2 \in V \setminus N, |N_s(s_2) \cap N| = 0 \not\geq 1 = |N_s(s_2) \cap (V \setminus N)| \\ \exists s_2 \in V \setminus N, |N_s(s_2) \cap N| \not\geq |N_s(s_2) \cap (V \setminus N)|; \end{aligned}$$

(v) $\Gamma_s = 0.7$ and corresponded set is $S = \{s_1\}$;

(vi) $\Gamma = 1$ and corresponded set is $S = \{s_1\}$.

Proposition 4.3. Let \mathcal{G} be a m -family of odd complete graphs with common neutrosophic vertex set. Then

(i) the set $S = \{v_i\}_{i=1}^{\lfloor \frac{n}{2} \rfloor + 1}$ is minimal-global-offensive alliance for \mathcal{G} ;

(ii) $\Gamma = \lfloor \frac{n}{2} \rfloor + 1$ for \mathcal{G} ;

(iii) $\Gamma_s = \min\{\sum_{s \in S} \sum_{i=1}^3 \sigma_i(s)\}_{S=\{v_i\}_{i=1}^{\lfloor \frac{n}{2} \rfloor + 1}}$ for \mathcal{G} ;

(iv) the sets $S = \{v_i\}_{i=1}^{\lfloor \frac{n}{2} \rfloor + 1}$ are only minimal-global-offensive alliances for \mathcal{G} .

Proof. (i). Suppose $NTG : (V, E, \sigma, \mu)$ is odd complete. Let $S = \{v_i\}_{i=1}^{\lfloor \frac{n}{2} \rfloor + 1}$. Thus

$$\forall z \in V \setminus S, |N_s(z) \cap S| = \lfloor \frac{n}{2} \rfloor + 1 > \lfloor \frac{n}{2} \rfloor - 1 = |N_s(z) \cap (V \setminus S)|$$

$$\forall z \in V \setminus S, |N_s(z) \cap S| > |N_s(z) \cap (V \setminus S)|$$

It implies $S = \{v_i\}_{i=1}^{\lfloor \frac{n}{2} \rfloor + 1}$ is global-offensive alliance for \mathcal{G} . If $S' = \{v_i\}_{i=1}^{\lfloor \frac{n}{2} \rfloor + 1} - \{z\}$ where $z \in S = \{v_i\}_{i=1}^{\lfloor \frac{n}{2} \rfloor + 1}$, then

$$\forall z \in V \setminus S, |N_s(z) \cap S| = \lfloor \frac{n}{2} \rfloor = \lfloor \frac{n}{2} \rfloor = |N_s(z) \cap (V \setminus S)|$$

$$\forall z \in V \setminus S, |N_s(z) \cap S| \not\geq |N_s(z) \cap (V \setminus S)|$$

So $S' = \{v_i\}_{i=1}^{\lfloor \frac{n}{2} \rfloor + 1} - \{z\}$ where $z \in S = \{v_i\}_{i=1}^{\lfloor \frac{n}{2} \rfloor + 1}$ isn't global-offensive alliance for \mathcal{G} . It induces $S = \{v_i\}_{i=1}^{\lfloor \frac{n}{2} \rfloor + 1}$ is minimal-global-offensive alliance for \mathcal{G} .

(ii), (iii) and (iv) are obvious. \square

Example 4.4. Consider Figure (14).

(i) $S_1 = \{s_1, s_2, s_3\}, S_2 = \{s_1, s_2, s_4\}, S_3 = \{s_1, s_2, s_5\}, S_4 = \{s_1, s_3, s_4\}, S_5 = \{s_1, s_3, s_5\}, S_6 = \{s_2, s_3, s_4\}, S_7 = \{s_2, s_3, s_5\}, S_8 = \{s_3, s_4, s_5\}$ are only minimal-global-offensive alliances;

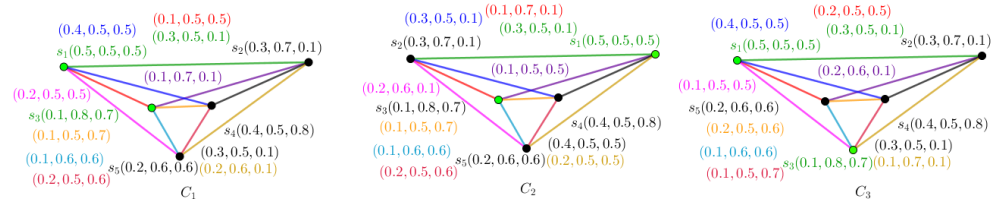


Figure 11. The set of black circles is minimal-global-offensive alliance.

- (ii) $S_3 = \{s_1, s_2, s_5\}$ is optimal such that forms both
minimal-global-offensive-alliance-neutrosophic number and
minimal-global-offensive-alliance number for \mathcal{G} ;
- (iii) $S_8 = \{s_3, s_4, s_5\}$ only forms minimal-global-offensive-alliance number but not
minimal-global-offensive-alliance-neutrosophic for \mathcal{G} ;
- (iv) $N = \{s_1, s_2\}$ isn't global-offensive alliance. Since there is three instances and only
one instance is enough for \mathcal{G} ;

- (a) First counterexample for the statement " $N = \{s_1, s_2\}$ is global-offensive
alliance." for \mathcal{G} ;

$$\begin{aligned} \exists s_3 \in V \setminus N, |N_s(s_3) \cap N| &= 2 = 2 = |N_s(s_3) \cap (V \setminus N)| \\ \exists s_3 \in V \setminus N, |N_s(s_3) \cap N| &= 2 \neq 2 = |N_s(s_3) \cap (V \setminus N)| \\ \exists s_3 \in V \setminus N, |N_s(s_3) \cap N| &\neq |N_s(s_3) \cap (V \setminus N)|; \end{aligned}$$

- (b) second counterexample for the statement " $N = \{s_1, s_2\}$ is global-offensive
alliance." for \mathcal{G} ;

$$\begin{aligned} \exists s_4 \in V \setminus N, |N_s(s_4) \cap N| &= 2 = 2 = |N_s(s_4) \cap (V \setminus N)| \\ \exists s_4 \in V \setminus N, |N_s(s_4) \cap N| &= 2 \neq 2 = |N_s(s_4) \cap (V \setminus N)| \\ \exists s_4 \in V \setminus N, |N_s(s_4) \cap N| &\neq |N_s(s_4) \cap (V \setminus N)|; \end{aligned}$$

- (c) third counterexample for the statement " $N = \{s_1, s_2\}$ is global-offensive
alliance." for \mathcal{G} .

$$\begin{aligned} \exists s_5 \in V \setminus N, |N_s(s_5) \cap N| &= 2 = 2 = |N_s(s_5) \cap (V \setminus N)| \\ \exists s_5 \in V \setminus N, |N_s(s_5) \cap N| &= 2 \neq 2 = |N_s(s_5) \cap (V \setminus N)| \\ \exists s_5 \in V \setminus N, |N_s(s_5) \cap N| &\neq |N_s(s_5) \cap (V \setminus N)|; \end{aligned}$$

- (v) $\Gamma_s = 4$ and corresponded set is $S_3 = \{s_1, s_2, s_5\}$ for \mathcal{G} ;

- (vi) $\Gamma = 3$ and corresponded sets are
 $S_1 = \{s_1, s_2, s_3\}, S_2 = \{s_1, s_2, s_4\}, S_3 = \{s_1, s_2, s_5\}, S_4 = \{s_1, s_3, s_4\}, S_5 =$
 $\{s_1, s_3, s_5\}, S_6 = \{s_2, s_3, s_4\}, S_7 = \{s_2, s_3, s_5\}, S_8 = \{s_3, s_4, s_5\}$ which are only
minimal-global-offensive alliances for \mathcal{G} .

Proposition 4.5. Let \mathcal{G} be a m -family of even complete graphs with common
neutrosophic vertex set. Then

- (i) the set $S = \{v_i\}_{i=1}^{\lfloor \frac{n}{2} \rfloor}$ is minimal-global-offensive alliance for \mathcal{G} ;
- (ii) $\Gamma = \lfloor \frac{n}{2} \rfloor$ for \mathcal{G} ;
- (iii) $\Gamma_s = \min\{\sum_{s \in S} \sum_{i=1}^3 \sigma_i(s)\}_{S=\{v_i\}_{i=1}^{\lfloor \frac{n}{2} \rfloor}}$ for \mathcal{G} ;
- (iv) the sets $S = \{v_i\}_{i=1}^{\lfloor \frac{n}{2} \rfloor}$ are only minimal-global-offensive alliances for \mathcal{G} .

Proof. (i). Suppose $NTG : (V, E, \sigma, \mu)$ is even complete. Let $S = \{v_i\}_{i=1}^{\lfloor \frac{n}{2} \rfloor}$. Thus

$$\forall z \in V \setminus S, |N_s(z) \cap S| = \lfloor \frac{n}{2} \rfloor > \lfloor \frac{n}{2} \rfloor - 1 = |N_s(z) \cap (V \setminus S)|$$

$$\forall z \in V \setminus S, |N_s(z) \cap S| > |N_s(z) \cap (V \setminus S)|$$

It implies $S = \{v_i\}_{i=1}^{\lfloor \frac{n}{2} \rfloor}$ is global-offensive alliance for \mathcal{G} . If $S' = \{v_i\}_{i=1}^{\lfloor \frac{n}{2} \rfloor} - \{z\}$ where $z \in S = \{v_i\}_{i=1}^{\lfloor \frac{n}{2} \rfloor}$, then

$$\forall z \in V \setminus S, |N_s(z) \cap S| = \lfloor \frac{n}{2} \rfloor - 1 < \lfloor \frac{n}{2} \rfloor + 1 = |N_s(z) \cap (V \setminus S)|$$

$$\forall z \in V \setminus S, |N_s(z) \cap S| \not> |N_s(z) \cap (V \setminus S)|$$

So $S' = \{v_i\}_{i=1}^{\lfloor \frac{n}{2} \rfloor} - \{z\}$ where $z \in S = \{v_i\}_{i=1}^{\lfloor \frac{n}{2} \rfloor}$ isn't global-offensive alliance for \mathcal{G} . It induces $S = \{v_i\}_{i=1}^{\lfloor \frac{n}{2} \rfloor}$ is minimal-global-offensive alliance for \mathcal{G} .

(ii), (iii) and (iv) are obvious. \square

Example 4.6. Consider Figure (12).

(i) $S_1 = \{s_1, s_2\}, S_2 = \{s_1, s_3\}, S_3 = \{s_1, s_4\}, S_4 = \{s_2, s_3\}, S_5 = \{s_2, s_4\}, S_6 = \{s_3, s_4\}$ are only minimal-global-offensive alliances for \mathcal{G} ;

(ii) $S_1 = \{s_1, s_2\}$ is optimal such that forms both minimal-global-offensive-alliance-neutrosophic number and minimal-global-offensive-alliance number for \mathcal{G} ;

(iii) $S = \{s_1, s_3\}$ only forms minimal-global-offensive-alliance number but not minimal-global-offensive-alliance-neutrosophic for \mathcal{G} ;

(iv) $N = \{s_1\}$ isn't global-offensive alliance. Since there is three instances and only one instance is enough for \mathcal{G} ;

(a) First counterexample for the statement " $N = \{s_1\}$ is global-offensive alliance." for \mathcal{G} ;

$$\exists s_2 \in V \setminus N, |N_s(s_2) \cap N| = 1 < 2 = |N_s(s_2) \cap (V \setminus N)|$$

$$\exists s_2 \in V \setminus N, |N_s(s_2) \cap N| = 1 \not> 2 = |N_s(s_2) \cap (V \setminus N)|$$

$$\exists s_2 \in V \setminus N, |N_s(s_2) \cap N| \not> |N_s(s_2) \cap (V \setminus N)|;$$

(b) second counterexample for the statement " $N = \{s_1\}$ is global-offensive alliance." for \mathcal{G} ;

$$\exists s_3 \in V \setminus N, |N_s(s_3) \cap N| = 1 < 2 = |N_s(s_3) \cap (V \setminus N)|$$

$$\exists s_3 \in V \setminus N, |N_s(s_3) \cap N| = 1 \not> 2 = |N_s(s_3) \cap (V \setminus N)|$$

$$\exists s_3 \in V \setminus N, |N_s(s_3) \cap N| \not> |N_s(s_3) \cap (V \setminus N)|;$$

(c) third counterexample for the statement " $N = \{s_1\}$ is global-offensive alliance." for \mathcal{G} .

$$\exists s_4 \in V \setminus N, |N_s(s_4) \cap N| = 1 < 2 = |N_s(s_4) \cap (V \setminus N)|$$

$$\exists s_4 \in V \setminus N, |N_s(s_4) \cap N| = 1 \not> 2 = |N_s(s_4) \cap (V \setminus N)|$$

$$\exists s_4 \in V \setminus N, |N_s(s_4) \cap N| \not> |N_s(s_4) \cap (V \setminus N)|;$$

(v) $\Gamma_s = 2.6$ and corresponded set is $S_1 = \{s_1, s_2\}$ for \mathcal{G} ;

(vi) $\Gamma = 2$ and corresponded sets are $S_1 = \{s_1, s_2\}, S_2 = \{s_1, s_3\}, S_3 = \{s_1, s_4\}, S_4 = \{s_2, s_3\}, S_5 = \{s_2, s_4\}, S_6 = \{s_3, s_4\}$ for \mathcal{G} .

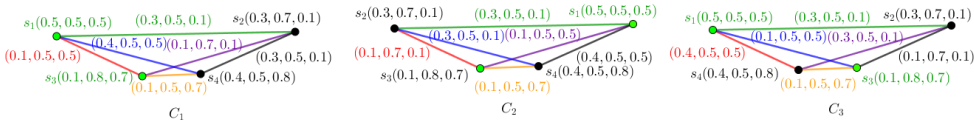


Figure 12. The set of black circles is minimal-global-offensive alliance.

5 Applications in Time Table and Scheduling

In this section, two applications for time table and scheduling are provided where the models are complete models which mean complete connections are formed as individual and family of complete models with common neutrosophic vertex set.

Designing the programs to achieve some goals is general approach to apply on some issues to function properly. Separation has key role in the context of this style. Separating the duration of work which are consecutive, is the matter and it has importance to avoid mixing up.

Step 1. (Definition) Time table is an approach to get some attributes to do the work fast and proper. The style of scheduling implies special attention to the tasks which are consecutive.

Step 2. (Issue) Scheduling of program has faced with difficulties to differ amid consecutive section. Beyond that, sometimes sections are not the same.

Step 3. (Model) The situation is designed as a model. The model uses data to assign every section and to assign to relation amid section, three numbers belong unit interval to state indeterminacy, possibilities and determinacy. There's one restriction in that, the numbers amid two sections are at least the number of the relation amid them. Table (1), clarifies about the assigned numbers to these situation.

Table 1. Scheduling concerns its Subjects and its Connections as a neutrosophic graph and its alliances in a Model.

Sections of NTG	n_1	$n_2 \cdots$	n_9
Values	$(0.99, 0.98, 0.55)$	$(0.74, 0.64, 0.46) \cdots$	$(0.99, 0.98, 0.55)$
Connections of NTG	E_1	E_2	E_3
Values	$(0.01, 0.01, 0.01)$	$(0.01, 0.01, 0.01)$	$(0.01, 0.01, 0.01)$

5.1 Case 1: Complete Model as Individual

Step 4. (Solution) The neutrosophic graph and its global offensive alliance as model, propose to use specific set. Every subject has connection with every given subject. Thus the connection is applied as possible and the model demonstrates full connections as possible. Using the notion of strong on the connection amid subjects, causes the importance of subject goes in the highest level such that the value amid two consecutive subjects, is determined by those subjects. If the configuration is complete, the set is different. Also, it holds for other types such that star, wheel, path, and cycle. The collection of situations is another application of global offensive alliance when the notion of family is applied in the way that all members of family are from same classes of neutrosophic graphs. As follows, There are four subjects which are represented as Figure (13). This model is strong. And the study proposes using specific set of objects which is called minimal-global-offensive alliance. There are also some analyses on other sets in

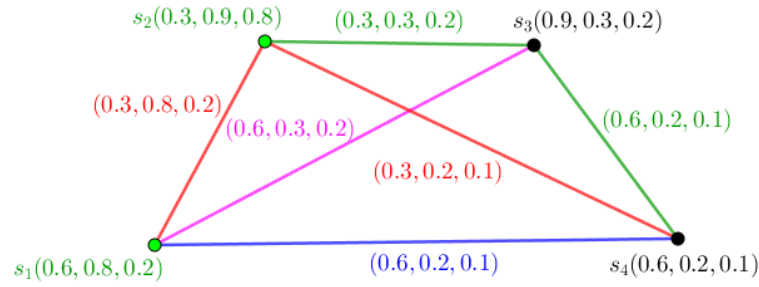


Figure 13. The set of black circles is minimal-global-offensive alliance.

the way that, the clarification is gained about being special set or not. Also, in the last part, there are two numbers to assign to this model and situation to compare them with same situations to get more precise. Consider Figure (13).

- (i) $S_1 = \{s_1, s_2\}, S_2 = \{s_1, s_3\}, S_3 = \{s_1, s_4\}, S_4 = \{s_2, s_3\}, S_5 = \{s_2, s_4\}, S_6 = \{s_3, s_4\}$ are only minimal-global-offensive alliances;
- (ii) $S_6 = \{s_3, s_4\}$ is optimal such that forms both minimal-global-offensive-alliance-neutrosophic number and minimal-global-offensive-alliance number;
- (iii) $S = \{s_1, s_3\}$ only forms minimal-global-offensive-alliance number but not minimal-global-offensive-alliance-neutrosophic;
- (iv) $N = \{s_1\}$ isn't global-offensive alliance. Since there is three instances and only one instance is enough;
 - (a) First counterexample for the statement " $N = \{s_1\}$ is global-offensive alliance.";

$$\exists s_2 \in V \setminus N, |N_s(s_2) \cap N| = 1 < 2 = |N_s(s_2) \cap (V \setminus N)|$$

$$\exists s_2 \in V \setminus N, |N_s(s_2) \cap N| = 1 \not\geq 2 = |N_s(s_2) \cap (V \setminus N)|$$

$$\exists s_2 \in V \setminus N, |N_s(s_2) \cap N| \not\geq |N_s(s_2) \cap (V \setminus N)|;$$
 - (b) second counterexample for the statement " $N = \{s_1\}$ is global-offensive alliance.";

$$\exists s_3 \in V \setminus N, |N_s(s_3) \cap N| = 1 < 2 = |N_s(s_3) \cap (V \setminus N)|$$

$$\exists s_3 \in V \setminus N, |N_s(s_3) \cap N| = 1 \not\geq 2 = |N_s(s_3) \cap (V \setminus N)|$$

$$\exists s_3 \in V \setminus N, |N_s(s_3) \cap N| \not\geq |N_s(s_3) \cap (V \setminus N)|;$$
 - (c) third counterexample for the statement " $N = \{s_1\}$ is global-offensive alliance."

$$\exists s_4 \in V \setminus N, |N_s(s_4) \cap N| = 1 < 2 = |N_s(s_4) \cap (V \setminus N)|$$

$$\exists s_4 \in V \setminus N, |N_s(s_4) \cap N| = 1 \not\geq 2 = |N_s(s_4) \cap (V \setminus N)|$$

$$\exists s_4 \in V \setminus N, |N_s(s_4) \cap N| \not\geq |N_s(s_4) \cap (V \setminus N)|;$$
- (v) $\Gamma_s = 2.3$ and corresponded set is $S_6 = \{s_3, s_4\}$;
- (vi) $\Gamma = 2$ and corresponded set is $S_6 = \{s_3, s_4\}$.

5.2 Case 2: Family of Complete Models

Step 4. (Solution) The neutrosophic graph and its global offensive alliance as model, propose to use specific set. Every subject has connection with every given subject. Thus the connection is applied as possible and the model demonstrates full connections as possible. Using the notion of strong on the connection amid subjects, causes the importance of subject goes in the highest level such that the

value amid two consecutive subjects, is determined by those subjects. If the configuration is complete, the set is different. Also, it holds for other types such that star, wheel, path, and cycle. The collection of situations is another application of global offensive alliance when the notion of family is applied in the way that all members of family are from same classes of neutrosophic graphs. As follows, There are five subjects which are represented in the formation of family of models as Figure (13). These models are strong in family. And the study proposes using specific set of objects which is called minimal-global-offensive alliance for this family of models. There are also some analyses on other sets in the way that, the clarification is gained about being special set or not. Also, in the last part, there are two numbers to assign to this family of models and collection of situations to compare them with collection of situations to get more precise. Consider Figure (14).

- (i) $S_1 = \{s_1, s_2, s_3\}, S_2 = \{s_1, s_2, s_4\}, S_3 = \{s_1, s_2, s_5\}, S_4 = \{s_1, s_3, s_4\}, S_5 = \{s_1, s_3, s_5\}, S_6 = \{s_2, s_3, s_4\}, S_7 = \{s_2, s_3, s_5\}, S_8 = \{s_3, s_4, s_5\}$ are only minimal-global-offensive alliances;
- (ii) $S_3 = \{s_1, s_2, s_5\}$ is optimal such that forms both minimal-global-offensive-alliance-neutrosophic number and minimal-global-offensive-alliance number for \mathcal{G} ;
- (iii) $S_8 = \{s_3, s_4, s_5\}$ only forms minimal-global-offensive-alliance number but not minimal-global-offensive-alliance-neutrosophic for \mathcal{G} ;
- (iv) $N = \{s_1, s_2\}$ isn't global-offensive alliance. Since there is three instances and only one instance is enough for \mathcal{G} ;
 - (a) First counterexample for the statement " $N = \{s_1, s_2\}$ is global-offensive alliance." for \mathcal{G} ;

$$\begin{aligned} \exists s_3 \in V \setminus N, |N_s(s_3) \cap N| = 2 = 2 = |N_s(s_3) \cap (V \setminus N)| \\ \exists s_3 \in V \setminus N, |N_s(s_3) \cap N| = 2 \neq 2 = |N_s(s_3) \cap (V \setminus N)| \\ \exists s_3 \in V \setminus N, |N_s(s_3) \cap N| \neq |N_s(s_3) \cap (V \setminus N)|; \end{aligned}$$
 - (b) second counterexample for the statement " $N = \{s_1, s_2\}$ is global-offensive alliance." for \mathcal{G} ;

$$\begin{aligned} \exists s_4 \in V \setminus N, |N_s(s_4) \cap N| = 2 = 2 = |N_s(s_4) \cap (V \setminus N)| \\ \exists s_4 \in V \setminus N, |N_s(s_4) \cap N| = 2 \neq 2 = |N_s(s_4) \cap (V \setminus N)| \\ \exists s_4 \in V \setminus N, |N_s(s_4) \cap N| \neq |N_s(s_4) \cap (V \setminus N)|; \end{aligned}$$
 - (c) third counterexample for the statement " $N = \{s_1, s_2\}$ is global-offensive alliance." for \mathcal{G} .

$$\begin{aligned} \exists s_5 \in V \setminus N, |N_s(s_5) \cap N| = 2 = 2 = |N_s(s_5) \cap (V \setminus N)| \\ \exists s_5 \in V \setminus N, |N_s(s_5) \cap N| = 2 \neq 2 = |N_s(s_5) \cap (V \setminus N)| \\ \exists s_5 \in V \setminus N, |N_s(s_5) \cap N| \neq |N_s(s_5) \cap (V \setminus N)|; \end{aligned}$$
- (v) $\Gamma_s = 4$ and corresponded set is $S_3 = \{s_1, s_2, s_5\}$ for \mathcal{G} ;
- (vi) $\Gamma = 3$ and corresponded sets are $S_1 = \{s_1, s_2, s_3\}, S_2 = \{s_1, s_2, s_4\}, S_3 = \{s_1, s_2, s_5\}, S_4 = \{s_1, s_3, s_4\}, S_5 = \{s_1, s_3, s_5\}, S_6 = \{s_2, s_3, s_4\}, S_7 = \{s_2, s_3, s_5\}, S_8 = \{s_3, s_4, s_5\}$ which are only minimal-global-offensive alliances for \mathcal{G} .

6 Open Problems

In this section, some questions and problems are proposed to give some avenues to pursue this study. The structures of the definitions and results give some ideas to make new settings which are eligible to extend and to create new study.

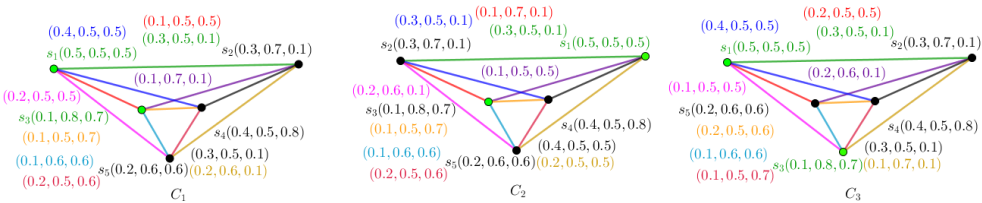


Figure 14. The set of black circles is minimal-global-offensive alliance.

Notion concerning alliance is defined in neutrosophic graphs. Neutrosophic number is also introduced. Thus,

Question 6.1. *Is it possible to use other types neighborhood arising from different types of edges to define new alliances?*

Question 6.2. *Are existed some connections amid different types of alliances in neutrosophic graphs?*

Question 6.3. *Is it possible to construct some classes of which have “nice” behavior?*

Question 6.4. *Which mathematical notions do make an independent study to apply these types in neutrosophic graphs?*

Problem 6.5. *Which parameters are related to this parameter?*

Problem 6.6. *Which approaches do work to construct applications to create independent study?*

Problem 6.7. *Which approaches do work to construct definitions which use all definitions and the relations amid them instead of separate definitions to create independent study?*

7 Conclusion and Closing Remarks

In this section, concluding remarks and closing remarks are represented. The drawbacks of this article are illustrated. Some benefits and advantages of this study are highlighted.

This study uses one definition concerning global offensive alliance to study neutrosophic graphs. New neutrosophic number is introduced which is too close to the notion of neutrosophic number but it’s different since it uses all values as type-summation on them. The connections of vertices which are clarified by general edges differ them from each other and put them in different categories to represent a set

Table 2. A Brief Overview about Advantages and Limitations of this study

Advantages	Limitations
1. Defining Global Offensive Alliances	1. General Results
2. Applying on Strong Neutrosophic Graphs	
3. Study on Complete Models	2. Deeply More Connections
4. Applying on Individuals	
5. Applying on Family	3. Same Models in Family

which is called global offensive alliance. Further studies could be about changes in the settings to compare this notion amid different settings of neutrosophic graphs theory. One way is finding some relations amid all definitions of notions to make sensible definitions. In Table (2), some limitations and advantages of this study are pointed out.

References

1. M. Akram, and G. Shahzadi, “Operations on Single-Valued Neutrosophic Graphs”, Journal of uncertain systems 11 (1) (2017) 1-26.

2. K. Atanassov, “Intuitionistic fuzzy sets”, Fuzzy Sets Syst. 20 (1986) 87-96.

3. S. Broumi, M. Talea, A. Bakali and F. Smarandache, “Single-valued neutrosophic graphs”, Journal of New Theory 10 (2016) 86-101.

4. N. Shah, and A. Hussain, “Neutrosophic soft graphs”, Neutrosophic Set and Systems 11 (2016) 31-44.

5. Henry Garrett, “Chromatic Number and Neutrosophic Chromatic Number”, Preprints 2021, 2021120177 (doi: 10.20944/preprints202112.0177.v1).

6. Henry Garrett, “Closing Numbers and Super-Closing Numbers as (Dual)Resolving and (Dual)Coloring alongside (Dual)Dominating in (Neutrosophic)n-SuperHyperGraph”, Preprints 2022, 2022010145 (doi: 10.20944/preprints202201.0145.v1).

7. Henry Garrett, “Co-degree and Degree of classes of Neutrosophic Hypergraphs”, Preprints 2022, 2022010027 (doi: 10.20944/preprints202201.0027.v1).

8. Henry Garrett, “Different Types of Neutrosophic Chromatic Number”, Preprints 2021, 2021120335 (doi: 10.20944/preprints202112.0335.v1).

9. Henry Garrett, “Dimension and Coloring alongside Domination in Neutrosophic Hypergraphs”, Preprints 2021, 2021120448 (doi: 10.20944/preprints202112.0448.v1).

10. Henry Garrett, “Metric Dimensions Of Graphs #12”, ResearchGate 2021 (doi: 10.13140/RG.2.2.20690.48322).

11. Henry Garrett, “Three Types of Neutrosophic Alliances based on Connectedness and (Strong) Edges”, Preprints 2022, 2022010239 (doi: 10.20944/preprints202201.0239.v1).

12. Henry Garrett, “Neutrosophic Chromatic Number Based on Connectedness”, Preprints 2021, 2021120226 (doi: 10.20944/preprints202112.0226.v1).

13. A. Shannon and K.T. Atanassov, “A first step to a theory of the intuitionistic fuzzy graphs”, Proceeding of FUBEST (Lakov, D., Ed.) Sofia (1994) 59-61.

14. F. Smarandache, “A Unifying field in logics neutrosophy: Neutrosophic probability, set and logic, Rehoboth: ” American Research Press (1998).

15. H. Wang, F. Smarandache, Y. Zhang, and R. Sunderraman, “Single-valued neutrosophic sets”, Multispace and Multistructure 4 (2010) 410-413.

16. L. A. Zadeh, “Fuzzy sets”, Information and Control 8 (1965) 338-353.