Metric Dimension in Fuzzy Graphs and Neutrosophic Graphs

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Abstract

In this study, the term dimension is introduced on fuzzy graphs and neutrosophic graphs. The classes of these specific graphs are chosen to obtain some results based on dimension. The types of crisp notions and fuzzy notions are used to make sense about the material of this study and the outline of this study uses some new notions which are crisp and fuzzy.

Keywords: Fuzzy Graphs, Neutrosophic Graphs, Dimension
AMS Subject Classification: 05C17, 05C22, 05E45

1   Background   1

Some literatures like cardinality of set, n-set, notation of fuzzy graphs, common vertex   2
set with same values for vertices and edges, permutation, fuzzy vertex set: vertices and   3
their values, fuzzy edge set: edges and their values. fixed-edge fuzzy graphs fixed-vertex   4
fuzzy graphs, fuzzy twin vertices are used. For using material look at [1–15].   5

2   Definitions   6

We use the notion of vertex in fuzzy graphs to define new notions which state the   7
relation amid vertices. In this way, the set of vertices are distinguished by another set of 8
vertices.   9

Definition 2.1. Let $G = (V, \sigma, \mu)$ be a fuzzy graph. A vertex m *fuzzy-resolves* vertices   10
f_1 and f_2 if $d(m, f_1) \neq d(m, f_2)$. A set M is *fuzzy resolving set* if for every couple of   11
vertices $f_1, f_2 \in V \setminus M$, there's a vertex $m \in M$ such that m fuzzy-resolves f_1 and f_2.   12
$|M|$ is called *fuzzy metric number* of G and $\min_M \Sigma_{m \in M} \sigma(m)$ is called *fuzzy metric*   13
dimension of G and if fuzzy metric number of set M equals fuzzy metric dimension,   14
then M is called *fuzzy metric set* of G.   15

Example 2.2. Let G be a fuzzy graph as figure (1). By applying Table (1), the 1-set is   16
explored which its cardinality is minimum. $\{f_6\}$ and $\{f_4\}$ are 1-set which has minimum   17
cardinality amid all sets of vertices but $\{f_4\}$ isn't fuzzy resolving set and $\{f_6\}$ is fuzzy   18
resolving set. Thus there's no fuzzy metric set but $\{f_6\}$. f_6 fuzzy-resolves all given   19
couple of vertices. Therefore one is fuzzy metric number of G and 0.13 is fuzzy metric   20
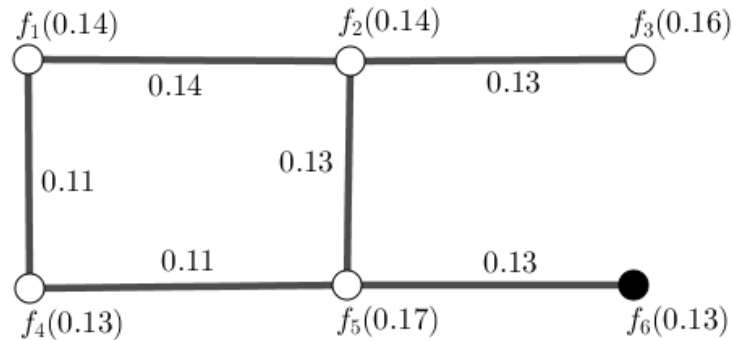
Figure 1. Black vertex $\{f_6\}$ is only fuzzy metric set amid all sets of vertices for fuzzy graph G.

dimension of G. By using Table (1), f_4 doesn't fuzzy-resolve f_2 and f_6. f_4 doesn't fuzzy-resolve f_1 and f_5, too.

Table 1. Distances of Vertices from sets of vertices $\{f_6\}$ and $\{f_4\}$ in Fuzzy Graph G.

Vertices	f_1	f_2	f_3	f_4	f_5	f_6
f_6	0.22	0.26	0.39	0.24	0.13	0
Vertices	f_1	f_2	f_3	f_4	f_5	f_6
f_4	0.11	0.24	0.37	0	0.11	0.24

Definition 2.3. Consider $\mathcal{G}$ as a family of fuzzy graphs on a common vertex set V. A vertex m *simultaneously fuzzy-resolves* vertices f_1 and f_2 if $d_G(m, f_1) \neq d_G(m, f_2)$, for all $G \in \mathcal{G}$. A set M is *simultaneously fuzzy resolving set* if for every couple of vertices $f_1, f_2 \in V \setminus M$, there's a vertex $m \in M$ such that m resolves f_1 and f_2, for all $G \in \mathcal{G}$. $|M|$ is called *simultaneously fuzzy metric number* of $\mathcal{G}$ and $\min \sigma_{m \in V} \sigma(m)$ is called *simultaneously fuzzy metric dimension* of $\mathcal{G}$ and if the simultaneously fuzzy cardinality of set M equals simultaneously fuzzy metric dimension, then M is called *simultaneously fuzzy metric set* of $\mathcal{G}$.

Example 2.4. Let $\mathcal{G} = \{G_1, G_2, G_3\}$ be a collection of fuzzy graphs with common fuzzy vertex set and a subset of fuzzy edge set as figure (2). By applying Table (2), the 1-set is explored which its cardinality is minimum. $\{f_2\}$ and $\{f_4\}$ are 1-set which has minimum cardinality amid all sets of vertices. $\{f_4\}$ is as fuzzy resolving set as $\{f_6\}$ is. Thus there's no fuzzy metric set but $\{f_4\}$ and $\{f_6\}$. f_6 as fuzzy-resolves all given couple of vertices as f_4. Therefore one is fuzzy metric number of $\mathcal{G}$ and 0.13 is fuzzy metric dimension of $\mathcal{G}$. By using Table (2), f_4 fuzzy-resolves all given couple of vertices.
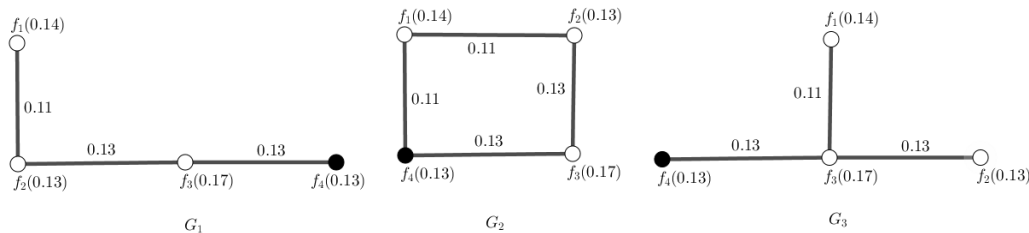
Figure 2. Black vertex $\{f_4\}$ and the set of vertices $\{f_2\}$ are simultaneously fuzzy metric set amid all sets of vertices for family of fuzzy graphs $\mathcal{G}$.

Table 2. Distances of Vertices from set of vertices $\{f_6\}$ in Family of Fuzzy Graphs $\mathcal{G}$.

Vertices of G_1	f_1	f_2	f_3	f_4
f_4	0.37	0.26	0.13	0
Vertices of G_2	f_1	f_2	f_3	f_4
f_4	0.11	0.22	0.13	0
Vertices of G_3	f_1	f_2	f_3	f_4
f_4	0.24	0.26	0.13	0

3   General Relationships

Proposition 3.1. *Let G be a path fuzzy graph. Then every leaf is fuzzy resolving set.*

Proof. Let l be a leaf. For every given a couple of vertices f_i and f_j, we get $d(l, f_i) \neq d(l, f_j)$. Since if we reassign indexes to vertices such that every vertex f_i and l have i vertices amid themselves, then $d(l, f_i) = \Sigma_{j \leq i} \mu(f_j f_i) \leq i$. Thus $j \leq i$ implies

$$\Sigma_{t \leq j} \mu(f_t f_j) + \Sigma_{j \leq s \leq i} \mu(f_s f_i) > \Sigma_{j \leq i} \mu(f f_i) \equiv d(l, f_j) + c = d(l, f_i) \equiv d(l, f_j) < d(l, f_i).$$

Therefore, by $d(l, f_j) < d(l, f_i)$, we get $d(l, f_i) \neq d(l, f_j)$. f_i and f_j are arbitrary so l fuzzy-resolves any given couple of vertices f_i and f_j which implies $\{l\}$ is a fuzzy resolving set. □

Corollary 3.2. *Let G be a fixed-edge path fuzzy graph. Then every leaf is fuzzy resolving set.*

Proof. Let l be a leaf. For every given couple of vertices, f_i and f_j, we get $d(l, f_i) = ci \neq d(l, f_j) = cj$. It implies l fuzzy-resolves any given couple of vertices f_i and f_j which implies $\{l\}$ is a fuzzy resolving set. □

Corollary 3.3. *Let G be a fixed-vertex path fuzzy graph. Then every leaf is fuzzy metric set, fuzzy metric number is one and fuzzy metric dimension is c where $c = \sigma(f)$, $f \in V$.*

Proof. By Proposition (3.1), every leaf is fuzzy resolving set. By $c = \sigma(f)$, $\forall f \in V$, every leaf is fuzzy metric set, fuzzy metric number is one and fuzzy metric dimension is c. □

Proposition 3.4. *Let G be a path fuzzy graph. Then a set including every couple of vertices is fuzzy resolving set.*

Proof. Let f and f' be a couple of vertices. For every given a couple of vertices f_i and f_j, we get either $d(f, f_i) \neq d(f, f_j)$ or $d(f', f_i) \neq d(f', f_j)$. □

Corollary 3.5. *Let G be a fixed-edge path fuzzy graph. Then every set containing couple of vertices is fuzzy resolving set.*

Proposition 3.6. *Let G be a fuzzy graph. An $(k-1)$-set from an k-set of fuzzy twin vertices is subset of a fuzzy resolving set.*

Proof. If t and t' are fuzzy twin vertices, then $N(t) = N(t')$ and $\mu(ts) = \mu(t's)$, for all $s \in N(t) = N(t')$. □

Corollary 3.7. *Let G be a fuzzy graph. The number of fuzzy twin vertices is $n-1$. Then fuzzy metric number is $n-2$.*

Corollary 3.8. *Let G be a fuzzy graph. The number of fuzzy twin vertices is $n-1$. Then G is fixed-edge fuzzy graph.*

Corollary 3.9. *Let G be a fixed-vertex fuzzy graph. The number of fuzzy twin vertices is $n-1$. Then fuzzy metric number is $n-2$, fuzzy metric dimension is $(n-2)\sigma(m)$ where m is fuzzy twin vertex with a vertex. Every $(n-2)$-set including fuzzy twin vertices is fuzzy metric set.*

Proposition 3.10. *Let G be a fixed-vertex fuzzy graph such that it's fuzzy complete. Then fuzzy metric number is $n-1$, fuzzy metric dimension is $(n-1)\sigma(m)$ where m is a given vertex. Every $(n-1)$-set is fuzzy metric set.*

Proof. In complete graph, every couple of vertices are twin vertices. By G is a fixed-vertex fuzzy graph and it's fuzzy complete, every couple of vertices are fuzzy twin vertices. Thus by Proposition (3.6), the result follows. □

Proposition 3.11. *Let $\mathcal{G}$ be a family of fuzzy graphs with common vertex set. Then simultaneously fuzzy metric number of $\mathcal{G}$ is $n-1$.*

Proof. Consider $(n-1)$-set. Thus there's no couple of vertices to be fuzzy-resolved. Therefore, every $(n-1)$-set is fuzzy resolving set for any given fuzzy graph. Then it holds for any fuzzy graph. It implies it's fuzzy resolving set and its cardinality is fuzzy metric number. $(n-1)$-set has the cardinality $n-1$. Then it holds for any fuzzy graph. It induces it's simultaneously fuzzy resolving set and its cardinality is simultaneously fuzzy metric number. □

Proposition 3.12. *Let $\mathcal{G}$ be a family of fuzzy graphs with common vertex set. Then simultaneously fuzzy metric number of $\mathcal{G}$ is greater than the maximum fuzzy metric number of $G \in \mathcal{G}$.*

Proof. Suppose t and t' are simultaneously fuzzy metric number of $\mathcal{G}$ and fuzzy metric number of $G \in \mathcal{G}$.. Thus t is fuzzy metric number for any $G \in \mathcal{G}$. Hence, $t \geq t'$. So simultaneously fuzzy metric number of $\mathcal{G}$ is greater than the maximum fuzzy metric number of $G \in \mathcal{G}$. □

Proposition 3.13. *Let $\mathcal{G}$ be a family of fuzzy graphs with common vertex set. Then simultaneously fuzzy metric number of $\mathcal{G}$ is greater than simultaneously fuzzy metric number of $\mathcal{H} \subseteq \mathcal{G}$.*

Proof. Suppose t and t' are simultaneously fuzzy metric number of $\mathcal{G}$ and $\mathcal{H}$. Thus t is fuzzy metric number for any $G \in \mathcal{G}$. It implies Thus t is fuzzy metric number for any $G \in \mathcal{H}$. So t is simultaneously fuzzy metric number of $\mathcal{H}$. By applying Definition about being the minimum number, $t \geq t'$. So simultaneously fuzzy metric number of $\mathcal{G}$ is greater than simultaneously fuzzy metric number of $\mathcal{H} \subseteq \mathcal{G}$. □

Theorem 3.14. *Fuzzy twin vertices aren't resolved in any given fuzzy graph.*

Proof. Let t and t' are fuzzy twin vertices. Then $N(t) = N(t')$ and $\mu(ts) = \mu(t's)$, for all edges $ts, t's \in E$. Thus for every given vertex $s' \in V$, $d_G(s', t) = d_G(s, t)$ where G is a given fuzzy graph. It means that t and t' aren't resolved in any given fuzzy graph. t and t' are arbitrary so fuzzy twin vertices aren't resolved in any given fuzzy graph. □

Proposition 3.15. *Let G be a fixed-vertex fuzzy graph. If G is fuzzy complete, then every couple of vertices are fuzzy twin vertices.*

Proof. Let t and t' be couple of given vertices. By G is fuzzy complete, $N(t) = N(t')$. By G is a fixed-vertex fuzzy graph, $\mu(ts) = \mu(t's)$, for all edges $ts, t's \in E$. Thus t and t' are fuzzy twin vertices. t and t' are arbitrary couple of vertices, hence every couple of vertices are fuzzy twin vertices. □

Theorem 3.16. *Let $\mathcal{G}$ be a family of fuzzy graphs with common vertex set and $G \in \mathcal{G}$ is a fixed-vertex fuzzy graph such that it's fuzzy complete. Then simultaneously fuzzy metric number is $n - 1$, simultaneously fuzzy metric dimension is $(n - 1)\sigma(m)$ where m is a given vertex. Every $(n - 1)$-set is simultaneously fuzzy metric set for $\mathcal{G}$.*

Proof. G is fixed-vertex fuzzy graph and it's fuzzy complete. So by Proposition (3.15), we get every couple of vertices in fuzzy complete are fuzzy twin vertices. So every couple of vertices, by Theorem (3.14), aren't resolved. □

Theorem 3.17. *Let $\mathcal{G}$ be a family of fuzzy graphs with common vertex set and for every given couple of vertices, there's a $G \in \mathcal{G}$ such that in that, they're fuzzy twin vertices. Then simultaneously fuzzy metric number is $n - 1$, simultaneously fuzzy metric dimension is $(n - 1)\sigma(m)$ where m is a given vertex. Every $(n - 1)$-set is simultaneously fuzzy metric set for $\mathcal{G}$.*

Theorem 3.18. *Let $\mathcal{G}$ be a family of fuzzy graphs with common vertex set. If $\mathcal{G}$ contains three fixed-vertex fuzzy stars with different center, then simultaneously fuzzy metric number is $n - 2$, simultaneously fuzzy metric dimension is $(n - 2)\sigma(m)$ where m is a given vertex. Every $(n - 2)$-set is simultaneously fuzzy metric set for $\mathcal{G}$.*

Proof. By Corollary (3.9), the result follows. □

References

1. M. Akram, and G. Shahzadi, *Operations on Single-Valued Neutrosophic Graphs*, Journal of uncertain systems 11 (1) (2017) 1-26.

2. K. Atanassov, *Intuitionistic fuzzy sets*, Fuzzy Sets Syst. 20 (1986) 87-96.

3. S. Broumi, M. Talea, A. Bakali and F. Smarandache, *Single-valued neutrosophic graphs*, Journal of New Theory 10 (2016) 86-101.

4. N. Shah, and A. Hussain, *Neutrosophic soft graphs*, Neutrosophic Set and Systems 11 (2016) 31-44.

5. Henry Garrett, *Big Sets Of Vertices*, Preprints 2021, 2021060189 (doi: 10.20944/preprints202106.0189.v1).

6. Henry Garrett, *Locating And Location Number*, Preprints 2021, 2021060206 (doi: 10.20944/preprints202106.0206.v1).

7. Henry Garrett, *Metric Dimensions Of Graphs*, Preprints 2021, 2021060392 (doi: 10.20944/preprints202106.0392.v1).

8. Henry Garrett, *New Graph Of Graph*, Preprints 2021, 2021060323 (doi: 10.20944/preprints202106.0323.v1).

9. Henry Garrett, *Numbers Based On Edges*, Preprints 2021, 2021060315 (doi: 10.20944/preprints202106.0315.v1).

10. Henry Garrett, *Matroid And Its Outlines*, Preprints 2021, 2021060146 (doi: 10.20944/preprints202106.0146.v1).

11. Henry Garrett, *Matroid And Its Relations*, Preprints 2021, 2021060080 (doi: 10.20944/preprints202106.0080.v1).

12. A. Shannon and K.T. Atanassov, *A first step to a theory of the intuitionistic fuzzy graphs*, Proceeding of FUBEST (Lakov, D., Ed.) Sofia (1994) 59-61.

13. F. Smarandache, *A Unifying field in logics neutrosophy: Neutrosophic probability, set and logic, Rehoboth:* American Research Press (1998).

14. H. Wang, F. Smarandache, Y. Zhang, and R. Sunderraman, *Single-valued neutrosophic sets*, Multispace and Multistructure 4 (2010) 410-413.

15. L. A. Zadeh, *Fuzzy sets*, Information and Control 8 (1965) 338-353.