

Models that link and suggest data about elementary particles, dark matter, and the cosmos

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Abstract—We suggest progress regarding the following six physics opportunities. List all elementary particles. Describe dark matter. Explain ratios of dark matter to ordinary matter. Explain eras in the history of the universe. Link properties of objects. Interrelate physics models. We use models based on Diophantine equations.

Keywords—Beyond the Standard Model, Concordance cosmology, Dark matter, Galaxy evolution, Inflation, Diophantine equations

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CONTENTS

I	Introduction	2
I-A	Overview - physics results	2
I-B	Overview - methods	2
I-C	Relationships between our work and other work	3
II	Methods	5
II-A	Charge, mass, other properties, and long-range forces	6
II-B	Elementary particles that do not carry long-range forces	8
II-C	Isomers and dark matter	9
III	Results	13
III-A	Explanations and suggestions regarding elementary particles	13
III-B	Explanations and suggestions regarding dark matter	13
III-C	Explanations and suggestions regarding the rate of expansion of the universe	16
III-D	Explanation regarding data about large-scale clumping	17
III-E	Explanation regarding data about interactions between neighboring galaxies	17
III-F	Suggestions regarding galaxy formation	17
III-G	Explanations regarding dark matter data	17
IV	Discussion	19
IV-A	Possibilities regarding elementary particles	19
IV-B	Possibilities regarding constraints regarding dark matter	19
IV-C	Possibilities regarding interactions involving the jay boson	20
IV-D	Possibilities regarding right-handed W bosons	21
IV-E	Relationships between our modeling and other modeling	21
V	Conclusion	22
	Acknowledgments	22
	References	23

I. INTRODUCTION

This unit previews physics results we propose, previews methods we use, and relates our work to other work in elementary particles, astrophysics, and cosmology.

A. Overview - physics results

This essay pursues the following two challenges. Describe new elementary particles and dark matter. Use descriptions of elementary particles and dark matter to explain astrophysics data and cosmology data.

Our explanations regarding large-scale data might help validate our descriptions of possible new elementary particles and our description of dark matter.

B. Overview - methods

One goal of our modeling is to match and extend a list of properties - of objects - that people infer or might infer based on observations based on so-called long-range interactions (or, so-called long-range forces). Long-range interactions include electromagnetism (which associates with notions of a spin-one boson - the photon), gravity (which associates with notions of a might-be spin-two boson - the graviton), possibly interactions that would associate with a spin-three boson, and possibly interactions that would associate with a spin-four boson.

We find it convenient to divide elementary particles into three sets - carriers of long-range interactions, other elementary bosons, and elementary fermions. We associate, respectively with the three sets, the symbols LRI (as in long-range interaction or as in elementary boson that associates with a long-range interaction), SRI (as in short-range interaction or as in elementary boson that does not associate with a long-range interaction), and ELF (as in elementary fermion).

We develop mathematics modeling that outputs characteristics of long-range interactions and properties - of objects - that long-range interactions measure. The modeling features solving Diophantine equations.

LRI solutions come in pairs. For example, regarding electromagnetism, one so-called PROP solution associates with the property of charge. That PROP solution has a so-called CURR partner solution that associates with a current of charge. We think that each LRI pair of one PROP solution and its CURR partner can adequately associate with special relativity.

Our LRI modeling for electromagnetism has some similarities to modeling based on charge-and-current 4-vectors and has some parallels to modeling based on an electric field, a magnetic field, and Maxwell's equations. However, some differences pertain. For example, LRI modeling includes a PROP solution that associates with a component of magnetic field that associates with the notion of charge dipoles. That PROP solution has a CURR partner that associates with a current of charge dipoles.

Our LRI modeling for gravitation has similarities and differences with respect to gravitoelectromagnetism. (Regarding gravitoelectromagnetism, see references [1] and [2].)

SRI solutions and ELF solutions come in PROP and CURR pairs. Each known SRI particle associates with a solution pair. Each not-yet-found SRI particle that this essay suggests associates with a solution pair. Each known ELF particle associates with a solution pair. Each not-yet-found ELF particle that this essay suggests associates with a solution pair.

We think that modeling based on the LRI, SRI, and ELF aspects discussed above suffices to point to possibly relevant new physics. For example, we interpret our modeling as describing aspects of two possible eras - in the evolution of the universe - before the possible inflationary epoch. And, we interpret our modeling as suggesting a mechanism that leads to the recent multi-billion-years era increases in the rate of expansion of the universe.

However, modeling just based on LRI, SRI, and ELF aspects discussed above does not suffice to explain some data about ratios of dark matter to ordinary matter and does not suffice to explain the magnitude of the recent multi-billion-years era increases in the rate of expansion of the universe. (Regarding ratios of dark matter to ordinary matter, perhaps preview table VII. Regarding the magnitude of the recent multi-billion-years era increases in the rate of expansion of the universe, perhaps preview the notion of reach - or ρ_I - in table XIV.)

To explain some data about ratios of dark matter to ordinary matter, we posit that nature includes six isomers of the set of SRI and ELF elementary particles. Ordinary matter associates with all of the SRI particles in so-called ELPI0 and all of the known ELF particles in ELPI0. The symbol ELPI denotes the three-word phrase elementary particle isomer. The integers l that associate with symbols of the form ELPI l range from zero to five. Dark matter associates with some yet-to-be-found ELF particles in EPLI0 and with all elementary particles in ELPI1 through ELPI5.

The symbol STUI l denotes stuff - such as hadron-like particles, atoms, and stars - made up of just (or essentially just) ELPI l elementary particles (plus LRI aspects that include electromagnetism and gravity).

We posit that, across isomers, the ELPI have similarities. We posit that the mass of each SRI particle in any one isomer is the same as the mass of a counterpart SRI elementary particle in each other isomer. We posit that the mass of each ELF particle in any one isomer is the same as the mass of a counterpart ELF elementary particle in each other isomer.

We posit that the six ELPI differ in at least one way. For each of isomer-zero and isomer-three, the flavour of the lowest-mass charged lepton equals the flavour for the two lowest-mass quarks. For each of the other four isomers, the flavour of the lowest-

mass charged lepton does not equal the flavour of the two lowest-mass quarks. (Perhaps, preview table XIII.) One possible other difference between isomers might be that ELPI0, ELPI2, and ELPI4 associate with left-handedness (for, for example, charged leptons) and ELPI1, ELPI3, and ELPI5 associate with right-handedness.

Regarding LRI, a so-called reach associates with each PROP-and-CURR pair. We use the symbol ρ_I to denote reach. Allowed values for ρ_I are one, two, and six. For relatively familiar physics - such as the physics of solar systems - the dominant gravitational PROP-and-CURR pair has a reach of six. The six STUI interact with each other via this component of gravity. Regarding electromagnetism, the reach that associates with the charge-and-charge-current PROP-and-CURR pair is one. The reach that associates with the charge-dipole-and-related-current PROP-and-CURR pair is one. Each one of the STUI has, in effect, its own instance of each of these two electromagnetic-centric PROP-and-CURR pairs. Each STUI does not interact with any other STUI via either of these two electromagnetic-centric PROP-and-CURR pairs.

The notion of six isomers and the notion of instances of LRI PROP-and-CURR pairs seem to suffice to explain ratios of dark matter to ordinary matter. (Perhaps, preview table VII.) The two notions might suffice to explain the size of the recent multi-billion-years era increases in the rate of expansion of the universe. (Perhaps, preview table XIV.)

C. Relationships between our work and other work

We discuss relationships between our work and other work. Here, other work includes observational research and modeling-centric research.

We discuss relationships - between our work and other work - regarding elementary particles, physics constants, and physics properties.

We discuss other work that tries to suggest new elementary particles.

Reference [3] lists some types of modeling that people have considered regarding trying to extend the elementary particle Standard Model, including trying to suggest elementary particles that people have yet to find. Types of models associate with terms such as large extra dimensions, Kaluza-Klein (which associates with notions of gravity in more than four dimensions), grand unification, supersymmetry, and superstrings. Reference [4] provides information about some of these types of modeling. References [5], [6], and [7] provide some information about modeling and about experimental results. Reference [8] provides other information about modeling and about experimental results. (Perhaps, see reviews numbered 86, 87, 88, 89, 90, and 94.)

We discuss possible elementary particles that people have yet to find and we suggest.

Reference [9] suggests the notions of dark matter charges and dark matter photons. We suggest dark matter isomers of charged elementary particles and, in effect, dark matter components - such as components associating with electrostatics and magnetostatics - of electromagnetism.

Reference [10] suggests the notion of a so-called inflaton field. We suggest an inflaton elementary particle. (Perhaps, preview table IX and note the OI boson.)

People suggest the notion of a graviton. (See, for example, reference [11].) We suggest a graviton. (Perhaps, preview table IX.)

Reference [12] discusses notions of sterile neutrinos and heavy neutrinos. We suggest possible elementary particles that might associate with notions of heavy neutrinos. (Perhaps, preview table IX.)

We discuss possible elementary particles that people have yet to find and our modeling seems not to suggest.

Reference [7] reviews modeling and experiments regarding so-called magnetic monopoles. Reference [7] notes that a symmetry regarding Maxwell's equations suggests that nature might include magnetic monopoles. We suggest that nature might not include an interaction that would associate with magnetic monopoles. (Perhaps, preview table III.)

Reference [5] reviews modeling and experiments regarding so-called axions. Reference [5] notes modeling that suggests that nature might include axions. We suggest that nature might not include axions. (Perhaps, preview table V.) We suggest that phenomena that people might attribute to axions might not associate with axions. One such phenomenon could be electromagnetic interactions between ordinary matter and dark matter based on, for example, the so-called 1G1'2'4 component of electromagnetism. (Perhaps, preview table III and table VIII.)

Reference [6] reviews modeling and experiments regarding so-called leptoquarks. We suggest that nature might not include leptoquarks. (Perhaps, preview table V.)

We discuss prospectively some aspects, assuming that our work gains attention.

We discuss neutrino masses and oscillations.

Reference [12] discusses modeling and data about neutrino masses and oscillations.

We suggest neutrino masses. (Perhaps, preview table XII.) We also suggest that, in effect, gravity measures neutrino masses and a spin-three analog (to electromagnetism and gravity) measures neutrino flavours. (Perhaps, preview table XII.) As far as we know, our modeling is not incompatible with data that reference [12] discusses. Future experimentation might help validate or refute aspects of our work regarding neutrinos.

We discuss gravitation.

Reference [13] discusses experimental tests of theories of gravity.

We suggest effects - associating with isomers of elementary particles and with reaches of components

of gravity - that suggest that other modeling regarding gravity would not be adequately accurate for some circumstances. (Perhaps, preview table XVIII.) This essay discusses some such circumstances. We are uncertain as to the extent to which aspects that reference [13] discusses would tend to validate or refute aspects of our modeling that pertains to gravitation.

We discuss physics constants and properties.

Our work seems to interrelate some physics constants. (Perhaps, preview table X and table XII.) Our work seems to interrelate some properties, including via modeling that catalogs physics properties. (Perhaps, preview table III.)

We might offer new approaches to estimating some physics properties. This essay points to masses - that would comport with recent experimental results and that would have smaller standard deviations than standard deviations that associate with recent experiments - for each of the tau elementary fermion and the Higgs boson. (Perhaps, preview respectively table XII and table X.) This essay notes - regarding the anomalous magnetic dipole moment of the tau elementary fermion - a possible estimate that might approximate a Standard Model estimate. (Perhaps, preview discussion related to table XVIII.) This essay notes - regarding the fraction of top quark decays that result in right-handed W bosons - a possible estimate that might approximate a Standard Model estimate.

We discuss relationships - between our work and other work - regarding cosmology.

We think that - with some exceptions - our work does not necessarily suggest significant changes - to concordance cosmology - regarding the large-scale evolution of the universe. (References [14], [15], and [16] review aspects of concordance cosmology.)

Each exception associates either with a possible aspect of nature for which people have no observations or with a known gap between observations and concordance cosmology.

One exception pertains regarding before inflation. One exception pertains regarding recent changes in the rate of expansion of the universe. In each case, we suggest noteworthy contributions by a gravitational force component for which each instance (of the component) has a reach that is greater than one isomer. (Perhaps, preview table VIII.) For times associating with between the two cases, we suggest dominance by gravitational force components that have reaches of one isomer. For times associating with between the two cases, we do not propose significant incompatibilities between our work and large-scale concordance cosmology.

We discuss a possibility regarding times before inflation. (Reference [15] discusses inflation.)

We think that no direct observations pertain. We suggest two eras before inflation. (Perhaps, preview table XIV.) The first of those two eras features aspects that the Standard Model and concordance cosmology do not include. One aspect is the so-called jay boson.

(Perhaps, preview table IX and table XIV.) The other aspect is the so-called $2G1'2'3'8'16$ component of gravity. (Perhaps, preview table XIV.) An instance of that component has a reach of six isomers. For purposes of discussion, we assume that the universe transited those two eras. We assume that concordance cosmology can embrace the jay boson. For the first of those two eras, an extrapolation of concordance cosmology techniques might underestimate the strength of the key driver - the $2G1'2'3'8'16$ component of gravity - by a factor of six.

We discuss phenomena during and after the lead-up to the current multi-billion-years era of increases in the rate of expansion of the universe.

Various people suggest that concordance cosmology underestimates increases in the rate of expansion. (References [16], [17], [18], [19], and [20] discuss relevant notions.)

We think that we point to a basis for the underestimates. Regarding times before that lead-up, we suggest dominance by instances of an attractive quadrupole gravitational force component (that is, $2G1'2'3$) with a reach of one isomer. (Perhaps, preview table XIV.) Before and during the recent multi-billion-years era, the $2G2'4$ gravitational force component gains prominence and then becomes dominant. Each instance of $2G2'4$ has a reach of two isomers. We suggest that concordance cosmology models that work well regarding times for which reach-one dominance pertains would not necessarily work well after those times. We suggest that extrapolating based on such concordance cosmology modeling would underestimate (conceptually by a factor of two) the strength of the driver for increases in the rate of expansion. We suggest that - to get good results via concordance cosmology modeling - people might adjust the equation of state. In general, for each relevant density, components of pressure that associate with repulsion need to increase.

Our suggested resolution regarding the underestimate seems to differ considerably from possible resolutions based on concordance cosmology modeling. Our suggested resolution focuses on phenomena that would pertain at the times for which concordance cosmology modeling seems not to be adequate. Other possible resolutions seem to focus on phenomena early in the history of the universe. (See reference [16].)

We discuss relationships - between our work and other work - regarding astrophysics.

We think that our modeling is not necessarily incompatible with astrophysics data or with results based on concordance cosmology modeling. (Here, we assume that the two-word term concordance cosmology includes aspects that associate with dark matter, astrophysics, and effects of gravity on scales as small as one galaxy.)

We discuss properties of dark matter.

Reference [21] suggests the following notions. Most dark matter comports with notions of cold dark matter.

Models that associate with the two-word term modified gravity might pertain; but - to the extent that the models suggest long-range astrophysical effects - such models might prove problematic. People suggest limits on the masses of basic dark matter objects. Observations suggest so-called small-scale challenges to the notion that all dark matter might be cold dark matter. People use laboratory techniques to try to detect dark matter. People use astrophysical techniques to try to infer properties of dark matter.

We think that our modeling regarding dark matter comports with such notions. For astrophysical phenomena (and not necessarily regarding the rate of expansion of the universe), components - that have reaches other than six - of gravity play roles locally; however, the impacts do not extend to cosmological scales. The dark matter isomer that might evolve similarly to ordinary matter might provide bases for resolving some of the so-called small-scale challenges.

We discuss observations and models regarding galaxy formation.

Reference [22] discusses galaxy formation and evolution, plus contexts in which galaxies form and evolve. Reference [22] discusses parameters by which people classify and describe galaxies.

We suggest that - regarding galaxies - observations of ratios of dark matter to ordinary matter might tend to cluster near some specific ratios. (Perhaps, preview table VII.) Our modeling seems to explain such ratios.

Our modeling suggests that ratios of dark matter to ordinary matter might reflect fundamental aspects - of nature - that concordance cosmology modeling does not include. Here, a key aspect is that of isomers. (Perhaps, preview table VII.)

Reference [22] seems not to preclude galaxies that have few ordinary matter stars. Reference [22] seems not to preclude galaxies that have little ordinary matter.

We think that dark matter to ordinary matter ratios that our modeling suggests are not necessarily incompatible with verified concordance cosmology modeling.

We discuss observations and models regarding interactions between galaxies.

Reference [23] suggests that concordance cosmology modeling might not adequately explain gravitational interactions between neighboring galaxies. We suggest that notions pertaining to reaches and isomers might bridge the gap between observations and concordance cosmology modeling.

We think that our work points to a possible opportunity to study harmony between results based on established kinematics models and results based on our notions of components of gravity.

II. METHODS

This unit develops and deploys modeling that matches all known elementary particles; suggests new elementary particles; interrelates elementary particles,

properties of individual objects, and properties of systems of objects; and provides specifications for dark matter.

The method that we develop here outputs solutions to equations that involve sums of integers.

Some solutions associate with modeling that has similarities to modeling based on Maxwell's equations, to modeling based on charge-and-current 4-vectors, or to modeling based on the notion of gravitoelectromagnetism. (References [1] and [2] discuss gravitoelectromagnetism.) Some solutions associate with electromagnetic fields - such as an electric field or a magnetic field - and with electromagnetic properties - such as charge and magnetic dipole moment - of systems. Some solutions associate with gravitational fields and with gravitational properties - such as mass.

Some solutions point to radial spatial dependences of potentials. (Regarding radial spatial dependences of potentials and forces, we use terminology that generally associates with Newtonian kinematics.) One such radial spatial dependence of potential is r^{-1} for a component of electromagnetism that associates with the charge of a system. Here, r denotes a distance away from the system that produces the component of electromagnetism. Another such radial spatial dependence of potential is r^{-2} for a component of electromagnetism that associates with the magnetic dipole moment of the system.

We associate the symbol 1G with solutions that associate with electromagnetism. Here, the one denotes the spin (in units of \hbar) of photons. We associate the symbol 2G with solutions that associate with gravitation. Here, the two denotes the spin (in units of \hbar) of (as yet not detected) gravitons. Some solutions associate with a would-be spin-three (or, 3G) elementary boson. Some solutions associate with a would-be spin-four (or, 4G) elementary boson. We associate the two-element term long-range force (or, the two-element term long-range interaction) with each one of 1G through 4G.

Other solutions associate mathematically with 0G. Some 0G solutions associate with elementary bosons, such as the Z boson and the W boson. Some 0G solutions associate with elementary fermions, such as quarks and charged leptons.

The method outputs solutions that seem to match all known elementary particles and that suggest new elementary particles.

We use the following method to catalog elementary particles. (Perhaps, preview table IX.) A symbol of the form $S\Phi$ associates with a so-called family of elementary particles. Each elementary particle associates with one family. Each family associates with one of one, three, or eight elementary particles. For a family, the value S denotes the spin (in units of \hbar) for each elementary particle in the family. S associates with the expression $S(S+1)\hbar^2$ that associates with angular momentum. Values of S include 0, 0.5, 1, 2, 3, and 4. Except regarding quarks and possible heavy neutrinos,

the family associates with a one-letter value of Φ . For quarks, $\Phi=Q^{1/3}$ associates with each quark for which the magnitude of the charge is one-third the magnitude of the charge of the electron. $\Phi=Q^{2/3}$ associates with each quark for which the magnitude of the charge is two-thirds the magnitude of the charge of the electron. For neutrino-like particles, $\Phi=N$ associates with the three known neutrinos. $\Phi=N'$ associates with possible particles for which we use the two-word term heavy neutrinos.

A. Charge, mass, other properties, and long-range forces

This unit develops and deploys modeling that interrelates elementary particles (such as the photon) that carry long-range forces, properties of individual objects, and properties of systems.

A thought experiment provides context for developing the modeling. We assume that gravitons exist. We consider the notion of combining the states of photons and the states of gravitons. For example, we consider a photon and graviton that, in effect, move together. We measure spin angular momenta (or, values of $s\hbar$) along the axis of motion. We assume that the graviton associates with $s\hbar$ being one of $-2\hbar$ and $+2\hbar$. We assume that the photon associates with $s\hbar$ being one of $-1\hbar$ and $+1\hbar$. Mathematically, combining the two angular momenta can result in a spin angular momentum of $-3\hbar$, $-1\hbar$, $+1\hbar$, or $+3\hbar$.

Another thought experiment provides more context regarding the modeling we develop. We consider a hypothetical (perhaps atom-like) object. The object exhibits orbitals. Each orbital associates with a unique magnitude $l_o\hbar$ of orbital angular momentum. Here, l_o is an integer in a range $1 \leq l_o \leq l_{max}$. The value of l_{max} associates with the object. Up to one entity can associate with (or, occupy) an orbital. Relative to some axis that is common to all orbitals, the angular momentum that associates with an occupied orbital is one of $-l_o\hbar$ and $+l_o\hbar$. (This thought experiment excludes - for the occupied orbital - values of l for which $-l_o < l < +l_o$.) The angular momentum that associates with an unoccupied orbital is $0\hbar$. (Regarding considering the object to be atom-like, the following notions pertain. The nucleus has zero spin. Entities that occupy orbitals have zero spin. Entities that occupy orbitals do not interact with each other.) Relative to the axis, the total angular momentum that associates with the object is the sum - over the occupied orbitals - of the respective $\pm l_o\hbar$.

We focus on mathematics that associates with the thought experiment about objects and orbitals. We do not explore the notion of direct physics relevance for such objects. We do not explore the notion of such objects. For convenience, we continue to use the word orbital.

The above thought experiment suggests expressions of the form $\sum_{o \in O} (\pm l_o)$. Here, O denotes the set of occupied orbitals. o denotes a member of O .

We define Σ to be the absolute value of the sum of the various values of $\pm l_o$. The equation $\Sigma = |\sum_{o \in O} (\pm l_o)|$ pertains. The term two-word Diophantine mathematics associates with the modeling that we pursue.

We define Γ to be the list of relevant values of l_o . We define n_Γ to be the number of elements in the list Γ . The symbol l_{max} denotes the maximum value of l_o in Γ . We use the symbol $\Sigma G \Gamma$ to denote the combination of a list Γ and a relevant value of Σ . The letter G anticipates an association with electromagnetism and an association with gravity. (Perhaps, think of G as in gamma rays and G as in gravity.) Within a list Γ , we separate values of l_o by using the symbol $'$. For example, for the Γ symbolized by $1'2$, $1 \in \Gamma$ and $2 \in \Gamma$.

Table I alludes to all $\Sigma G \Gamma$ solutions, for which $1 \leq l_o \leq l_{max} \leq 4$ and no two values of l_o are the same.

We associate the symbol ΣG with solutions of the form $\Sigma G \Gamma$. We associate the symbol $\Sigma G'$ with ΣG solutions for which $\Sigma \in \Gamma$. We associate the symbol $\Sigma G''$ with ΣG solutions for which $\Sigma \notin \Gamma$.

We explore the notion that solutions that table I lists associate with long-range interactions (or, LRI) and with properties - of objects - that people do infer or might infer via observations based on information carried by electromagnetic fields, gravitational fields, and possibly other similar fields.

Regarding observations - via electromagnetism - pertaining to an object with nonzero charge, people might infer both a magnitude of charge of the object and a velocity with which the object moves. For models based on special relativity, the notion of a charge-and-current 4-vector pertains.

We deploy the symbol PROP to associate with $\Sigma G \Gamma$ solutions that associate with properties. We deploy the symbol CURR to associate with $\Sigma G \Gamma$ solutions that associate with currents of properties.

We extend the notions of PROP and CURR to apply widely regarding modeling regarding LRI. We anticipate that, for each LRI PROP solution there is an LRI CURR solution.

We posit the following associations. 1G associates with electromagnetism. 2G associates with gravitation. 3G associates with interactions with a function of elementary fermion flavor. 4G associates with interactions with magnitude of internal angular momentum. $l_o = 4$ associates with rotation. (For a preview regarding the usefulness of these posits, see the Σ column and the properties column in table III.)

We posit that a solution associates with a so-called RDP of the form $\Xi^{-n_{\Sigma G \Gamma}}$. Here, we consider Newtonian modeling for potentials (as in potential energy) that associate with fields (such as the electromagnetic field and the gravitational field) that an

Table I: $\Sigma\Gamma$ solutions, assuming that $1 \leq l_o \leq l_{max} \leq 4$ and that no two values of l_o are the same. The symbol 0 is a placeholder for an unused pair, $-l_o$ and $+l_o$, of values. For each row, there are 2^{n_Γ} possible ways to assign signs regarding the set of n_Γ terms. There are 2^{n_Γ} expressions of the form $\sum_{o \in O} (\pm l_o)$. Thus, there are $2^{n_\Gamma-1}$ solutions for $\Sigma = |\sum_{o \in O} (\pm l_o)|$. The number $n_{\Sigma\Gamma}$ equals $2^{n_\Gamma-1}$ and states the number of solutions. For example, for $l_{max} = 2$ and $n_\Gamma = 2$, the solutions are 1G1'2 (as in $1 = |-1 + 2|$) and 3G1'2 (as in $3 = |+1 + 2|$). The notion column refers to the number of solutions. One solution pertains for monopole. Two solutions pertain for dipole. Four solutions pertain for quadrupole. Eight solutions pertain for octupole. The Σ column shows values of Σ that associate with solutions. For the case of octupole, each one of $\Sigma = 2$ and $\Sigma = 4$ associates with two solutions. Regarding $\Sigma = 2$, $|-1 + 2 - 3 + 4| = 2 = |-1 - 2 - 3 + 4|$. Regarding $\Sigma = 4$, $|-1 - 2 + 3 + 4| = 4 = |+1 + 2 - 3 + 4|$. The symbol n_{o0} denotes the number of times the symbol 0 associates with an l_o for which $1 \leq l_o \leq l_{max}$.

l_{max}	n_Γ	$l_o = 1$	$l_o = 2$	$l_o = 3$	$l_o = 4$	Γ	$n_{\Sigma\Gamma}$	Notion	Σ	n_{o0}
1	1	± 1	-	-	-	1	1	Monopole	1	0
2	1	0	± 2	-	-	2	1	Monopole	2	1
2	2	± 1	± 2	-	-	1'2	2	Dipole	1,3	0
3	1	0	0	± 3	-	3	1	Monopole	3	2
3	2	± 1	0	± 3	-	1'3	2	Dipole	2,4	1
3	2	0	± 2	± 3	-	2'3	2	Dipole	1,5	1
3	3	± 1	± 2	± 3	-	1'2'3	4	Quadrupole	0,2,4,6	0
4	1	0	0	0	± 4	4	1	Monopole	4	3
4	2	± 1	0	0	± 4	1'4	2	Dipole	3,5	2
4	2	0	± 2	0	± 4	2'4	2	Dipole	2,6	2
4	2	0	0	± 3	± 4	3'4	2	Dipole	1,7	2
4	3	± 1	± 2	0	± 4	1'2'4	4	Quadrupole	1,3,5,7	1
4	3	± 1	0	± 3	± 4	1'3'4	4	Quadrupole	0,2,6,8	1
4	3	0	± 2	± 3	± 4	2'3'4	4	Quadrupole	1,3,5,9	1
4	4	± 1	± 2	± 3	± 4	1'2'3'4	8	Octupole	0,2,2,4,4,6,8,10	0

object produces. RDP stands for radial dependence of potential. For a solution other than a monopole solution, the potential can (and generally does) vary based on angular coordinates (as well as based on a radial coordinate). We posit that $\Xi^{-1} = r^{-1}$, in which r is the spatial distance from the object.

We note, but do not comment further regarding, the notions that the posited RDP of the form Ξ^{-1} associates with a physics use of the word monopole and that mentions in table I of the word monopole associate with a mathematics use of the word monopole. Similar parallel notions pertain regarding each of the words dipole, quadrupole, and so forth. (Also, we provide a cautionary note regarding terminology. Per table II, we associate the solution for which Σ is one and Γ is 1'2'4 with each one of the following: Ξ^{-3} and hence mathematical quadrupole, r^{-3} and hence a behavior of potential that associates with a notion of quadrupole, and a physics object that associates with a magnetic dipole that rotates around an axis that does not equal the axis that associates with the magnetic dipole. One way to think about the seeming tension between quadrupole and dipole is to associate the factor Ξ^{-1} that associates with $l_o = 4$ with $(ct)^{-1}$ instead of with r^{-1} . Here, c denotes the speed of light and t denotes the time that light takes to go from the magnetic-dipole object to the distance r from the object. This interpretation has consistency with the notion that the relevant quadrupole component of the electromagnetic field associates with an object that people might characterize as having the properties of a magnetic dipole.)

For the moment, we ignore solutions for which $\Sigma =$

0 and solutions for which $\Sigma \geq 5$.

Table II explores the notion that solutions have meaning regarding physics.

We suggest that table II points to a useful basis for modeling regarding, at least, electromagnetism and gravity.

Notions of three degrees of freedom seem to pertain regarding solutions that table II shows.

The following examples - of three degrees of freedom - pertain regarding 1G' solutions. Regarding 1G1'2, three degrees of freedom pertain. Two degrees of freedom associate with the orientation of the (magnetic moment) 3-vector. One degree of freedom associates with the magnitude of the 3-vector. Compared to 1G1'2, 1G1'2'4 has three more degrees of freedom. Two degrees of freedom associate with the orientation of the angular momentum 3-vector. One degree of freedom associates with the magnitude of the 3-vector.

Regarding each of the solutions that table II shows, $l_o = 4$ seems to associate - regarding rotation - with three degrees of freedom. However, we are careful to note that - for each of the 3G2'3'4, 4G1'2'3'4v, and 4G1'2'3'4w solutions - there is no associated same- Σ $\Sigma\Gamma$ other solution to which one can add $l_o = 4$ to obtain the subject solution.

We suggest that - for some aspects of our modeling - three degrees of freedom, mathematics associating with two one-dimensional harmonic oscillators, and mathematics associating with the group $SU(2)$ associate with each other. (For integers l such that $l \geq 2$, reference [24] interrelates mathematics associating with l one-dimensional harmonic oscillators and mathematics associating with the group $SU(l)$.) Here, we consider

Table II: PROP $\Sigma G'$ solutions, for $l_{max} \leq 4$. Here, the allowed values of l_o are 1, 2, 3, and 4. The table suggests possible contributions - from one object - to each of four ΣG . 1G1 associates with a component - of the electromagnetic field that the object produces - that associates with the object's charge. The word scalar associates with this solution. 1G1'2 associates with the object's magnetic field. An axis associates with that field. The one-element term 3-vector associates with this solution. 1G1'2'4 associates with a combination of magnetic field and rotation (over time) of the axis of the magnetic field. (The Earth is an object for which the axis of rotation does not equal the axis of the magnetic field.) 2G2 associates with the object's mass. The word scalar associates with this solution. 2G2'4 associates with rotation of the object's mass. An axis associates with that rotation. The one-element term 3-vector associates with this solution. (Regarding general relativity, this solution seems to associate with aspects of rotational frame dragging.) 2G1'2'3 associates with a non-spherically symmetric distribution of mass. 2G1'2'3'4v associates with rotation (of a non-spherically symmetric distribution of mass) around a minor axis of moment of inertia. 2G1'2'3'4w associates with rotation (of a non-spherically symmetric distribution of mass) around a major axis of moment of inertia. For gravity produced by an object like the Sun, PROP 2G' solutions other than 2G2 associate with adjustments with respect to the gravity that associates with 2G2. Regarding large-scale gravitation, PROP 2G' solutions other than 2G2 can associate with gravitational effects that dominate gravitational effects that associate with 2G2. We postpone discussing 3G' solutions and 4G' solutions.

Σ	Monopole	Dipole	Quadrupole	Octupole
1	1G1	1G1'2	1G1'2'4	-
2	2G2	2G2'4	2G1'2'3	2G1'2'3'4v, 2G1'2'3'4w
3	3G3	-	3G2'3'4	-
4	4G4	-	-	4G1'2'3'4v, 4G1'2'3'4w

that one oscillator might associate with boson-like excitations regarding a relevant aspect that associates with the relevant value of $-l_o$. The other oscillator might associate with boson-like excitations regarding a relevant aspect that associates with the relevant value of $+l_o$. The number of generators of the group $SU(2)$ is three.

We explore notions of CURR solutions and notions of $l_o \geq 5$.

Regarding PROP solutions, the following notions pertain. The combination of $\Sigma = 1$ and $l_o = 1$ associates with charge. The combination of $\Sigma = 2$ and $l_o = 2$ associates with mass. The combination of $\Sigma = 4$ and $l_o = 4$ associates with angular momentum.

To explore CURR solutions, we want to add three degrees of freedom that associate with the CURR aspects of PROP-and-CURR 4-vectors.

Regarding CURR solutions that associate with PROP modeling for which $l_{max} \leq 4$, we posit the following. $8 \in \Gamma$ associates with momentum. $l_o = 7$ associates (as does $l_o = 1$ for PROP modeling) with charge. $l_o = 6$ associates (as does $l_o = 2$ for PROP modeling) with mass. $l_o = 5$ associates (as does $l_o = 3$ for PROP modeling) with a function of elementary fermion flavour.

For PROP solutions we allow $l_o = 16 \in \Gamma$. For CURR solutions we allow $l_o = 16 \in \Gamma$ and $l_o = 32 \in \Gamma$.

Table III extends table II and lists PROP and CURR $\Sigma G'$ solutions, for which $l_{max} \leq 16$ for PROP $\Sigma G'$ solutions.

Table IV lists some $\Sigma G''$ solutions for which we posit relevance. Table IV previews the notion that 1G3'4 associates with - at least - depletion (via interactions with hydrogen-like atoms) of CMB (or,

cosmic microwave background radiation) by ordinary matter and some dark matter. (Perhaps, preview table VII.) Table IV previews the notion that 3G1'2 associates with modeling regarding anomalous magnetic moments. (Perhaps, preview table VIII.) For 3G1'2, two CURR solutions pertain.

B. Elementary particles that do not carry long-range forces

This unit matches and suggests elementary particles that do not carry long-range forces.

We explore the notion that solutions for which $\Sigma = 0$ associate with known and possible non-G-family elementary particles. Based on arithmetic, for each $\Sigma = 0$ solution, n_Γ is at least three. We associate the symbol SRI (as in short-range interaction or as in elementary boson that does not associate with a long-range interaction) with non-G-family elementary bosons. We associate the symbol ELF (as in elementary fermion) with fermion elementary particles.

Table V shows 0G Γ solutions that might associate with elementary bosons that are not G-family bosons. (Reference [10] discusses the inflaton particle.)

Table VI shows 0G Γ solutions that might associate with elementary fermions.

We discuss a contrast - for non-G-family elementary particles and for more complicated objects - between a notion for which this essay uses the word free and a notion for which this essay uses the word interlaced.

Each SRI and ELF elementary particle associates with just one of free and interlaced. Free pertains if $l_{max} = 8$ for the CURR solution. Interlaced pertains if $l_{max} = 32$ for the CURR solution.

We suggest that the notion of $l_{max} = 8$ for the CURR solution associates with the possibility that an

Table III: PROP and CURR $\Sigma G'$ solutions, for which $l_{max} \leq 16$ for PROP $\Sigma G'$ solutions. Table III extends table II so as to include - for each PROP $\Sigma G'$ solution - a CURR $\Sigma G'$ solution. For example 1G7'8 satisfies $\Sigma = 1$ and associates with - for an object that produces an electromagnetic field - the charge-current 3-vector that complements the scalar charge that associates with PROP and with 1G1. The symbol $n_{l_o,PROP}$ denotes the number of elements in the Γ that associates with PROP. For a CURR $\Sigma G'$ solution, the RDP (or, radial dependence of potential) equals Ξ^{-1} times the RDP for the associated PROP $\Sigma G'$ solution. Table III lists an RDF - or radial dependence of force - for each PROP solution. For each one of PROP and CURR, the RDF equals Ξ^{-1} times the RDP. For example, for each of 1G1 and 2G2, the RDF is Ξ^{-2} , which is r^{-2} . Table III shows properties - of objects that produce $\Sigma G'$ components of ΣG - that associate with the PROP solution. The table attempts to use familiar symbols. The table associates the symbols with phrases. The symbol NYN abbreviates the three-word phrase not yet named and denotes a property for which people might have yet to assign a name. The symbol TBD denotes the three-word phrase to be determined. No one $\Sigma G'$ solution associates with both PROP and CURR.

Σ	$\Sigma G'$ PROP	$\Sigma G'$ CURR	$n_{l_o,PROP}$	RDF PROP	Properties associating with PROP
1	1G1	1G7'8	1	Ξ^{-2}	Q - Charge
1	1G1'2	1G2'7'8	2	Ξ^{-3}	μ - Magnetic dipole moment
1	1G1'2'4	1G1'2'4'8	3	Ξ^{-4}	μ, s - Magnetic dipole moment and angular momentum
1	1G1'2'4'8'16	1G1'2'4'8'16'32	5	Ξ^{-6}	NYN - TBD
2	2G2	2G6'8	1	Ξ^{-2}	m - Mass
2	2G2'4	2G2'4'8	2	Ξ^{-3}	m, s - Rotating mass
2	2G1'2'3	2G1'2'3'8	3	Ξ^{-4}	m, NYN - Non-spherical aspects of mass
2	2G1'2'3'4v, 2G1'2'3'4w	2G1'2'3'4'8v, 2G1'2'3'4'8w	4	Ξ^{-5}	m, NYN, s - Rotating non-spherical aspects of mass
2	2G1'2'3'8'16	2G1'2'3'8'16'32	5	Ξ^{-6}	m, NYN - TBD
2	2G1'2'3'4'8'16	2G1'2'3'4'8'16'32	6	Ξ^{-7}	m, NYN - TBD
3	3G3	3G5'8	1	Ξ^{-2}	NYN - TBD (a function of elementary fermion flavour)
3	3G2'3'4	3G2'3'4'8	3	Ξ^{-4}	NYN - TBD
3	3G2'3'4'8'16	3G2'3'4'8'16'32	5	Ξ^{-6}	NYN - TBD
4	4G4	4G4'8	1	Ξ^{-2}	S - Angular momentum
4	4G1'2'3'4v, 4G1'2'3'4w	4G1'2'3'4'8v, 4G1'2'3'4'8w	4	Ξ^{-5}	NYN - TBD
4	4G1'2'3'4'8'16	4G1'2'3'4'8'16'32	6	Ξ^{-7}	NYN - TBD

Table IV: Some G-family solutions that associate with $\Sigma G''$

Σ	PROP $\Sigma G'$	PROP $\Sigma = \dots$	CURR $\Sigma = \dots$	$n_{l_o,PROP}$	Association
1	1G3'4	$ -3+4 $	$ -3-4+8 $	2	Hyperfine interactions
3	3G1'2	$ +1+2 $	$ +1-6+8 , +2-7+8 $	2	Anomalous magnetic moments

object models as having a well-defined and constant momentum. We suggest that the notion of $l_{max} = 32$ for the CURR solution associates with the possibility that an object models as existing within a system (of more than one object) that models as having a well-defined and constant momentum. For $l_{max} = 32$ for the CURR solution, the object does not necessarily model as having a well-defined and constant momentum. This essay does not emphasize the combined notion of the extents to which modeling might feature notions of free and interlaced for each of momentum ($l_o = 8$), angular momentum ($l_o = 4$), and mass ($l_o = 2$).

C. Isomers and dark matter

This unit suggests - based on astrophysics data - that most dark matter consists of five isomers of most of the ordinary matter elementary particles that are not G-family elementary particles.

Discussion above points to two types of elementary particles that would measure as dark matter or that would provide a basis for dark matter. 0.5N' fermions associate with the notion of free and would measure

as dark matter. 0.5R fermions associate with the notion of interlaced. Hadron-like particles containing gluons and 0.5R fermions would contain no charged particles and would measure as dark matter.

We use the term DMAI to denote stuff that has bases in 0.5N' particles or 0.5R elementary fermions. DM abbreviates the two-word term dark matter. AI abbreviates the two-word term all isomers. (Here, we allude to a notion of multiple isomers of some elementary particles. For the moment we assume that nature includes just one isomer.)

We use notation of the form DM:OM to denote an inferred ratio of DM effects to OM effects. OM abbreviates the two-word term ordinary matter.

Table VII lists some observed ratios of dark matter effects to ordinary matter effects. (For data and discussion regarding densities of the universe, see reference [8]. For data and discussion regarding galaxy clusters, see references [25], [26], [27], and [28]. For data and discussion regarding absorption of CMB, see references [29], [30], and [31]. For data and discussion regarding observed early galaxies, see references [32] and [33]. For data and discussion regarding later galaxies for which ratios of 5⁺:1 pertain, see reference [34].

Table V: Solutions that might associate with non-G-family elementary (or, with SRI) bosons. Each column with label $0 = \dots$ shows a calculation that produces the $\Sigma = 0$ that associates with a $0G\Gamma$ solution. Each integer that the calculation includes is a member of Γ . No other integer is a member of Γ . The symbol $n_{l_o,PROP}$ denotes the number of l_o that appear in the Γ for the PROP solution. The symbol n_{EP} denotes the number of elementary particles. We posit that the aye (or, OI boson) associates with notions of an inflaton. Inflatons would be zero-mass zero-charge bosons that might have played key roles during a hypothesized inflationary epoch, early in the evolution of the universe. The W boson is the only charged elementary boson. In the table, only one PROP solution does not have - as members of Γ - all three of 1, 3, and 4. Based on the previous two sentences, we associate $0G1'2'3$ with the W boson and we associate $0G1'3'4$ with the Z boson. We posit that, paralleling aspects of some $1G'$ CURR solutions, for the CURR solution that associates with the W boson, $l_o = 7$ associates with charge. We posit that, paralleling aspects of some $1G'$ CURR solutions, for the CURR solution that associates with the gluons, $l_o = 5$ associates with color charge. We posit that the jay (or, 1J) boson associates with notions of Pauli repulsion. Pauli repulsion associates with the notion that two fermions (whether elementary fermions or not elementary fermions) cannot occupy the same state. Pauli repulsion associates with repulsive aspects of the residual strong force. We suggest (but do not require) that - with respect to the Standard Model notion that $SU(3)$ symmetry associates with the strong interaction - the jay boson might (for some modeling) associate with the identity operator that $SU(3)$ representations for the gluons (which transmit color charge) exclude. We suggest that mutual - for gluons and jay bosons - association with the strong force associates with the notion that the 1J PROP solution equals the 1U PROP solution. We suggest that the jay boson can interact with each fermion elementary particle, regardless of whether the notion of color charge pertains for the elementary fermion. We suggest that the jay boson can interact with each fermion non-elementary particle. Each one of two uses of a pair of the symbol \dagger points to dual use of a solution.

$0 = \dots$, re $0G\Gamma$ for PROP	$0 = \dots$, re $0G\Gamma$ for CURR	$n_{l_o,PROP}$	Family	Bosons	n_{EP}
$ +1-2-3+4 $	$ +1-2-3-4+8 $	4	0H	Higgs	1
$ +1-2-3-4-8+16 $	$ +1-2-3-4-8-16+32 ^\dagger$	6	0I	Aye	1
$ -1-3+4 $	$ -1-3-4+8 $	3	1Z	Z	1
$ -1-2+3 $	$ +2-3-7+8 $	3	1W	W	1
$ -1-3-4-8+16 ^\dagger$	$ -1+2-4-5-8-16+32 $	5	1U	Gluons	8
$ -1-3-4-8+16 ^\dagger$	$ +1-2-3-4-8-16+32 ^\dagger$	5	1J	Jay	1

Table VI: Solutions that might associate with elementary fermions (or, ELF elementary particles). We posit, regarding PROP $0G$ solutions, that $6 \in \Gamma$ associates with elementary fermions. (Regarding bosons and the CURR $2G6'8$ solution, $6 \in \Gamma$ associates with mass. Otherwise, regarding boson $\Sigma G\Gamma$ solutions for which $\Sigma \geq 1$, $6 \notin \Gamma$ pertains. We suggest that each elementary fermion has positive mass.) We posit that, in effect, each PROP solution associates with three flavours and thus with three elementary fermions. Paralleling notions pertaining to non-G-family elementary bosons, if, and only if, each of 1, 3, and 4 is a member of Γ , an elementary particle that associates with table VI has zero charge. We discuss solutions that associate with quarks. $0.5Q$ particles are the only known particles for which $0 < Q < 1$. Here, Q denotes the magnitude of the charge, in units of $|q_e|$. q_e denotes the charge of the electron. The notion of PROP solution associates with each of the following two solutions: $0=|+1-3-6-8+16|$ and $0=|+2-4-6-8+16|$. The notion of CURR solution associates with each of the following two solutions: $0=|+1-3-6-8-16+32|$ and $|+2-4-6-8-16+32|$. Two possibilities exist. For one possibility, the following two sentences pertain. Quarks with one magnitude of charge associate with one of the PROP solutions and one of the CURR solutions. Quarks with the other magnitude of charge associate with the other one of the PROP solutions and the other one of the CURR solutions. For the other possibility, the following three sentences pertain. One of the PROP solutions and one of the CURR solutions associate (for modeling purposes) with $Q = 1/2$. The other one of the PROP solutions and the other one of the CURR solutions associate (for modeling purposes) with $Q = 1/6$. One linear combination of the two pairs associates with $Q = (1/2) + (1/6) = 2/3$ and another linear combination of the two pairs associates with $Q = (1/2) - (1/6) = 1/3$. (Perhaps, preview aspects, such as $Q = 1/2$, of table XI and table XII.)

$0 = \dots$, re $0G\Gamma$ for PROP	$0 = \dots$, re $0G\Gamma$ for CURR	$n_{l_o,PROP}$	Families	Fermions	n_{EP}
$ -1+2-3-4+6 $	$ +1-2+3-4-6+8 $	5	0.5N	Neutrinos	3
$ -1-2-3+6 $	$ +2+3-6-7+8 $	4	0.5C	Charged leptons	3
$ +1-3-4+6 $	$ -1+3-4-6+8 $	4	0.5N'	Heavy neutrinos	3
$ +1-3-6-8+16 $	$ +1-3-6-8-16+32 $	5	$0.5Q^{1/3}, 0.5Q^{2/3}$	Quarks	6
$ +2-4-6-8+16 $	$ +2-4-6-8-16+32 $				
$ -1+3-4-6-8+16 $	$ -1+3-4-6-8-16+32 $	6	0.5R	Arcs	3

For data and discussion regarding galaxies for which ratios of $\sim 4:1$ pertain, see references [35] and [36]. For data and discussion regarding the combination of $0^+:1$ and later, see references [37], [38], [39], [40], and [41]. For data and discussion regarding observed dark matter galaxies, see references [34], [42], and [43]. References [44] and [45] suggest, regarding galaxy clusters, the existence of clumps of dark matter that might be individual galaxies. Current techniques might not be capable of observing early dark matter galaxies.)

While there might not be all that much data regarding ratios of other than $5^+:1$ and $1:0^+$, table VII might point to seemingly prevalent approximate DM:OM ratios of $1:1$, $0^+:1$, and $\sim 4:1$. We suggest that, if DMAI is the only type of dark matter, DMAI might not suffice to explain ratios of dark matter to ordinary matter. We suggest that the notion of DMAI might not suffice to explain dark matter.

We suggest that nature includes six isomers of the SRI and ELF elementary particles - or, six isomers of the set of elementary particles that associates with all non-G-family elementary bosons and all elementary fermions. (See table V and table VI.) One isomer associates with ordinary matter plus one isomer of DMAI. That one isomer of DMAI measures as dark matter. Each one of the other five isomers of the set of non-G-family elementary particles measures as dark matter. Regarding densities of the universe, the five isomers of non-DMAI that measure as dark matter associate with the 5 in the DM:OM ratio of $5^+:1$. The six isomers of DMAI associate with the $^+$ in the DM:OM ratio of $5^+:1$.

We use a two-word phrase isomer *number* to denote one isomer. Here, *number* can be any one of zero, one, ..., and five. We associate the two-word term isomer zero with the isomer that includes ordinary matter. We use the two-word phrase alt isomer to denote any one of the five isomers that does not associate with ordinary matter.

All six isomers produce and interact with a common notion of gravity. We suggest that one instance of 2G2 mediates interactions between all six isomers. We say that one instance of 2G2 has a reach of six, as in six isomers. We suggest that each isomer associates with its own instance of 1G1 and its own instance of 1G1'2. We say that each instance of 1G1 has a reach of one, as in one isomer. Each instance of 1G1'2 has a reach of one. Each isomer - including the ordinary matter isomer - scarcely interacts with any other isomer via electromagnetism.

We address the topic of reach for each $\Sigma G\Gamma$ to which table I alludes. Based on the reach of 1G1 and the reach of 1G1'2, we suggest that $n_{o0} = 0$ associates with a reach of one. Based on the reach of 2G2, we suggest that $n_{o0} = 1$ associates with a reach of six. We posit that, for $n_{o0} \geq 1$, the reach (of one instance of a relevant PROP $\Sigma G\Gamma$) equals the number of generators of the group $SU(7)$ divided by

the number of generators of the group $SU(2n_{o0} + 1)$. For an integer l that is at least two, the number of generators of the group $SU(l)$ is $l^2 - 1$. The reach that associates with $n_{o0} = 2$ is two. The reach that associates with $n_{o0} = 3$ is one. The number of instances of a PROP $\Sigma G\Gamma$ component of a ΣG is six divided by the reach that associates with the PROP $\Sigma G\Gamma$ solution.

We assume that the reach of a CURR counterpart solution to a PROP $\Sigma G\Gamma$ solution is the same as the reach of the PROP $\Sigma G\Gamma$ solution.

We address the reach of the 2G1'2'3'8'16 PROP solution, which table III lists and table I does not list. For 2G1'2'3'8'16, each of 1, 2, and 3 appears in Γ and 4 does not appear in Γ . We assume that we can use the n_{o0} that pertains for 2G1'2'3. (See table I.) The reach for 2G1'2'3'8'16 is six.

Table VIII shows the reach (ρ_I) for - and other information about - each one of some solutions that table II, table III, and table IV list.

Regarding the notion of a reach, ρ_I , of two, there are three instances of the PROP solution. We number the isomers so that one instance of the solution intermediates interactions between isomer zero and isomer three. One instance of the solution intermediates interactions between isomer one and isomer four. One instance of the solution intermediates interactions between isomer two and isomer five.

We use notation of the form $\Sigma(\rho_I)G\Gamma$ to denote a $\Sigma G\Gamma$ solution and the reach ρ_I that associates with one modeling use that features an instance of the solution. For example, 2(2)G2'4 pertains regarding 2G2'4. We extend use of such notation to non-LRI elementary particles. For non-LRI elementary particles, the reach is one and notation of the form $S(1)\Phi$ pertains.

If the stuff that associates with each of the five all-dark-matter isomers evolved similarly to ordinary matter, our suggestions regarding dark matter might not adequately comport with observations regarding the Bullet Cluster collision of two galaxy clusters. Elsewhere, we suggest that the isomers of ELF elementary particles differ sufficiently that our suggestions regarding dark matter do not necessarily disagree with observations pertaining to the Bullet Cluster. (Perhaps, preview discussion related to table XIII.)

We discuss notions regarding excitation and de-excitation of LRI fields (or, of G-family elementary particles).

An excitation associates with a value of Σ and with a set of isomers. For example, consider an excitation that associates with active-gravitational properties of an ordinary matter star. The word active associates with the notion that the star generates gravity. The word gravitational associates with $\Sigma = 2$. The excitation might associate with the 2G2 solution, with the 2G2'4 solution, or with another 2G solution. Because the star consists just of ordinary matter stuff, the set of isomers consists just of isomer zero.

Table VII: Ratios of dark matter to ordinary matter. DM denotes dark matter. OM denotes ordinary matter. The notion of DM:OM pertains. Inferences come from interpreting data. The word early associates with the notion that people consider the data to pertain regarding early in the history of the universe. (Redshifts z tend to exceed seven, but could be somewhat less.) The word later associates with the notion that observations pertain to objects later in the history of the universe. In table VII, the four-word phrase some absorption of CMB associates with the notion that people measured some specific depletion of CMB (or, cosmic microwave background radiation) and inferred twice as much depletion as people expected based solely on hyperfine interactions with hydrogen atoms. Possibly, half of the depletion associates with DM effects. The three-word phrase dark matter galaxy denotes a galaxy that contains much less ordinary matter than dark matter.

DM	OM	Phenomena	Status	Terminology
5 ⁺	1	Densities of the universe	Observed	-
5 ⁺	1	Some galaxy clusters	Observed	-
1	1	Some absorption of CMB	Possibly observed	-
0 ⁺	1	Galaxies (early)	Observed	-
5 ⁺	1	Many (later) galaxies	Observed	-
~4	1	Some (later) galaxies	Observed	-
0 ⁺	1	Some (later) galaxies	Observed	-
1	0 ⁺	Some (later) galaxies	Observed	Dark matter galaxies
1	0 ⁺	Some (early) galaxies	Not (yet) observed	(Would be) dark matter galaxies

Table VIII: Reaches and other information regarding some solutions that associate with electromagnetism, gravity, 3G, and 4G. ρ_I denotes reach. $\Sigma \in \Gamma$ associates with the symbol G'. $\Sigma \notin \Gamma$ associates with the symbol G''. NYN denotes not yet named. Discussion related to table XVIII provides information regarding the notion of anomalous magnetic moment.

S	Σ	PROP solution	ρ_I	Solution type	PROP RDF	Object property
1	1	1G1	1	$\Sigma \in \Gamma$	Ξ^{-2}	Charge
1	1	1G1'2	1	$\Sigma \in \Gamma$	Ξ^{-3}	Magnetic dipole moment
1	1	1G1'2'4	6	$\Sigma \in \Gamma$	Ξ^{-4}	NYN
1	1	1G3'4	2	$\Sigma \notin \Gamma$	Ξ^{-3}	NYN
2	2	2G2	6	$\Sigma \in \Gamma$	Ξ^{-2}	Mass
2	2	2G2'4	2	$\Sigma \in \Gamma$	Ξ^{-3}	NYN
2	2	2G1'2'3	1	$\Sigma \in \Gamma$	Ξ^{-4}	NYN
2	2	2G1'2'3'4v	1	$\Sigma \in \Gamma$	Ξ^{-5}	NYN
2	2	2G1'2'3'4w	1	$\Sigma \in \Gamma$	Ξ^{-5}	NYN
2	2	2G1'2'3'8'16	6	$\Sigma \in \Gamma$	Ξ^{-6}	NYN
3	3	3G3	2	$\Sigma \in \Gamma$	Ξ^{-2}	NYN
3	3	3G2'3'4	6	$\Sigma \in \Gamma$	Ξ^{-4}	NYN
3	3	3G1'2	1	$\Sigma \notin \Gamma$	Ξ^{-3}	Anomalous magnetic moment
4	4	4G4	1	$\Sigma \in \Gamma$	Ξ^{-2}	Angular momentum
4	4	4G1'2'3'4v	1	$\Sigma \in \Gamma$	Ξ^{-5}	NYN
4	4	4G1'2'3'4w	1	$\Sigma \in \Gamma$	Ξ^{-5}	NYN

A de-excitation associates with the notion of passive properties, with any same- Σ G-family solution and with a set of isomers that associates with the original excitation. We continue the previous example. Regarding $\Sigma = 2$, the word passive associates with the notion that an object interacts with gravity that other objects actively produce. Because 2G2 has a reach of six, any object can de-excite, via 2G2, the excitation that the example features. Because 2G1'2'3 has a reach of one, only isomer zero stuff can de-excite, via 2G1'2'3, the excitation. Because 2G2'4 has a reach of two, only isomer zero stuff or isomer three stuff can de-excite the excitation via 2G2'4.

Generally, ten types of de-excitations exist. One type consists of de-excitations that associate with reach-six solutions. Three types consist of de-excitations that associate with reach-two solutions. One of the three types associates with isomer zero and isomer three. Another one of the three types associates with isomer one and isomer four. The other one of the three types associates with isomer two and isomer five. Six types

consist of de-excitations that associate with reach-one solutions. Each one of the six types associates with exactly one isomer.

We discuss excitations and de-excitations that associate with long-range forces produced by a galaxy that consists mainly of stuff that associates with five isomers. (Perhaps, see table VII.) We discuss electromagnetism. A one-isomer distant observer would sense mostly aspects that associate with the distant observer's isomer. Here, most detection of photons would associate with the reach-one solutions 1G1 and 1G1'2. Via aspects that associate with solutions such as 1G1'2'4 (for which the reach is six) and 1G3'4 (for which the reach is two), the one-isomer observer might sense aspects that associate with isomers other than the observer's isomer. We discuss gravitation. The one-isomer distant observer would sense all of the galaxy's stuff via 2G2 (for which the reach is six). But the sensing of subtleties (such as rotation of stuff or irregular distributions of stuff) associates with solutions (such as 2G2'4 and 2G1'2'3) for which the reaches are

less than six. (Perhaps, see table VIII.) The one-isomer observer would not necessarily sense directly via 2G all of the subtleties.

III. RESULTS

This unit discusses explanations of known data and discusses suggestions regarding possible data that people have not yet measured. The discussion includes explanations and suggestions regarding elementary particles, dark matter, galaxies, and the cosmos.

A. Explanations and suggestions regarding elementary particles

This unit lists elementary particles that associate with our modeling and discusses relationships between properties of elementary particles.

Table IX consolidates and summarizes information about all elementary particles of which people know or which this essay suggests. (See table III, table V, and table VI.)

We explore notions that point to relationships among properties of objects, elementary particles, and long-range interactions.

Table X discusses relationships between properties of elementary bosons. (Regarding the masses of the Higgs, Z, and W bosons, we used data that reference [8] provides.)

Table X points to possibly deeper (than people might otherwise suggest) relationships between the physics properties of spin, mass, and charge.

We turn our attention to properties of elementary fermions.

Work immediately below has roots in a thought experiment. For the thought experiment, we consider hypothetical elementary fermions for which $Q = 1$. For some value of mass, the gravitational attraction between two identical such hypothetical elementary fermions would equal the electrostatic repulsion between the two fermions. Our work shows that a mass - so-called $m(18, 3)$ - seems to have meaning beyond the notion that - for the mass $m(18, 3)$ - gravitational attraction between two $Q = 1$ identical elementary fermions would be three-quarters of the electrostatic repulsion between the two identical elementary fermions. (Perhaps, preview table XII.)

Table XI discusses relationships between properties of known charged elementary fermions. (Reference [8] provides the data that underlies table XI.)

Table XII shows equations that underlie aspects of table XI and suggests extensions - for example, regarding 0.5N neutrinos and 0.5R arcs - to table XI. (Reference [8] provides the data that underlies table XII.)

Table XI and table XII point to possibly deeper (than people might otherwise suggest) relationships between the physics properties of spin, mass, and charge.

We explore two alternatives regarding values of $d'(0)$, $d'(1)$, and $d'(2)$. (See table XII.) Changing those

numbers would impact only the calculated masses for quarks and the calculated suggested masses for arcs. In both cases, if one excludes one of three methods for estimating the mass of the top quark, the calculated mass for each of the six quarks is within five standard deviations of the experimental mass. (Reference [8] discusses the three methods.) For the third method for estimating the mass of the top quark, the value that we calculate for the mass of the top quark would be less than eleven standard deviations below the mass people have calculated.

One alternative has bases in the notions of $d'(-1) = 0^2/2^2$, $d'(0) = 1^2/2^2$, $d'(1) = -2^2/2^2$, and $d'(2) = -(2 \times 3)/2^2$. For this alternative, the three arc rest energies would, respectively, be ≈ 8.14 MeV, $m(1, 3)c^2$, and $m(2, 3)c^2$.

The other alternative has bases in the notions of $d'(0) \approx 0.264825$, $d'(1) = -2^2/2^2$, and $d'(2) = -(2 \times 3)/2^2$. For this alternative, the three arc rest energies would, respectively, equal $m(1, 3)c^2$, $m(1, 3)c^2$, and $m(2, 3)c^2$. Across the three 0.5C elementary fermions and the three 0.5R elementary fermions, $m(0, 3)c^2$ would pertain once, $m(1, 3)c^2$ would pertain twice, $m(2, 3)c^2$ would pertain twice, and $m(3, 3)c^2$ would pertain once.

We speculate regarding possible masses for heavy neutrinos.

For purposes of estimating or calculating masses, neutrinos associate with a value of M'' for which $-6 \leq M'' \leq -3$. Charged leptons associate with $0 \leq M'' \leq 3$. If heavy neutrinos associate with $6 \leq M'' \leq 9$, a lower bound on rest energies for heavy neutrinos might be $m(6, 3)c^2 \sim 6 \times 10^3 \text{ GeV}$, which might be large enough to comport with limits that associate with observations. (References [46] and [47] discuss limits that observations may set. People have not detected 1N' particles.)

B. Explanations and suggestions regarding dark matter

This unit suggests specifications for dark matter.

We consider a thought experiment.

Regarding each l that is at least one, we assume that the elementary particles in isomer l match - with respect to mass - the elementary particles in isomer zero.

For the purposes of this thought experiment and for $0 \leq l \leq 5$, we associate the quarks in isomer l with three values of M'' . (See table XI and table XII.) The values are $3l + 0$, $3l + 1$, and $3l + 2$. Across the six isomers, quarks associate with each value of M'' that is in the range $0 \leq M'' \leq 17$. Regarding quarks and flavours, we assume that - within isomer l - flavour 1 associates with $M'' = 3l$, flavour 2 associates with $M'' = 3l + 1$, and flavour 3 associates with $M'' = 3l + 2$.

For the purposes of this thought experiment, aspects of table XI and table XII point to the possibility that

Table IX: Elementary particles. The symbol Q associates with magnitude of charge. The columns labeled $Q > 0$ and $Q = 0$ have entries in the form of a name of one particle or a name of a set of more than one particle, followed by (in parentheses) a number of particles, followed by a family. NYN denotes not yet named. NYD denotes not yet detected. One might assert that people know of some NYD particles, at least indirectly. For purposes of this essay, $\text{PROP } 8 \notin \Gamma$ associates with a notion of can model as not interlaced (or, as free) and $\text{PROP } 8 \in \Gamma$ associates with a notion of always models as interlaced (or, bound).

S	m	$Q > 0$	$Q = 0$	Status	Σ	$\text{PROP } 8 \in \Gamma$
0	>0	-	Higgs boson (1) - 0H	Known	0	No
1/2	>0	Charged leptons (3) - 0.5C	Neutrinos (3) - 0.5N	Known	0	No
1/2	>0	-	Heavy neutrinos (3) - 0.5N'	NYD	0	No
1	>0	W boson (1) - 1W	Z boson (1) - 1Z	Known	0	No
1	$=0$	-	Photon (1) - 1G	Known	1	Some components
2	$=0$	-	Graviton (1) - 2G	NYD	2	Some components
3	$=0$	-	NYN (1) - 3G	NYD	3	Some components
4	$=0$	-	NYN (1) - 1G	NYD	4	Some components
0	$=0$	-	Aye boson (1) - 0I	NYD	0	Yes
1/2	>0	Quarks (3) - $0.5Q^{1/3}$	-	Known	0	Yes
1/2	>0	Quarks (3) - $0.5Q^{2/3}$	-	Known	0	Yes
1/2	>0	-	Arcs (3) - 0.5R	NYD	0	Yes
1	$=0$	-	Jay boson (1) - 1J	NYD	0	Yes
1	$=0$	-	Gluons (8) - 1U	Known	0	Yes

Table X: Relationships between properties of elementary bosons. Q denotes the magnitude of charge, in units of $|q_e|$. m denotes mass, in units of $m_{\text{Higgs}}/17^{1/2}$ or in units of $m_Z/9^{1/2}$. S denotes spin, as in the expression $S(S+1)\hbar^2$. l_m equals -1 for $m > 0$ and equals 0 for $m = 0$. The sum is the sum of the numbers in the preceding four columns. Each sum is the square of an integer. For each nonzero mass particle, the integer equals $n_{l_o, \text{PROP}}$. There are no nonzero mass elementary bosons for which the integer equals one or two. (For a Γ that includes just one value of l_o or that includes just two values of l_o , $\Sigma \neq 0$ pertains.) NYN denotes the three-word phrase not yet named. Of the non-zero masses to which table X alludes, the most accurately known mass is that of the Z boson. Using the mass of the Z boson and numbers in table X, one can calculate a nominal mass for the Higgs boson and a nominal mass for the W boson. The calculated mass for the Higgs boson differs from the experimentally determined mass by less than two (experimental) standard deviations. The calculated mass for the W boson differs from the experimentally determined mass by less than four (experimental) standard deviations. To the extent that one uses the notion that ruling out an equality requires a difference of at least five standard deviations, experimental results do not seem to rule out relationships that table X states. NYN denotes not yet named.

Bosons	Family	$Q(Q+1)$	m^2	S^2	l_m	Sum
Higgs	0H	0	17	0	-1	16
Aye	0I	0	0	0	0	0
Z	1Z	0	9	1	-1	9
W	1W	2	7	1	-1	9
Jay	1J	0	0	1	0	1
Gluons	1U	0	0	1	0	1
Photon	1G	0	0	1	0	1
Graviton	2G	0	0	4	0	4
NYN	3G	0	0	9	0	9
NYN	4G	0	0	16	0	16

Table XI: $\log_{10}(m_{\text{particle}}/m_e)$ for known charged elementary fermions. Regarding “flavour,” this table generalizes, based on terminology that associates with charged leptons and neutrinos. For example, people use the term electron-neutrino. The “Flavour (0.5C)” terms pertain for fermions in the 0.5C family. The “Flavour (0.5Q)” terms pertain for quarks (or, elementary particles in the two families $0.5Q^{2/3}$ and $0.5Q^{1/3}$). M'' is an integer parameter. The domain $-6 \leq M'' \leq 18$ might have relevance regarding modeling. Q denotes the magnitude of charge, in units of $|q_e|$. Regarding the rightmost four columns, items show (conceptually) $\log_{10}(m_{\text{particle}}/m_e)$ and the name of an elementary fermion. For each Calc-D case, no particle pertains. For the Calc-F case, no particle pertains. Regarding $Q = 1/2$ (\dagger), no elementary particles associate with this column. Each number in this column equals the average of the numbers in the $Q = 2/3$ column and the $Q = 1/3$ column. The notion of geometric mean pertains regarding the mass of the $Q = 2/3$ particle and the mass of the $Q = 1/3$ particle. Regarding Calc-F, a formula for $m(M'', M')$ calculates this number. Table XII shows the formula.

“Flavour” (0.5C)	“Flavour” (0.5Q)	M''	$Q = 1$ (0.5C)	$Q = 2/3$ ($0.5Q^{2/3}$)	$Q = 1/2$ (\dagger)	$Q = 1/3$ ($0.5Q^{1/3}$)
1 (Electron)	1 (Electron)	0	0.00 Electron	0.66 Up	0.80 (Calc-D)	0.94 Down
-	2 (Mu)	1	1.23 (Calc-F)	3.36 Charm	2.83 (Calc-D)	2.29 Strange
2 (Mu)	3 (Tau)	2	2.32 Muon	5.52 Top	4.72 (Calc-D)	3.92 Bottom
3 (Tau)	-	3	3.54 Tau	-	-	-

Table XII: Equations that underlie aspects of table XI and possible extensions to table XI. This table shows equations that may pertain regarding all known charged elementary fermions, the known 0.5N neutrinos, and the suggested 0.5R arcs. This table does not discuss the suggested 0.5N' heavy neutrinos.

Topic	Note
Preliminary calculation	$\beta' = m_\tau/m_e$ - Defines β' . m_τ equals the mass of the tau particle (which is a charged lepton). m_e equals the mass of the electron. $(4/3) \times (\beta^2)^6 = ((q_e)^2/(4\pi\epsilon_0))/(G_N(m_e)^2)$ - Defines β . The right-hand side of the equation is the ratio of the electrostatic repulsion between two electrons to the gravitational attraction between the two electrons. The ratio does not depend on the distance between the two electrons. $\beta \approx 3477.1891 \pm 0.0226$ - This number results from data and the formula that defines β . The standard deviation reflects the standard deviation for G_N , the gravitational constant. $\beta' = \beta$ - We posit this equation. $m_{\tau, \text{calculated}} \approx 1776.8400 \pm 0.0115 \text{ MeV}/c^2$ - This number results from data and from $\beta' = \beta$.
Main calculation	<p>These calculations produce numbers that table XI shows.</p> $M' = 3Q$. $m(M'', M') = m_e \times (\beta^{1/3})^{M'' + (j_{M''}')d''} \times (\alpha^{-1/4})^{g(M') \cdot (1 + M'') + j_{M'}' d'(M'')}$. $\alpha = ((q_e)^2/(4\pi\epsilon_0))/(\hbar c)$ - Expression for α , the fine-structure constant. $j_{M''}' = 0, +1, 0, -1$ for, respectively, $M'' \bmod 3 = 0, 1, 3/2, 2$; with $3/2 \bmod 3 \equiv 3/2$. $d'' = (2 - (\log(m_\mu/m_e)/\log(\beta^{1/3}))) \approx 3.840679 \times 10^{-2}$. $g(M') = 0, 3/2, 3/2, 3/2, 3/2$, for, respectively, $M' = 3, 2, 3/2, 1, 0$. $j_{M'}' = 0, -1, 0, +1, +3$ for, respectively, $M' = 3, 2, 3/2, 1, 0$. $d'(0) \sim 0.324$, $d'(1) \sim -1.062$, $d'(2) \sim -1.509$ - Based on attempting to fit data.
Extensions	
Arcs	<p>Our work suggests (but does not require) some specific masses for the three arc particles.</p> $M' = 0$ - This notion comports with the notion - for arcs - that $Q = 0$. $m(M'', 0) = m(M'', 1) \cdot (m(M'', 1)/m(M'', 2))$ - This essay assumes this equation. $m(0, 0)c^2 \approx 10.7 \text{ MeV}$, $m(1, 0)c^2 \approx 6.8 \text{ MeV}$, $m(2, 0)c^2 \approx 102 \text{ MeV}$. $m(-1, 3) = m(-1, 3/2)$ - Comports with the equation underlying the main calculation. $m(M'', 3/2)$ pertains - regarding elementary fermions - for $M'' \leq -1$. We suggest masses for the three 0.5N neutrinos.
$M'' = -1$ Assumption Neutrinos	<p>People suggest - based on observations - that the sum of the three neutrino rest energies is at least approximately $0.06\text{eV}/c^2$ and not more than approximately $0.12\text{eV}/c^2$. People suggest that astrophysics data suggests that at least two distinct masses pertain. We offer two possibilities.</p> <ul style="list-style-type: none"> $mc^2 = m(-4, 3/2)c^2 \approx 3.4 \times 10^{-2} \text{ eV}$ pertains for each of the three neutrinos. Aspects related to 3G and 4G might account for the perception that more than one mass must pertain. (Interactions between 2G and an elementary fermion conserve the mass of the elementary fermion, but do not necessarily conserve the flavour of the elementary fermion. Interactions between 3G and an elementary fermion conserve the flavour of the elementary fermion, but do not necessarily conserve the mass of the elementary fermion. Interactions between 4G and an elementary fermion do not necessarily conserve the mass of the elementary fermion or the flavour of the elementary fermion.) $mc^2 = m(-4, 3/2)c^2 \approx 3.4 \times 10^{-2} \text{ eV}$ pertains for each of two neutrinos. For one neutrino, one of $m(-6, 3/2)c^2 \approx 4.2 \times 10^{-6} \text{ eV}$ and $m(-5, 3/2)c^2 \approx 4.4 \times 10^{-4} \text{ eV}$ might pertain.
$M'' = 18$ Monopole properties	$((q_e)^2/(4\pi\epsilon_0))/(G_N(m(18, 3))^2) = 4/3$ - Based on the formula underlying the main calculation. A force strength factor of 4 seems to associate with 1G1 and a force strength factor of 3 seems to associate with 2G2. (See, above, the equation $(4/3) \times (\beta^2)^6 = ((q_e)^2/(4\pi\epsilon_0))/(G_N(m_e)^2)$.) Possibly, other force strength factors would be 2 for 3G3, 1 for 4G4, and zero for 5G5. Possibly, the notion of zero force strength regarding 5G5 associates with a lack of relevance for (and a lack of monopole properties that would associate with) solutions $\Sigma G \Sigma$ for which $\Sigma \geq 5$.

means for matching flavours and masses for charged leptons do not match means for matching flavours and masses for quarks. For charged leptons, isomer zero does not have a charged lepton that associates with $M'' = 1$ and does have a charged lepton that associates with $M'' = 3$. We assume that - for each l - a charged lepton associates with each of $M'' = 3l + 0$, $M'' = 3l + 2$, and $M'' = 3l + 3$.

We assume that - for each isomer l such that $1 \leq l \leq 5$ - the charged-lepton flavour that associates with $M'' = 3(l) + 0$ equals the flavour that associates with the isomer $l - 1$ charged lepton that associates with the same value of M'' and - thus - with $M'' = 3(l - 1) + 3$. We assume that across the six isomers, one cyclical order pertains regarding flavours for charged leptons.

Table XIII shows, for isomers of charged elementary fermions, matches between masses and flavours.

Beyond the topic of flavours, the topic of handedness exists. Ordinary matter associates with left-handedness. We suggest the possibility (but we do not necessarily require) that isomers 0, 2, and 4 associate with left-handedness and that isomers 1, 3, and 5 associate with right-handedness.

We prepare to discuss the evolution of stuff that associates with each isomer.

We associate the symbol OMSE with all SRI elementary particles and all ELF elementary particles except 0.5N' and 0.5R elementary particles. OMSE abbreviates the three-element phrase ordinary-matter-similar elementary particles. We associate the symbol DMAI with the 0.5N' and 0.5R elementary particles. DMAI abbreviates the five-word phrase dark matter regarding all isomers. DMAI associates with the notion that - regarding isomer zero - these particles measure as

Table XIII: Matches between masses and flavours, for isomers of charged elementary fermions. The symbol $0.5Q$ denotes the pair $0.5Q^{1/3}$ and $0.5Q^{2/3}$.

Isomer	M'' (0.5Q)	Respective flavours (0.5Q)	M'' (0.5C)	Respective flavours (0.5C)
0	0, 1, 2	1,2,3	0, 2, 3	1,2,3
1	3, 4, 5	1,2,3	3, 5, 6	3,1,2
2	6, 7, 8	1,2,3	6, 8, 9	2,3,1
3	9, 10, 11	1,2,3	9, 11, 12	1,2,3
4	12, 13, 14	1,2,3	12, 14, 15	3,1,2
5	15, 16, 17	1,2,3	15, 17, 18	2,3,1

being dark matter and do not measure as being ordinary matter.

We use the three-element term isomer *number* stuff to denote objects (including SRI elementary particles, ELF elementary particles, hadron-like particles, clumps of stuff, and stars) that associate with the isomer *number* set of other-than-SG elementary particles.

We discuss - for each isomer - the evolution of the stuff associating with that isomer.

0.5R particles model as interlaced. (See table VI.) We suggest that - at least after the inflationary epoch - 0.5R-based stuff consists of hadron-like particles. Each 0.5R-based-stuff hadron-like particle includes gluons and at least two arcs. (We de-emphasize discussing roles that jay bosons might play.) Our work does not suggest an extent to which 0.5R-based stuff might form primordial black holes.

0.5N' particles model as free. (See table VI.)

Regarding each one of the six isomers, we suggest that stuff made from DMAI behaves as cold dark matter.

We discuss the evolution of isomer 1, 2, 4, and 5 OMSE stuff.

Here, we use the two-word term alt isomer to designate an isomer other than isomer zero and isomer three.

A charged baryon that includes exactly three flavour 3 quarks is more massive than the counterpart zero-charge baryon that includes exactly three flavour 3 quarks. (For example, two tops and a bottom have a larger total mass than do one top and two bottoms.) Alt isomer flavour 3 charged leptons are less massive than isomer zero flavour 3 charged leptons. When flavour 3 quark states are much populated (and based on interactions mediated by W bosons), the alt isomer converts more charged baryons to zero-charge baryons than does isomer zero. Eventually, in the alt isomer, interactions that entangle multiple W bosons result in the alt isomer having more neutrons and fewer protons than does isomer zero. The sum of the mass of a proton and the mass of an alt isomer flavour 1 charged lepton exceeds the mass of a neutron. Compared to isomer zero neutrons, alt isomer neutrons scarcely decay. The IGM (or, intergalactic medium) that associates with the alt isomer scarcely interacts with itself via electromagnetism.

We discuss the evolution of isomer three OMSE stuff.

The following possibilities pertain. The evolution of isomer three OMSE stuff parallels the evolution of ordinary matter (or, isomer zero OMSE stuff). The evolution of isomer three OMSE stuff does not parallel the evolution of ordinary matter (or, isomer zero OMSE stuff). The second possibility might associate with - for example - a difference in handedness - with respect to charged leptons or with respect to W bosons - between isomer three and isomer zero. (Perhaps note that - regarding SRI and ELF elementary particles - 7 is a member of the CURR Γ for, and only for, the W boson and charged leptons. See table V and table VI.)

C. Explanations and suggestions regarding the rate of expansion of the universe

This unit suggests eras in the rate of expansion of the universe and suggests mechanisms that associate with the eras.

Concordance cosmology points to three eras in the so-called rate of expansion of the universe. The eras feature, respectively, rapid expansion; continued expansion, with the rate of expansion decreasing; and continued expansion, with the rate of expansion increasing.

This essay suggests using the notion of eras regarding the separating of clumps - that, today, people would consider to be large - of stuff. Examples of such clumps might include galaxy clusters and possibly even larger clumps.

We provide perspective. As two objects move away from each other, the relative effect of an RDF $\Xi^{-(k+1)}$ component decreases compared to the effect of an RDF Ξ^{-k} component. One might associate the two-word phrase time period with a time range in which an RDF Ξ^{-l} component provides dominant effects. Assuming that objects move away from each other and that one time period associates with $\Xi^{-(k+1)}$ and another time period associates with Ξ^{-k} , the time period that associates with $\Xi^{-(k+1)}$ comes before the time period that associates with Ξ^{-k} . Two smaller objects (such as galaxies) transit similar time periods more quickly than do two larger objects (such as galaxy clusters).

We consider a thought experiment. We consider two objects that are some distance apart. We consider doubling linear dimensions - that is doubling the distance between the objects and doubling the diameters of the objects - while maintaining a constant mass per unit volume - of each object. A PROP RDF Ξ^{-6}

force after the doubling of linear dimensions equals the PROP RDF Ξ^{-6} force before the doubling of linear dimensions.

Table XIV discusses eras in the rate of separating of large clumps. (For discussion about the possible inflationary epoch, see references [48] and [10]. For data and discussion about the two multi-billion-years eras, see references [49], [50], [51], and [52]. For data and discussion about the possibility that concordance cosmology modeling underestimates - for the second multi-billion-years era - increases in the rate of separation, see references [17], [18], [19], [20], [53], [54], and [55].)

Table XV suggests details regarding eras to which table XIV alludes.

Before inflation, boson PROP solutions for which $\Sigma \geq 2$ and $8 \in \Gamma$ associate with dominant long-range effects. The word interlaced associates with those PROP solutions. After inflation, compared to boson PROP solutions for which $\Sigma \geq 2$ and $8 \notin \Gamma$, boson PROP solutions for which $\Sigma \geq 2$ and $8 \in \Gamma$ do not associate with significant long-range effects. Boson PROP solutions for which $\Sigma = 0$ and $8 \in \Gamma$ continue to associate with relevant effects, but just on small (distance) scales. The word free associates with PROP solutions for which $\Sigma \geq 2$ and $8 \notin \Gamma$. Perhaps, a notion of a phase change - for the universe - pertains regarding times around inflation.

D. Explanation regarding data about large-scale clumping

This unit suggests an explanation for the notion that concordance cosmology overestimates large-scale clumping of matter.

People suggest that concordance cosmology modeling overestimates large-scale clumping of matter - ordinary matter and dark matter. (For data and discussion, see references [56], [57], [58], and [20].)

We suggest that concordance cosmology modeling associates with a repulsive component - $2(1)2^4$ - of gravity. Our modeling suggests that $2(2)2^4$ pertains. (That is, for each instance of $2G2^4$, a reach of two isomers pertains.) The additional (compared to concordance cosmology modeling) repulsion might explain the overestimating - of clumping, per concordance cosmology modeling - that people suggest.

E. Explanation regarding data about interactions between neighboring galaxies

This unit suggests an explanation for the notion that concordance cosmology might not account for some observations about effects - within individual galaxies - of the gravity associated with nearby galaxies.

People suggest that concordance cosmology modeling might not account for some observations about effects - within individual galaxies - of the gravity associated with nearby galaxies. (For data and discussion, see reference [23].)

We suggest that concordance cosmology modeling associates with a repulsive component - $2(1)2^4$ - of gravity. Our modeling suggests that $2(2)2^4$ pertains. The additional (compared to concordance cosmology modeling) repulsion might explain at least some aspects of the observations that people report.

F. Suggestions regarding galaxy formation

This unit suggests that our notions regarding long-range interactions and our specifications for dark matter combine to provide insight regarding galaxy formation.

We suggest aspects regarding events leading to the formation of a galaxy.

Reference [34] suggests that galaxies form around early clumps of stuff. The reference associates the word halo with such clumps.

Table XIV suggests that single-isomer stuff - such as stuff that features $0.5R$ particles - forms during an era in which the PROP solution $2G1^23^816$ - which associates with attraction - dominates regarding prototype large clumps. Smaller-scale clumps might form before larger-scale clumps. Effects that associate with the PROP solution $2G1^23$ - which is attractive might contribute to the formation of smaller-scale clumps. The reach that associates with $2G1^23$ is one.

We suggest that each one of many early halos associates with one isomer. We associate with such early halos the three-element term one-isomer original clump. We know of no reason why the six isomers would not form such clumps approximately equally. (Concordance cosmology suggests that known elementary fermions form early in the era in which effects that associate with $2G1^23$ dominate regarding large-scale phenomena. Per remarks above, we suggest that that era starts after the formation of halos. Also, we suggest that our scenario does not depend on whether or when $0.5N'$ particles first form.)

Table XVI discusses suggestions regarding the formation and early evolution of a galaxy for which a notion of a one-isomer original clump pertains.

Presumably, some galaxies form based on two or more clumps, for which all of the clumps associate with just one isomer. Presumably, some galaxies form based on two or more clumps, for which some clumps associate with isomers that are not the same as the isomers that associate with some other clumps.

G. Explanations regarding dark matter data

This unit shows that our specifications for dark matter seem to explain data about various phenomena.

Table XVII extends aspects of table VII and provides explanations for data regarding phenomena that might involve dark matter. (Reference [32] influenced our choice of a time range to associate with the word early. References [59] and [60] provide data about collisions. Regarding early galaxies, see table XVI.

Table XIV: Eras regarding the rate of separating of large clumps. The rightmost four columns suggest eras. Subsequent rows associate with later eras. Regarding eras that would precede inflation, our modeling points to the possibility for the two eras that the table discusses. Concordance cosmology suggests inflation and the next two eras. Regarding inflation, people hypothesize this era. Possibly, no direct evidence exists for this era. Observations support the notions of the two billions-of-years eras. Concordance cosmology seems to underestimate the rate of separating for the more recent of the two eras. NYN denotes not yet named. The word speculative denotes that our modeling does not necessarily address the relevant duration or era. The leftmost four columns describe phenomena that our modeling suggests as noteworthy causes for the eras. An RDF associates with the PROP solution. Generally, a noteworthy cause associates with notions of acceleration. Generally, an era associates with a range of velocities. A noteworthy cause may gain prominence before an era starts.

Force	PROP solution	RDF	ρ_I	Rate of separating	Era name	Duration	Comment
Attractive	2G1'2'3'8'16	Ξ^{-6}	6	Is negative	NYN	Speculative	Isomers of 0.5R and 1J form
Repulsive	0G1'3'4'8'16	-	1	Turns positive	NYN	Speculative	Isomers of 0I form
Repulsive	2G1'2'3'4x	Ξ^{-5}	1	Increases rapidly	Inflation	Fraction of a second	Inflatons (or, 0I) participate
Attractive	2G1'2'3	Ξ^{-4}	1	Decreases	NYN	Billions of years	Most known particles form
Repulsive	2G2'4	Ξ^{-3}	2	Increases	NYN	Billions of years	-
Attractive	2G2	Ξ^{-2}	6	Would decrease	NYN	-	Speculative

Table XV: Details regarding eras regarding the rate of separating of large clumps. Table XIV discusses the eras. Table XV de-emphasizes the notion that 0.5N' elementary fermions might form before the beginning of the first multi-billion-years era. Each of the symbols 2G1'2'3'4x and 2G1'2'3'4y denotes either or both of 2G1'2'3'4v and 2G1'2'3'4w. A thought experiment regarding PROP RDF Ξ^{-6} forces might associate with reasons not to pursue - based on aspects that this essay suggests - modeling for possible eras before the eras that this table discusses.

Rate of separating	Note
Is negative	<p>Possibility: 2G1'2'3'8'16 and its compacting of "some form of energy" lead to conditions suitable for the universe to form and evolve.</p> <p>Possibility: The value of six for ρ_I associates with setting up a system for which roughly equal creation of isomers pertains.</p> <p>Possibility: The following interactions might characterize this era. For each interaction, the net circular polarization for each of before and after the interaction might be zero. Presumably, the formation of gluons (or, 1(1)U) could associate with the formation of arcs (or, 0.5(1)R)).</p> <ul style="list-style-type: none"> • 2(6)G1'2'3'8'16 + 2(6)G1'2'3'8'16 \rightarrow 0.5(1)R + 0.5(1)R. • 2(6)G1'2'3'8'16 + 2(6)G1'2'3'8'16 \rightarrow 1(1)J + 1(1)J. <p>Possibility: The six isomers of 0.5R populate approximately equally.</p>
Turns positive	<p>Possibility: Some clumps of 0.5R stuff serve - eventually - as seeds for galaxies.</p> <p>0G1'3'4'8 associates with the 1J (or, jay) boson. The jay boson associates with the notion of Pauli repulsion.</p> <p>Possibility: 1J bosons stop the implosion of stuff that is significantly 0.5R particles.</p> <p>Possibility: The following interaction might characterize this era. Here, the net circular polarization for each of before and after the interaction would be two. 1(1)J + 1(1)J \rightarrow 2(1)G1'2'3'4x + 0(1)I.</p> <p>Possibility: The six isomers of 0I populate approximately equally.</p>
Increases rapidly	<p>Some concordance cosmology modeling suggests that inflatons provide the major component of stuff.</p> <p>Possibility: The following interaction might characterize this era. Here, the net circular polarization for each of before and after the interaction would be two. 0(1)I + 2(1)G1'2'3'4x \rightarrow 0(1)I + 2(1)G1'2'3'4y.</p>
Decreases	<p>Some concordance cosmology modeling suggests that the first significant appearance of known elementary particles occurs early in this era.</p>
Increases	<p>Our modeling suggests a reason why concordance cosmology modeling seems to underestimate increases in the rate of separation. People, when using modeling based on general relativity, might try to extend the use of an equation of state that works well regarding the first multi-billion-years era. Our modeling notes that, in effect, ρ_I equals one (based on 2(1)G1'2'3) pertains during the first multi-billion-years era and ρ_I equals two (based on 2(2)G2'4) pertains during the second multi-billion-years era. Our modeling suggests, in effect, changing (regarding the stress-energy tensor) the main repulsive component of gravity to double (compared to concordance cosmology modeling) the would-be effects of a supposed (based on concordance cosmology modeling) 2(1)G2'4.</p>
Would decrease	<p>This essay does not try to explore the possibility that (or to estimate a time at which) a transition - for the largest observable objects - from repulsion based on 2G2'4 to attraction based on 2G2 might occur.</p>

Table XVI: Eras and other information regarding the evolution of a galaxy for which a notion of a one-isomer original clump pertains. The table suggests eras, with subsequent rows associating with later eras. The leftmost four columns in the table describe a component of 2G that is a noteworthy cause for the era. The table associates with a scenario in which a galaxy forms based on one original clump and does not significantly collide with other galaxies. The galaxy might retain some stuff that associates with the repelled isomer.

Force	PROP solution	RDF	ρ_I	Comment
Attractive	2G1'2'3	Ξ^{-4}	1	A one-isomer original clump forms.
Repulsive	2G2'4	Ξ^{-3}	2	The original clump repels (some) stuff that associates with the isomer that associates with the original clump and (most) stuff that associates with one other isomer.
Attractive	2G2	Ξ^{-2}	6	The original clump attracts stuff that associates with the four not-repelled isomers and stuff that associates with the isomer that associates with the original clump.

Extrapolating from results that references [34] and [61] discuss regarding ultrafaint dwarf galaxies that orbit the Milky Way galaxy might suggest that the universe contains many DM:OM 1 : 0⁺ later galaxies. References [62] and [63] provide data and discussion regarding the undetected object.)

We discuss other observations regarding galaxy evolution.

People report the notion that some galaxies seem to stop forming stars. (See reference [64] and reference [65].) Such so-called quenching might take place within three billion years after the big Bang, might associate with a relative lack of hydrogen atoms, and might pertain to half of a certain type of galaxy. (See reference [65].) Reference [66] discusses a galaxy that seems to have stopped accruing both ordinary matter and dark matter about four billion years after the Big Bang.

We suggest that the quenching and the stopping of accruing nearby matter might associate with repulsion that associates with 2(2)G2'4. Quenching might associate with galaxies for which original clumps featured isomer zero stuff or isomer three stuff. The galaxy that reference [66] discusses might (or might not) associate with the notion of significant presence early on of one of isomers zero and three, one of isomers one and four, and one of isomers two and five. Such early presences might associate with a later lack of nearby stuff for the galaxy to accrue.

IV. DISCUSSION

This unit discusses some possibilities regarding specific possible elementary particles and regarding dark matter and discusses relationships between our modeling and other modeling.

We associate the two-word term extant modeling with modeling - including Standard Model modeling and concordance cosmology modeling - that other people developed.

A. Possibilities regarding elementary particles

This unit discusses possibilities regarding properties and existence of hypothesized elementary particles.

This essay seemingly does not suggest an elementary boson that would associate with notions of an axion. Observations that people might associate with effects of axions might associate instead with the difference between our notion of 1(6)G1'2'4 and extant modeling notions that might associate with notions of 1(1)G1'2'4.

This essay does not suggest an elementary particle that would associate with notions of a magnetic monopole. Table I and table III seem not to suggest a 1G interaction with a monopole other than an electric monopole.

This essay does not necessarily suggest precluding notions of elementary particles that would associate with PROP solutions for which $\log_4(l_{max})$ is an integer that exceeds two.

B. Possibilities regarding constraints regarding dark matter

This unit discusses the extent to which our notion of dark matter comports with constraints - about the nature of dark matter - that people associate with data about dark matter or with outputs from extant models that have bases in assumptions about dark matter.

We discuss aspects related to cosmological models.

Reference [34] summarizes some thinking about constraints on dark matter and about notions of dark matter. The article notes that so-called CDM (or, cold dark matter) might comport well with various models. Some models associate with the one-element term Λ CDM. The article notes that people have yet to determine directly whether nature includes CDM stuff. The article notes that people consider that notions of self-interacting dark matter might be appropriate regarding nature. We use the acronym SIDM for the three-element phrase self-interacting dark matter.

We suggest that isomer zero 0.5R-based stuff, isomer zero 0.5N' stuff, and all stuff associating with isomers one, two, four, and five might comport with notions of CDM. We suggest that the remaining dark matter stuff (or, isomer three OMSE stuff) might associate with notions of SIDM.

We suggest that our notion of dark matter is not necessarily incompatible with constraints - that have bases in cosmological models - on dark matter.

We discuss aspects related to collisions of pairs of galaxy clusters. In particular we discuss the Bullet Cluster collision of two galaxy clusters. (Reference [67] discusses the Bullet Cluster.) Presumably, observations regarding other such collisions might pertain.

Observations suggest two general types of trajectories for stuff. Most dark matter - from either one of the clusters - exits the collision with trajectories consistent with having interacted just gravitationally with the other cluster. Also, ordinary matter stars - from the same cluster - exit the collision with trajectories consistent with having interacted just gravitationally with the other cluster. However, ordinary matter IGM (or, intergalactic medium) - from the one cluster - lags behind the cluster's ordinary matter stars and dark matter. That ordinary matter IGM interacted electromagnetically with the other cluster's ordinary matter IGM, as well as gravitationally with the other cluster.

We are uncertain as to the extent to which observational data suggests that essentially no dark matter lags the bulk of dark matter. Should the fraction of lagging dark matter be too small, we might need to reconsider the extent to which isomer three differs from isomer one. For one example, possibly isomer three has right-handed elementary fermions but interactions involving such fermions model as retaining aspects of left-handed-centric interactions that associate with isomer zero. For another example, possibly isomer three does not evolve adequately similarly to isomer zero. To the extent that isomer three adequately differs

Table XVII: Explanations for dark matter ratios and other phenomena. DM:OM denotes a ratio of dark matter effects to ordinary matter effects. Regarding densities of the universe, we posit that DMAI stuff associates with the plus in DM:OM $5^+ : 1$. Stuff - other than DMAI stuff - that associates with isomers one through five associates with the five in DM:OM $5^+ : 1$. Regarding some galaxy clusters, we posit that galaxy clusters (that have not collided with other galaxy clusters) associate with DM:OM ratios that are similar to DM:OM ratios for densities of the universe. Regarding some absorption of CMB, we posit that isomer three hydrogen-like atoms account for the half of the absorption for which isomer zero (or, ordinary matter) hydrogen atoms do not account. (See table IV.) The reach of an instance of $1G3^4$ is two isomers. (See table VIII.) Presumably, the $4 \in \Gamma$ associates with spin and, hence, with hyperfine transitions. Regarding galaxies, the notion of early associates with observations that pertain to galaxies that people associate with (or, would, if people could detect the galaxies, associate with) high redshifts. High might associate with $z > 7$ and possibly with smaller values of z . Here, z denotes redshift. Regarding galaxies for which the DM:OM is $5^+ : 1$, DMAI stuff associates with the plus in DM:OM $5^+ : 1$. Regarding the undetected object, we posit that the undetected object is a clump of dark matter. Data regarding stellar stream GD-1 suggest the possibility of effects from a yet-to-be-detected non-ordinary-matter clump with a mass of 10^6 to 10^8 solar masses.

Aspect	Comment
DM:OM $5^+ : 1$ - Densities of the universe	-
DM:OM $5^+ : 1$ - Some galaxy clusters	-
DM:OM $1 : 1$ - Some absorption of CMB	Half of the absorption might be via DM.
DM:OM $0^+ : 1$ - Some early galaxies	For each of some early galaxies, each original clump associates with isomer zero. Later, the galaxy might accumulate DM.
DM:OM $0^+ : 1$ - Some later galaxies	Some early DM:OM $0^+ : 1$ galaxies survive (without significant collisions with galaxies for which DM:OM is not $0^+ : 1$) until later times.
DM:OM $1 : 0^+$ - Some early galaxies	For each of some early galaxies, each original clump associates with an isomer other than isomer zero. Early on, the density of OM stars is small and people do not detect the galaxy. Later, the galaxy might accumulate enough OM to be visible.
DM:OM $1 : 0^+$ - Some later galaxies	Some early DM:OM $1 : 0^+$ galaxies survive (without significant collisions with galaxies for which the DM:OM is not $1 : 0^+$) until later times.
DM:OM $\sim 4 : 1$ - Some later galaxies	An original clump might associate with any isomer other than isomer three. (Isomer three repels OM stuff.) Eventually, the galaxy accumulates enough stuff (that does not associate with the isomer that associates with the original clump) to have a DM:OM ratio that is somewhat near $4 : 1$.
DM:OM $5^+ : 1$ - Many later galaxies	Over time, galaxies collide. Collisions tend to result in the formation of larger galaxies that include much stuff from smaller galaxies. A later galaxy that results from enough collisions is likely to associate with somewhat similar - across the six isomers - amounts of stuff from originally one- (or few-) isomer original clump galaxies.
Undetected object in the Milky Way galaxy	The object might be DM.

from or does not evolve similarly to isomer zero, our explanation regarding CMB depletion via - in part - interactions with dark matter hydrogen-like atoms might be inaccurate (for example, based on an inaccurate estimate of the number of isomer three hydrogen-like atoms).

We suggest that our notion of dark matter is not necessarily incompatible with constraints - that have bases in observations of collisions of galaxy clusters - on dark matter.

C. Possibilities regarding interactions involving the jay boson

This unit discusses interactions that involve jay bosons.

We discuss interactions - that involve jay bosons - that might take place before or during inflation.

We consider a thought experiment in which two jay bosons move in parallel, interact, and produce one aye boson plus something else. Here, we assume that conservation of angular momentum pertains and that one can de-emphasize orbital angular momentum. We consider two cases. In the first case, the

two jay bosons have the same (one of either right or left) circular polarization. Conservation of angular momentum allows an outgoing combination of one 2G particle and one 0I particle. Conservation of angular momentum precludes producing one 1G particle and one 0I particle. In the second case, one jay boson has left circular polarization and the other jay boson has right circular polarization. Conservation of angular momentum allows the production of two 0I particles and prohibits the production of one 1G particle and one 0I particle.

The two cases might comport with notions that gravitation can be significant during inflation and that electromagnetism might become significant essentially only after inflation. The three cases might comport with the notion that jay bosons form before aye bosons form. (See table XIV.)

We discuss so-called Pauli crystals.

Reference [68] and reference [69] report detection of Pauli crystals. We suggest that modeling based on the notion of jay bosons might help explain relevant phenomena.

We discuss the notion of Pauli repulsion.

Extant modeling includes the notion that two identi-

cal fermions cannot occupy the same state. Regarding extant modeling, one notion is that repulsion between identical fermions associates with overlaps of wave functions. Another notion features wave functions that are anti-symmetric with respect to the exchange of two identical fermions.

Our modeling might be compatible with such aspects of extant modeling and, yet, not necessitate - for kinematics modeling - the use of wave functions. Modeling based on jay bosons might suffice.

Modeling based on jay bosons might suggest that prevention of two identical fermions from occupying the same state might associate with, in effect, trying to change aspects related to the fermions. Notions of changing a spin orientation or, for elementary fermions, changing a flavour might pertain.

We discuss a possible discrepancy - regarding energy levels in positronium - between extant modeling and observation.

Reference [70] and reference [71] discuss the transition - between two states of positronium - characterized by the expression $2^3S_1 \rightarrow 2^3P_0$. People discuss the energy that associates with the transition. Four standard deviations below the nominal observed value of energy approximately equals four standard deviations above the nominal value of energy that extant modeling suggests.

Perhaps, notions regarding jay bosons extend to explain the might-be discrepancy regarding positronium. (For example, thinking of extant modeling based on the Dirac equation, a notion of virtual charge exchange or virtual flavour change might pertain.)

To the extent that extant modeling does not suffice, modeling related to the jay boson might close the gap between observation and modeling.

D. Possibilities regarding right-handed W bosons

This unit discusses the possibility that our modeling suggests approximately the fraction of W bosons that are right-handed that result from decays of top quarks.

We consider a thought experiment.

Aspects related to table XII and table XIII suggest values of calculated masses that do not associate with masses of known or suggested elementary particles. For example, our modeling does not suggest that $m(5,3)$ associates with the inertial mass of an isomer one charged lepton. However, perhaps such masses associate with some measurable aspects of nature. For charged leptons and $0 \leq l \leq 4$ and $0 \leq l_2 \leq 2$, $m(3(l+1) + l_2, 3) = \beta m(3(l+0) + l_2, 3)$. One might conjecture that isomer zero observations of some aspects of isomer one phenomena associate with notions of non-inertial masses that are β times the inertial masses for isomer zero elementary particles (and that are β times inertial masses for the counterpart isomer one elementary particles).

Furthermore, isomer one might associate with right-handedness in a manner similar to the association with

isomer zero of left-handedness. (See discussion related to table XIII.)

Reference [72] discusses the fraction of decays - of ordinary matter top quarks for which the decay products include W bosons - that might produce right-handed W bosons. The fraction, f_+ , is 3.6×10^{-4} . Reference [8] provides a confidence level of 90 percent that the rest energy of a W_R (or, right-handed W boson) exceeds 715 GeV. (Perhaps, note also, reference [73].)

Based on notions of scaling that might calculate non-inertial mass-like quantities, one might conjecture that our modeling suggests that $f_+ \sim e^{(\beta^{-1})} - 1 \approx \beta^{-1} \approx 2.9 \times 10^{-4}$. This estimate might not be incompatible with results that reference [72] discusses. A notion of $m_{\text{non-inertial}, W_R \text{ isomer one}} c^2 = \beta m_W c^2 \approx 2.8 \times 10^5 \text{ GeV}$ might pertain. Here, the notion of non-inertial mass might associate with inferences that associate with 1G or 1W and do not associate directly with 2G.

E. Relationships between our modeling and other modeling

This unit discusses relationships between our modeling and types of extant models.

We suggest perspective about our modeling.

Our modeling features two bases.

One basis unifies and decomposes aspects of electromagnetism and gravity. For each of electromagnetism and gravity, the decomposition seems to associate well with properties - of objects - that people can measure and that extant modeling features. For electromagnetism, the properties include charge and magnetic moment. For gravity, the properties include mass and moments of inertia.

One basis features isomers of elementary particles that do not intermediate long-range interactions and instances of components of long-range interactions.

Our modeling extends from the two bases to do the following. Match all known elementary particles and suggest possible other elementary particles. Describe dark matter. Point to explanations for data that extant modeling seems not to explain. Suggest data that might associate with future observations.

We suggest the possibility that the notion that our work explains phenomena that extant modeling does not explain points to usefulness for our work. Some explanations have quantitative bases but - to the extent that this essay uses the explanations - are qualitative. Presumably, people can use simulations to help verify or refute some of our qualitative explanations. At least one explanation - regarding depletion of CMB - is quantitative. Generally, we know of no cases in which our suggestions that address possible gaps between extant modeling and observations point - compared to extant modeling - in a wrong direction regarding closing gaps.

We suggest that the small set of bases for our modeling, the breadth of seemingly coherent scope of our

modeling, the simplicity of Diophantine mathematics, and the possible ease of integrating our modeling and extant modeling point to possible usefulness for our work.

We suggest approximate relationships between aspects of our modeling and types of kinematics models.

Table XVIII discusses approximate relationships between aspects of our modeling and types of kinematics models.

We discuss the case of QED. In particular, we explore modeling regarding anomalous magnetic moments for 0.5C elementary particles (or, charged leptons).

Table IV associates two CURR solutions with the relevant (or, 3G1'2) PROP solution. The 3G1'6'8 CURR solution includes 6 in Γ . We posit that the strength of 3G1'6'8 can vary based on mass, but not based on charge. The 3G2'7'8 CURR solution includes 7 in Γ . We posit that the strength of 3G2'7'8 can vary based on charge, but not based on mass.

We explore the notion that one can express a_{cl} , the anomalous magnetic moment for the cl charged lepton, via the expression $a_7 + a_6 t_{cl}$. Here, a_7 might vary only with charge and would be a constant with respect to a choice between $cl = e$ (for the electron), $cl = \mu$ (for the muon), and $cl = \tau$ (for the tau). Here, a_6 might vary only with mass. We assume that t_{cl} is $(\log(m_{cl}/m_e))^2$. (Perhaps, compare with table XI and with aspects - that comport with squares of properties - of table XII. The notion of squares of properties might associate with notions of self-interactions.) Based on data that reference [8] provides regarding the electron and the muon, we calculate a_7 and a_6 . Then, we calculate a value, $a_{\tau,PM}$, for a_τ . Here, PM denotes the two-word term proposed modeling. PM associates with our work. Reference [74] provides, based on Standard Model modeling techniques, a first-order result - which we call $a_{\tau,SM}$ - for a_τ . Here, SM denotes the two-word term Standard Model. The value of $a_{\tau,PM}$ results in a value of $(a_{\tau,PM} - a_{\tau,SM})/a_{\tau,SM}$ of approximately -0.00228 . Each of $a_{\tau,PM}$ and $a_{\tau,SM}$ comports with experimental data that reference [8] provides.

We discuss aspects related to the value of two for reach (or, ρ_I).

This essay suggests that $\rho_I = 2$ pertains for some components of long-range interactions (or, LRI). This essay suggests that the notion of $\rho_I = 2$ might have importance regarding explaining data regarding the following - some depletion of CMB, large-scale clumping, the recent multi-billion-years era of increases regarding the rate of separation of large clumps, gravitational interactions between neighboring galaxies, and galaxy formation.

V. CONCLUSION

Our work suggests augmentations - to physics modeling - that produce results that may provide progress

regarding the following physics opportunities. Complete the list of elementary particles. Describe dark matter. Explain ratios of dark matter to ordinary matter. Explain eras in the history of the universe. Link properties of objects. Interrelate physics models.

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Table XVIII: Approximate relationships between aspects of our modeling and types of kinematics models. k_I denotes a number - one or six - of isomers. Extant modeling associates with $k_I = 1$. Each of some of the items in the symbol column does not associate with an extant modeling notion or symbol. Regarding NEW, the row in the table assumes that electromagnetism is not relevant. Regarding EAM, 1G1 associates with non-moving charge and 1G7'8 associates with moving charge. No other components have relevance. This modeling associates with charge-current 4-vectors and with Maxwell's equations. Regarding GNR, the notions that l_{max} PROP is 4 (or is 16) and l_{max} CURR is 8 (or is 32) might not pertain exactly regarding general relativity. Also regarding GNR and the possible case of $k_I = 6$, the notion of geodesic motion would not necessarily pertain. For example, consider an isomer zero star and three possible planets. The planets are identical except that one planet associates with isomer zero, one planet associates with isomer one, and one planet associates with isomer three. The planets start out on identical orbits. We consider six cases. First assume that - out of the 2G' components - only 2(6)G2 pertains. The planets traverse identical orbits. Second, assume that 2(2)G2'4 associates with nonzero effects. The isomer one planet orbits as if 2G2'4 does not pertain. The isomer zero planet and the isomer three planet traverse a trajectory that differs from the trajectory that is common for the previous four cases. QCD associates with 1U, $0.5Q^{1/3}$, and $0.5Q^{2/3}$. We suggest the possibility that QCD also associates with 0.5R. WIP associates with 1W and 1Z. PEP associates with 1J, each 0.5Φ family, and fermions that are not elementary particles.

Kinematics modeling	Range of Σ	l_{max} PROP	l_{max} CURR	k_I	Symbol
Newtonian gravity	2	2	2	1	NEW
Special relativity	1,2	2	8	1	SPR
Electrostatics	1	1	1	1	EST
Electromagnetism	1	1	8	1	EAM
Quantum electrodynamics	1,3	2	8	1	QED
General relativity	2	4 (or 16)	8 (or 32)	1	GNR
Quantum chromodynamics	0	16	32	1	QCD
Weak-interaction phenomena	0	4	8	1	WIP
Pauli repulsion phenomena	0	16	32	1	PEP
Suggested by our modeling	0,1,2,3,4	16	32	6	PRM

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