Metric Dimension in fuzzy(neutrosophic) Graphs-VII

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Abstract

New notion of dimension as set, as two optimal numbers including metric number, dimension number and as optimal set are introduced in individual framework and in formation of family. Behaviors of twin and antipodal are explored in fuzzy(neutrosophic) graphs. Fuzzy(neutrosophic) graphs, under conditions, fixed-edges, fixed-vertex and strong fixed-vertex are studied. Some classes as path, cycle, complete, strong, t-partite, bipartite, star and wheel in the formation of individual case and in the case, they form a family are studied in the term of dimension. Fuzzification (neutrosofication) of twin vertices but using crisp concept of antipodal vertices are another approaches of this study. Thus defining two notions concerning vertices which one of them is fuzzy(neutrosophic) titled twin and another is crisp titled antipodal to study the behaviors of cycles which are partitioned into even and odd, are concluded. Classes of cycles according to antipodal vertices are divided into two classes as even and odd. Parity of the number of edges in cycle causes to have two subsections under the section is entitled to antipodal vertices. In this study, the term dimension is introduced on fuzzy(neutrosophic) graphs. The locations of objects by a set of some junctions which have distinct distance from any couple of objects out of the set, are determined. Thus it's possible to have the locations of objects outside of this set by assigning partial number to any objects. The classes of these specific graphs are chosen to obtain some results based on dimension. The types of crisp notions and fuzzy(neutrosophic) notions are used to make sense about the material of this study and the outline of this study uses some new notions which are crisp and fuzzy(neutrosophic). Some questions and problems are posed concerning ways to do further studies on this topic. Basic familiarities with fuzzy(neutrosophic) graph theory and graph theory are proposed for this article.

Keywords: Fuzzy Graphs, Neutrosophic Graphs, Dimension
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1 Background

Fuzzy set in **Ref.** [15], neutrosophic set in **Ref.** [2], related definitions of other sets in **Refs.** [2, 13, 14], graphs and new notions on them in **Refs.** [5–11], neutrosophic graphs in **Ref.** [3], studies on neutrosophic graphs in **Ref.** [1], relevant definitions of other graphs based on fuzzy graphs in **Ref.** [12], related definitions of other graphs based on neutrosophic graphs in **Ref.** [4], are proposed.

In this section, I use two subsections to illustrate a perspective about the background of this study.

1.1 Motivation and Contributions

In this study, there's an idea which could be considered as a motivation.

Question 1.1. *Is it possible to use mixed versions of ideas concerning "crisp", "fuzzy" and "neutrosophic" to define some notions which are applied to fuzzy(neutrosophic) graphs?*

It's motivation to find notions to use in any classes of fuzzy(neutrosophic) graphs. Real-world applications about locating the item, are another thoughts which lead to be considered as motivation. Distance and path amid two items have key roles to locate. Thus they're used to define new ideas which conclude to the structure of metric dimension. The concept of connectedness inspire to study the behavior of path and distance in the way that, both individual fuzzy(neutrosophic) graphs and family of them are the cases of study.

The framework of this study is as follows. In section (2), I introduce main definitions alongside some examples to clarify about them. In section (3), general attributes are given. In section (4), one idea about specific fuzzy(neutrosophic) vertices is used to form the results for fuzzy(neutrosophic) graphs and family of them. In section (5), one idea about specific crisp vertices is used to form the results for fuzzy(neutrosophic) graphs and family of them especially fuzzy(neutrosophic) cycles as two subsections, In section (6), the results are extended for fuzzy(neutrosophic) graphs and family of them. In section (7), two applications are posed for fuzzy(neutrosophic) graphs and family of them. In section (8), some problems and questions for further studies are proposed. In section (9), gentle discussion about results and applications are featured. In section (10), a brief overview concerning advantages and limitations of this study alongside conclusions are formed.

1.2 Preliminaries

To clarify about the models, I use some definitions and results, and in this way, results have a key role to make sense about the definitions and to introduce new ways to use on these models in the terms of new notions. For instance, the concept of complete is used to specialize a graph in every environment. To differentiate, I use an adjective or prefix in every definition. Two adjectives "fuzzy" and "neutrosophic" are used to distinguish every graph or classes of graph or any notion on them.

$G : (V, E)$ is called a **crisp graph** where V is a set of objects and E is a subset of $V \times V$ such that this subset is symmetric. A crisp graph $G : (V, E)$ is called a **fuzzy graph** $G : (\sigma, \mu)$ where $\sigma : V \to [0, 1]$ and $\mu : E \to [0, 1]$ such that $\mu(xy) \leq \sigma(x) \wedge \sigma(y)$ for all $xy \in E$. A crisp graph $G : (V, E)$ is called a **neutrosophic graph** $G : (\sigma, \mu)$ where $\sigma = (\sigma_1, \sigma_2, \sigma_3) : V \to [0, 1]$ and $\mu = (\mu_1, \mu_2, \mu_3) : E \to [0, 1]$ such that $\mu(xy) \leq \sigma(x) \wedge \sigma(y)$ for all $xy \in E$. A crisp graph $G : (V, E)$ is called a **crisp complete** where $\forall x \in V, \ \forall y \in V, \ xy \in E$. A fuzzy graph $G : (\sigma, \mu)$ is called **fuzzy complete** where it's complete and $\mu(xy) = \sigma(x) \wedge \sigma(y)$ for all $xy \in E$. A neutrosophic graph $G : (\sigma, \mu)$ is called a **neutrosophic complete** where it's complete and $\mu(xy) = \sigma(x) \wedge \sigma(y)$ for all $xy \in E$. An N which is a set of vertices, is called **fuzzy(neutrosophic) cardinality** and it's denoted by $|N|$ such that $|N| = \Sigma_{n \in N} \sigma(n)$. A crisp graph $G : (V, E)$ is called a **crisp strong**. A fuzzy graph $G : (\sigma, \mu)$ is called **fuzzy strong** where $\mu(xy) = \sigma(x) \wedge \sigma(y)$ for all $xy \in E$. A neutrosophic graph $G : (\sigma, \mu)$ is called a **neutrosophic strong** where $\mu(xy) = \sigma(x) \wedge \sigma(y)$ for all $xy \in E$. A distinct sequence of vertices $v_0, v_1, \cdots, v_n$ in a

crisp graph $G : (V, E)$ is called **crisp path** with length n from v_0 to v_n where $v_i v_{i+1} \in E, \ i = 0, 1, \cdots, n - 1$. If one edge is incident to a vertex, the vertex is called **leaf**. A path $v_0, v_1, \cdots, v_n$ is called **fuzzy path** where $\mu(v_i v_{i+1}) > 0, \ i = 0, 1, \cdots, n - 1$. A path $v_0, v_1, \cdots, v_n$ is called **neutrosophic path** where $\mu(v_i v_{i+1}) > 0, \ i = 0, 1, \cdots, n - 1$. Let $P : v_0, v_1, \cdots, v_n$ be fuzzy(neutrosophic) path from v_0 to v_n such that it has minimum number of vertices as possible, then $d(v_0, v_n)$ is defined as $\Sigma_{i=0}^n \mu(v_{i-1} v_i)$. A path $v_0, v_1, \cdots, v_n$ with exception of v_0 and v_n in a crisp graph $G : (V, E)$ is called **crisp cycle** with length n for v_0 where $v_0 = v_n$. A cycle $v_0, v_1, \cdots, v_0$ is called **fuzzy cycle** where there are two edges xy and uv such that $\mu(xy) = \mu(uv) = \bigwedge_{i=0,1,\cdots,n-1} \mu(v_i v_{i+1})$. A cycle $v_0, v_1, \cdots, v_0$ is called **neutrosophic cycle** where there are two edges xy and uv such that $\mu(xy) = \mu(uv) = \bigwedge_{i=0,1,\cdots,n-1} \mu(v_i v_{i+1})$. A fuzzy(neutrosophic) cycle is called **odd** if the number of its vertices is odd. Similarly, a fuzzy(neutrosophic) cycle is called **even** if the number of its vertices is even. A fuzzy(neutrosophic) graph is called **fuzzy(neutrosophic) t-partite** if V is partitioned to t parts, $V_1, V_2, \cdots, V_t$ and the edge xy implies $x \in V_i$ and $y \in V_j$ where $i \neq j$. If it's fuzzy(neutrosophic) complete, then it's denoted by $K_{\sigma_1, \sigma_2, \cdots, \sigma_t}$ where σ_i is σ on V_i instead V which mean $x \notin V_i$ induces $\sigma_i(x) = 0$. If $t = 2$, then it's called **fuzzy(neutrosophic) complete bipartite** and it's denoted by K_{σ_1, σ_2} especially, if $|V_1| = 1$, then it's called **fuzzy(neutrosophic) star** and it's denoted by S_{1, σ_2}. In this case, the vertex in V_1 is called **center** and if a vertex joins to all vertices of fuzzy(neutrosophic), it's called **fuzzy(neutrosophic) wheel** and it's denoted by W_{1, σ_2}. A set is **n-set** if its

Table 1. Crisp-fying, Fuzzy-fying and Neutrosophic-fying

Crisp Graphs	Fuzzy Graphs	Neutrosophic Graphs	
Crisp Complete	Fuzzy Complete	Neutrosophic Complete	
Crisp Strong	Fuzzy Strong	Neutrosophic Strong	
Crisp Path	Fuzzy Path	Neutrosophic Path	
Crisp Cycle	Fuzzy Cycle	Neutrosophic Cycle	
Crisp t-partite	Fuzzy t-partite	Neutrosophic t-partite	
Crisp Bipartite	Fuzzy Bipartite	Neutrosophic Bipartite	
Crisp Star	Fuzzy Star	Neutrosophic Star	
Crisp Wheel	Fuzzy Wheel	Neutrosophic Wheel	

cardinality is n. A **fuzzy vertex set** is the subset of vertex set of (neutrosophic) fuzzy graph such that the values of these vertices are considered. A **fuzzy edge set** is the subset of edge set of (neutrosophic) fuzzy graph such that the values of these edges are considered. Let $\mathcal{G}$ be a family of fuzzy graphs or neutrosophic graphs. This family have **fuzzy(neutrosophic) common** vertex set if all graphs have same vertex set and its values but edges set is subset of fuzzy edge set. A (neutrosophic) fuzzy graph is called **fixed-edge fuzzy(neutrosophic) graph** if all edges have same values. A (neutrosophic) fuzzy graph is called **fixed-vertex fuzzy(neutrosophic) graph** if all vertices have same values. A couple of vertices x and y is called **crisp twin** vertices if either $N(x) = N(y)$ or $N[x] = N[y]$ where $\forall x \in V, \ N(x) = \{y| \ xy \in E\}, \ N[x] = N(x) \cup \{x\}$. Two vertices t and t' are called **fuzzy(neutrosophic) twin** vertices if $N(t) = N(t')$ and $\mu(ts) = \mu(t's)$, for all $s \in N(t) = N(t')$. $\max_{x,y \in V(G)} |E(P(x, y))|$ is called **diameter** of G and it's denoted by $D(G)$ where $|E(P(x, y))|$ is the number of edges on the path from x to y. For any given vertex x if there's exactly one vertex y such that $\min_{P(x,y)} |E(P(x, y))| = D(G)$, then a couple of vertices x and y are called **antipodal** vertices.

Table 2. Crisp-fying, Fuzzy-fying and Neutrosophic-fying

Crisp Vertex Set	Fuzzy Vertex Set	Neutrosophic Vertex Set	
Crisp Edge Set	Fuzzy Edge Set	Neutrosophic Edge Set	
Crisp Common	Fuzzy Common	Neutrosophic Common	
Crisp Fixed-edge	Fuzzy Fixed-edge	Neutrosophic Fixed-edge	
Crisp Fixed-vertex	Fuzzy Fixed-vertex	Neutrosophic Fixed-vertex	
Crisp Twin	Fuzzy Twin	Neutrosophic Twin	

2   Definitions

I use the notion of vertex in fuzzy(neutrosophic) graphs to define new notions which state the relation amid vertices. In this way, the set of vertices are distinguished by another set of vertices.

Definition 2.1. Let $G = (V, \sigma, \mu)$ be a fuzzy(neutrosophic) graph. A vertex m *fuzzy(neutrosophic)-resolves* vertices f_1 and f_2 if $d(m, f_1) \neq d(m, f_2)$. A set M is *fuzzy(neutrosophic)-resolving set* if for every couple of vertices $f_1, f_2 \in V \setminus M$, there's a vertex $m \in M$ such that m fuzzy(neutrosophic)-resolves f_1 and f_2. $|M|$ is called *fuzzy(neutrosophic)-metric number* of G and

$$\min_{S \text{ is fuzzy(neutrosophic)-resolving set}} \Sigma_{s \in S} \sigma(s) = \Sigma_{m \in M} \sigma(m)$$

is called *fuzzy(neutrosophic)-metric dimension* of G and if

$$\min_{S \text{ is fuzzy(neutrosophic)-resolving set}} \Sigma_{s \in S} \sigma(s) = \Sigma_{m \in M} \sigma(m)$$

where M is fuzzy(neutrosophic)-resolving set, then M is called *fuzzy(neutrosophic)-metric set* of G.

Example 2.2. Let G be a fuzzy(neutrosophic) graph as figure (1). By applying Table (3), the 1-set is explored which its cardinality is minimum. $\{f_6\}$ and $\{f_4\}$ are 1-set which has minimum cardinality amid all sets of vertices but $\{f_4\}$ isn't fuzzy(neutrosophic)-resolving set and $\{f_6\}$ is fuzzy(neutrosophic)-resolving set. Thus there's no fuzzy(neutrosophic)-metric set but $\{f_6\}$. f_6 fuzzy(neutrosophic)-resolves all given couple of vertices. Therefore one is fuzzy(neutrosophic)-metric number of G and 0.13 is fuzzy(neutrosophic)-metric dimension of G. By using Table (3), f_4 doesn't fuzzy(neutrosophic)-resolve f_2 and f_6. f_4 doesn't fuzzy(neutrosophic)-resolve f_1 and f_5, too.

Table 3. Distances of Vertices from sets of vertices $\{f_6\}$ and $\{f_4\}$ in fuzzy(neutrosophic) Graph G.

Vertices	f_1	f_2	f_3	f_4	f_5	f_6
f_6	0.22	0.26	0.39	0.24	0.13	0
Vertices	f_1	f_2	f_3	f_4	f_5	f_6
f_4	0.11	0.24	0.37	0	0.11	0.24

Definition 2.3. Consider $\mathcal{G}$ as a family of fuzzy(neutrosophic) graphs on a common vertex set V. A vertex m *simultaneously fuzzy(neutrosophic)-resolves* vertices f_1 and f_2 if $d_G(m, f_1) \neq d_G(m, f_2)$, for all $G \in \mathcal{G}$. A set M is *simultaneously fuzzy(neutrosophic)-resolving set* if for every couple of vertices $f_1, f_2 \in V \setminus M$, there's a
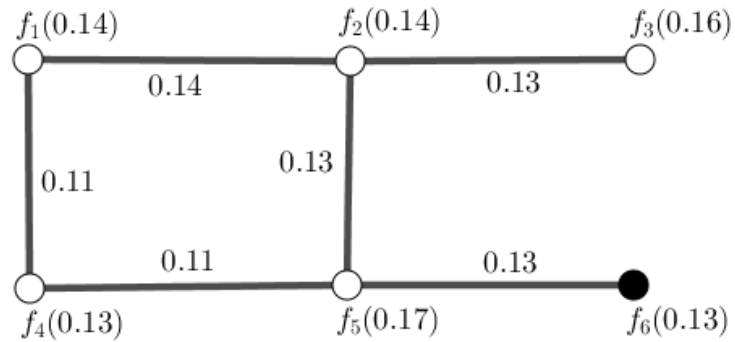
Figure 1. Black vertex $\{f_6\}$ is only fuzzy(neutrosophic)-metric set amid all sets of vertices for fuzzy(neutrosophic) graph G.

vertex $m \in M$ such that m resolves f_1 and f_2, for all $G \in \mathcal{G}$. $|M|$ is called *simultaneously fuzzy(neutrosophic)-metric number* of $\mathcal{G}$ and

$$\min_{S \text{ is fuzzy(neutrosophic)-resolving set}} \Sigma_{s \in S} \sigma(s) = \Sigma_{m \in M} \sigma(m)$$

is called *simultaneously fuzzy(neutrosophic)-metric dimension* of $\mathcal{G}$ and if

$$\min_{S \text{ is fuzzy(neutrosophic)-resolving set}} \Sigma_{s \in S} \sigma(s) = \Sigma_{m \in M} \sigma(m)$$

where M is fuzzy(neutrosophic)-resolving set, then M is called *simultaneously fuzzy(neutrosophic)-metric set* of $\mathcal{G}$.

Example 2.4. Let $\mathcal{G} = \{G_1, G_2, G_3\}$ be a collection of fuzzy(neutrosophic) graphs with common fuzzy(neutrosophic) vertex set and a subset of fuzzy(neutrosophic) edge set as figure (2). By applying Table (4), the 1-set is explored which its cardinality is minimum. $\{f_2\}$ and $\{f_4\}$ are 1-set which has minimum cardinality amid all sets of vertices. $\{f_4\}$ is as fuzzy(neutrosophic)-resolving set as $\{f_6\}$ is. Thus there's no fuzzy(neutrosophic)-metric set but $\{f_4\}$ and $\{f_6\}$. f_6 as fuzzy(neutrosophic)-resolves all given couple of vertices as f_4. Therefore one is fuzzy(neutrosophic)-metric number of $\mathcal{G}$ and 0.13 is fuzzy(neutrosophic)-metric dimension of $\mathcal{G}$. By using Table (4), f_4 fuzzy(neutrosophic)-resolves all given couple of vertices.
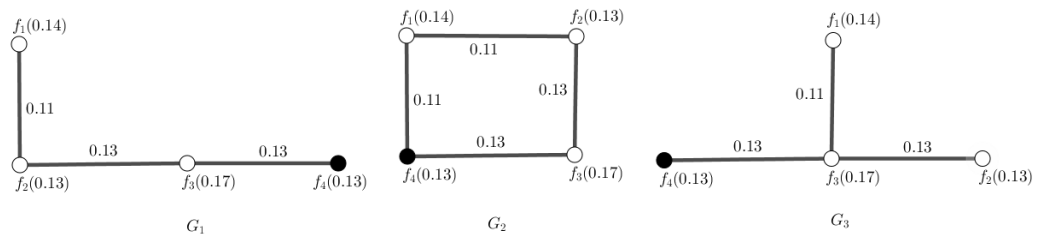
Figure 2. Black vertex $\{f_4\}$ and the set of vertices $\{f_2\}$ are simultaneously fuzzy(neutrosophic)-metric set amid all sets of vertices for family of fuzzy(neutrosophic) graphs $\mathcal{G}$.

3   General Relationships

Proposition 3.1. *Let G be a fuzzy(neutrosophic) path. Then every leaf is fuzzy(neutrosophic)-resolving set.*

Table 4. Distances of Vertices from set of vertices $\{f_6\}$ in Family of fuzzy(neutrosophic) Graphs $\mathcal{G}$.

Vertices of G_1	f_1	f_2	f_3	f_4
f_4	0.37	0.26	0.13	0
Vertices of G_2	f_1	f_2	f_3	f_4
f_4	0.11	0.22	0.13	0
Vertices of G_3	f_1	f_2	f_3	f_4
f_4	0.24	0.26	0.13	0

Proof. Let l be a leaf. For every given a couple of vertices f_i and f_j, I get $d(l, f_i) \neq d(l, f_j)$. Since if I reassign indexes to vertices such that every vertex f_i and l have i vertices amid themselves, then $d(l, f_i) = \Sigma_{j \leq i} \mu(f_j f_i) \leq i$. Thus $j \leq i$ implies

$$\Sigma_{t \leq j} \mu(f_t f_j) + \Sigma_{j \leq s \leq i} \mu(f_s f_i) > \Sigma_{j \leq i} \mu(f f_i) \equiv d(l, f_j) + c = d(l, f_i) \equiv d(l, f_j) < d(l, f_i).$$

Therefore, by $d(l, f_j) < d(l, f_i)$, I get $d(l, f_i) \neq d(l, f_j)$. f_i and f_j are arbitrary so l fuzzy(neutrosophic)-resolves any given couple of vertices f_i and f_j which implies $\{l\}$ is a fuzzy(neutrosophic)-resolving set. $\square$

Corollary 3.2. *Let G be a fixed-edge fuzzy(neutrosophic) path. Then every leaf is fuzzy(neutrosophic)-resolving set.*

Proof. Let l be a leaf. For every given couple of vertices, f_i and f_j, I get $d(l, f_i) = ci \neq d(l, f_j) = cj$. It implies l fuzzy(neutrosophic)-resolves any given couple of vertices f_i and f_j which implies $\{l\}$ is a fuzzy(neutrosophic)-resolving set. $\square$

Corollary 3.3. *Let G be a fixed-vertex fuzzy(neutrosophic) path. Then every leaf is fuzzy(neutrosophic)-metric set, fuzzy(neutrosophic)-metric number is one and fuzzy(neutrosophic)-metric dimension is c where $c = \sigma(f)$, $f \in V$.*

Proof. By Proposition (3.1), every leaf is fuzzy(neutrosophic)-resolving set. By $c = \sigma(f)$, $\forall f \in V$, every leaf is fuzzy(neutrosophic)-metric set, fuzzy(neutrosophic)-metric number is one and fuzzy(neutrosophic)-metric dimension is c. $\square$

Proposition 3.4. *Let G be a fuzzy(neutrosophic) path. Then a set including every couple of vertices is fuzzy(neutrosophic)-resolving set.*

Proof. Let f and f' be a couple of vertices. For every given a couple of vertices f_i and f_j, I get either $d(f, f_i) \neq d(f, f_j)$ or $d(f', f_i) \neq d(f', f_j)$. $\square$

Corollary 3.5. *Let G be a fixed-edge fuzzy(neutrosophic) path. Then every set containing couple of vertices is fuzzy(neutrosophic)-resolving set.*

Proof. Consider G is a fuzzy(neutrosophic) path. Thus by Proposition (3.4), every set containing couple of vertices is fuzzy(neutrosophic)-resolving set. So it holds for any given fixed-edge path fuzzy(neutrosophic) graph. $\square$

4   Fuzzy(Neutrosophic) Twin Vertices

Proposition 4.1. *Let G be a fuzzy(neutrosophic) graph. An $(k-1)$-set from an k-set of fuzzy(neutrosophic) twin vertices is subset of a fuzzy(neutrosophic)-resolving set.*

Proof. If t and t' are fuzzy(neutrosophic) twin vertices, then $N(t) = N(t')$ and $\mu(ts) = \mu(t's)$, for all $s \in N(t) = N(t')$. $\square$

Corollary 4.2. *Let G be a fuzzy(neutrosophic) graph. The number of fuzzy(neutrosophic) twin vertices is $n - 1$. Then fuzzy(neutrosophic)-metric number is $n - 2$.*

Proof. Let f and f' be two vertices. By supposition, the cardinality of set of fuzzy(neutrosophic) twin vertices is $n - 2$. Thus there are two cases. If both are fuzzy(neutrosophic) twin vertices, then $N(f) = N(f')$ and $\mu(fs) = \mu(f's')$, $\forall s \in N(f)$, $\forall s' \in N(f')$. It implies $d(f, t) = d(f, t)$ for all $t \in V$. Thus suppose if not, then let f be a vertex which isn't fuzzy(neutrosophic) twin vertices with any given vertex and let f' be a vertex which is fuzzy(neutrosophic) twin vertices with any given vertex but not f. By supposition, it's possible and this is only case. Therefore, any given distinct vertex fuzzy(neutrosophic)-resolves f and f'. Then $V \setminus \{f, f'\}$ is fuzzy(neutrosophic)-resolving set. It implies fuzzy(neutrosophic)-metric number is $n - 2$. □

Corollary 4.3. *Let G be a fuzzy(neutrosophic) graph. The number of fuzzy(neutrosophic) twin vertices is n. Then G is fixed-edge fuzzy(neutrosophic) graph.*

Proof. Suppose f and f' are two given edges. By supposition, every couple of vertices are fuzzy(neutrosophic) twin vertices. It implies $\mu(f) = \mu(f')$. f and f' are arbitrary so every couple of edges have same values. It induces G is fixed-edge fuzzy(neutrosophic) graph. □

Corollary 4.4. *Let G be a fixed-vertex fuzzy(neutrosophic) graph. The number of fuzzy(neutrosophic) twin vertices is $n - 1$. Then fuzzy(neutrosophic)-metric number is $n - 2$, fuzzy(neutrosophic)-metric dimension is $(n - 2)\sigma(m)$ where m is fuzzy(neutrosophic) twin vertex with a vertex. Every $(n - 2)$-set including fuzzy(neutrosophic) twin vertices is fuzzy(neutrosophic)-metric set.*

Proof. By Corollary (4.2), fuzzy(neutrosophic)-metric number is $n - 2$. By G is a fixed-vertex fuzzy(neutrosophic) graph, fuzzy metric dimension is $(n - 2)\sigma(m)$ where m is fuzzy(neutrosophic) twin vertex with a vertex. One vertex doesn't belong to set of fuzzy(neutrosophic) twin vertices and a vertex from that set, are out of fuzzy metric set. It induces every $(n - 2)$-set including fuzzy(neutrosophic) twin vertices is fuzzy metric set. □

Proposition 4.5. *Let G be a fixed-vertex fuzzy(neutrosophic) graph such that it's fuzzy(neutrosophic) complete. Then fuzzy(neutrosophic)-metric number is $n - 1$, fuzzy(neutrosophic)-metric dimension is $(n - 1)\sigma(m)$ where m is a given vertex. Every $(n - 1)$-set is fuzzy(neutrosophic)-metric set.*

Proof. In fuzzy(neutrosophic) complete, every couple of vertices are twin vertices. By G is a fixed-vertex fuzzy(neutrosophic) graph and it's fuzzy(neutrosophic) complete, every couple of vertices are fuzzy(neutrosophic) twin vertices. Thus by Proposition (4.1), the result follows. □

Proposition 4.6. *Let $\mathcal{G}$ be a family of fuzzy(neutrosophic) graphs with common vertex set. Then simultaneously fuzzy(neutrosophic)-metric number of $\mathcal{G}$ is $n - 1$.*

Proof. Consider $(n - 1)$-set. Thus there's no couple of vertices to be fuzzy(neutrosophic)-resolved. Therefore, every $(n - 1)$-set is fuzzy(neutrosophic)-resolving set for any given fuzzy(neutrosophic) graph. Then it holds for any fuzzy(neutrosophic) graph. It implies it's fuzzy(neutrosophic)-resolving set and its cardinality is fuzzy(neutrosophic)-metric number. $(n - 1)$-set has the cardinality $n - 1$. Then it holds for any fuzzy(neutrosophic) graph. It induces it's simultaneously fuzzy(neutrosophic)-resolving set and its cardinality is simultaneously fuzzy(neutrosophic)-metric number. □

Proposition 4.7. *Let $\mathcal{G}$ be a family of fuzzy(neutrosophic) graphs with common vertex set. Then simultaneously fuzzy(neutrosophic)-metric number of $\mathcal{G}$ is greater than the maximum fuzzy(neutrosophic)-metric number of $G \in \mathcal{G}$.*

Proof. Suppose t and t' are simultaneously fuzzy(neutrosophic)-metric number of $\mathcal{G}$ and fuzzy(neutrosophic)-metric number of $G \in \mathcal{G}$. Thus t is fuzzy(neutrosophic)-metric number for any $G \in \mathcal{G}$. Hence, $t \geq t'$. So simultaneously fuzzy(neutrosophic)-metric number of $\mathcal{G}$ is greater than the maximum fuzzy(neutrosophic)-metric number of $G \in \mathcal{G}$. □

Proposition 4.8. *Let $\mathcal{G}$ be a family of fuzzy(neutrosophic) graphs with common vertex set. Then simultaneously fuzzy(neutrosophic)-metric number of $\mathcal{G}$ is greater than simultaneously fuzzy(neutrosophic)-metric number of $\mathcal{H} \subseteq \mathcal{G}$.*

Proof. Suppose t and t' are simultaneously fuzzy(neutrosophic)-metric number of $\mathcal{G}$ and $\mathcal{H}$. Thus t is fuzzy(neutrosophic)-metric number for any $G \in \mathcal{G}$. It implies t is fuzzy(neutrosophic)-metric number for any $G \in \mathcal{H}$. So t is simultaneously fuzzy(neutrosophic)-metric number of $\mathcal{H}$. By applying Definition about being the minimum number, $t \geq t'$. So simultaneously fuzzy(neutrosophic)-metric number of $\mathcal{G}$ is greater than simultaneously fuzzy(neutrosophic)-metric number of $\mathcal{H} \subseteq \mathcal{G}$. □

Theorem 4.9. *Fuzzy(neutrosophic) twin vertices aren't fuzzy(neutrosophic)-resolved in any given fuzzy(neutrosophic) graph.*

Proof. Let t and t' be fuzzy(neutrosophic) twin vertices. Then $N(t) = N(t')$ and $\mu(ts) = \mu(t's)$, for all $s, s' \in V$ such that $ts, t's \in E$. Thus for every given vertex $s' \in V$, $d_G(s', t) = d_G(s, t)$ where G is a given fuzzy(neutrosophic) graph. It means that t and t' aren't resolved in any given fuzzy(neutrosophic) graph. t and t' are arbitrary so fuzzy(neutrosophic) twin vertices aren't resolved in any given fuzzy(neutrosophic) graph. □

Proposition 4.10. *Let G be a fixed-vertex fuzzy(neutrosophic) graph. If G is fuzzy(neutrosophic) complete, then every couple of vertices are fuzzy(neutrosophic) twin vertices.*

Proof. Let t and t' be couple of given vertices. By G is fuzzy(neutrosophic) complete, $N(t) = N(t')$. By G is a fixed-vertex fuzzy(neutrosophic) graph, $\mu(ts) = \mu(t's)$, for all edges $ts, t's \in E$. Thus t and t' are fuzzy(neutrosophic) twin vertices. t and t' are arbitrary couple of vertices, hence every couple of vertices are fuzzy(neutrosophic) twin vertices. □

Theorem 4.11. *Let $\mathcal{G}$ be a family of fuzzy(neutrosophic) graphs with common vertex set and $G \in \mathcal{G}$ is a fixed-vertex fuzzy(neutrosophic) graph such that it's fuzzy(neutrosophic) complete. Then simultaneously fuzzy(neutrosophic)-metric number is $n-1$, simultaneously fuzzy(neutrosophic)-metric dimension is $(n-1)\sigma(m)$ where m is a given vertex. Every $(n-1)$-set is simultaneously fuzzy(neutrosophic)-metric set for $\mathcal{G}$.*

Proof. G is fixed-vertex fuzzy(neutrosophic) graph and it's fuzzy(neutrosophic) complete. So by Theorem (4.10), I get every couple of vertices in fuzzy(neutrosophic) complete are fuzzy(neutrosophic) twin vertices. So every couple of vertices, by Theorem (4.9), aren't resolved. □

Corollary 4.12. *Let $\mathcal{G}$ be a family of fuzzy(neutrosophic) graphs with fuzzy(neutrosophic) common vertex set and $G \in \mathcal{G}$ is a fuzzy(neutrosophic) complete. Then simultaneously fuzzy(neutrosophic)-metric number is $n-1$, simultaneously fuzzy(neutrosophic)-metric dimension is $(n-1)\sigma(m)$ where m is a given vertex. Every $(n-1)$-set is simultaneously fuzzy(neutrosophic)-metric set for $\mathcal{G}$.*

Proof. By fuzzy(neutrosophic) graphs with fuzzy(neutrosophic) common vertex set, G 245
is fixed-vertex fuzzy(neutrosophic) graph. It's fuzzy(neutrosophic) complete. So by   246
Theorem (4.11), I get intended result.   □   247

Theorem 4.13. *Let $\mathcal{G}$ be a family of fuzzy(neutrosophic) graphs with common vertex*   248
set and for every given couple of vertices, there's a $G \in \mathcal{G}$ such that in that, they're   249
fuzzy(neutrosophic) twin vertices. Then simultaneously fuzzy(neutrosophic)-metric   250
number is $n - 1$, simultaneously fuzzy(neutrosophic)-metric dimension is $(n-1)\sigma(m)$ 251
where m is a given vertex. Every $(n-1)$-set is simultaneously   252
fuzzy(neutrosophic)-metric set for $\mathcal{G}$.   253

Proof. By Proposition (4.6), simultaneously fuzzy(neutrosophic)-metric number is   254
$n - 1$. By Theorem (4.9), simultaneously fuzzy(neutrosophic)-metric dimension is   255
$(n-1)\sigma(m)$ where m is a given vertex. Also, every $(n-1)$-set is simultaneously   256
fuzzy(neutrosophic)-metric set for $\mathcal{G}$.   □   257

Theorem 4.14. *Let $\mathcal{G}$ be a family of fuzzy(neutrosophic) graphs with common vertex*   258
set. If $\mathcal{G}$ contains three fixed-vertex fuzzy(neutrosophic) stars with different center, then   259
simultaneously fuzzy(neutrosophic)-metric number is $n - 2$, simultaneously   260
fuzzy(neutrosophic)-metric dimension is $(n-2)\sigma(m)$ where m is a given vertex. Every   261
$(n-2)$-set is simultaneously fuzzy(neutrosophic)-metric set for $\mathcal{G}$.   262

Proof. The cardinality of set of fuzzy(neutrosophic) twin vertices is $n - 1$. Thus by   263
Corollary (4.4), the result follows.   □   264

Corollary 4.15. *Let $\mathcal{G}$ be a family of fuzzy(neutrosophic) graphs with*   265
fuzzy(neutrosophic) common vertex set. If $\mathcal{G}$ contains three fuzzy(neutrosophic) stars   266
with different center, then simultaneously fuzzy(neutrosophic)-metric number is $n - 2$,   267
simultaneously fuzzy(neutrosophic)-metric dimension is $(n-2)\sigma(m)$ where m is a given   268
vertex. Every $(n-2)$-set is simultaneously fuzzy(neutrosophic)-metric set for $\mathcal{G}$.   269

Proof. By fuzzy(neutrosophic) graphs with fuzzy(neutrosophic) common vertex set, G 270
is fixed-vertex fuzzy(neutrosophic) graph. It's fuzzy(neutrosophic) complete. So by   271
Theorem (4.14), I get intended result.   □   272

5   Antipodal Vertices   273

5.1   Even Fuzzy(Neutrosophic) Cycle   274

Proposition 5.1. *Consider two antipodal vertices x and y in any given fixed-edge even*   275
fuzzy(neutrosophic) cycle. Let u and v be given vertices. Then $d(x, u) \neq d(x, v)$ if and   276
only if $d(y, u) \neq d(y, v)$.   277

Proof. ($\Rightarrow$). Consider $d(x, u) \neq d(x, v)$. By   278
$d(x, u) + d(u, y) = d(x, y) = D(G)$, $D(G) - d(x, u) \neq D(G) - d(x, v)$. It implies   279
$d(y, u) \neq d(y, v)$.   280
   ($\Leftarrow$). Consider $d(y, u) \neq d(y, v)$. By   281
$d(y, u) + d(u, x) = d(x, y) = D(G)$, $D(G) - d(y, u) \neq D(G) - d(y, v)$. It implies   282
$d(x, u) \neq d(x, v)$.   □   283

Proposition 5.2. *Consider two antipodal vertices x and y in any given fixed-edge even*   284
fuzzy(neutrosophic) cycle. Let u and v be given vertices. Then $d(x, u) = d(x, v)$ if and   285
only if $d(y, u) = d(y, v)$.   286

Proof. (⇒). Consider $d(x, u) = d(x, v)$. By   287
$d(x, u) + d(u, y) = d(x, y) = D(G)$, $D(G) - d(x, u) = D(G) - d(x, v)$. It implies   288
$d(y, u) = d(y, v)$.   289
   (⇐). Consider $d(y, u) = d(y, v)$. By   290
$d(y, u) + d(u, x) = d(x, y) = D(G)$, $D(G) - d(y, u) = D(G) - d(y, v)$. It implies   291
$d(x, u) = d(x, v)$.   □   292

Proposition 5.3. *The set contains two antipodal vertices, isn't*   293
fuzzy(neutrosophic)-metric set in any given fixed-edge even fuzzy(neutrosophic) cycle. 294

Proof. Let x and y be two given antipodal vertices in any given even   295
fuzzy(neutrosophic) cycle. By Proposition (5.1), $d(x, u) \neq d(x, v)$ if and only if 296
$d(y, u) \neq d(y, v)$. It implies that if x fuzzy(neutrosophic)-resolves a couple of vertices, 297
then y fuzzy(neutrosophic)-resolves them, too. Thus either x is in   298
fuzzy(neutrosophic)-metric set or y is. It induces the set contains two antipodal vertices, 299
isn't fuzzy(neutrosophic)-metric set in any given even fuzzy(neutrosophic) cycle.   □ 300

Proposition 5.4. *Consider two antipodal vertices x and y in any given fixed-edge even* 301
fuzzy(neutrosophic) cycle. x fuzzy(neutrosophic)-resolves a given couple of vertices, z 302
and z', if and only if y does.   303

Proof. (⇒). x fuzzy(neutrosophic)-resolves a given couple of vertices, z and z', then 304
$d(x, z) \neq d(x, z')$. By Proposition (5.1), $d(x, z) \neq d(x, z')$ if and only if $d(y, z) \neq d(y, z')$. 305
Thus y fuzzy(neutrosophic)-resolves a given couple of vertices z and z'.   306
   (⇐). y fuzzy(neutrosophic)-resolves a given couple of vertices, z and z', then 307
$d(y, z) \neq d(y, z')$. By Proposition (5.1), $d(y, z) \neq d(y, z')$ if and only if $d(x, z) \neq d(x, z')$. 308
Thus x fuzzy(neutrosophic)-resolves a given couple of vertices z and z'.   □ 309

Proposition 5.5. *There are two antipodal vertices aren't fuzzy(neutrosophic)-resolved* 310
by other two antipodal vertices in any given fixed-edge even fuzzy(neutrosophic) cycle. 311

Proof. Suppose x and y are a couple of vertices. It implies $d(x, y) = D(G)$. Consider u 312
and v are another couple of vertices such that $d(x, u) = \frac{D(G)}{2}$. It implies $d(y, u) = \frac{D(G)}{2}$. 313
Thus $d(x, u) = d(y, u)$. Therefore, u doesn't fuzzy(neutrosophic)-resolve a given couple 314
of vertices x and y. By $D(G) = d(u, v) = d(u, x) + d(x, v) = \frac{D(G)}{2} + d(x, v)$, 315
$d(x, v) = \frac{D(G)}{2}$. It implies $d(y, v) = \frac{D(G)}{2}$. Thus $d(x, v) = d(y, v)$. Therefore, v doesn't 316
fuzzy(neutrosophic)-resolve a given couple of vertices x and y.   □   317

Proposition 5.6. *For any two antipodal vertices in any given fixed-edge even*   318
fuzzy(neutrosophic) cycle, there are only two antipodal vertices don't   319
fuzzy(neutrosophic)-resolve them   320

Proof. Suppose x and y are a couple of vertices such that they're antipodal vertices. 321
Let u be a vertex such that $d(x, u) = \frac{D(G)}{2}$. It implies $d(y, u) = \frac{D(G)}{2}$. Thus 322
$d(x, u) = d(y, u)$. Therefore, u doesn't fuzzy(neutrosophic)-resolve a given couple of 323
vertices x and y. Let v be a antipodal vertex for u such that u and v are antipodal 324
vertices. Thus v $d(x, v) = \frac{D(G)}{2}$. It implies $d(y, v) = \frac{D(G)}{2}$. Therefore, v doesn't 325
fuzzy(neutrosophic)-resolve a given couple of vertices x and y. If u is a vertex such that 326
$d(x, u) \neq \frac{D(G)}{2}$ and v is a vertex such that u and v are antipodal vertices. Thus 327
$d(x, v) \neq \frac{D(G)}{2}$ It induces either $d(x, u) \neq d(y, u)$ or $d(x, v) \neq d(y, v)$. It means either u 328
fuzzy(neutrosophic)-resolves a given couple of vertices x and y or v 329
fuzzy(neutrosophic)-resolves a given couple of vertices x and y.   □   330

Proposition 5.7. *In any given fixed-edge even fuzzy(neutrosophic) cycle, for any* 331
vertex, there's only one vertex such that they're antipodal vertices.   332

Proof. If $d(x,y) = D(G)$, then x and y are antipodal vertices. □   333

Proposition 5.8. *Let G be a fixed-edge even fuzzy(neutrosophic) cycle. Then every*   334
couple of vertices are fuzzy(neutrosophic)-resolving set if and only if they aren't   335
antipodal vertices.   336

Proof. If x and y are antipodal vertices, then they don't fuzzy(neutrosophic)-resolve a   337
given couple of vertices u and v such that they're antipodal vertices and $d(x,u) = \frac{D(G)}{2}$.   338
Since $d(x,u) = d(x,v) = d(y,u) = d(y,v) = \frac{D(G)}{2}$. □   339

Corollary 5.9. *Let G be a fixed-edge even fuzzy(neutrosophic) cycle. Then*   340
fuzzy(neutrosophic)-metric number is two.   341

Proof. A set contains one vertex x isn't fuzzy(neutrosophic)-resolving set. Since it   342
doesn't fuzzy(neutrosophic)-resolve a given couple of vertices u and v such that   343
$d(x,u) = d(x,v) = 1$. Thus fuzzy(neutrosophic)-metric number ≥ 2. By Proposition   344
(5.8), every couple of vertices such that they aren't antipodal vertices, are   345
fuzzy(neutrosophic)-resolving set. Therefore, fuzzy(neutrosophic)-metric number is   346
2. □   347

Corollary 5.10. *Let G be a fixed-edge even fuzzy(neutrosophic) cycle. Then*   348
fuzzy(neutrosophic)-metric set contains couple of vertices such that they aren't antipodal   349
vertices.   350

Proof. By Corollary (5.9), fuzzy(neutrosophic)-metric number is two. By Proposition   351
(5.8), every couple of vertices such that they aren't antipodal vertices, are   352
fuzzy(neutrosophic)-resolving set. Therefore, fuzzy(neutrosophic)-metric set contains   353
couple of vertices such that they aren't antipodal vertices. □   354

Corollary 5.11. *Let $\mathcal{G}$ be a family of fixed-edge odd fuzzy(neutrosophic) cycles with*   355
fuzzy(neutrosophic) common vertex set. Then simultaneously fuzzy(neutrosophic)-metric   356
set contains couple of vertices such that they aren't antipodal vertices and   357
fuzzy(neutrosophic)-metric number is two.   358

5.2   Odd Fuzzy(Neutrosophic) Cycle   359

Proposition 5.12. *In any given fixed-edge odd fuzzy(neutrosophic) cycle, for any*   360
vertex, there's no vertex such that they're antipodal vertices.   361

Proof. Let G be a fixed-edge odd fuzzy(neutrosophic) cycle. if x is a given vertex. Then   362
there are two vertices u and v such that $d(x,u) = d(x,v) = D(G)$. It implies they aren't   363
antipodal vertices. □   364

Proposition 5.13. *Let G be a fixed-edge odd fuzzy(neutrosophic) cycle. Then every*   365
couple of vertices are fuzzy(neutrosophic)-resolving set.   366

Proof. Let l and l' be couple of vertices. Thus, by Proposition (5.12), l and l' aren't   367
antipodal vertices. It implies for every given couple of vertices f_i and f_j, I get either   368
$d(l, f_i) \neq d(l, f_j)$ or $d(l', f_i) \neq d(l', f_j)$. Therefore, f_i and f_j are   369
fuzzy(neutrosophic)-resolved by either l or l'. It induces the set $\{l, l'\}$ is   370
fuzzy(neutrosophic)-resolving set. □   371

Proposition 5.14. *Let G be a fixed-edge odd fuzzy(neutrosophic) cycle. Then*   372
fuzzy(neutrosophic)-metric number is two.   373

Proof. Let l and l' be couple of vertices. Thus, by Proposition (5.12), l and l' aren't antipodal vertices. It implies for every given couple of vertices f_i and f_j, I get either $d(l, f_i) \neq d(l, f_j)$ or $d(l', f_i) \neq d(l', f_j)$. Therefore, f_i and f_j are fuzzy(neutrosophic)-resolved by either l or l'. It induces the set $\{l, l'\}$ is fuzzy(neutrosophic)-resolving set. □

Corollary 5.15. *Let G be a fixed-edge odd fuzzy(neutrosophic) cycle. Then fuzzy(neutrosophic)-metric set contains couple of vertices.*

Proof. By Proposition (5.14), fuzzy(neutrosophic)-metric number is two. By Proposition (5.13), every couple of vertices are fuzzy(neutrosophic)-resolving set. Therefore, fuzzy(neutrosophic)-metric set contains couple of vertices. □

Corollary 5.16. *Let $\mathcal{G}$ be a family of fixed-edge odd fuzzy(neutrosophic) cycles with fuzzy(neutrosophic) common vertex set. Then simultaneously fuzzy(neutrosophic)-metric set contains couple of vertices and fuzzy(neutrosophic)-metric number is two.*

6   Extended Results

Proposition 6.1. *If I use fixed-vertex strong fuzzy(neutrosophic) cycles instead of fixed-edge fuzzy(neutrosophic) cycles, then all results of Section (5) hold.*

Proof. Let G be a fixed-vertex strong fuzzy(neutrosophic) cycles. By G is fuzzy(neutrosophic) strong and it's fixed-vertex, G is fixed-edge fuzzy(neutrosophic). □

Proposition 6.2. *Let G be a fixed-vertex strong fuzzy(neutrosophic) path. Then an 1-set contains leaf, is fuzzy(neutrosophic)-resolving set. An 1-set contains leaf, is fuzzy(neutrosophic)-metric set. Fuzzy(neutrosophic)-metric number is one. Fuzzy(neutrosophic)-metric dimension is $\sigma(m)$ where m is a given vertex.*

Corollary 6.3. *Let $\mathcal{G}$ be a family of fuzzy(neutrosophic) paths with common vertex set such that they've a common leaf. Then simultaneously fuzzy(neutrosophic)-metric number is 1, simultaneously fuzzy(neutrosophic)-metric dimension is $\sigma(m)$ where m is a given vertex. 1-set contains common leaf, is simultaneously fuzzy(neutrosophic)-metric set for $\mathcal{G}$.*

Proposition 6.4. *Let G be a fixed-vertex strong fuzzy(neutrosophic) path. Then an 2-set contains every couple of vertices, is fuzzy(neutrosophic)-resolving set. An 2-set contains every couple of vertices,, is fuzzy(neutrosophic)-metric set. Fuzzy(neutrosophic)-metric number is two. Fuzzy(neutrosophic)-metric dimension is $2\sigma(m)$ where m is a given vertex.*

Corollary 6.5. *Let $\mathcal{G}$ be a family of fuzzy(neutrosophic) paths with common vertex set such that they've no common leaf. Then an 2-set is simultaneously fuzzy(neutrosophic)-resolving set, simultaneously fuzzy(neutrosophic)-metric number is 2, simultaneously fuzzy(neutrosophic)-metric dimension is $2\sigma(m)$ where m is given vertices. Every 2-set is simultaneously fuzzy(neutrosophic)-metric set for $\mathcal{G}$.*

Proposition 6.6. *Let G be a fixed-edge fuzzy(neutrosophic) t-partite. Then every set contains couple of vertices in different parts, is fuzzy(neutrosophic)-resolving set.*

Corollary 6.7. *Let G be a fixed-vertex strong fuzzy(neutrosophic) t-partite. Then every $(n-2)$-set excludes two vertices from different parts, is fuzzy(neutrosophic)-resolving set. Every $(n-2)$-set excludes two vertices from different parts, is fuzzy(neutrosophic)-metric set. Fuzzy(neutrosophic)-metric number is $n-2$. Fuzzy(neutrosophic)-metric dimension is $(n-2)\sigma(m)$ where m is a given vertex.*

Corollary 6.8. *Let G be a fixed-vertex strong fuzzy(neutrosophic) bipartite. Then every $(n-2)$-set excludes two vertices from different parts, is fuzzy(neutrosophic)-resolving set. Every $(n-2)$-set excludes two vertices from different parts, is fuzzy(neutrosophic)-metric set. Fuzzy(neutrosophic)-metric number is $n-2$. Fuzzy(neutrosophic)-metric dimension is $(n-2)\sigma(m)$ where m is a given vertex.*

Corollary 6.9. *Let G be a fixed-vertex strong fuzzy(neutrosophic) star. Then every $(n-2)$-set excludes center and a given vertex, is fuzzy(neutrosophic)-resolving set. An $(n-2)$-set excludes center and a given vertex, is fuzzy(neutrosophic)-metric set. Fuzzy(neutrosophic)-metric number is $(n-2)$. Fuzzy(neutrosophic)-metric dimension is $(n-2)\sigma(m)$ where m is a given vertex.*

Corollary 6.10. *Let G be a fixed-vertex strong fuzzy(neutrosophic) wheel. Then every $(n-2)$-set excludes center and a given vertex, is fuzzy(neutrosophic)-resolving set. Every $(n-2)$-set excludes center and a given vertex, is fuzzy(neutrosophic)-metric set. Fuzzy(neutrosophic)-metric number is $n-2$. Fuzzy(neutrosophic)-metric dimension is $(n-2)\sigma(m)$ where m is a given vertex.*

Corollary 6.11. *Let $\mathcal{G}$ be a family of fixed-vertex strong fuzzy(neutrosophic) t-partite with common vertex set. Then simultaneously fuzzy(neutrosophic)-metric number is $n-2$, simultaneously fuzzy(neutrosophic)-metric dimension is $(n-2)\sigma(m)$ Every $(n-2)$-set excludes two vertices from different parts, is simultaneously fuzzy(neutrosophic)-resolving set for $\mathcal{G}$. There's an $(n-2)$-set which is simultaneously fuzzy(neutrosophic)-metric set for $\mathcal{G}$.*

Corollary 6.12. *Let $\mathcal{G}$ be a family of fixed-vertex strong fuzzy(neutrosophic) bipartite with common vertex set. Then simultaneously fuzzy(neutrosophic)-metric number is $n-2$, simultaneously fuzzy(neutrosophic)-metric dimension is $(n-2)\sigma(m)$ Every $(n-2)$-set excludes two vertices from different parts, is simultaneously fuzzy(neutrosophic)-resolving set for $\mathcal{G}$. There's an $(n-2)$-set which is simultaneously fuzzy(neutrosophic)-metric set for $\mathcal{G}$.*

Corollary 6.13. *Let $\mathcal{G}$ be a family of fixed-vertex strong fuzzy(neutrosophic) star with common vertex set. Then simultaneously fuzzy(neutrosophic)-metric number is $n-2$, simultaneously fuzzy(neutrosophic)-metric dimension is $(n-2)\sigma(m)$ Every $(n-2)$-set excludes center and a given vertex, is simultaneously fuzzy(neutrosophic)-resolving set for $\mathcal{G}$. There's an $(n-2)$-set which is simultaneously fuzzy(neutrosophic)-metric set for $\mathcal{G}$.*

Corollary 6.14. *Let $\mathcal{G}$ be a family of fixed-vertex strong fuzzy(neutrosophic) wheel with common vertex set. Then simultaneously fuzzy(neutrosophic)-metric number is $n-2$, simultaneously fuzzy(neutrosophic)-metric dimension is $(n-2)\sigma(m)$ Every $(n-2)$-set excludes center and a given vertex, is simultaneously fuzzy(neutrosophic)-resolving set for $\mathcal{G}$. There's an $(n-2)$-set which is simultaneously fuzzy(neutrosophic)-metric set for $\mathcal{G}$.*

7   Applications

Two applications are posed as follow.

7.1   Located Places

A program is devised for a robot to locate every couple of given places, separately. The number which this program assigns to any place from a given couple of places are unique. Thus every place has a unique number when a couple of places are given.

7.2   Covid-19 and Identifying Infected People   463

Dark network is description for infected people who are anonymous in the matter of   464
Covid-19. Virus and its anonymously style to transmit the virus from one person to   465
another person, could make a dark network involving people. Consider everyone as   466
network titled fuzzy(neutrosophic). It means that the person and his networks   467
containing his connections make two models, fixed-edge fuzzy(neutrosophic) and   468
fixed-vertex strong fuzzy(neutrosophic). Now, I have a family of people which everyone   469
is a model in the terms of Covid-19.   470

8   Open Problems   471

The crisp notion of dimension is defined on fuzzy(neutrosophic) graphs. Thus   472

Question 8.1. *Is it possible to define fuzzy(neutrosophic) notion of dimension on*   473
fuzzy(neutrosophic) graphs?   474

There are too many limitations on the classes of fuzzy(neutrosophic) graphs by using   475
fixed-edge fuzzy(neutrosophic) graphs and fixed-vertex strong fuzzy(neutrosophic)   476
graphs.   477

Question 8.2. *Is an approach existed to compute current dimension for specific classes*   478
of fuzzy(neutrosophic) graphs?   479

Question 8.3. *What are basic attributes of current dimension for general classes of*   480
fuzzy(neutrosophic) graphs?   481

Finding other classes of fuzzy(neutrosophic) graphs has an ordinary approach to   482
develop this study.   483

Question 8.4. *Which new classes of fuzzy(neutrosophic) graphs are existed to develop*   484
this notion of current dimension?   485

Question 8.5. *Which new classes of fuzzy(neutrosophic) graphs are existed to compute*   486
this notion of current dimension?   487

Question 8.6. *Which general approaches are existed to study this notion of current*   488
dimension in fuzzy(neutrosophic) graphs?   489

Question 8.7. *Which specific approaches are existed to study this notion of current*   490
dimension in fuzzy(neutrosophic) graphs?   491

Problem 8.8. *Are there special crisp sets of vertices, e.g. antipodal vertices for*   492
fuzzy(neutrosophic) cycles, which have key role to study this notion of current   493
dimension in fuzzy(neutrosophic) graphs?   494

Problem 8.9. *Are there fuzzy(neutrosophic) special sets of vertices, e.g.*   495
fuzzy(neutrosophic) twin vertices for general classes, which have key role to study this   496
notion of current dimension in fuzzy(neutrosophic) graphs?   497

9   Discussion   498

10   Conclusion and Closing Remarks   499

This study uses mixed combinations of fuzzy concepts and crisp concepts to explore new   500
notion of crisp dimension in fuzzy(neutrosophic) graphs as individual and as family. In   501

this way, some crisp notions like antipodal vertices are defined to use as a tool to study 502
fuzzy(neutrosophic) cycles as individual and as family. Also, some fuzzy(neutrosophic) 503
notions like fuzzy(neutrosophic) twin vertices are defined to use as a tool to study 504
general classes of fuzzy(neutrosophic) graphs as individual and as family. Mixed family 505
of fuzzy(neutrosophic) graphs are slightly studied by using fuzzy(neutrosophic) twin 506
vertices and other ideas as individual and as family. In Table (5), I mention some 507
advantages and limitations concerning this article and its proposed notions.

Table 5. A Brief Overview about Advantages and Limitations of this study

Advantages	Limitations
1. Using crisp and fuzzy(neutrosophic) notions in one framework together simultaneously. 2. Study on fuzzy(neutrosophic) as individual and as family. 3. Involved classes as complete, strong, path, cycle, t-partite, bipartite, star, wheel.	1. The most usages of fixed-edge fuzzy(neutrosophic) graphs and fixed-vertex strong fuzzy(neutrosophic) graphs.
4. Characterizing classes of fuzzy(neutrosophic) graphs with smallest metric number and largest metric number.	2. Characterizing classes of fuzzy(neutrosophic) graphs with smallest dimension number and largest dimension number.

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