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Metric Dimension in fuzzy(neutrosophic) Graphs-IV

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Abstract

In this study, the term dimension is introduced on fuzzy(neutrosophic) graphs. The classes of these specific graphs are chosen to obtain some results based on dimension. The types of crisp notions and fuzzy(neutrosophic) notions are used to make sense about the material of this study and the outline of this study uses some new notions which are crisp and fuzzy(neutrosophic).

Keywords: Fuzzy Graphs, Neutrosophic Graphs, Dimension
AMS Subject Classification: 05C17, 05C22, 05E45

1 Background

To clarify about the definitions, I use some examples and in this way, exemplifying has key role to make sense about the definitions and to introduce new ways to use on these models in the terms of new notions. The concept of complete is used to classify specific graph in every environment. To differentiate, I use an adjective or prefix in every definition. Two adjectives "fuzzy" and "neutrosophic" are used to distinguish every graph or classes of graph or any notion on them.

$G:(V,E)$ is called a **crisp graph** where V is a set of objects and E is a subset of $V\times V$ such that this subset is symmetric. A crisp graph $G:(V,E)$ is called a **fuzzy graph** $G:(\sigma,\mu)$ where $\sigma:V\to[0,1]$ and $\mu:E\to[0,1]$ such that $\mu(xy)\le\sigma(x)\wedge\sigma(y)$ for all $xy\in E$. A crisp graph $G:(V,E)$ is called a **neutrosophic graph** $G:(\sigma,\mu)$ where $\sigma=(\sigma_1,\sigma_2,\sigma_3):V\to[0,1]$ and $\mu=(\mu_1,\mu_2.\mu_3):E\to[0,1]$ such that $\mu(xy)\le\sigma(x)\wedge\sigma(y)$ for all $xy\in E$. A crisp graph $G:(V,E)$ is called a **crisp complete** where $\forall x\in V,\ \forall y\in V,\ xy\in E$. A fuzzy graph $G:(\sigma,\mu)$ is called **fuzzy complete** where it's complete and $\mu(xy)=\sigma(x)\wedge\sigma(y)$ for all $xy\in E$. A neutrosophic graph $G:(\sigma,\mu)$ is called a **neutrosophic complete** where it's complete and $\mu(xy)=\sigma(x)\wedge\sigma(y)$ for all $xy\in E$. An N which is a set of vertices, is called **fuzzy(neutrosophic) cardinality** and it's denoted by $|N|$ such that $|N|=\Sigma_{n\in N}\sigma(n)$. A crisp graph $G:(V,E)$ is called a **crisp strong**. A fuzzy graph $G:(\sigma,\mu)$ is called **fuzzy strong** where $\mu(xy)=\sigma(x)\wedge\sigma(y)$ for all $xy\in E$. A neutrosophic graph $G:(\sigma,\mu)$ is called a **neutrosophic strong** where $\mu(xy)=\sigma(x)\wedge\sigma(y)$ for all $xy\in E$. A distinct sequence of vertices $v_0,v_1,\cdots,v_n$ in a crisp graph $G:(V,E)$ is called **crisp path** with length n from v_0 to v_n where $v_iv_{i+1}\in E,\ i=0,1,\cdots,n-1$. If one edge is incident to a vertex, the vertex is called **leaf**. A path $v_0,v_1,\cdots,v_n$ is called **fuzzy path** where

$\mu(v_i v_{i+1}) > 0,\ i = 0, 1, \cdots, n-1$. A path $v_0, v_1, \cdots, v_n$ is called **neutrosophic path** where $\mu(v_i v_{i+1}) > 0,\ i = 0, 1, \cdots, n-1$. Let $P : v_0, v_1, \cdots, v_n$ be fuzzy(neutrosophic) path from v_0 to v_n such that it has minimum number of vertices as possible, then $d(v_0, v_n)$ is defined as $\Sigma_{i=0}^{n} \mu(v_{i-1} v_i)$. A path $v_0, v_1, \cdots, v_n$ with exception of v_0 and v_n in a crisp graph $G : (V, E)$ is called **crisp cycle** with length n for v_0 where $v_0 = v_n$. A cycle $v_0, v_1, \cdots, v_0$ is called **fuzzy cycle** where there are two edges xy and uv such that $\mu(xy) = \mu(uv) = \bigwedge_{i=0,1,\cdots,n-1} \mu(v_i v_{i+1})$. A cycle $v_0, v_1, \cdots, v_0$ is called **neutrosophic cycle** where there are two edges xy and uv such that $\mu(xy) = \mu(uv) = \bigwedge_{i=0,1,\cdots,n-1} \mu(v_i v_{i+1})$. A fuzzy(neutrosophic) cycle is called **odd** if the number of its vertices is odd. Similarly, a fuzzy(neutrosophic) cycle is called **even** if the number of its vertices is even. A set is **n-set** if its cardinality is n. A **fuzzy vertex**

Table 1. Crisp-fying, Fuzzy-fying and Neutrosophic-fying

Crisp Graphs	Fuzzy Graphs	Neutrosophic Graphs
Crisp Complete	Fuzzy Complete	Neutrosophic Complete
Crisp Strong	Fuzzy Strong	Neutrosophic Strong
Crisp Path	Fuzzy Path	Neutrosophic Path
Crisp Cycle	Fuzzy Cycle	Neutrosophic Cycle

set is the subset of vertex set of (neutrosophic) fuzzy graph such that the values of these vertices are considered. A **fuzzy edge set** is the subset of edge set of (neutrosophic) fuzzy graph such that the values of these edges are considered. Let $\mathcal{G}$ be a family of fuzzy graphs or neutrosophic graphs. This family have **fuzzy(neutrosophic) common** vertex set if all graphs have same vertex set and its values but edges set is subset of fuzzy edge set. A (neutrosophic) fuzzy graph is called **fixed-edge fuzzy(neutrosophic) graph** if all edges have same values. A (neutrosophic) fuzzy graph is called **fixed-vertex fuzzy(neutrosophic) graph** if all vertices have same values. A couple of vertices x and y is called **crisp twin** vertices if either $N(x) = N(y)$ or $N[x] = N[y]$ where $\forall x \in V,\ N(x) = \{y|\ xy \in E\},\ N[x] = N(x) \cup \{x\}$. Two vertices t and t' are called **fuzzy(neutrosophic) twin** vertices if $N(t) = N(t')$ and $\mu(ts) = \mu(t's)$, for all $s \in N(t) = N(t')$. $\max_{x,y \in V(G)} |E(P(x,y))|$ is called **diameter** of

Table 2. Crisp-fying, Fuzzy-fying and Neutrosophic-fying

Crisp Vertex Set	Fuzzy Vertex Set	Neutrosophic Vertex Set
Crisp Edge Set	Fuzzy Edge Set	Neutrosophic Edge Set
Crisp Common	Fuzzy Common	Neutrosophic Common
Crisp Fixed-edge	Fuzzy Fixed-edge	Neutrosophic Fixed-edge
Crisp Fixed-vertex	Fuzzy Fixed-vertex	Neutrosophic Fixed-vertex
Crisp Twin	Fuzzy Twin	Neutrosophic Twin

G and it's denoted by $D(G)$ where $|E(P(x,y))|$ is the number of edges on the path from x to y. A couple of vertices x and y is called **antipodal** vertices if $\min_{P(x,y)} |E(P(x,y))| = D(G)$. For using material look at [1–15].

2 Definitions

We use the notion of vertex in fuzzy(neutrosophic) graphs to define new notions which state the relation amid vertices. In this way, the set of vertices are distinguished by another set of vertices.

Definition 2.1. Let $G = (V, \sigma, \mu)$ be a fuzzy(neutrosophic) graph. A vertex m *fuzzy(neutrosophic)-resolves* vertices f_1 and f_2 if $d(m, f_1) \neq d(m, f_2)$. A set M is

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fuzzy(neutrosophic)-resolving set if for every couple of vertices $f_1, f_2 \in V \setminus M$, there's a vertex $m \in M$ such that m fuzzy(neutrosophic)-resolves f_1 and f_2. $|M|$ is called *fuzzy(neutrosophic)-metric number* of G and

$$\min_{S \text{ is fuzzy(neutrosophic)-resolving set}} \Sigma_{s\in S}\sigma(s) = \Sigma_{m\in M}\sigma(m)$$

is called *fuzzy(neutrosophic)-metric dimension* of G and if

$$\min_{S \text{ is fuzzy(neutrosophic)-resolving set}} \Sigma_{s\in S}\sigma(s) = \Sigma_{m\in M}\sigma(m)$$

where M is fuzzy(neutrosophic)-resolving set, then M is called *fuzzy(neutrosophic)-metric set* of G.

Example 2.2. Let G be a fuzzy(neutrosophic) graph as figure (1). By applying Table (3), the 1-set is explored which its cardinality is minimum. $\{f_6\}$ and $\{f_4\}$ are 1-set which has minimum cardinality amid all sets of vertices but $\{f_4\}$ isn't fuzzy(neutrosophic)-resolving set and $\{f_6\}$ is fuzzy(neutrosophic)-resolving set. Thus there's no fuzzy(neutrosophic)-metric set but $\{f_6\}$. f_6 fuzzy(neutrosophic)-resolves all given couple of vertices. Therefore one is fuzzy(neutrosophic)-metric number of G and 0.13 is fuzzy(neutrosophic)-metric dimension of G. By using Table (3), f_4 doesn't fuzzy(neutrosophic)-resolve f_2 and f_6. f_4 doesn't fuzzy(neutrosophic)-resolve f_1 and f_5, too.

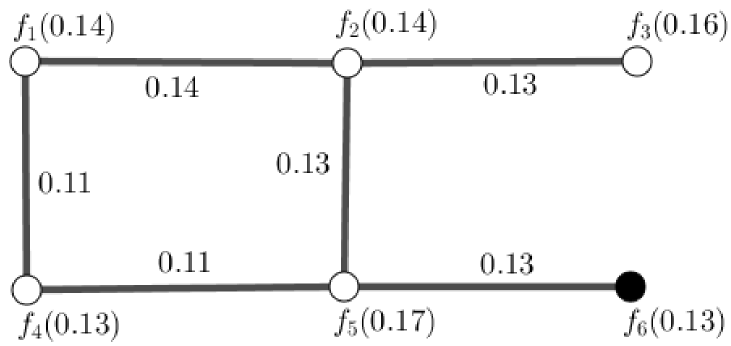

Figure 1. Black vertex $\{f_6\}$ is only fuzzy(neutrosophic)-metric set amid all sets of vertices for fuzzy(neutrosophic) graph G.

Table 3. Distances of Vertices from sets of vertices $\{f_6\}$ and $\{f_4\}$ in fuzzy(neutrosophic) Graph G.

Vertices	f_1	f_2	f_3	f_4	f_5	f_6
f_6	0.22	0.26	0.39	0.24	0.13	0
Vertices	f_1	f_2	f_3	f_4	f_5	f_6
f_4	0.11	0.24	0.37	0	0.11	0.24

Definition 2.3. Consider $\mathcal{G}$ as a family of fuzzy(neutrosophic) graphs on a common vertex set V. A vertex m *simultaneously fuzzy(neutrosophic)-resolves* vertices f_1 and f_2 if $d_G(m, f_1) \neq d_G(m, f_2)$, for all $G \in \mathcal{G}$. A set M is *simultaneously fuzzy(neutrosophic)-resolving set* if for every couple of vertices $f_1, f_2 \in V \setminus M$, there's a vertex $m \in M$ such that m resolves f_1 and f_2, for all $G \in \mathcal{G}$. $|M|$ is called *simultaneously fuzzy(neutrosophic)-metric number* of $\mathcal{G}$ and

$$\min_{S \text{ is fuzzy(neutrosophic)-resolving set}} \Sigma_{s\in S}\sigma(s) = \Sigma_{m\in M}\sigma(m)$$

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is called *simultaneously fuzzy(neutrosophic)-metric dimension* of $\mathcal{G}$ and if

$$\min_{S \text{ is fuzzy(neutrosophic)-resolving set}} \Sigma_{s\in S}\sigma(s) = \Sigma_{m\in M}\sigma(m)$$

where M is fuzzy(neutrosophic)-resolving set, then M is called *simultaneously fuzzy(neutrosophic)-metric set* of $\mathcal{G}$.

Example 2.4. Let $\mathcal{G} = \{G_1, G_2, G_3\}$ be a collection of fuzzy(neutrosophic) graphs with common fuzzy(neutrosophic) vertex set and a subset of fuzzy(neutrosophic) edge set as figure (2). By applying Table (4), the 1-set is explored which its cardinality is minimum. $\{f_2\}$ and $\{f_4\}$ are 1-set which has minimum cardinality amid all sets of vertices. $\{f_4\}$ is as fuzzy(neutrosophic)-resolving set as $\{f_6\}$ is. Thus there's no fuzzy(neutrosophic)-metric set but $\{f_4\}$ and $\{f_6\}$. f_6 as fuzzy(neutrosophic)-resolves all given couple of vertices as f_4. Therefore one is fuzzy(neutrosophic)-metric number of $\mathcal{G}$ and 0.13 is fuzzy(neutrosophic)-metric dimension of $\mathcal{G}$. By using Table (4), f_4 fuzzy(neutrosophic)-resolves all given couple of vertices.

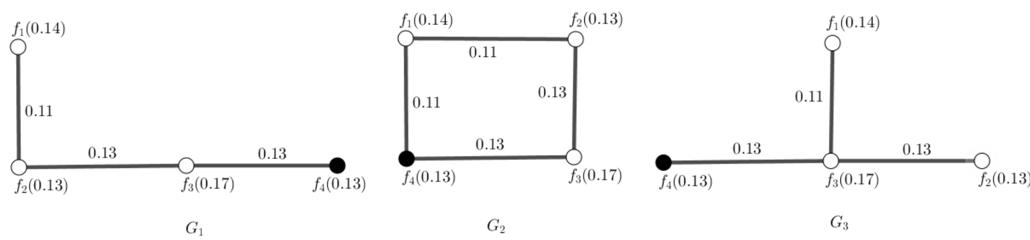

Figure 2. Black vertex $\{f_4\}$ and the set of vertices $\{f_2\}$ are simultaneously fuzzy(neutrosophic)-metric set amid all sets of vertices for family of fuzzy(neutrosophic) graphs $\mathcal{G}$.

Table 4. Distances of Vertices from set of vertices $\{f_6\}$ in Family of fuzzy(neutrosophic) Graphs $\mathcal{G}$.

Vertices of G_1	f_1	f_2	f_3	f_4
f_4	0.37	0.26	0.13	0
Vertices of G_2	f_1	f_2	f_3	f_4
f_4	0.11	0.22	0.13	0
Vertices of G_3	f_1	f_2	f_3	f_4
f_4	0.24	0.26	0.13	0

3 General Relationships

Proposition 3.1. *Let G be a fuzzy(neutrosophic) path. Then every leaf is fuzzy(neutrosophic)-resolving set.*

Proof. Let l be a leaf. For every given a couple of vertices f_i and f_j, we get $d(l, f_i) \neq d(l, f_j)$. Since if we reassign indexes to vertices such that every vertex f_i and l have i vertices amid themselves, then $d(l, f_i) = \Sigma_{j\leq i}\mu(f_j f_i) \leq i$. Thus $j \leq i$ implies

$$\Sigma_{t\leq j}\mu(f_t f_j) + \Sigma_{j\leq s\leq i}\mu(f_s f_i) > \Sigma_{j\leq i}\mu(f f_i) \equiv d(l, f_j) + c = d(l, f_i) \equiv d(l, f_j) < d(l, f_i).$$

Therefore, by $d(l, f_j) < d(l, f_i)$, we get $d(l, f_i) \neq d(l, f_j)$. f_i and f_j are arbitrary so l fuzzy(neutrosophic)-resolves any given couple of vertices f_i and f_j which implies $\{l\}$ is a fuzzy(neutrosophic)-resolving set. □

Corollary 3.2. *Let G be a fixed-edge fuzzy(neutrosophic) path. Then every leaf is fuzzy(neutrosophic)-resolving set.*

Proof. Let l be a leaf. For every given couple of vertices, f_i and f_j, we get $d(l, f_i) = ci \neq d(l, f_j) = cj$. It implies l fuzzy(neutrosophic)-resolves any given couple of vertices f_i and f_j which implies $\{l\}$ is a fuzzy(neutrosophic)-resolving set. □

Corollary 3.3. *Let G be a fixed-vertex fuzzy(neutrosophic) path. Then every leaf is fuzzy(neutrosophic)-metric set, fuzzy(neutrosophic)-metric number is one and fuzzy(neutrosophic)-metric dimension is c where $c = \sigma(f),\ f \in V$.*

Proof. By Proposition (3.1), every leaf is fuzzy(neutrosophic)-resolving set. By $c = \sigma(f),\ \forall f \in V$, every leaf is fuzzy(neutrosophic)-metric set, fuzzy(neutrosophic)-metric number is one and fuzzy(neutrosophic)-metric dimension is c. □

Proposition 3.4. *Let G be a fuzzy(neutrosophic) path. Then a set including every couple of vertices is fuzzy(neutrosophic)-resolving set.*

Proof. Let f and f' be a couple of vertices. For every given a couple of vertices f_i and f_j, we get either $d(f, f_i) \neq d(f, f_j)$ or $d(f', f_i) \neq d(f', f_j)$. □

Corollary 3.5. *Let G be a fixed-edge fuzzy(neutrosophic) path. Then every set containing couple of vertices is fuzzy(neutrosophic)-resolving set.*

Proof. Consider G is a fuzzy(neutrosophic) path. Thus by Proposition (3.4), every set containing couple of vertices is fuzzy(neutrosophic)-resolving set. So it holds for any given fixed-edge path fuzzy(neutrosophic) graph. □

4 Fuzzy(Neutrosophic) Twin Vertices

Proposition 4.1. *Let G be a fuzzy(neutrosophic) graph. An $(k-1)$-set from an k-set of fuzzy(neutrosophic) twin vertices is subset of a fuzzy(neutrosophic)-resolving set.*

Proof. If t and t' are fuzzy(neutrosophic) twin vertices, then $N(t) = N(t')$ and $\mu(ts) = \mu(t's)$, for all $s \in N(t) = N(t')$. □

Corollary 4.2. *Let G be a fuzzy(neutrosophic) graph. The number of fuzzy(neutrosophic) twin vertices is $n-1$. Then fuzzy(neutrosophic)-metric number is $n-2$.*

Proof. Let f and f' be two vertices. By supposition, the cardinality of set of fuzzy(neutrosophic) twin vertices is $n-2$. Thus there are two cases. If both are fuzzy(neutrosophic) twin vertices, then $N(f) = N(f')$ and $\mu(fs) = \mu(f's'),\ \forall s \in N(f)$, $\forall s' \in N(f')$. It implies $d(f,t) = d(f,t)$ for all $t \in V$. Thus suppose if not, then let f be a vertex which isn't fuzzy(neutrosophic) twin vertices with any given vertex and let f' be a vertex which is fuzzy(neutrosophic) twin vertices with any given vertex but not f. By supposition, it's possible and this is only case. Therefore, any given distinct vertex fuzzy(neutrosophic)-resolves f and f'. Then $V \setminus \{f, f'\}$ is fuzzy(neutrosophic)-resolving set. It implies fuzzy(neutrosophic)-metric number is $n-2$. □

Corollary 4.3. *Let G be a fuzzy(neutrosophic) graph. The number of fuzzy(neutrosophic) twin vertices is n. Then G is fixed-edge fuzzy(neutrosophic) graph.*

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Proof. Suppose f and f' are two given edges. By supposition, every couple of vertices are fuzzy(neutrosophic) twin vertices. It implies $\mu(f) = \mu(f')$. f and f' are arbitrary so every couple of edges have same values. It induces G is fixed-edge fuzzy(neutrosophic) graph. □

Corollary 4.4. *Let G be a fixed-vertex fuzzy(neutrosophic) graph. The number of fuzzy(neutrosophic) twin vertices is $n-1$. Then fuzzy(neutrosophic)-metric number is $n-2$, fuzzy(neutrosophic)-metric dimension is $(n-2)\sigma(m)$ where m is fuzzy(neutrosophic) twin vertex with a vertex. Every $(n-2)$-set including fuzzy(neutrosophic) twin vertices is fuzzy(neutrosophic)-metric set.*

Proof. By Corollary (4.2), fuzzy(neutrosophic)-metric number is $n-2$. By G is a fixed-vertex fuzzy(neutrosophic) graph, fuzzy metric dimension is $(n-2)\sigma(m)$ where m is fuzzy(neutrosophic) twin vertex with a vertex. One vertex doesn't belong to set of fuzzy(neutrosophic) twin vertices and a vertex from that set, are out of fuzzy metric set. It induces every $(n-2)$-set including fuzzy(neutrosophic) twin vertices is fuzzy metric set. □

Proposition 4.5. *Let G be a fixed-vertex fuzzy(neutrosophic) graph such that it's fuzzy(neutrosophic) complete. Then fuzzy(neutrosophic)-metric number is $n-1$, fuzzy(neutrosophic)-metric dimension is $(n-1)\sigma(m)$ where m is a given vertex. Every $(n-1)$-set is fuzzy(neutrosophic)-metric set.*

Proof. In fuzzy(neutrosophic) complete, every couple of vertices are twin vertices. By G is a fixed-vertex fuzzy(neutrosophic) graph and it's fuzzy(neutrosophic) complete, every couple of vertices are fuzzy(neutrosophic) twin vertices. Thus by Proposition (4.1), the result follows. □

Proposition 4.6. *Let $\mathcal{G}$ be a family of fuzzy(neutrosophic) graphs with common vertex set. Then simultaneously fuzzy(neutrosophic)-metric number of $\mathcal{G}$ is $n-1$.*

Proof. Consider $(n-1)$-set. Thus there's no couple of vertices to be fuzzy(neutrosophic)-resolved. Therefore, every $(n-1)$-set is fuzzy(neutrosophic)-resolving set for any given fuzzy(neutrosophic) graph. Then it holds for any fuzzy(neutrosophic) graph. It implies it's fuzzy(neutrosophic)-resolving set and its cardinality is fuzzy(neutrosophic)-metric number. $(n-1)$-set has the cardinality $n-1$. Then it holds for any fuzzy(neutrosophic) graph. It induces it's simultaneously fuzzy(neutrosophic)-resolving set and its cardinality is simultaneously fuzzy(neutrosophic)-metric number. □

Proposition 4.7. *Let $\mathcal{G}$ be a family of fuzzy(neutrosophic) graphs with common vertex set. Then simultaneously fuzzy(neutrosophic)-metric number of $\mathcal{G}$ is greater than the maximum fuzzy(neutrosophic)-metric number of $G \in \mathcal{G}$.*

Proof. Suppose t and t' are simultaneously fuzzy(neutrosophic)-metric number of $\mathcal{G}$ and fuzzy(neutrosophic)-metric number of $G \in \mathcal{G}$. Thus t is fuzzy(neutrosophic)-metric number for any $G \in \mathcal{G}$. Hence, $t \geq t'$. So simultaneously fuzzy(neutrosophic)-metric number of $\mathcal{G}$ is greater than the maximum fuzzy(neutrosophic)-metric number of $G \in \mathcal{G}$. □

Proposition 4.8. *Let $\mathcal{G}$ be a family of fuzzy(neutrosophic) graphs with common vertex set. Then simultaneously fuzzy(neutrosophic)-metric number of $\mathcal{G}$ is greater than simultaneously fuzzy(neutrosophic)-metric number of $\mathcal{H} \subseteq \mathcal{G}$.*

Proof. Suppose t and t' are simultaneously fuzzy(neutrosophic)-metric number of $\mathcal{G}$ and $\mathcal{H}$. Thus t is fuzzy(neutrosophic)-metric number for any $G \in \mathcal{G}$. It implies t is fuzzy(neutrosophic)-metric number for any $G \in \mathcal{H}$. So t is simultaneously fuzzy(neutrosophic)-metric number of $\mathcal{H}$. By applying Definition about being the minimum number, $t \geq t'$. So simultaneously fuzzy(neutrosophic)-metric number of $\mathcal{G}$ is greater than simultaneously fuzzy(neutrosophic)-metric number of $\mathcal{H} \subseteq \mathcal{G}$. □

Theorem 4.9. *Fuzzy(neutrosophic) twin vertices aren't fuzzy(neutrosophic)-resolved in any given fuzzy(neutrosophic) graph.*

Proof. Let t and t' be fuzzy(neutrosophic) twin vertices. Then $N(t) = N(t')$ and $\mu(ts) = \mu(t's)$, for all $s, s' \in V$ such that $ts, t's \in E$. Thus for every given vertex $s' \in V$, $d_G(s', t) = d_G(s, t)$ where G is a given fuzzy(neutrosophic) graph. It means that t and t' aren't resolved in any given fuzzy(neutrosophic) graph. t and t' are arbitrary so fuzzy(neutrosophic) twin vertices aren't resolved in any given fuzzy(neutrosophic) graph. □

Proposition 4.10. *Let G be a fixed-vertex fuzzy(neutrosophic) graph. If G is fuzzy(neutrosophic) complete, then every couple of vertices are fuzzy(neutrosophic) twin vertices.*

Proof. Let t and t' be couple of given vertices. By G is fuzzy(neutrosophic) complete, $N(t) = N(t')$. By G is a fixed-vertex fuzzy(neutrosophic) graph, $\mu(ts) = \mu(t's)$, for all edges $ts, t's \in E$. Thus t and t' are fuzzy(neutrosophic) twin vertices. t and t' are arbitrary couple of vertices, hence every couple of vertices are fuzzy(neutrosophic) twin vertices. □

Theorem 4.11. *Let $\mathcal{G}$ be a family of fuzzy(neutrosophic) graphs with common vertex set and $G \in \mathcal{G}$ is a fixed-vertex fuzzy(neutrosophic) graph such that it's fuzzy(neutrosophic) complete. Then simultaneously fuzzy(neutrosophic)-metric number is $n - 1$, simultaneously fuzzy(neutrosophic)-metric dimension is $(n - 1)\sigma(m)$ where m is a given vertex. Every $(n - 1)$-set is simultaneously fuzzy(neutrosophic)-metric set for $\mathcal{G}$.*

Proof. G is fixed-vertex fuzzy(neutrosophic) graph and it's fuzzy(neutrosophic) complete. So by Theorem (4.10), we get every couple of vertices in fuzzy(neutrosophic) complete are fuzzy(neutrosophic) twin vertices. So every couple of vertices, by Theorem (4.9), aren't resolved. □

Corollary 4.12. *Let $\mathcal{G}$ be a family of fuzzy(neutrosophic) graphs with fuzzy(neutrosophic) common vertex set and $G \in \mathcal{G}$ is a fuzzy(neutrosophic) complete. Then simultaneously fuzzy(neutrosophic)-metric number is $n - 1$, simultaneously fuzzy(neutrosophic)-metric dimension is $(n - 1)\sigma(m)$ where m is a given vertex. Every $(n - 1)$-set is simultaneously fuzzy(neutrosophic)-metric set for $\mathcal{G}$.*

Proof. By fuzzy(neutrosophic) graphs with fuzzy(neutrosophic) common vertex set, G is fixed-vertex fuzzy(neutrosophic) graph. It's fuzzy(neutrosophic) complete. So by Theorem (4.11), we get intended result. □

Theorem 4.13. *Let $\mathcal{G}$ be a family of fuzzy(neutrosophic) graphs with common vertex set and for every given couple of vertices, there's a $G \in \mathcal{G}$ such that in that, they're fuzzy(neutrosophic) twin vertices. Then simultaneously fuzzy(neutrosophic)-metric number is $n - 1$, simultaneously fuzzy(neutrosophic)-metric dimension is $(n - 1)\sigma(m)$ where m is a given vertex. Every $(n - 1)$-set is simultaneously fuzzy(neutrosophic)-metric set for $\mathcal{G}$.*

Proof. By Proposition (4.6), simultaneously fuzzy(neutrosophic)-metric number is $n-1$. By Theorem (4.9), simultaneously fuzzy(neutrosophic)-metric dimension is $(n-1)\sigma(m)$ where m is a given vertex. Also, every $(n-1)$-set is simultaneously fuzzy(neutrosophic)-metric set for $\mathcal{G}$. □

Theorem 4.14. *Let $\mathcal{G}$ be a family of fuzzy(neutrosophic) graphs with common vertex set. If $\mathcal{G}$ contains three fixed-vertex fuzzy(neutrosophic) stars with different center, then simultaneously fuzzy(neutrosophic)-metric number is $n-2$, simultaneously fuzzy(neutrosophic)-metric dimension is $(n-2)\sigma(m)$ where m is a given vertex. Every $(n-2)$-set is simultaneously fuzzy(neutrosophic)-metric set for $\mathcal{G}$.*

Proof. The cardinality of set of fuzzy(neutrosophic) twin vertices is $n-1$. Thus by Corollary (4.4), the result follows. □

Corollary 4.15. *Let $\mathcal{G}$ be a family of fuzzy(neutrosophic) graphs with fuzzy(neutrosophic) common vertex set. If $\mathcal{G}$ contains three fuzzy(neutrosophic) stars with different center, then simultaneously fuzzy(neutrosophic)-metric number is $n-2$, simultaneously fuzzy(neutrosophic)-metric dimension is $(n-2)\sigma(m)$ where m is a given vertex. Every $(n-2)$-set is simultaneously fuzzy(neutrosophic)-metric set for $\mathcal{G}$.*

Proof. By fuzzy(neutrosophic) graphs with fuzzy(neutrosophic) common vertex set, G is fixed-vertex fuzzy(neutrosophic) graph. It's fuzzy(neutrosophic) complete. So by Theorem (4.14), we get intended result. □

5 Antipodal Vertices

Proposition 5.1. *Consider two antipodal vertices x and y in any given fuzzy(neutrosophic) cycle. Let u and v be given vertices. Then $d(x,u) \neq d(x,v)$ if and only if $d(y,u) \neq d(y,v)$.*

Proof. $(\Rightarrow)$. Consider $d(x,u) \neq d(x,v)$. By $d(x,u) + d(u,y) = d(x,y) = D(G),\ D(G) - d(x,u) \neq D(G) - d(x,v)$. It implies $d(y,u) \neq d(y,v)$.

$(\Leftarrow)$. Consider $d(y,u) \neq d(y,v)$. By $d(y,u) + d(u,x) = d(x,y) = D(G),\ D(G) - d(y,u) \neq D(G) - d(y,v)$. It implies $d(x,u) \neq d(x,v)$. □

Proposition 5.2. *Consider two antipodal vertices x and y in any given even fuzzy(neutrosophic) cycle. Let u and v be given vertices. Then $d(x,u) = d(x,v)$ if and only if $d(y,u) = d(y,v)$.*

Proof. $(\Rightarrow)$. Consider $d(x,u) = d(x,v)$. By $d(x,u) + d(u,y) = d(x,y) = D(G),\ D(G) - d(x,u) = D(G) - d(x,v)$. It implies $d(y,u) = d(y,v)$.

$(\Leftarrow)$. Consider $d(y,u) = d(y,v)$. By $d(y,u) + d(u,x) = d(x,y) = D(G),\ D(G) - d(y,u) = D(G) - d(y,v)$. It implies $d(x,u) = d(x,v)$. □

Proposition 5.3. *The set contains two antipodal vertices, isn't fuzzy(neutrosophic)-metric set in any given even fuzzy(neutrosophic) cycle.*

Proof. Let x and y be two given antipodal vertices in any given even fuzzy(neutrosophic) cycle. By Proposition (5.1), $d(x,u) \neq d(x,v)$ if and only if $d(y,u) \neq d(y,v)$. It implies that if x fuzzy(neutrosophic)-resolves a couple of vertices, then y fuzzy(neutrosophic)-resolves them, too. Thus either x is in fuzzy(neutrosophic)-metric set or y is. It induces the set contains two antipodal vertices, isn't fuzzy(neutrosophic)-metric set in any given even fuzzy(neutrosophic) cycle. □

Proposition 5.4. *Consider two antipodal vertices x and y in any given even fuzzy(neutrosophic) cycle. x fuzzy(neutrosophic)-resolves a given couple of vertices, z and z', if and only if y does.*

Proof. (⇒). x fuzzy(neutrosophic)-resolves a given couple of vertices, z and z', then $d(x,z) \neq d(x,z')$. By Proposition (5.1), $d(x,z) \neq d(x,z')$ if and only if $d(y,z) \neq d(y,z')$. Thus y fuzzy(neutrosophic)-resolves a given couple of vertices z and z'.

(⇐). y fuzzy(neutrosophic)-resolves a given couple of vertices, z and z', then $d(y,z) \neq d(y,z')$. By Proposition (5.1), $d(y,z) \neq d(y,z')$ if and only if $d(x,z) \neq d(x,z')$. Thus x fuzzy(neutrosophic)-resolves a given couple of vertices z and z'. □

Proposition 5.5. *There are two antipodal vertices aren't fuzzy(neutrosophic)-resolved by other two antipodal vertices in any given even fuzzy(neutrosophic) cycle.*

Proof. Suppose x and y are a couple of vertices. It implies $d(x,y) = D(G)$. Consider u and v are another couple of vertices such that $d(x,u) = \frac{D(G)}{2}$. It implies $d(y,u) = \frac{D(G)}{2}$. Thus $d(x,u) = d(y,u)$. Therefore, u doesn't fuzzy(neutrosophic)-resolve a given couple of vertices x and y. By $D(G) = d(u,v) = d(u,x) + d(x,v) = \frac{D(G)}{2} + d(x,v)$, $d(x,v) = \frac{D(G)}{2}$. It implies $d(y,v) = \frac{D(G)}{2}$. Thus $d(x,v) = d(y,v)$. Therefore, v doesn't fuzzy(neutrosophic)-resolve a given couple of vertices x and y. □

Proposition 5.6. *Let G be a fixed-edge odd fuzzy(neutrosophic) cycle. Then every couple of vertices are fuzzy(neutrosophic)-resolving set.*

Proof. Let l and l' be couple of vertices. Thus, by G is odd cycle, l and l' aren't antipodal vertices. It implies for every given couple of vertices f_i and f_j, we get either $d(l,f_i) \neq d(l,f_j)$ or $d(l',f_i) \neq d(l',f_j)$. Therefore, f_i and f_j are fuzzy(neutrosophic)-resolved by either l or l'. It induces the set $\{l,l'\}$ is fuzzy(neutrosophic)-resolving set. □

Proposition 5.7. *Let G be a fixed-edge odd fuzzy(neutrosophic) cycle. Then fuzzy(neutrosophic)-metric number is two.*

Proof. Let l and l' be couple of vertices. Thus, by G is odd cycle, l and l' aren't antipodal vertices. It implies for every given couple of vertices f_i and f_j, we get either $d(l,f_i) \neq d(l,f_j)$ or $d(l',f_i) \neq d(l',f_j)$. Therefore, f_i and f_j are fuzzy(neutrosophic)-resolved by either l or l'. It induces the set $\{l,l'\}$ is fuzzy(neutrosophic)-resolving set. □

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