

Article

Multimodal Carbonates: Distribution of Oil Saturation in the Microporous Rock Regions

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Abstract: Improved oil recovery from tight carbonate formations may provide the world with a major source of lower-rate power over several decades. Here we provide an overview of the Arab D formation in the largest oil field on earth, the Ghawar. We investigate the occurrence of microporosity of different origins and sizes using scanning electron microscopy (SEM) and pore casting techniques. Then, we present a robust calculation of the probability of invasion and oil saturation distribution in the nested micropores using mercury injection capillary pressure data available in the literature. We show that large portions of the micropores in Arab D formation would have been bypassed during primary drainage unless the invading crude oil ganglia were sufficiently long. Considering the asphaltenic nature of oil in the Ghawar, we expect the invaded portions of the pores to turn mixed-wet, thus becoming inaccessible to waterflooding until further measures are taken to modify the system's chemistry.

Keywords: microporous carbonates; multimodal porosity; primary drainage; capillary invasion; mixed wettability

1. Introduction

There are no isotropic, homogeneous carbonate reservoirs made of one rock type. Gravity and countercurrent water imbibition drive oil production from the large blocks of fractured, permeable macroporous carbonates. However, the occluded microporous micritic carbonates [1] (solidified and diagenetically altered muds) remain largely inaccessible to waterflooding, especially when the crude oil is asphaltenic and rock mixed-wet. Worldwide, about 60% of remaining oil is locked in carbonates, and 7 of the 10 largest oil fields have carbonate reservoirs [2,3]. Most of the world's largest carbonate oilfields are located in the Middle East, Russia, Kazakhstan, and the Americas. In particular, the carbonate mudrocks and limestones in the Permian Basin, Bakken, and Eagle Ford in the U.S. hold giant amounts of oil. Most carbonate fields are mature and in need of improved/enhanced oil recovery by fluid injection.

Here we focus on the Ghawar Field [4] in the Jubaila Arab D limestone formations, simply because it has been investigated in great detail over the last 60-70 years and many measurements of rock and fluid properties are publicly available. The oil in place in the Ghawar is likely to exceed 250 billion barrels ([4] supported by our calculations), making it 10× the Prudhoe Bay, the largest oil field in North America. Microporosity occurs throughout Arab Formation carbonate reservoirs of Saudi Arabia and elsewhere, and it affects the fluid flow properties and ultimate recovery of hydrocarbons. Examination of Arab D samples indicates that microporosity occurs as four major types: (1) microporous grains, (2) microporous matrix, (3) microporous fibrous to bladed cement, and (4) microporous equant cements [5]. Even in the best reservoir quality rocks, 25% to 50% of the total pore volume of the sample occurs as microporosity. Cantrell and Hagerty, and Clerke et al. [6] established that there are three unique sets of micropores in the Arab D carbonates, termed "porositons." These pore sets are nested:

$$\mathcal{P}_3 \subset \mathcal{P}_2 \subset \mathcal{P}_1 \subset \mathcal{M} \quad (1)$$



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Equation (1) reads: The lowest permeability micropore regions, termed \mathcal{P}_3 , reside inside the more permeable ones, termed \mathcal{P}_2 , which in turn are occluded in the most permeable micropore regions, termed \mathcal{P}_1 . Since the porosities are nested from the most to the least permeable one (are in series), the shapes of the rescaled mercury injection capillary pressures should not be too far from the actual water drainage capillary pressures. Examples of the typical oil/water capillary pressures in \mathcal{P}_1 - \mathcal{P}_3 are shown in **Figure 1**. The microporous porosity \mathcal{P}_1 is in turn occluded in the high-permeability, well-connected and fractured macroporosity, termed \mathcal{P}_M . Not all reservoir rock layers have all three micropore porosities: some have none, just \mathcal{P}_M , some have only \mathcal{P}_1 in \mathcal{P}_M , and others \mathcal{P}_2 in \mathcal{P}_1 in \mathcal{P}_M , or \mathcal{P}_3 in \mathcal{P}_2 in \mathcal{P}_1 in \mathcal{P}_M .

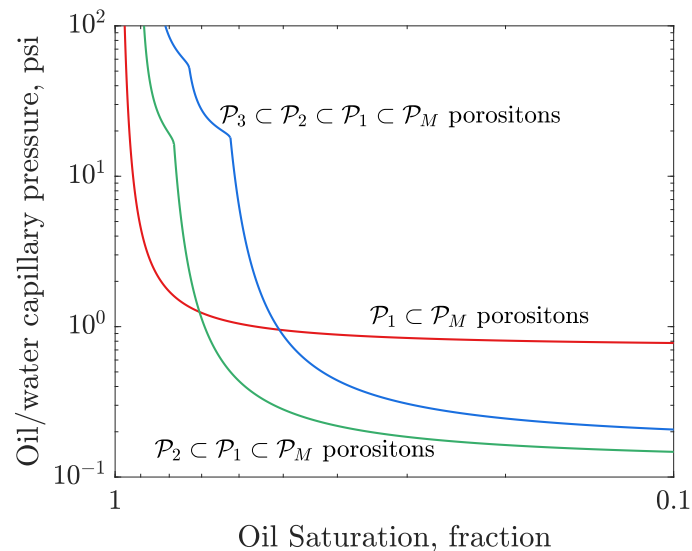


Figure 1. The 480 carbonate core samples in the database in Appendix 3 of [6] can be described by one, two or three nested macro- and micro-pore systems, each characterised by a unique Thomeer [7,8] hyperbola. The capillary entry pressures into the macroporosity that surrounds the microporous regions are significantly lower than the capillary pressure plateaus that span 30 to 60 plus percent of the total porosity.

1.1. Petrophysical Properties of Multimodal Carbonates in the Ghawar

The most thorough summary of the pore systems, facies, and reservoir properties in the Ghawar core samples can be found in [6], and for background, we refer the readers to this paper and the references therein. The petrophysical carbonate database in Appendix 3 of Clerke et al.'s paper consists of 480 samples from two different data sets.

The first set includes 125 samples with mercury injection capillary pressures (MICP) compiled by Hagerty and Cantrell (1990, unpublished report). The MICP results in this set reveal one, two, or three nested pore systems, each characterized by a unique Thomeer hyperbola, see [7,8]. 35% of the samples require a single hyperbola, 62% two hyperbolas, and 3% three hyperbolas.

The second, *Rosetta Stone* data set comes from 10 cored wells in an NS-transect along the spine of the Ghawar. All of the 3,500 core plugs from these wells were cataloged with the facies of [9]. Later seven to nine samples from each of up to six facies per well were selected at random to give a uniform density of 90 samples per facies. MICP experiments and [7,8] hyperbola fits were performed on these samples, and the resulting data were combined with the 125 samples in the first data set yielding 480 unique samples published in Appendix 3 of [6]. The Thomeer hyperbola fits of three random samples described by one, two, or three micropore systems inside macroporosity are shown in **Figure 1**.

1.2. The Depositional Environment

A bird's eye view of the Ghawar and its oil and brine was given in the 1959 AAPG Bulletin paper by the *Arabian Oil Company Staff* [10]. Ghawar oil was found in the shallow-water carbonate deposits of the upper Jubaila and lower Arab formations in the Upper Jurassic. It is confined above by an anhydrite layer. Most production comes from calcarenites, where a lot of the original pore space is preserved. Fine-grained limestone, dolomitic limestone, calcarenitic limestone, and dolomite are among the other rock types that make up the reservoir. The accumulated oil has a maximum vertical column of around 1,300 feet and is conservatively estimated to cover 875 square miles in area. The oil is undersaturated, with saturation pressures decreasing and oil-water contact rising to the south.

The depositional structure was augmented by the inevitable near-vertical fractures created after burial. Therefore, there were multiple high-permeability pathways (fractures, macroporosity layers, and vertical burrow channels) that delivered tall oil ganglia to the inter-dispersed micropore occlusions. If the buoyancy force was sufficient for the oil ganglia to overcome the capillary entry pressure to the microporous grains, the initial pore-filling brine was drained, and these grains were saturated with oil to the extent that is quantified later using the petrophysical description published in Appendix 3 in Clerke *et al.* [6]. **Figure 2** shows SEM images of the etched epoxy pore cast of an Arab-D sample, illustrating the shape of each microporosity type.

1.3. Classification of Macro- and Microporosity in the Carbonate Samples

Clerke *et al.* [6] adopted the microporosity definition by Cantrell and Hagerty [5], in which the carbonate microporosity consists of all pores that are approximately 10 microns in diameter or smaller. This definition is empirical and is based on the idea that microporosity is the difference between total measured porosity and porosity visible from the microscopic examination of thin sections (point count porosity). Since the optical resolution limit of most petrographic microscopes is approximately 10 microns, by default, microporosity includes all pores less than 10 microns in size. MICP data reveal that micropores much smaller than 10 microns (as small as 0.01 microns in diameter) are common in most Arab carbonates, although none of these tiny micropores are optically resolvable with a standard petrographic microscope. We have to keep this engineering definition in mind when we compare contributions from different authors around the world.

There are several key observations in favor of the multimodal porosity and permeability carbonates in the Ghawar saturating with oil to a high degree:

1. The "connate" water saturation measured in a Hessler cell subjected to high-speed centrifugation to mimic the 700 – 1300 ft tall oil columns in the Arab D reservoirs, ranges from 2.8% to 35%, with a majority of the saturations in the teens and twenties of porosity units, [11]. In the present day Ghawar, the oil column is 200 – 240 ft thick in the Upper Jubaila-Arab-D carbonates below the C-D anhydrite, [10].
2. The initial oil fill-up of the Arab D carbonates created oil columns in the macroporous rock. These columns could be several hundred feet in thickness (z_o) and could generate the buoyancy force (capillary pressure), $(\rho_w - \rho_o)gz_o$, that with time would drain the initially bypassed, water-filled microporous regions nested inside the oil-filled macroporosity.
3. As we show here, the oil ganglia 10 – 100 ft tall, flowing up by buoyancy, would create enough of the capillary pressure to invade some of the occluded microporous regions they were slowly passing by.
4. The oil-water capillary pressure would be highest at the top of the oil column, and the multimodal carbonates there be most fully drained of water. This is good news for the continuing or new oil recovery schemes in the Ghawar, which must be custom-designed to produce most of the oil in the microporous rock regions [12–18].

In this paper, we start by guessing a couple of plausible scenarios of filling up the microporous parts of the Ghawar with oil. We then investigate how the different ways of

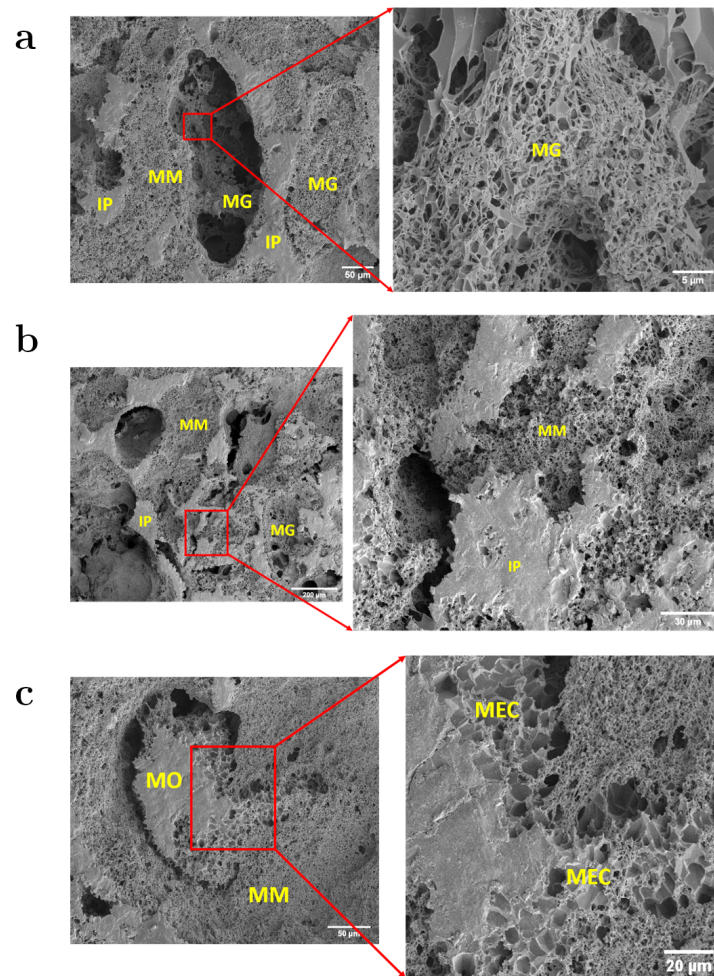


Figure 2. SEM images of the etched Arab-D pore casts showing the three carbonate microporosity types identified by Cantrell and Hagerty [5]; the solid represents the pore space and the etched away grains are black voids. **(a)** Interparticle (IP) macropores appear as solid epoxy, while type I microporosity in microporous grains (MG) appears as a fine network of sponge-like micropores. **(b)** The heavily micritized matrix type II microporosity, is abundant throughout the sample. MM and MG seem to have very similar pore morphologies. **(c)** Intraparticle moldic pore (MO) appears as solid epoxy within a micritized grain. Type III microporosity in intercrystalline micropores (MEC) seems to interconnect the MO to the rest of the matrix pore network. Images by Ahmed Hassan.

filling up the reservoir result in more or less efficient drainage of water-filled micropores by the invading oil.

2. Methods

2.1. SEM imaging of pore casts

We used high-resolution SEM images of carbonates pore casts of Arab-D samples to identify microporosity types summarized in Table 1 [19,20]. The rock samples were impregnated by epoxy in two stages: vacuum impregnation in a desiccator, followed by high-pressure (up to 1000 psi) impregnation. After the epoxy cures, the epoxy pore casts were acid etched with 0.25M HCl at room temperature for 2 hours. Afterwards, the acid was gently discarded from the samples and replaced with DI water. In order to remove traces of HCl, the water was replaced several times over the course of 6 hours. After rinsing with water, the samples were rinsed with ethyl alcohol and dried in air at room temperature. The high-resolution 2D images of the etched pore casts were captured using a FEI Nova Nano Scanning Electron Microscope. The samples were sputter-coated with a ~ 5 nm-thick

platinum/palladium layer to reduce charge effects. For all images acquired in this study, the SEM was operated in secondary electron mode at 3–4 kV and with spot size of 2.5.

Table 1. Major types of microporosity in Arab-D

Microporosity type	Description
Type I	Microporous grain (MG)
Type II	Microporous matrix (MM)
Type III	Microporous equant intercrystalline between cement crystals (MEC)

2.2. The Lognormal and Generalized Extreme Value (GEV) Distributions

In probability theory, a lognormal distribution is a continuous probability distribution of a random variable whose logarithm is normally distributed. Here we will use normal distributions to fit the logarithms of pore throat sizes of the three major classes of microporosity in carbonates.

In a log-normal distribution of a random variable $X > 0$ that has outcomes $\{x\}$, μ is the mean and σ is the standard deviation, respectively, of $\ln x$ (the variable's logarithm is normally distributed). In cartesian coordinates, μ and σ are the *location* and *scale* parameters, respectively.

$$\text{pdf}(x; \mu, \sigma) = f(x; \mu, \sigma) = \frac{1}{x} \frac{1}{\sqrt{2\pi\sigma^2}} \exp \left[-\frac{(\ln x - \mu)^2}{2\sigma^2} \right] \tag{2}$$

where pdf = f is the probability density function.

The Generalized Extreme Value (GEV) distribution models the distributions of extrema (minima or maxima) of data blocks. In this particular application, we apply GEV to model the distribution of pore throats in carbonate macroporosity.

In a generalized extreme value distribution of a random variable X that has outcomes $\{x\}$, $\mu \in \mathbb{R}$ is the *location parameter*, $\sigma > 0$ the *scale parameter*, and $\xi \in \mathbb{R}$ (often denoted by k) is the *shape parameter*. Always

$$1 + \xi(x - \mu)/\sigma > 0 \tag{3}$$

The probability density function is

$$\begin{aligned} f(x; \mu, \sigma, \xi) = & \frac{1}{\sigma} \left[1 + \xi \left(\frac{x - \mu}{\sigma} \right)^{(-1/\xi)-1} \right] \times \\ & \times \exp \left\{ - \left[1 + \xi \left(\frac{x - \mu}{\sigma} \right) \right]^{-1/\xi} \right\} \end{aligned} \tag{4}$$

We will use GEV statistics to fit the pdf of the logarithm of macropore sizes, because the lognormal distribution fails to capture the logarithm's right fat tail.

2.3. The Heights of Oil Columns Necessary to Drain the Ghawar Carbonates with Nested Microporous Porositons

The drainage mercury/air capillary pressures in Appendix 3 in Clerke *et al.* [6] were approximated with the Thomeer [7] hyperbolas, one each for up to the three porositons measured the in core samples. These pressures are plotted in **Figure A1**, from which one can see the broad and complex distribution of pore throat diameters in the samples.

The MICP data have been converted to the oil/water drainage capillary pressures as follows:

$$\begin{aligned}
P_{c, \text{Hg/air}} &= \frac{2\sigma_{\text{Hg/air}} \cos(\theta_{\text{Hg/air}})}{r} \\
P_{c, \text{oil/water}} &= P_{c, \text{ow}} = \frac{2\sigma_{\text{oil/water}} \cos(\theta_{\text{oil/water}})}{r} \\
P_{c, \text{oil/water}} &= \frac{\sigma_{\text{oil/water}} \cos(\theta_{\text{oil/water}})}{\sigma_{\text{Hg/air}} |\cos(\theta_{\text{Hg/air}})|} P_{c, \text{Hg/air}}
\end{aligned} \tag{5}$$

The contact angle for mercury invasion is assumed to be $\theta_{\text{Hg/air}} = 140^\circ$. Since the Ghawar carbonates are weakly water wet to neutrally water wet, see e.g., Verma *et al.* [11], we assume that the oil/water contact angle is $\theta_{\text{oil/water}} = 40^\circ$, which may be conservative, i.e., the receding contact angles [21] may actually be higher. The interfacial tensions are assumed to be $\sigma_{\text{Hg/air}} = 485.5 \text{ mN/m}$, and $\sigma_{\text{oil/water}} = 30 \text{ mN/m}$. The latter number takes into the account the correction for high reservoir temperature in the Ghawar.

The MICP pressures are translated into the oil-water capillary pressures using Eq. (5)₃ and then converted into the equivalent vertical extents of the oil ganglia displacing water:

$$P_{c, \text{oil/water}} = \underbrace{(\rho_w - \rho_o)gh}_{\text{Buoyancy pressure}} \tag{6}$$

At the reservoir temperature and pressure, the live oil density is assumed to be $\rho_o = 710 \text{ kg/m}^3$, and the brine density, $\rho_w = 1070 \text{ kg/m}^3$. The resulting $\Delta\rho g = 0.156 \text{ psi/ft}$. All of these numbers can be adjusted in the future. Thus calculated heights of vertical oil columns necessary to drain the nested porosities in the database rock samples are shown in Figure A2.

The probability of invasion, Pr and the corresponding porosity weighted oil saturation, $\langle S_o \rangle$ are obtained as follows

$$\begin{aligned}
Pr(H_o) &= \frac{\sum_j^n \mathbb{I}(S_{o,j}(H_o) > 0)}{n}, & \langle S_o(H_o) \rangle &= \frac{\sum_j^n \phi_j S_{o,j}(H_o)}{\sum_j^n \phi_j}
\end{aligned} \tag{7}$$

Where the summation is over the admissible samples j in the database, where n is the total number of samples. H_o is the height of the hydrostatic oil column. \mathbb{I} is an indicator function returning 1 when the condition is met and 0 otherwise. ϕ_j are the sample porosities, and $S_{o,j}$ are the oil saturations calculated for each admissible sample given an oil column height.

3. Results and Discussion

3.1. The Distribution of Pore Throat Diameters from the Capillary Entry Pressures

All of the mercury-air capillary pressures in the database were converted to pore throat diameters as follows

$$\begin{aligned}
d_{\text{pore throat}} &= \frac{4\sigma_{\text{Hg/air}} |\cos(\theta)|}{P_{d_i}} = \frac{4 \times 485.5 \times 10^{-3} |\cos(140^\circ)| \text{ N/m}}{P_{d_i} \times 1.01325 \times 10^5 / 14.69595 \text{ N/m}^2} \times 10^6 \text{ micron} \\
d_{\text{pore throat}} &= \frac{216}{P_{d_i}} \quad i = \text{Porositon } \mathcal{M}, \text{ Porositon } \mathcal{P}_1, \text{ Porositon } \mathcal{P}_2, \text{ Porositon } \mathcal{P}_3
\end{aligned} \tag{8}$$

All P_{d_i} s in the database are in psi.

We use the description of the pore systems in Arab D carbonate by Clerke *et al.* [6] that is built upon the intrinsic, fundamental and separate maximum pore-throat diameter modal elements named *porositons*. Porositons are stable, recurring, and intrinsic modes in

the maximum pore-throat diameter of the nested carbonate pore systems. These universal nested pore systems are shown in **Figure 3**.

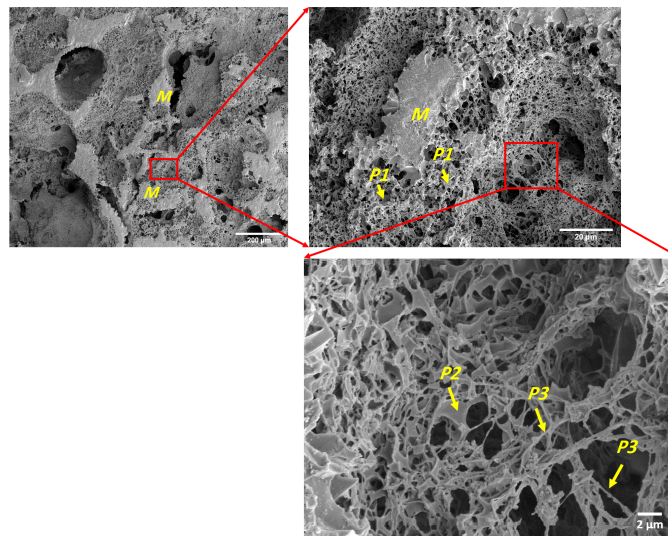


Figure 3. SEM images of the Arab-D pore casts showing the four nested porositons identified by Cantrell and Hagerty [5], namely, M , P_1 , P_2 , and P_3 . The grey solid represents the pore space. Images by Ahmed Hassan.

The discrete and unique porositons emerge from the following procedure. The logarithms of the pore throat diameters are binned into 75 equal classes and fit with the Generalized Extreme Value (GEV) distribution function for the macroporous porosity, M , and with the normal distributions for the three microporous porositons, P_1 , P_2 , and P_3 . The respective distributions are described in **Appendix A**. The results are shown in **Figure 4** as a multimodal probability distribution and summarized in **Table A1**. The macroporosity pore throats are not fit well with a lognormal distribution. Note the clear valleys at about 6 and 0.4 microns.

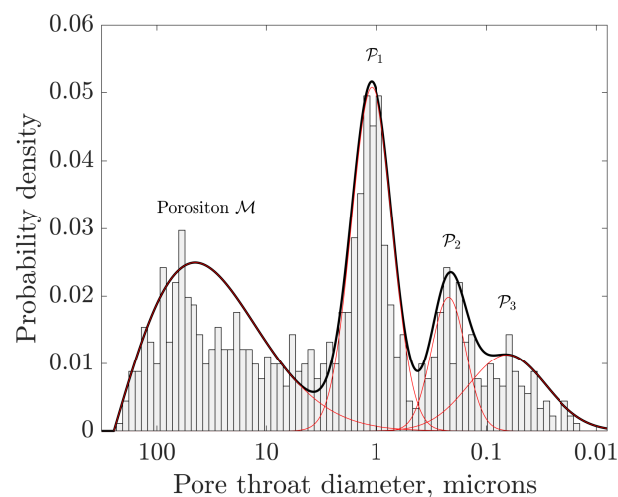


Figure 4. The semilogarithmic plot of the multimodal probability distribution for all pore throat diameters in the carbonate database in Clerke *et al.* [6]. Each red curve is a fit of a part of the distribution of the logarithm of pore sizes with a Generalized Extreme Value or GEV pdf for the macroporosity and three normal distributions for the microporosity. The black curve is the sum of the four distributions.

3.2. The probability of draining the nested porosities

The somewhat counterintuitive summary of the probabilities of invasion of the nested porosities is shown in **Figure 5 a**. For example, there is only a 41% probability of draining Porosity 1inM (P1) with no other discernible porosities nested in 1. On the other hand, the probabilities P12 or P13 of invading Porosity 1inM are 80-90% with a 20-30 ft oil column when this porosity contains at least one more porosity (2in1 or 3in2in1). Amazingly, the probability of draining Porosity 2in1inM (P22) is 40% when the oil column is 100 ft thick, and 80% when there is also Porosity 3 (P23). As expected, Porosity 3in2in1inM (P33), which has the smallest controlling pore throat size, has only a 10% probability of being drained by 100 ft of oil. The details of draining Porosities 1inM, 2in1inM and 3in2in1inM in the multimodal carbonate rock samples are discussed and shown in **Appendix B**.

The corresponding oil saturations that are porosity weighted are plotted in **Figure 5 b**. The porosity-weighted saturations and invasion probabilities show that it is increasingly more difficult to invade all porosities from 2in1inM, through 1inM, to 3in2in1inM.

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Figure 5. (a) The probabilities of invading different nested porosities in the carbonate database from Clerke *et al.* [6]. **(b)** The corresponding porosity-weighted oil saturations in the nested porosities. P1 denotes primary drainage of cores with only P1inM. P12 is drainage of only porosity 1inM in the cores with two porosities 1 and 2. P22 is drainage of porosity 2 in the cores with 2in1inM. P13 is drainage of porosity 1inM in the cores with three porosities. P23 is drainage of porosity 2in1inM and P33 is drainage of porosity 3in2in1inM in the cores with three porosities.

Our assumption is that some or most of the nested porosities were bypassed to some degree when oil was filling up the reservoir (flowing up), unless the oil ganglia were very long. However, the bypassed pockets of water in the higher-order porosities were slowly drained when the macroscopic oil-water interface passed them on the way down. The always-connected water in these pockets sank, making room for oil in the counter-current drainage. Crude oil in the Ghawar contains about 5% of asphaltenes and the formation brine up to 22% of dissolved solids [22,23]. Therefore the parts of the pores in contact with oil will become more oil-wet through asphaltenic deposition, while the corners and asperities, masked by water, will remain water-wet. A sequence of drainage events in the nested (in series) porosities in [Equation 1](#) would then make the pores in each porosity mixed-wet [24–26]. With such a wettability configuration, spontaneous imbibition is unlikely to occur during water flooding due to the very high advancing contact angles [27]. Since the 1960s, most of the Ghawar reservoirs have been put on peripheral waterfloods [4], where seawater injected downdip along the flanks has been pushing oil up towards horizontal producers in a gravity-stable mode. Injected water can easily flow through the highly permeable macroporosity, bypassing the occluded microporosity.

We are working on providing experimental and theoretical evidence for the possibility of spontaneously imbibing mixed-wet pores with high advancing contact angles ($> 90^\circ$) through piston-type imbibition, given the contact angle is lowered below a specific threshold value we establish from the Mayer–Stowe–Princen (MSP) theory. Such reduction of the contact angle will require injection of chemically-tuned brines. Therefore, we are investigating the effects of brine chemistry on the physicochemical properties of the crude oil/brine interfacial films [28], which are responsible for the development of mixed wettability. A better understanding of the underlying mechanisms would help maximize oil extraction through the recovery of bypassed oil [13–15].

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supervision, T.P.; project administration, T.P.; funding acquisition, T.P.. All authors have read and agreed to the published version of the manuscript.

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Conflicts of Interest: The authors declare no conflict of interest.

Appendix A The Lognormal and Generalized Extreme Value (GEV) Distributions

The probability density function of a **log-normal distribution** is expressed as,

$$\text{pdf}(x; \mu, \sigma) = f(x; \mu, \sigma) = \frac{1}{x} \frac{1}{\sqrt{2\pi\sigma^2}} \exp\left[-\frac{(\ln x - \mu)^2}{2\sigma^2}\right] \quad (\text{A1})$$

where $\text{pdf} = f$ is the probability density function

The distribution *mode* (pdf's peak value) is

$$\text{Mode} = \exp(\mu - \sigma^2) \quad (\text{A2})$$

Median is defined as

$$\text{Median}(\mu, \sigma) = \int_0^{m^*} f(x; \mu, \sigma) dx = \int_{m^*}^{\infty} f(x; \mu, \sigma) dx = \frac{1}{2} \quad (\text{A3})$$

The lognormal distribution's *median* is equal to

$$\text{Median}(\mu) = \exp(\mu) \quad (\text{A4})$$

The most important *mean* or *expected value* of the lognormal distribution is

$$E(X) = m = \int_0^{\infty} x f(x; \mu, \sigma) dx = \exp\left[\mu + \frac{\sigma^2}{2}\right] \quad (\text{A5})$$

The *variance* of a lognormal distribution is

$$V(X) = E[(X - \mu)^2] = (e^{\sigma^2} - 1)E^2(X) \quad (\text{A6})$$

and its *standard deviation* is

$$s = \sqrt{V(X)} \quad (\text{A7})$$

In a **generalized extreme value** distribution of a random variable X that has outcomes $\{x\}$, $\mu \in \mathbb{R}$ is the *location parameter*, $\sigma > 0$ the *scale parameter*, and $\xi \in \mathbb{R}$ (often denoted by k) is the *shape parameter*. Always

$$1 + \xi(x - \mu)/\sigma > 0 \quad (\text{A8})$$

The probability density function is

$$f(x; \mu, \sigma, \xi) = \frac{1}{\sigma} \left[1 + \xi \left(\frac{x - \mu}{\sigma} \right)^{(-1/\xi)-1} \right] \times \exp \left\{ - \left[1 + \xi \left(\frac{x - \mu}{\sigma} \right) \right]^{-1/\xi} \right\} \quad (\text{A9})$$

The distribution's *mode* (pdf's peak value) is

$$\text{Mode}(\mu, \sigma, \xi) = \mu + \frac{\sigma}{\xi} \left[(1 + \xi)^{-\xi} - 1 \right] \quad (\text{A10})$$

The *median* is

$$\text{Median}(\mu, \sigma, \xi) = \mu + \sigma \frac{(\ln 2)^{-\xi} - 1}{\xi}, \quad \xi \neq 0 \quad (\text{A11})$$

The most important *mean* or *expected value* of the GEV distribution is

$$E(X) = m = \mu - \frac{\sigma}{\xi} + \frac{\sigma}{\xi} \Gamma(1 - \xi) \quad (\text{A12})$$

The *variance* of the GEV distribution is

$$V(X) = \frac{\sigma^2}{\xi^2} \left[\Gamma(1 - 2\xi) - \Gamma^2(1 - \xi) \right], \quad \xi \neq 0, \xi < \frac{1}{2} \quad (\text{A13})$$

if $\xi = 0$

$$V(X) = \sigma^2 \frac{\pi^2}{6} \quad (\text{A14})$$

and V does not exist otherwise. The *standard deviation* is

$$s = \sqrt{V} \quad (\text{A15})$$

Table A1. A summary of the mean, mode, and median pore diameters for the four individual distributions in Figure 4.

Macroporosity GEV	$\xi_M = -0.50$	$\mu_M = 1.35$	$\sigma_M = 0.52$		
Macroporosity	$E_M = 29$	$\text{Mode}_M = 45$	$\text{Median}_M = 33$	$\text{STD}_M = 3.04$	μm
Gauss \mathcal{P}_1	$\mu_1 = \log(1.10\mu\text{m})$	$\sigma_1 = -\log(0.65\mu\text{m})$			
\mathcal{P}_1	$E_1 = 1.06$	$\text{Mode}_1 = 1.01$	$\text{Median}_1 = 1.04$	$\text{STD}_1 = 0.20$	μm
Gauss \mathcal{P}_2	$\mu_2 = \log(0.22\mu\text{m})$	$\sigma_2 = -\log(0.70\mu\text{m})$			
\mathcal{P}_2	$E_2 = 0.52$	$\text{Mode}_2 = 0.51$	$\text{Median}_2 = 0.52$	$\text{STD}_2 = 0.08$	μm
Gauss \mathcal{P}_3	$\mu_3 = \log(0.07\mu\text{m})$	$\sigma_3 = -\log(0.45\mu\text{m})$			
\mathcal{P}_3	$E_3 = 0.32$	$\text{Mode}_3 = 0.27$	$\text{Median}_3 = 0.31$	$\text{STD}_3 = 0.12$	μm

Appendix B The Heights of Oil Columns Necessary to Drain the Ghawar Carbonates with Nested Microporous Porositons

The drainage mercury/air capillary pressures in Appendix 3 in Clerke *et al.* [6], characterized by the Thomeer [7] hyperbolas, are plotted in Figure A1.

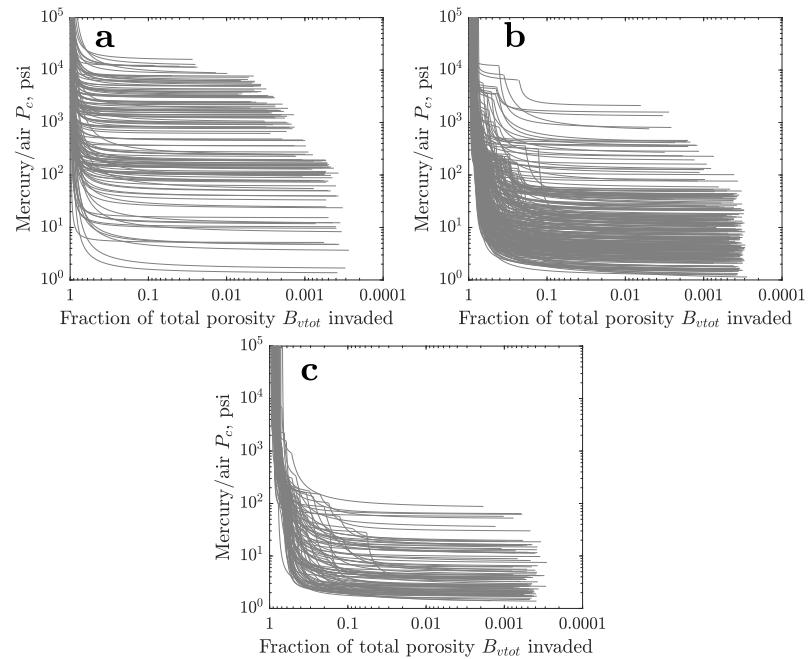


Figure A1. (a) The MICP curves for the rock samples the are described by a single Thomeer hyperbola (Porositon 1 in Porositon M). (b) The MICP curves for the rock samples the are described by two Thomeer hyperbolas (Porositon 2 in Porositon 1 in Porositon M). (c) The MICP curves for the rock samples the are described by three Thomeer hyperbolas (Porositon 3 in Porositon 2 in Porositon 1 in Porositon M).

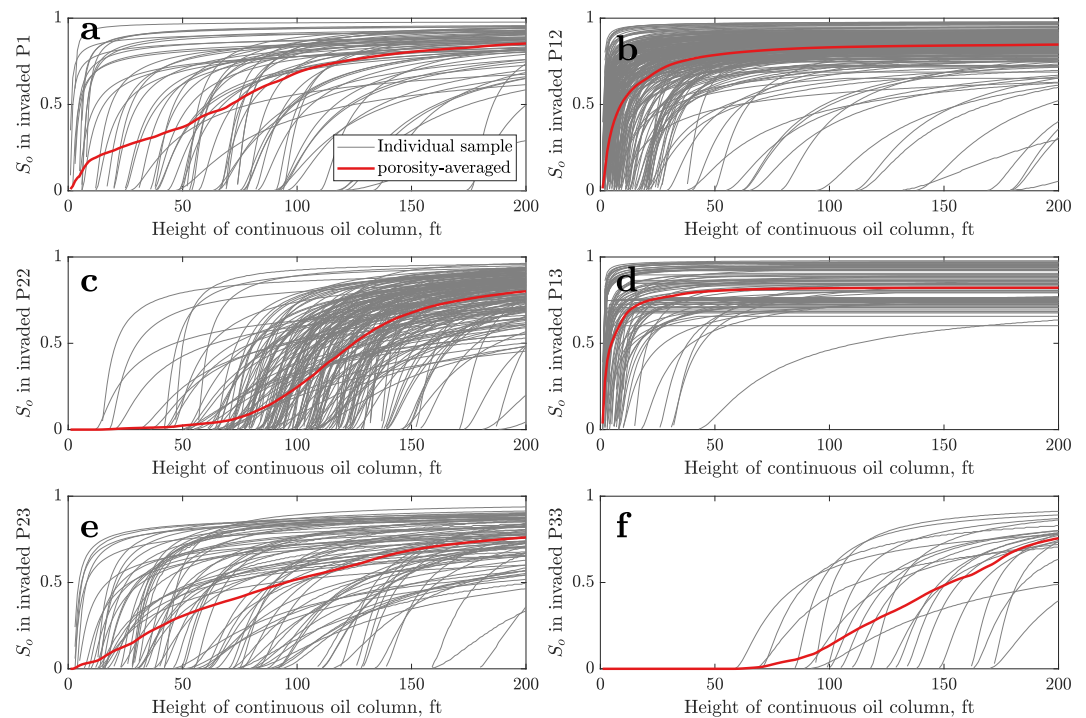


Figure A2. The heights of vertical oil columns necessary to drain the carbonate rock samples with different sets of porosities.

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