Distribution of composite numbers and determination of prime numbers

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Abstract

Integer is either a composite number or a prime number. Therefore, detecting composite numbers is important for solving prime numbers. The study of prime numbers, apart from satisfying human curiosity, can be very important. In this article, the order of composite numbers has been detected. And explained with a simple method and a simple function. And, a method has been developed in which all composite numbers and therefore prime numbers can be determined by using the specified methods, functions and formulas.

Keywords: composite number; composite numbers distribution; composite numbers generation; prime number;

1   Introduction

To begin, let's define the composite number and prime number: Composite number is the name given to numbers with more than one prime factor. A prime number is a number that has no factors other than 1 and itself. Also, prime numbers is infinite. [EW81] An integer is either a composite number or a prime number. Therefore, detecting composite numbers is important for solving prime numbers. The study of prime numbers, apart from satisfying human curiosity, can be very important because, regardless of the choice of counting basis and independent of mathematical notation, prime numbers will have a clear and unambiguous meaning among intelligent beings. Despite the efforts of many curious and smart people, the mystery of prime numbers, specifically their distribution, remains an unsolved problem in mathematics. [Bah21] Also, one of the importance of prime numbers is: Cryptography has gained much more importance today. Asymmetric cryptography as a method is more favored as it is more difficult to break. Asymmetric cryptography is based on prime numbers. The mystery of the prime numbers keeps drawing attention on the subject. [YK17] This article will explain the order of composite numbers. Many formulas will be written to obtain a composite number. And, by detecting composite numbers, prime numbers will be tried to be obtained.

2   Theorem and Proofs

2.1   Yildgenn's Theorem

This theorem states that composite numbers are in an order.

Let's start by explaining the order simply:

$n = N$

$P_1 = P_2$:

$P^2 = $ composite number

$P_1{}^2 + (P_2 \cdot P_3) = $ composite number

Example:

Let $n = 1; P_1 = P_2 = 2; P_3 = 3$:

$P_1{}^2 = P_2{}^2 = 2^2 = 4 = $ composite number

$P_1{}^2 + (P_2 \cdot P_3) = 2^2 + (3 \cdot 2) = 10 = $ composite number

Let's continue by explaining with function to make it more understandable:

$n = N$

$x_1 = x_2$:

$f(x_1): P \implies N$

$g(x_2): P \implies N$

$b(x_3): P \implies N$

$f(x_1) = x_1^2 = $ composite number

$b(x_3) = x_3^n$

$g(x_2) = x_2 \cdot b$

$(f + g)(x_1) = $ composite number

Example:

Let $n = 0; x_1 = x_2 = 7; x_3 = 11$:

$f(7) = 7^2 = 49 = $ composite number

$b(11) = 11^0 = 1$

$g(7) = 7 \cdot 1 = 7$

$(f + g)(7) = 56 = $ composite number

2.2   Proofs

Now it's time to prove the validity of these methods and function. To prove this, let's first remind the definition of composite number. A composite number is the name given to numbers with more than one prime factor. For example, 14 has prime factors of 2 and 7. Therefore, it is a composite number. Let's focus again on proving the validity of this method and function.

$n = N$

$P_1 = P_2$:

The proof that P^2 forms a composite number is as follows:

Since $\frac{P^2}{P} = P$, our result has at least two prime factors, P and P.

Therefore, P^2 forms a composite number.

The proof that $P_1{}^2 + (P_2 \cdot P_3{}^n)$ forms a composite number is as follows:

Since $\frac{P_1{}^2 + (P_2 \cdot P_3{}^n)}{P_1} = P_1 + (P_2 \cdot P_3{}^n)$ and $P_1 = P_2$, our results has at least two prime factors, P_1 and P_2.

Therefore, $P_1{}^2 + (P_2 \cdot P_3{}^n)$ forms a composite number.

$n = N$

$x_1 = x_2$

$f(x_1): P \implies N$

$g(x_2): P \implies N$

$b(x_3): P \implies N$

$f(x_1) = x_1^2$

$b(x_3) = x_3^n$

$g(x_2) = x_2 \cdot b$:

The proof that x_1^2 forms a composite number is as follows:

Since $\frac{x_1^2}{x_1} = x$, our results has at least two prime factors, x and x.

Therefore, x_1^2 forms a composite number.

The proof that $(f + g)(x_1)$ forms a composite number is as follows:

Since $(f + g)(x_1) = x_1^2 + (x_2 \cdot x_3^n)$, $\frac{x_1^2 + x_2 \cdot x_3^n}{x_1} = x_1 + (x_2 \cdot x_3^n)$ and $x_1 = x_2$, our results has at least two prime factors, x_1 and x_2.

Therefore, $(f + g)(x_1)$ forms a composite number.

2.3   Other Formulas

Apart from these methods and function,

$n = N+$

$q(x_1): P \implies N+$

$b(x_2): P \implies N$

$p(y): N+ \implies N+$

$x_1 = x_2$

$b(x_2) = x_2^n$

$$q(x_1) = \begin{cases} x_1 + b, & \log_{x_1} n \geq 2 \\ x_1^2, & \log_{x_1} n < 2 \end{cases} = y_1$$

$$p(y) = \begin{cases} y + b, & \log_y n \geq 2 \\ y^2, & \log_y n < 2 \end{cases} = y_2$$

$y = $ composite number

in its form, a piecewise function can be written.

Example:

Let $n = 1; x_1 = x_2 = 7$:

$$q(7) = \begin{cases} 7 + b, & \log_7 1 \geq 2 \\ 7^2, & \log_7 1 < 2 \end{cases} = y_1$$

$\log_7 1 \geq 2 = 7^2 \geq 1 \implies 7 + b$

$b(7) = 7^1 = 7$

$7 + b = 7 + 7 = 14 = y_1 = $ composite number

$$p(14) = \begin{cases} 14 + b, & \log_1 14 \geq 2 \\ 14^2, & \log_1 14 < 2 \end{cases} = y_2$$

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$\log_1 14 \geq 2 = 14^2 \geq 1 \implies 14 + 7 = 21 = y_2 = \text{composite number}$

$$p(21) = \begin{cases} 21 + b, & \log_1 21 \geq 2 \\ 21^2, & \log_1 21 < 2 \end{cases} = y_3$$

$\log_1 21 \geq 2 = 21^2 \geq 1 \implies 21 + 7 = 28 = y_2 = \text{composite number}$

To give another example:
Let $n = 10; x_1 = x_2 = 3$:

$$q(3) = \begin{cases} 3 + b, & \log_3 10 \geq 2 \\ 3^2, & \log_3 10 < 2 \end{cases} = y_1$$

$\log_3 10 < 2 = 9^2 < 10 \implies 3^2 = 9 = y_1 = \text{composite number}$

$$p(9) = \begin{cases} 9 + b, & \log_9 10 \geq 2 \\ 9^2, & \log_9 10 < 2 \end{cases} = y_2$$

$\log_9 10 \geq 2 = 9^2 \geq 81 \implies 9 + b$
$b(3) = 3^{10} = 59049$
$9 + b = 9 + 59049 = 59058 = y_2 = \text{composite number}$

The proof of the validity of this piecewise function is:
$n = N+$
$q(x_1): P \implies N+$
$b(x_2): P \implies N$
$p(y): N+ \implies N+$
$x_1 = x_2$
$b(x_2) = x_2^n$

$$q(x_1) = \begin{cases} x_1 + b, & \log_{x_1} n \geq 2 \\ x_1^2, & \log_{x_1} n < 2 \end{cases} = y_1$$

$$p(y) = \begin{cases} y + b, & \log_y n \geq 2 \\ y^2, & \log_y n \geq 2 \end{cases} = y_2 :$$

The proof that x^2 forms a composite number is as follows:
Since $\frac{x^2}{x} = x$, our results has at least two prime factors, x and x.
Therefore, x^2 forms a composite number.
The proof that $x_1^2 + x_2^n$ forms a composite number is as follows:
Since $\frac{x_1 + x_2^n}{x_1} = x_2^n$ and $x_1 = x_2$, our results has at least two prime factors, x_1 and x_2.
The proof that y^2 forms a composite number is as follows:
Since $\frac{y^2}{y} = y$, our results has at least two prime factors, y and y.
Therefore, y^2 forms a composite number.
The proof that $y + x_2^n$ forms a composite number is as follows:
Since $\frac{y + x_2^n}{x_2} = y + x_2^n - 1$ and $x_1 = x_2$, our results has at least two prime factors, x_1 and x_2.
Therefore, $y + x_2^n$ forms a composite number.

Also, to get larger composite numbers, a formula can be written like this:
$n_1 = N+$
$n_2 = P$
$f(x_1): P \implies N$
$g(x_2): P \implies N$
$b(x_3): P \implies N$
$x_1 = x_2$
$f(x_1) = x_1^2$

$$g(x_2) = x_2 \cdot b$$
$$b(x_3) = x_3^{n_1}$$

$$\sum_{x_1=P}^{n_2=P} f + g = k$$

$k =$ composite number

Example:

Let $n_1 = 1; n_2 = 5; x_1 = x_2 = 2; x_3 = 3$:

$$\sum_2^5 f+g = (2^2+(2\cdot3^1))+(3^2+(2\cdot3^1))+(5^2+(2\cdot3^1)) = (4+6)+(9+6)+(25+6) = 10+15+31 = 56 = k$$

$k =$ composite number

The proof that $\sum_{x_1=P}^{n_2=P} f + g$ forms a composite number is as follows:

$$\sum_{x_1=P}^{n_2=P} f + g = (x_1^2 + (x_2 \cdot x_3^{n_1})) + (x_4^2 + (x_5 \cdot x_6^{n_1})) + \cdots + (x_{n_2-2}^2 + (x_{n_2-1} \cdot x_{n_2}^{n_1}))$$

Since $\frac{\sum_{x_1=P}^{n_2=P} f+g}{x_1} = \frac{(x_1^2+(x_2\cdot x_3^{n_1}))+(x_4^2+(x_5\cdot x_6^{n_1}))+\cdots+(x_{n_2-2}^2+(x_{n_2-1}\cdot x_{n_2}^{n_1}))}{x_1} = (x_1 + (x_2 \cdot x_3^{n_1})) + (x_4^2 + (x_5 \cdot x_6^{n_1})) + \cdots + (x_{n_2-2}^2 + (x_{n_2-1} \cdot x_{n_2}^{n_1}))$ and $x_1 = x_2$, our results has at least two prime factors, x_1 and x_2. Therefore, $\sum_{x_1=P}^{n_2=P} f + g$ forms a composite number.

To get even larger composite numbers, a formula can be written like this:

$n_1 = N+$
$n_2 = P$
$f(x_1): P \implies N$
$g(x_2): P \implies N$
$b(x_3): P \implies N$
$x_1 = x_2$
$f(x_1) = x_1^2$
$g(x_2) = x_2 \cdot b$
$b(x_3) = x_3^{n_1}$

$$\prod_{x_1=P}^{n_2=P} f + g = l$$

$l =$ composite number

Example:

Let $n_1 = 1; n_2 = 5; x_1 = x_2 = 2; x_3 = 3$:

$$\prod_2^5 f+g = (2^2+(2\cdot3^1))\cdot(3^2+(2\cdot3^1))\cdot(5^2+(2\cdot3^1)) = (4+6)\cdot(9+6)\cdot(25+6) = 10\cdot15\cdot31 = 4650 = l$$

$l =$ composite number

The proof that $\prod_{x_1=P}^{n_2=P} f + g$ forms a composite number is as follows:

$$\prod_{x_1=P}^{n_2=P} f + g = (x_1^2 + (x_2 \cdot x_3^{n_1})) \cdot (x_4^2 + (x_5 \cdot x_6^{n_1})) \cdot \cdots \cdot (x_{n_2-2}^2 + (x_{n_2-1} \cdot x_{n_2}^{n_1}))$$

Since $\frac{\prod_{x_1=P}^{n_2=P} f+g}{x_1} = \frac{(x_1^2+(x_2 \cdot x_3^{n_1})) \cdot (x_4^2+(x_5 \cdot x_6^{n_1})) \cdot \cdots \cdot (x_{n_2-2}^2 +(x_{n_2-1} \cdot x_{n_2}^{n_1}))}{x_1} = (x_1 + (x_2 \cdot x_3^{n_1})) \cdot (x_4^2 + (x_5 \cdot x_6^{n_1})) \cdot \cdots \cdot (x_{n_2-2}^2 + (x_{n_2-1} \cdot x_{n_2}^{n_1}))$ and $x_1 = x_2$, our result has at least two prime factors, x_1 and x_2.

Therefore, $\prod_{x_1=P}^{n_2=P} f + g$ forms a composite number.

3   Inferences

Regarding the Yildgenn Theorem, the following inferences can be made from the methods and functions stated so far:

As mentioned earlier, an integer is either a composite number or a prime number.

This means that detecting all composite numbers means detecting all prime numbers.

As stated in the Yildgenn Theorem:

$n = N$

$P_1 = P_2$:

$P^2 = z_1$

$P_1{}^2 + (P_2 \cdot P_3) = z_2$

$z = $ composite number

Let's edit this method so that it works better for us: P_1, changed to $x_1 = $ odd prime (Except for 2, since it is not an even prime number, it has been changed to a single prime to avoid computational congestion.).

P_2, changed to $x_2 = $ odd prime (Except for 2, since it is not an even prime number, it has been changed to a single prime to avoid computational congestion.).

P_3, changed to 2 (Since we want to detect all prime numbers, we aim to detect all odd (the reason it is specified as odd, as mentioned before, there is no even prime number other than 2, that is to prevent processing density) composite numbers. To detect all odd composite numbers, and therefore all prime numbers, we replace it with the smallest prime number, (composite number: the name given to numbers with more than one prime factor) so 2.):

$n = N$

$x_1 = $ odd prime

$x_2 = $ odd prime

$x_1 = x_2$

$x_1^2 = z_1$

$z_1 + (x_2 \cdot 2) = z_2$

$z_2 + (x_2 \cdot 2) = z_3$

$\ldots$

$z = $ odd composite number

If x is given all odd prime values linearly from the smallest odd prime to the largest odd prime, the resulting odd numbers between any two z-values or between 0-smallest z-values and not equal to any z-values are prime.

Example:

Suppose the only, odd prime number we know is 3:

$3^2 = 9 = z_1$

$9 + (3 \cdot 2) = 15 = z_2$

We know that odd numbers between any two z-values and not equal to any z-values are prime. In this case, odd numbers between z_1 and z_2 that do not equal any z value are prime.

The odd numbers that do not equal any z value between z_1 and z_2 are 11 and 13.

So 11 and 13 are prime numbers.

Also, all odd numbers with the between 0-smallest z-value are also prime. Because there cannot be a z-value smaller than the smallest z-value. In this case, it is also impossible for any value less than the smallest z value to be equal to any z value.

So, 5 and 7 are also prime numbers.

$15 + (3 \cdot 2) = 21 = z_3$

The numbers between z_2 and z_3 that do not equal any z value are: 17, 19.

So, 17 and 19 are prime numbers.

$5^2 = 25 = z_4$

The number between z_3 and z_4 that is not equal to any z-value is 23. So 23 is prime number.

$21 + (3 \cdot 2) = 27 = z_5$

There is no number between z_4 and z_5 that does not equal any z value.

$27 + (3 \cdot 2) = 33 = z_6$

The numbers between z_5 and z_6 that do not equal any z value are 29 and 31. So 29 and 31 are prime numbers.

. . .

The example can be continued in this way.

4   Results

The order of composite numbers has been detected. And explained with a simple method and a simple function. The validity of the method and function has been proven. Using this simple method and function, other formulas were written in which composite numbers could be determined. The validity of these formulas was then proven. Finally, a method has been developed in which all composite numbers and therefore prime numbers can be determined by using the specified methods, functions and formulas.

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