THE BIPOLAR LOGIC OF FEEDFORWARD AND FEEDBACK CIRCUITS

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ABSTRACT

Instead of the conventional 0 and 1 values, bipolar reasoning uses -1, 0, +1 to describe double-sided judgements in which neutral elements are halfway between positive and negative evaluations (e.g., “uncertain” lies between “impossible” and “totally sure”). We discuss the state-of-the-art in bipolar logics and recall two medieval forerunners, i.e., William of Ockham and Nicholas of Autrecourt, who embodied a bipolar mode of thought that is eminently modern. Starting from the trivial observation that “once a wheat sheaf is sealed and tied up, the packed down straws display the same orientation”, we work up a new theory of the bipolar nature of networks, suggesting that orthodromic (i.e., feedforward, bottom-up) projections might be functionally coupled with antidromic (i.e., feedback, top-down) projections via the mathematical apparatus of presheaves/globular sets. When an entrained oscillation such as a neuronal spike propagates from A to B, changes in B might lead to changes in A, providing unexpected antidromic effects. Our account points towards the methodological feasibility of novel neural networks in which message feedback is guaranteed by backpropagation mechanisms endowed in the same feedforward circuits. Bottom-up/top-down transmission at various coarse-grained network levels provides fresh insights in far-flung scientific fields such as object persistence, memory reinforcement, visual recognition, Bayesian inferential circuits and multidimensional activity of the brain. Implying that axonal stimulation by external sources might backpropagate and modify neuronal electric oscillations, our theory also suggests testable previsions concerning the optimal location of transcranial magnetic stimulation’s coils in patients affected by drug-resistant epilepsy.

Keywords: bipolar fuzzy set; decision making problem; non-classical logic; scholasticism; transcranial magnetic stimulation.

INTRODUCTION

The notion of bipolarity relies on the presence of two kinds of entities of diametrically opposed nature, i.e., a positive and a negative entity (Amgoud et al., 2008). Bipolar reasoning considers a positive side, a negative side and their close relationships (Zhang 1998). Whereas the logical space of classical logics is based on the dyadic relationships 0,1 which does not support computational assessment of unknown propositions, the logical space of bipolar logics (BL) is based on the triadic relationships -1, 0, +1 which supports computational coexistence of the two bipolar variables. Double-sided judgements, widely utilized in biophysical and social phenomena such as top-down/bottom-up, cooperation/competition and interests/conflict of interests, have been especially used in argumentation frameworks (Amgoud et al., 2008; Cayrol and Lagasquie-Schiex 2013). Countless subfields of engineering, economic, social disciplines, decision making, etc. display uncertainty problems that cannot be portrayed through the unipolar 0,1 information of the standard two-valued logic. In turn, multipolar information allows the description of systems in which information is either incomplete, or not fully reliable, or vague and fuzzy, or contradictory, or deficient (Dalkili and Demirtas, 2021; Tozzi and Peters, 2020).

Here we focus on a peculiar kind of bipolarity, i.e., the bipolarity between feedforward and feedback projections in biological and artificial neural networks. The orthodromic (i.e., feedforward, bottom-up) and antidromic (i.e., feedback, top-down) brain activity has been tackled via mathematical/topological weapons such as algebraic topology, near set theory and category theory (Tozzi et al., 2017a; Peters et al., 2017). Nevertheless, these theoretical approaches to nervous functions are constrained by soaring abstraction and vagueness in defining regular structures and nearness relations, thus preventing the formulation of sharp experimental previsions (Wolski 2013). The main goal of this paper is to overcome these difficulties, showing how the mathematical apparatus of presheaves /globular sets provides theoretical examples of applications in feedforward/feedback bipolar networks.

We will proceed as follows. At first, we will discuss the different types of bipolar judgement and their subtending logics. Then, we will tackle the past and the future developments of bipolar logics. At first, we will provide an historical overview of two medieval precursors of bipolar logics, i.e., as William of Ockham and Nicholas of Autrecourt. Next, we will show how double-sided judgements can be very useful to tackle current and next-to-come methodological issues such as orthodromic/antidromic feedback networks. Then, we will provide a mathematical

description of presheaves/sheaves/globular sets, using their intricate mathematical apparatus to analyze orthodromic and antidromic activities of the nervous system. After that, we will provide operational and theoretical examples of feasible applications in different disciplines, suggesting experimental previsions that are testable with the current available technologies.

THE PRESENT: STATE-OF-THE-ART IN BIPOLAR LOGICS

To tackle uncertainty problems, various types of logical approaches involving bipolar judgements have been introduced. Here we describe two of the most successful logical accounts of bipolar reasoning, i.e., the bipolar fuzzy set and the paraconsistent logics.

Various Bipolar Logics (BL) have been introduced in the very last decades, such as, e.g., the bipolar fuzzy set theory for bipolar reasoning (Zhang 1998) and its variants/extensions (Akram 2013; Chen et al., 2014; Zhang 2017; Lee J-G and Hur, 2019; Riaz and Tehrim, 2020; Dalkili and Demirtas, 2021). Fuzzy concrete domains bring together qualitative and quantitative reasoning and are therefore appropriate for representing bipolarity and imprecision (Hudelot et al, 2008). A bipolar scale is a totally ordered set with an interior element 0 termed “neutral” which divides positive from negative evaluations (Dubois and Prade H. 2008). Different types of bipolar judgements have been described in the literature of bipolar fuzzy sets. Here we, following Dubois and Prade (2008), will provide an effort to unify these types in a broader, simpler and more manageable framework. We will consider two cases of bivariate polarity: symmetric and asymmetric.

- 1) The first and simplest case involves **symmetric bipolarity**, in which the negative is the inverse mirror of the positive and the positive is the inverse mirror of the negative. The values can be ordered into a unidimensional linear list, i.e., a straight line equipped with two extremities -1 and +1 and a middle point 0 (**Table 1, upper part**). Unlike two-valued logics where 0 and 1 stands for true and false, here -1 stands for false, +1 stands for true and 0 stands for half-true. The two mirror images are comparable and mutually exclusive. Their relation is bipolar, reflexive, symmetric, transitive crisp or fuzzy, commutative, associative, distributive and monotonic (Palladino and Palladino, 2007).
- 2) The second case involves **asymmetric bipolarity**, in which the totally ordered negative scale is distinct from the totally ordered positive scale, since they separately evaluate positive and negative information. In this case two dimensions (i.e., the positive and the negative) are involved, such that the values can be distributed within a two-dimensional space (**Table 1, lower part**). Note that the neutral level is located at one extremity of the scale in both the positive and the negative scale. To make an example, the negative pole of a bipolar relation is the unipolar relation formed with all the negative and zero values in the negative side of the bipolar relation. The asymmetric bipolarity is the union of the positive and the negative scales and no associativity may occur for finite scales. Unlike the symmetric case, each bipolar comparison operator leads to a different resolution. Two general cases of asymmetric bipolarity can be described:
 - a) A case characterized by the coexistence of positive and negative poles with duality relation. An item is judged according to two independent evaluations performed on the same data, one in favour and one against. This is the case of formal argumentation and paraconsistent logics. In turn, in uncertainty theory characterized by incomplete information, the weak evaluation of an item may be the complement of the positive evaluation of the contrary item.
 - b) A “strongest” case, characterized by the fact that the negative and the positive parts of the information are of different nature and just partially related through minimum consistency requirements. In this case, the positive image is not a mirror image of the negative image, since the negative and the positive pieces of information cannot be aggregated using the standard principles of information merging. For example, two separate possibility distributions can be disjoint in knowledge representation. A situation that is not impossible is not necessarily guaranteed as possible if not explicitly permitted; in turn, what is guaranteed possible should not be impossible. When observations accumulate, background knowledge increases, positive information aggregates disjunctively and negative information aggregates conjunctively.

Table 2 shows that the three values -1, 0, +1 have a different meaning according to the scientific field under evaluation, such as, e.g., probability theory, 3-valued logic with neutral truth value, uncertainty and boundary problems, information theory, and so on.

Paraconsistent logics (henceforward PCLs) suggest that not every contradiction entails arbitrary absurdities (Jaśkowski 1948; da Costa 1974). PCLs reject two tenets of the classical Aristotelian logic: the principle of explosion (henceforward PEX) and the law of noncontradiction (henceforward LNC).

PEX asserts that *ex falso quodlibet*, i.e., from falsehood -or from contradiction- anything follows:

$$p \wedge \neg p \rightarrow q$$

i.e., for any statements p and q , if p and not- p are both true, then it logically follows that q is true. Once a contradiction has been asserted, both a positive statement and its own negation can be proven true. The inference from p and $\neg p$ to an arbitrary conclusion leads to inconsistencies in formal axiomatic systems, making it impossible to distinguish truth from falsehood. To avoid the danger of meaningless statements implicit in PEX, the law of noncontradiction (LNC) comes to the rescue:

$$\neg(p \wedge \neg p)$$

Since contradictories cannot be simultaneously true, LNC removes the inconsistency from the beginning and neutralizes PEX.

PCLs, together with other non-classical logics such as relevance and minimal logic (Johansson 1973), suggest that the classical *ex falso quodlibet* is incorrect (Priest and Routley, 1989) since the inference from p and $\neg p$ to an arbitrary conclusion does not hold (Varzi 2000). A logic is paraconsistent iff it is not the case for all sentences p, q that $p, \neg p \vdash q$. This means that we cannot be certain of the truth of any proposition which is irreducible to PEX (Copleston 1974).

As an example, during argumentation one states “I said this”, while another “no. I said that”, even though something is clear to both. Everything which appears to be true is true according to the two contenders, even at the risk to predicate the truth and the falsity of the same thing in the logical discourse. People taking part in the discussion may disagree while being (self-) consistent, using some vague terms either purposefully or unintentionally (Ciuciura 2013). Conclusions are not really demonstrated, since the opposite conclusions can be drawn from the evidence of each one of the contenders. This approach is very similar to what suggested by the PCLs’ non-adjunctive discursive logic: “from the fact that a thesis P and a thesis Q have been advanced in a discourse it does not follow that the thesis $P \vee Q$ has been advanced, because it may happen that P and Q have been advanced by different persons” (Jaśkowski 1999).

Nowadays BL and PCLs are largely used in disparate fields such as bipolar reasoning, bipolar clustering, conflict resolution coordination, equilibrium relations, protocol design for cooperation and coordination in multiagent decision analysis (Zhang 1998), multivalued social graphs, multivalued social network models (Chen et al., 2014), representation of preferences in artificial intelligence or in cognitive psychology (Amgoud et al., 2008), inclusion degrees of a rough set (Chen et al., 2014), evaluation of brain imaging (Hudelot et al., 2008). Da Silva and Livet (2008) proposed that bivariate bipolarity of emotions (negative, neutral, positive) might provide the tools for emotional reasoning that helps to discriminate between deduction, induction and abduction. Also, BT has been used in decision making. e.g., when a group of friends chooses the movie to watch, when the products of a magazine are ordered, when three professors vote against a candidate applying for an academic job (Chen et al., 2014). BL has also been utilized to create algorithms for multiple attribute decision making problems in planning a children hospital (Hashim et al., 2018). The last, but not the least, bipolarity is involved in choices concerning spatial relations such as above and below, left and right, etc (Hudelot et al, 2008).

	<h2 style="text-align: center;">SYMMETRIC BIPOLARITY</h2>
General framework of bipolar relationships	negative	neutral	positive
Three-valued logic with neutral truth value	false	half-true	true
Probability theory	impossible	uncertain	totally sure

	<h2 style="text-align: center;">ASYMMETRIC BIPOLARITY</h2>
Argumentation in favour and against ill-known events	in disfavour	doubtful	in favour
Plausibility	impossible	non-impossible	guaranteed possible
Belief	believe $\neg p$	Believe neither p nor $\neg p$, due to our ignorance	believe p
Confidence in an event	background knowledge: constraints to how the world behaves (e.g., physical laws and common sense decreasing the possible worlds)	uncertain	data: empirical observation of the world leads to positive token of support
When observations accumulate	negative information: conjunctive aggregation	missing information	positive information: disjunctive aggregation
Bipolar reasoning in information theory	negative information: what is considered impossible	uncertain	positive information: what is granted as possible
Incomplete information	possible	doubtful	necessary
Rough set theory	elements not belonging to the subset A	elements with unknown membership	elements belonging to the subset A

Table 1. Examples of symmetric and asymmetric bipolarity in different scientific fields. Text modified from: Dubois and Prade, 2008.

THE PAST: MEDIEVAL FORERUNNERS OF BIPOLAR LOGICS

Middle-ages thinkers provided accounts of oppositions, antitheses and contrasts that, rather astonishingly, match up well with the most advanced logical bipolar theories. One of the foremost descriptions of the concept of contraries can be found in the Medieval Quodlibetal Questions by William of Ockham (Ockham, 1991). He states that the name “contraries” can be taken in different ways, since every distinction can be either real, formal or of reason (Ockham, Quodlibetal 1.3). The opposition can be either (5.17, 5.24):

- I. Between real things outside the mind,
- II. Or between noncomplex (i.e., simple) and complex (i.e., combined) signs of things.

Noncomplex and complex signs of contraries are real mental qualities which correspond to opposition of reason. Thoughts are expressed by mental words which are signs of true things (1.6). The names (either mental, spoken or written predicable) are conceptually distinct from one another, although they are not really diverse (3.2).

The mental opposition between noncomplex signs has four modes (5.17):

- 1) Contraries. They are signs of those things that signify positively and affirmatively, since no negation should be posited in their definition.
 - a) Some contraries cannot be predicated on the same thing simultaneously, but only successively. To make an example, “a human being is white” and “a human being is black” may occur one after the other but cannot exist at the same moment in the same subject. The same God does not have the power to produce infinitely many things simultaneously, rather He virtually and simultaneously contains infinitely many effects that can be produced successively (3.1).
 - b) Other contraries cannot be truly predicated either at the same time or successively (e.g., “whiteness” and “blackness”).
- 2) Privative. One of the signs signifies positively (e.g., the disposition), while the other signs signifies both what is positively/affirmatively and what is opposite privatively (e.g., the privation). In the nominal definition of privation, a negation precedes the disposition that is opposed to it, so that the privation signifies the disposition negatively. For example, blindness is not a real thing in the eye, since “blind” is a human being (or an eye) that does not have the vision it ought to have by nature. Therefore, some signs signify by positing and affirming, while others signify by taking away and denying. While which is not an entity is altogether nothing, privations are truly something, since the latter are either concepts of the mind, or spoken things, or things outside the soul.
- 3) Relative. There are relative names that cannot be predicated of the same thing in the same respect relative to the same thing (e.g., equal and unequal, similar and dissimilar).
- 4) Contradictories. These signs cannot be predicated of the same thing at the same time and are such that one of them signifies something affirmatively (e.g., human being) and the other precisely the same thing negatively while indeterminately signifying anything affirmatively (e.g., nonhuman being).

The mental opposition between complex signs has three modes:

- 1) The signs have the same subject and predicate, but one is affirmative and the other is negative (e.g., “Socrates is an animal” and “Socrates is not an animal”). One must be universal and the other particular, or both must be particular.
- 2) Contrary complex signs have the same subject and predicate, but one is a universal affirmative and the other a universal negative. The terms must be taken significatively, otherwise they are not opposed. For example, “the expression *every human being* is a common term with a universal sign” and “the expression *no human being* is a common term with a universal sign” are both true, hence are not contraries.
- 3) Virtual (not formal) contradictories. These complex signs must have the same subject and cannot be both true at the same time. “No animal is running” and “some human being is running” are not opposed as contraries or contradictories, since they have not the same subject. However, they are opposed since “some human being is running” implies the contradictory “no animal is running”. Therefore, subalternates (e.g., “every A is B” and “some A is B”) and subcontraries (e.g., “some A is B” and “some A is not B”) are not opposed.

Furthermore, Ockham makes subtle distinctions between slightly different cases involving the linguistic concepts of “false” and “true”, suggesting how a proposition can be first true and afterward false, or vice versa (Ockham, Quodlibetal 4.4, 5.17):

- a) “Socrates talks”. This proposition is true in the instant in which Socrates talks. If Socrates does not talk, this proposition is false.
- b) “Socrates will talk tomorrow”. This proposition is invariably true from always until tomorrow, but it will not be true anymore by tomorrow. Before tomorrow nothing can modify the truth value of this proposition, since it is true now and will never become false before tomorrow. The truth of this proposition does not have a beginning, but has an end, i.e., tomorrow. A proposition cannot change from being true to being false before the time at which the revealed thing is to occur. If a proposition is true at a given time, then it has been true at every past time. Indeed, every true present-tense proposition has a necessary past-tense counterpart.
- c) “Socrates will not talk tomorrow”. This proposition is invariably false from always until tomorrow, but is invariably true after tomorrow. The truth of this proposition does have a beginning (i.e., tomorrow), but lacks an end.
- d) “Caesar once existed and does not exist now” is true and false in this instant. Caesar both exists and does not exist in this place, since before or after cannot exist into a single instant (4.11).
- e) “The priest on the altar starts to utter: *hoc est...*”. Until the priest does not fully utter *hoc est Corpus Christi*, the body of Christ does not exist yet in the Eucharist. The priest’s utterance in the middle of the phrase is true as mental proposition, but it is not yet true as spoken proposition, because the words *Corpus Christi* have not yet been pronounced (2.19). Reality itself can be different at the beginning and at the end of an utterance. A mental proposition is true at the beginning and at the end since it exists all at once as a whole (*ens permanens*). In turn, a spoken proposition exists just successively and not all at once (*ens successivum*), so that it is true at the end and not at the beginning (2.19). For further details, see Tozzi (2021).

True and false predicated of a whole proposition are distinct in the way that a term is distinct from a proposition of which is not part: e.g., true is not a part of the proposition “Socrates is wise”, even though the term true is truly predicated of this proposition. In turn, the (abstract names) “truth” and “falsity” are not things distinct from true and the false propositions. The names “truth” and “falsity” are connotative names, not absolute names (a connotative name signifies one thing primarily and another secondarily, e.g., “white” signifies the white subject primarily and the whiteness itself secondarily). If a proposition has truth, then it holds for absolute terms, but not for connotative terms. This means that truth can be falsity in the same way as a proposition can be false. But the proposition “truth is falsity” should be denied, in just the way that “a true proposition is a false proposition” should be denied. If no proposition exists, a human being is still truly an animal, while “a human being is an animal” is not true.

Note that the notion of bipolarity in Ockham’s accounts of contraries frequently displays asymmetric bipolarity. To provide a few examples, Ockham describes privation (blind) and disposition (sight), dissimilar and similar, beliefs and evident knowledge (1.1), nonhuman being and human being, “which is not an entity is altogether nothing” and “privations are truly something”. Ockham’s account of contraries is related with bipolar logics also in another subtle way. In touch with the three values -1, 0, +1 of bipolar reasoning, Ockham suggests that there is a priority between the opposition of the two contrary extremes and the opposition of the middle and of the contrary extremes, since an extreme is opposed to an extreme prior to the middle’s being opposed to an extreme (3.7).

In conclusion, Ockham’s foresight concerning the modern bipolar logics can be summarized by his own proposition “if two things are not sufficient to truth, a third is needed” (1.8).

Nicholas Of Autrecourt: A forerunner of paraconsistent logics. Paraconsistent logics (henceforward PCLs), first put forward in 1910-1929 by the Russians Vasil’ev and Oreov, were formalized around the 50s by the Polish Jaśkowski (1948) and the Brazilian da Costa (1974). Leaving apart the logical treatment tackled by the French Parvipontians Adam Balsham and William of Soissons in the 12th-century (Priest 2011), PEX has been accepted as self-evident throughout the centuries. However, the critique to PEX (and to the law of noncontradiction LNC) put forward by PCLs has been anticipated in writings dating back to the first half of the 14th century. The 50-year-old Nicholas of Autrecourt, a Master of Arts in the Parisian arts faculty and *baccalaureus* and *licentiatus* in theology, was condemned in 1347 by the papal court in Avignon after allegations of false teaching. About 60 articles of either “false, or erroneous, suspect, presumptuous, dangerous, or heretic” propositions were collected and publicly recanted in Avignone and Paris (Kaluza 1995).

Here we focus on Autrecourt’s condemned articles concerning LNC, which he calls the *primum principium*. The Latin articles reproduced here are numbered according to the Chartularium Universitatis Parisiensis (Deniflis and Chatelain, 1891):

Item, dixi, proh dolor, in primo principio³ quando legi Sententias et in epistola secunda et sexta, quas scripsi contra Bernardum⁴, quod ex eo quod una res est, non potest evidenter evidētia deducta ex primo principio inferri, quod alia res sit (5). — *Falsam et revocandam*.

Item, dixi in locis predictis quod ex eo, quod una res est, non potest evidenter inferri quod alia res non sit (6). — *Falsam et revocandam*.

Item, dixi in locis predictis quod ex eo, quod una res non est, non potest evidenter evidētia deducta ex primo principio inferri, quod alia res non sit (8). — *Falsam et revocandam*.

Item, dixi locis predictis quod ex eo, quod una res non est, non potest evidenter inferri, quod alia res sit (7). — *Falsam et revocandam, quanquam aliqui dicant eam veram esse*.

It roughly translates to:

Starting from LNC, it follows that:

- From the fact that one thing is, it cannot be inferred the fact that another thing is (5)
- From the fact that one thing is, it cannot be inferred that another thing is not (6)
- From the fact that one thing is not, it cannot be inferred that another thing is (7)
- From the fact that one thing is not, it cannot be inferred that another thing is not (8)''

Therefore, Autrecourt suggests that the following logical propositions cannot be inferred from LNC:

$p \rightarrow q$
 $p \rightarrow \neg q$
 $\neg p \rightarrow \neg q$
 $\neg p \rightarrow q$

This means that:

$\neg(p \vee \neg p) \rightarrow \neg(q \vee \neg q)$
 Consequently,
 $\neg(p \wedge \neg p) \rightarrow \neg(q \vee \neg q)$

Since $p \wedge \neg p \rightarrow q$ is false, Autrecourt rejects PEX, in touch with paraconsistent logics (Fitch 2013).

The logical and epistemological strength of LNC in Autrecourt's writings is matter of controversy. It has been suggested that he bears a dual interpretation of the certitude provided by LNC (Groarke 1984). On the one hand, Autrecourt holds the conventional scholastic view that LNC stands for the foremost principle ensuring firm ground upon which to build the truth. For example, in the second letter to Bernard, he writes that there is no other certitude but the certitude of LNC, except for the certitudes of faith (De Rijk 1994). He asserts that every syllogism can be reduced to the truth of LNC through immediate or mediate operations. On the other hand, Autrecourt raises elsewhere questions about the steadiness of his own belief in LNC. The last chapters of the Tractatus (conceivably written later) appear to somewhat deny the absolute certitude previously guaranteed by LNC. The most contentious paragraphs (O'Donnell 1939, pg. 237, chapter: *an omne illud quod apparet sit*) are reproduced here:

Octava conclusio est quod non omnes propositiones possunt ostendi esse evidentes per primum principium; nam cum primo principio oportet assumere aliquid quod contineat conclusionem in actu; minor non, quemadmodum major et principium primum, videlicet in potentia.

Nona conclusio est, quod aliquis posset per consuetudinem aut alias resilire, ut non assentiret indubitanter ei, quod est primum principium esse verum utpote, si sic nutriretur, ut sibi diceretur, quod est aliquod agens omnipotens, quod potest facere contrarium et quod non debet ipsum movere evidētia, quia oppositum potest stare cum ea ut declararetur in multis.

Decima conclusio est quod non posset non assentire quin esset sibi clarum et evidens.

Here an objection is raised, put forward in Giles’ Letter to Nicholas too: the fact that God can make miracles seems to deny the existence of LNC (De Rijk 1994). God can annihilate every proposition by miracle, and, if He should do so, LNC would not be valid, since it would not even exist. This means that the possibility to distinguish, e.g., between the propositions “God exists” and “God does not exist” fades away. If contradictories signify the same, the firmness of LNC is undermined beyond repair. Autrecourt suggests elsewhere in the Treatise that the only thing which guarantees the certitude of the first principles is our firm belief that we know them clearly and evidently. In Autrecourt’s opinion, the knowledge of appearances cannot rely on LNC, since what is known through LNC is analytic and devoid of factual information (Beuchot, 2005).

In touch with the critique to PNC, it is noteworthy that a variant of PCLs, i.e., the relevance logic, asserts that antecedent and consequent of implications must be relevantly related. It is remarkable that this correlation is explicitly required by the same Autrecourt, who plainly states that the antecedent and the subsequent must share their contents. The consequent cannot be inferred from a doubtful antecedent, since antecedent and consequent obey the principle of identity and must be identical. According to Autrecourt, a priori demonstrations hold just when the identity $A=A$ occurs between the consequent and the antecedent (dal Pra 1952; Maccagnolo 1953). In Autrecourt the cognitive borders between logical and psychological approaches are rather fuzzy. The dichotomy lies in the fact that he sometimes stresses empiricism, another times Revelation. For example, he writes in a letter to Giles that it is unclear whether the consequent is equal to the antecedent, since it is not discernible whether a perceived thing is simple and indivisible (De Rijk 1994). Only God comprehends all the things in a single, simple apprehension. The strongest way to provide a bridge between mental contents and their supposed noumenal correlates is to build a probability, though *probable* in the mere sense of being of worthier assent than its contradictory opposite (McDermott 1973). In conclusion, Autrecourt’s account is in touch with paraconsistent logics: they both aim to remove not just PEX, but also LNC. Also, it is noteworthy that the unfeasibility of $p \wedge \neg p \rightarrow q$ is frequently observed also in the argumentation of natural language, e.g., when two men hold opposite conclusions.

THE FUTURE: BACKPROPAGATION OF ANTIDROMIC PATHS

In the sequel, we will examine directional paths equipped with different degrees of feedforward and feedback trajectories. Feedforward/feedback trajectories are well fit to describe a widespread array of neural networks, either artificial or natural. To make an example, the study of biofeedback circuits in the nervous system never ceases to surprise. It has been recently discovered that fine-tune limb movements in mice are modulated not only by signals from the motor system, but also by tactile feedbacks targeting the brainstem cuneate nucleus (Conner et al., 2021). Various examples of directional pathways exhibiting bipolarity are illustrated in **Table 2**.

Table 2. Different types of directional paths.

Note that all the triads in **Table 2** display the two-dimensional asymmetric type of bivariate polarity. In terms of bipolar fuzzy sets, the membership degree $[0,+1]$ implies that the element satisfies the property (in our case, the occurrence of feedforward trajectories), while $[-1,0]$ implies that the element satisfies the implicit counter-property (in our case, the occurrence of feedback trajectories) (Chen et al., 2014). See **Figure 1A**. In turn, the membership degree $[0,0]$ implies that the path does exhibit neither orthodromic nor antidromic features (in our case, the path stands still). Further, the membership degree $[-1,+1]$ implies a bidirectional path that exhibits both orthodromic and antidromic features (in our case, the path simultaneously takes two opposite trajectories) (**Figure 1A**).

Researchers are still trying to figure out how the brain might perform backpropagation to solve credit assignment problems. Here the pre-sheaf category from category theory and algebraic topology comes into play (Bredon, 1997), providing a structure rich enough to convey a feasible explanation of orthodromic and antidromic paths in neural networks. As an example of presheaf, think to a sheaf of wheat, sealed and tied up with twine supply: in this case, the countless straws with various orientation are compacted in well-ordered tiny sections, where they display coherent behavior and matching description (**Figure 1B**). The same happens in tractography, the recently introduced neuroanatomic technique which allows the identification of well-defined anatomical white matter structures resembling presheaves (**Figure 1C**).

We will describe here the mathematical apparatus that will be used in the sequel to describe the feedforward and backpropagation in neural circuits. We will talk at first about composition of functions. When working with a specific input value (named either $x \in X$, or another letter), the composition of functions (denoted by the symbol $\circ$):

$$g \circ f(x) \quad (1)$$

is defined by:

$$(g \circ f)(x) = g(f(x)) \quad (2)$$

In other words, take the function $f(x)$ and put it in place of x in the function g , so that the output of f is used as the input of g (Bergman, 2011). In case of two functions:

$$f: X \rightarrow Y \text{ and } g: Y \rightarrow Z, \quad (3)$$

then $g \circ f$ is a function from X to Z , so that:

$$g \circ f: X \rightarrow Z \quad (4)$$

This is where globular sets and presheaves come into play. A globular set is a geometric shape for higher structures, standing to simplicial sets as globes to simplices. The category of globes is a category whose objects are natural numbers, and whose morphisms are generated from:

$$\sigma n: [n] \rightarrow [n+1] \quad (5)$$

$$\tau n: [n] \rightarrow [n+1] \quad (6)$$

A globular set is a presheaf on the category of globes (Kashiwara and Schapira, 1994; Bredon, 1997), i.e., a particular kind of contravariant functor which helps to capture sections:

$$gSet := PSh(G) \quad (7)$$

To achieve a presheaf F in the topological space W (standing for our universe of discourse), two requirements are needed: gluability and uniqueness (Street, 2000; Simpson, 2011).

Once these requirements are satisfied, for each open $X \subset W$, the restriction map is a homeomorphism

$$\rho_{w,X}: F(W) \rightarrow F(X) \quad (8)$$

such that $\rho_{w,w}$ is an identity.

Therefore, in terms of the category of globes, the globular set $X \in gSet$ is a sequence of sets $X_0, X_1, X_2, \dots, X_n$, i.e., a collection of the set of n -globes $\{X_n\}_{n \in \mathbb{N}} \subset W$, that is equipped with pairs of functions:

$$\{s_n, t_n: X_{n+1} \rightarrow X_n\}_{n \in \mathbb{N}} \quad (9)$$

such that the following globular identities hold:

$$s_n \circ s_{n+1} = s_n \circ t_{n+1} \quad (10)$$

and

$$t_n \circ s_{n+1} = t_n \circ t_{n+1} \quad (11)$$

where s and t stand for the two sections termed, respectively, source and target (**Figure 2A**).

Note that X_n may also stand for directed edges at level n .

A further constraint is required: no reflexive globular sets do exist, or, in other words, do not exist degeneracy maps going back in the opposite direction $X_1 \rightarrow X_2$ (Simpson, 2011).

Our next step will be to use globular sets and presheaves to sketch a neural model able to describe the nervous orthodromic and antidromic activities taking place in cortical tissues. Sheaf theory is subtle enough that seemingly similar constructions yield vastly different invariants. Slight changes in the definition of stalks, restriction maps and the underlying topological base spaces lead to fully different results. Since our aim is just to provide network theorists with a novel methodological tool for the evaluation of bipolar networks, the explicit construction of the proper presheaf is beyond the scope of this paper.

Consider a brain area W , equipped with two functions termed s and t (**Figure 2A**). They diverge everywhere, apart from a cortical subarea (i.e., a subset of W) which encompasses X_1 and X_2 , with $X_1 \cap X_2$. If in subarea $X_1 \cap X_2$ the above-mentioned constraints are satisfied and the topological concept of presheaf holds, the two functions s and t have an unusual behavior, i.e., they do not diverge inside the subsets X_1 and X_2 , rather display coherence, equality and matching description. Globular sets and presheaf theory require that the composition of functions occurring inside the subset $X_1 \cap X_2$ does not diverge as it does in other subsets. Furthermore, the functions s_2 and t_2 in X_2 precede the functions s_1 and t_1 in X_1 , so that the output of the cortical subarea X_2 is used as the input of the cortical subarea X_1 . This means that changes in functions occurring in X_2 lead to corresponding antidromic changes in the orthodromic functions occurring in X_1 . In general terms, we can state that source and targets in X_1 follow sources and targets in X_2 . Also, there is an equivalence between:

- 1) X_1 sources that follow X_2 sources and X_1 sources that follow X_2 targets.
- 2) X_1 targets that follow X_2 sources and X_1 targets that follow X_2 targets.
- 3) X_1 targets that follow X_2 sources and X_1 targets that follow X_2 targets.

In brief, X_2 functions (source and target) point towards and modify X_1 functions (source and target). Therefore, X_2 controls X_1 , and not vice versa.

Summarizing, the brain activity assessed in terms of presheaves/globular sets might explain different features of neural functions (**Figures 2B-2C**):

- 1) It elucidates why brain oscillations equipped with different spike frequency and amplitude tend to converge and coalesce towards single functional areas, where overlapping and matching synchronized frequencies/amplitudes contribute to keep the signal steady (**Figure 2B**).
- 2) It suggests that a hierarchical oscillatory activation occurs inside single functional areas, where the oscillations are constrained to follow strict temporal sequences.
- 3) It explains the occurrence of feedforward flows and backpropagation: when an entrained oscillation propagates from A to B , changes in B frequency leads to changes in A frequency (**Figure 2C**). This means that orthodromic (i.e., feedforward, bottom-up) projections are functionally coupled with antidromic (i.e., feedback, top-down) projections.

This last, counterintuitive observation has foremost implications in the study of the brain activity and of artificial neural networks, as we will see in the sequel.

Figure 1A. Bipolar relationships between propagation and backpropagation. See text for further details. **Figures 1B-1C.** Examples of presheaves-like structures in the real world. **Figure 1B:** a wheat sheaf displays a cross-section in which the straws, rather being scattered, are remarkably close one each other. **Figure 1C:** diffusion MRI maps white matter bundles and assesses their structural integrity. Note that the white matter bundles display close analogy with the wheat straws depicted in Figure 1B. Modified from: <http://brain.labsolver.org/>.

PRESHEAVES/GLOBULAR SETS FOR THE EVALUATION OF FEEDFORWARD/FEEDBACK TRAJECTORIES: TESTABLE PREVISIONS

In the previous sections, we showed how the mathematical apparatus of globular sets/presheaves might provide a well-grounded mathematical framework to assess network phenomena characterized by propagation and backpropagation. In this Section we are going to provide testable experimental and theoretical examples of feasible applications in neuroscience and network theory.

Orthodromic and antidromic currents in the human brain. Neuronal feedforward pathways are associated with enhanced gamma (30 to 70 Hz) oscillations, whereas feedback projections, which account for a large portion of all connections between neurons in the thalamocortical system, selectively modulate alpha/low-beta (8 to 15 Hz) oscillations (Mejias et al., 2016; Michalareas et al., 2016; Sikkens et al., 2019). The interplay between hierarchically organized feedforward (bottom-up) and feedback (top-down) signaling involves intricate dynamics across multiple (intralaminar, interlaminar, interareal, and whole cortex) scales involved in attentional processes, predictive coding, executive control (Mejias et al., 2016). Information propagates in opposite directions through visual areas during bottom-up intake of current stimuli and top-down processes of memory or attention (Bastos et al., 2015; Zheng and Colgin, 2015; Michalareas et al., 2016). In working memory and its volitional control, an interplay occurs between network gamma oscillations in superficial cortical layers and alpha/beta in deep cortical layers (Miller et al., 2018). To provide an example, **Figure 3A** describes the case of prediction and errors signals in the Bayesian brain. In terms of presheaf theory, a change in frequency occurring in the cortical location X_2 leads also to a change in frequency in the cortical location X_1 . This means that the brain electric frequency might be controlled antidromically, so that distal frequencies might be able to modify proximal frequencies. Therefore, presheaf models would help to describe the neurophysiology of brain stimulation via external sources such as transcranial magnetic stimulation (henceforward TMS), i.e., a focal electric brain stimulation induced by powerful magnetic fields (Chung et al., 2016; Jan et al., 2017; Kohli and Casson, 2019).

TMS protocols are divided into single-pulse, paired-pulse and repetitive TMS (henceforward rTMS). Extracellular electromagnetic stimulations produce electrical fields in both myelinated and unmyelinated axons (Rattay, 1986; Basser and Roth, 1991), generating hyperpolarized regions in cell membranes (Rattay et al., 2012). Intracranial cortical stimulation induces selective activation across cell types and layers (Komarov et al., 2019), while external brain gamma stimulation modulates motor performance/GABA-A-ergic interneuron activity (Guerra et al., 2018; McDermott et al., 2018). The origins, cellular mechanisms and functional effects of rTMS are unidentified (Benali et al., 2011). At present, there is no systematic method to predict which neural elements will be activated by a given stimulation regime (Komarov et al., 2019), since spike initiation site may change for short and long pulses (Rattay et al., 2012) and every individual has different rhythmic firing pattern in response to burst stimulation (Chung et al., 2019).

Our model sheds new light on the mechanisms of rTMS, suggesting that axonal stimulations from external sources might modify the frequency charges of the involved neurons. **Figures 3B-E** illustrate a theoretical example of a neuron firing at, say, 30 Hz. If we apply on the axon an external 10-Hz current, our model predicts that the neuron will synchronize its output frequency towards 10-Hz. Due to the mathematical constraints of presheaves, we suggest that the 10-Hz external axonal stimulation might be able to antidromically backpropagate from the neuronal axon to the body. Despite our scarce knowledge of antidromic oscillations' effects on neural information processing, experimental data point towards the possibility that external axon modulation affects frequency encoding in neurons, before and after it is conveyed. Axons are not only information carriers, rather their trunks are endowed with ionotropic and metabotropic receptors for transmitters/neuromodulators (DeMaegd and Stein, 2018). Neuromodulator effects on axons generate ectopic spiking, characterized by ion potentials backpropagating towards different sites independent of incoming synaptic events. Increasing ectopic spike frequency leads to delayed peripheral bursts, reductions in spike number/burst duration, and modifications in sensory firing frequency. Computational models demonstrate that antidromic action potentials can alter sensory encoding in neurons with slow ionic conductances (DeMaegd and Stein, 2018). During γ -tACS over the right temporal lobe, Santamecchi et al. (2019) observed functional connectivity of bilateral temporal lobes after stimulation, caused by EEG increase in γ spectral power over bilateral temporal lobes.

In sum, in the context of a globular set theory-framed approach, axonal neuromodulation might have the role to produce rapid frequency changes also in sites far from the stimulated axon. This paves the way to the application of presheaves in different contexts regarding feedforward/feedback paths, such as the use of rTMS in human disease. Specific high frequency rTMS has been already approved for the treatment of major depression. Low frequency rTMS (0.3-1 Hz) are less studied, but its ability to decrease cortical excitability suggests a potential therapeutic effect in patients affected by drug-resistant epilepsy (Jan et al., 2017). Even though conventional rTMS stimulators activate only superficial cortical areas, it is feasible to reach deep epileptic foci (such as, for example, in temporal lobe epilepsy) using specially designed H-coils which provide low frequency rTMS (Gersner et al., 2016). Chen et al. (2016) retrospectively analyzed the evidence for the efficacy of TMS in drug-resistant epilepsy compared with available treatments. They evaluated studies that used rTMS of any frequency, duration, intensity, and setup (focal or vertex treatment) on patients of

different ages. Given the extreme variability in techniques and outcome reporting, the evidence for efficacy of rTMS for reduction in seizure rate, seizure frequency and mean number of epileptic discharges is still lacking (Jan et al., 2017). The main problem is that there is still no agreement on optimized stimulation parameters and patterns of rTMS for epilepsy. TMS effects are erratic across individuals and depend on frequency, number of stimuli within a train, stimulation intensity, type of coil, coil position, duration of stimulation and inter-train interval.

Our approach suggests the proper coil location for transcranial magnetic stimulation in epilepsy. In case of epileptic seizures, our model hypothesizes that the onset area of pathological spikes might extend to other regions through a mechanism of synchronization linked with the transient formation of globular sets. Once hypothesized that synchronous pathological spikes might occur inside functional globular sets, a novel mathematical framework for epilepsy is achieved that is worth to be investigated. If the presheaf framework holds true, the starting epileptic focus (standing for X_1) propagates to the surrounding brain areas (standing for X_2), giving rise to synchronized pathological waves with matching description/zero divergence (**Figure 4A**). The activation of areas X_2 (e.g., through transcranial stimulation with coils located in X_2) are therefore able to modify the oscillatory activity of the initial locus of epilepsy onset X_1 . This means that the areas surrounding the locus of seizure onset keep the seizure active, and not vice versa. This suggests an easily testable prevision: to achieve the best therapeutic effect and confirm that the mathematical apparatus of globular sets holds true in epilepsy, rTMS treatments must not focus on the first focus of onset of epilepsy, rather on the surrounding areas where the pathological spikes propagate (**Figure 4**).

The occurrence of back-propagation is a widespread phenomenon identified in different systems and at different levels of observation. For example, a chemical counterpart of the above-described mathematical retrograde path defined by globular sets has been implicitly described in literature. Wang et al. (2020) studied catalyzed chemical reactions (such as the ubiquitous click reactions in living cells) which provoke increases in free-energy/enthalpy in the medium. They discovered that these energetic increases generate long-range mechanical perturbations that permit to speed the mobility of the Brownian diffusion of reactants and nearby solvent. In other words, contrary to the chemistry dogma that molecular diffusion and chemical reaction are unrelated, some reactions can boost the surrounding reactants and solvents, increasing their speed and mobility. The Authors also elucidated a noteworthy issue: if you want to keep a stable concentration gradient and a constant fluid density, you need to counterbalance the above-described effects of boosted diffusion molecules/anterograde diffusion by using backflow of solvent molecules. This new boosting mechanism in chemistry corresponds to the same backpropagation processes described by presheaves and globular sets. The boosting mechanism is remarkable also for another reason: we speculate that, when the chemical boost is prolonged and self-sustained enough, the backpropagation might contribute to provide the energy required to start life from chemical reactions and preserve the living organism's efforts to survive via homeostatic balance. Back-propagation can be found also in the brain (Smith et al., 2013). During the sensitive cue period, "feed-forward" signals flow from granular to supragranular/infragranular layers in the primate temporal cortex. In turn, during the sensitive delay period, reverse "feed-back" signals flow from infragranular to supragranular layers (Takeuchi et al, 2011). This reversal of signal flow suggests that the temporal cortex differentially recruits its laminar circuits for sensory and mnemonic processing.

The last, but not the least, we stated that source and targets in X_n follow sources and targets in X_{n+1} . This suggests another intriguing possibility. If we take the value "n" in terms of "spatial" dimensions of the subtending manifold, we are allowed to investigate a recently raised hypothesis, i.e., that the brain activity takes place in multidimensional spaces (Tozzi and Peters 2016, Tozzi, 2019). In the context of presheaf theory, the cortical activity in three dimensions (plus time) depends on the cortical activity in higher dimensions (plus time). When a change in three-dimensions occurs (either in source or target maps), we need to look for the foregoing changes of source and target maps located in higher dimensions (**Figure 5B**). For example, when a change takes place in the three-dimensional source S_3 , we need to look for changes with matching description in the higher dimension S_4 , because:

$$S_n \circ S_{n+1} = S_n \circ t_{n+1}$$

Therefore, a change in S_3 is correlated with previous changes either in S_4 or in T_4 .

Figure 3. Orthodromic and antidromic currents in the human brain. **Figure 3A.** An example of feedforward and feedback projections in the brain. The globular sets are superimposed to the corresponding gamma feedforward spikes. **Figure 3B-E.** Procedure to experimentally test whether presheaves do exist in the brain. **Figure 3B:** according to presheaf models, the neuronal axon must be equipped with globular set's intersecting subsets. The soma is on the left of the depicted neuron. The yellow circles superimposed to the axon illustrate the intersecting subsets of a globular set. **Figures 3C-E** provide an example of the proposed procedure. Take a neuron that produces gamma bursts (**Figure 3C**). The red arrow superimposed to the axon illustrates the standard, orthodromic axonal propagation of gamma-spikes. According to presheaf theories, if an external and continuous beta spike is applied to the distal axon (blue arrow in **Figure 3D**), the neuron will generate entrained beta bursts, instead of the previous gamma bursts (**Figure 3E**).

Figure 4. Application of presheaves in different contexts of feedforward and feedback paths. **Figure 4A, left.** Recorded surface EEG (displayed in a bipolar transverse montage) showing a left parietal EEG seizure onset. The red circle stands for the area of onset of the seizure X_1 , while the yellow circles for the areas of seizure propagation X_2 and X_3 . Modified from: Beleza et al. (2010). **Figure 4A, right.** EEG mount illustrating the globular sets involved in the seizure depicted in **Figure 4A, left**. Starting from the area X_1 of onset, the seizure propagates through the areas X_2 and X_3 . Our model predicts that a transcranial stimulation with coils located in X_2 (dark cylinder) might counteract the seizure. **Figures 4B-C.** Globular sets encompassing manifolds of different spatial dimensions (d_4 , d_3 , d_2). The Figures illustrate how interdimensional projections map from higher dimensional (d_4) to lower dimensional manifolds (d_2). Note that functions in higher dimensions precede functions in lower dimensions.

CONCLUSIONS

We showed that the mathematical apparatus of presheaves/globular sets gives us the possibility to treat feedforward paths and backpropagation in terms of category theory. The key question is: what is the rationale for choosing the intricate mathematical apparatus of presheaves to approach bipolarity in network theory? The local-to-global machinery of algebraic topology seems poised to provide the perfect tools for gathering disparate neural activities into a coherent whole. To provide an example concerning the human central nervous system, presheaves offer invaluable benefits for the analysis of nervous activities: a) they generalize the local systems that are so ubiquitous in the brain areas and subareas; b) they have powerful applications to the topology of algebraic and the analytic complex varieties of singular spaces such as the nervous phase spaces (Dimca, 2004); c) they make available a suitable notion of “general coefficient systems” that could be useful for the assessment of nonlinear nervous dynamics; d) they stand for common methods of comparison between different cohomology theories of general topological spaces (Bredon, 1997); e) they provide foundations of near sets that can be used to describe brain areas and subareas (Wolski, 2013). Also, globular sets allow to analyze the stability of synchronous states in dynamical systems such as the brain (Papo and Buldú, 2019), shedding new light of the still unknown mechanisms of spike synchronization. Presheaves permit us to draw previsions that can be experimentally tested with the current neuro-technologies, such as, e.g., a counterintuitive claim: when an entrained oscillation propagates from the brain area A to a brain area B, changes in spike frequency in B lead to changes in spike frequency in A.

Since the size of the globular sets may vary, presheaves can be used for the assessment of different coarse-grained nervous structures at the micro-, meso- and macro-levels of observation. The human ability to re-identify objects at different times and across observation gaps leads to the phenomenon termed “object persistence” (Fields, 2013). The computational tasks required by object persistence can be tackled in terms of presheaf theory both at the microscopic level of histological microcolumns (Tozzi et al., 2017b) and at the macroscopic level of brain functional/anatomical areas. Concerning the micro-levels of observation, we suggest that the globular sets could be located inside the multilayer arrangement of the cortex, in particular in the layer IV of the primary sensory occipital areas of the Primates, where large amounts of granular cells process the visual information coming from the thalamus. In this sub-cortical milieu, the role of the globular sets’ enduring coalescence of functions would be threefold:

1. Synchronize oscillatory patterns.
2. Stabilize information content.
3. Produce unambiguous, long-lasting percepts.

Concerning the macro-levels of brain analysis, the face-selective response to static images of famous and unfamiliar faces is mediated by neural activity in the occipital/posterior temporal cortex, across the fusiform face area (henceforward FFA) (Axelrod et al., 2019). FFA recognizes faces despite the underlying noise and the huge number of conflicting inputs, so that the brain will be always able to recognize Jennifer Aniston, independent of her dresses, make-up or haircut. If we consider FFA in terms of a sheaf F equipped with, say, two globular sets $X_1 \cap X_2$, a change occurring in X_2 will also involve X_1 . Since the FFA subareas X_1 and X_2 are synchronized, they must always display matching description: this ensures stability to the percept and explains why face recognition is always constant, despite the changes in environmental inputs. Also, a presheaf approach might help to elucidate the issue of memory. We speculate that memory could be stored in globular sets that allow preservation and flexibility of the information. If memories are scattered throughout the brain as suggested by the sparse code theory, they could be encompassed in dispersed globular sets that display the same content in different locations (in $X_1, X_2, \dots, X_n$). Therefore, the labile stability of information content might be granted by the globular set apparatus endowed in brain structures, in which the ubiquitous divergence among functions equals zero.

Our approach paves the way to novel attitudes towards issues at the border between biophysics and philosophy. The treatment and transmission of information is a physical process that must be regulated by the Landauer Principle, regardless of the underlying computational logic (Bormashenko, 2019). Physical Bayesian theories consider the brain as an inferential machine which requires priors. These priors can be mathematically assessed in terms of presheaves that keep information stable. Two subsets $X_1 \cap X_2$ could stand not just for two “spatially” separated structures, but also for two “temporally” separated events, i.e., two ensuing temporal windows. The dictates of globular sets theory require that the event X_1 at time t_1 must follow the event X_2 at time t_2 . This means that events in t_2 can influence events in t_1 . This counterintuitive claim does not break any physical law, if we think to priors in terms of abstract mental activities such as predictions, volitions, goals. X_1 becomes real only when the event predicted by mental activities occurs in the external world. Therefore, we speculate that real events X_1 occurring in the world are generated by the predictions X_2 created by our mental Bayesian priors. This mathematical approach provides a response to one of the most controversial philosophical issues concerning the relationships between the “mental” and the “real world”: presheaves might explain how the elusive, immaterial mental activity provokes physical effects in the real world.

Apart from the above-described paraconsistent and bipolar fuzzy logics, globular sets also display affinities with conditional logic, i.e., a type of modal logic in which validity is defined as truth preservation over all worlds of all

interpretations (Priest 2008). In touch with recent topological claims (Ahmad and Peters, 2018; Peters 2019), there are similarities between globular sets and conditional logic C. Stalnaker (1968) stated that the worlds accessible to the world w through R_A (that is, the worlds essentially the same as w , except that A is true there) should be thought of as the worlds that are the most similar to w at which A is true, with similarity coming by degrees. This suggests that nervous presheaves might be investigated in terms of conditional logic C. Incidentally, this could also clarify why different perceptions come together in a single, whole percept.

We described how and why Nicholas of Autrecourt can be considered to have pioneered the current paraconsistent logics. Here we suggest novel lines of research involving Autrecourt and PCLs. He states that the ultimate reason why the evident truths (including LNC) are to be accepted is that they please our minds, since a being is nobler than another if naturally pleases men more (Kennedy et al., 1971). In touch with Aristotle, the optimistic concept of *cosmic goodness* is the pivotal point of Autrecourt's thought. Since the universe has complete goodness, falsehood is the evil of intellect. The intellect is not made for being pleased with the false, so that it can be stated that "what appears is, what is evident is true" (McDermott 1973). The concept of cosmic goodness recalls the modern notion of the anthropic principle, asserting that the scientific scrutiny of the Universe would not even be possible if the laws of the universe had been incompatible with the development of life (Dicke 1961). Since humans have survived and are still here, this means that the evolution did not let our senses to believe to false cues. This takes us in the logical world of PCLs, where our intellect provides an effort to grab not just the being, but also the non-being (Maccagnolo 1953). In touch with Autrecourt's cosmic goodness, PCLs hold that no true theory would ever contain inconsistencies. PCLs and the gnoseological belief in truth provided by the human evolution allow the intrinsic mechanisms of damage control to restore the contradictions generated by the unavoidable errors intrinsic in every information system. A further connection can be found among Autrecourt, PCLs and quantum mechanics (Brown 1993). Autrecourt writes in the *Tractatus* that two points can touch with each retaining its own different position. This seemingly weird statement is in touch with the quantum concept of bosons' superposition (i.e., identical bosons can occupy the same quantum state), since bosons are not subject to the Pauli exclusion principle. Autrecourt declares in the *Condemnations* that a transition occurs from one state to the contradictory one in the absence of a real intrinsic change of any of the terms. Connectives such as "—" do not mean anything, since they are syncategorematic terms lacking denotation and ontological status (Thijssen 1990). In touch with Ockham, Autrecourt seems to support the thesis that we have no knowledge of things outside the word, but only of terms: God and creatures become nothing. In accordance with this medieval insight, recent approaches interpret quantum mechanics as a reference-frame theory pertaining to observer-dependent relational properties (Yang 2018). Remarkably, these radical formulations of quantum mechanics have been experimentally supported by recent authoritative papers (The BIG Bell Test Collaboration 2018). In touch with PCLs and Autrecourt's Ockhamism, The BIG Bell Test Collaboration seems to contradict the tenets of the local realism and suggests that the properties of the physical world are dependent from the observer.

In conclusion, we provided feasible examples of applications of presheaves/globular sets in network theory. Many natural and artificial neural networks (e.g., Hopfield 1982) are equipped with bipolar paths: a feedforward path which guarantees the anterograde propagation of the message and a feedback path which regulates the anterograde message and ensures its retrograde backpropagation. Our theoretical approach provides the first step towards a fully novel mathematical type of neural network in which message backpropagation is provided by the same feedforward fibers. This would allow computational neural networks to spare the bulky occurrence of dedicated feedback circuits.

STATEMENT

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