Class 2 heating cycles: A new class of thermodynamic cycles

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Supplementary Information

Figure S1 shows three typical examples of class 2 heating cycles (HC-2s), which have been presented in the main text. Without loss of generality, they are all in basic form A (HC-2A). Here we derive their coefficients of performance (COPs, the ratio of the cycle's output to input) for high-temperature heating when the cycle's net work output $W_{net} = 0$. The former two cycles are regarded as internally reversible [1].

1. The HC-2A with isothermal heat transfer processes (Figure S1a)

According to the law of conservation of energy, we have

$$W_{net} = Q_{in} - Q_{out,\,H} - Q_{out,\,L} = 0 \tag{S1}$$

where Q_{in} is the heat absorbed from the medium-temperature heat source by the cycle, $Q_{out,\,H}$ is the heat rejected to the high-temperature heat sink by the cycle, and $Q_{out,\,L}$ is the heat rejected to the low-temperature heat sink by the cycle.

The cycle's net entropy change should be zero. Thus,

$$\oint \mathrm{d}S = \frac{Q_{\mathrm{in}}}{T_{\mathrm{M}}} - \frac{Q_{\mathrm{out,\,H}}}{T_{\mathrm{H}}} - \frac{Q_{\mathrm{out,\,L}}}{T_{\mathrm{L}}} = 0 \tag{S2}$$

Combining Eq. (S1) and Eq. (S2), we can express the COPs as

$$\mathrm{COP}_{\mathrm{HD}} = \frac{Q_{\mathrm{out,\,H}}}{Q_{\mathrm{in}}} = \frac{T_{\mathrm{H}}(T_{\mathrm{M}} - T_{\mathrm{L}})}{T_{\mathrm{M}}(T_{\mathrm{H}} - T_{\mathrm{L}})} \tag{S3}$$

$$\mathrm{COP}_{\mathrm{CD}} = \frac{Q_{\mathrm{out,\,H}}}{Q_{\mathrm{out,\,L}}} = \frac{T_{\mathrm{H}}(T_{\mathrm{M}} - T_{\mathrm{L}})}{T_{\mathrm{L}}(T_{\mathrm{H}} - T_{\mathrm{M}})} \tag{S4}$$

where $\mathrm{COP}_{\mathrm{HD}}$ and $\mathrm{COP}_{\mathrm{CD}}$ are the COPs when the cycle is driven by medium-temperature heat and by low-temperature cold energy respectively.

2. The HC-2A with isobaric heat transfer processes, employing an ideal gas as its working medium (Figure S1b)

Notice that state point 7 follows the rules of both heat absorption and depressurization. Dividing the medium-temperature heat Q_{in} into two parts, and regarding the ideal gas's isobaric specific heat c_p as constant [1], we obtain

$$W_{\mathrm{net}} = Q_{\mathrm{in}} - Q_{\mathrm{out,\,H}} - Q_{\mathrm{out,\,L}} = \left(Q_{\mathrm{in}}^{(1)} + Q_{\mathrm{in}}^{(2)}\right) - Q_{\mathrm{out,\,H}} - Q_{\mathrm{out,\,L}} = 0 \tag{S5}$$

$$\begin{aligned} Q_{\mathrm{in}}^{(1)} &= H_7 - H_2 = mc_p(T_7 - T_2) \\ Q_{\mathrm{in}}^{(2)} &= H_3 - H_7 = mc_p(T_3 - T_7) \\ Q_{\mathrm{out,\,H}} &= H_4 - H_5 = mc_p(T_4 - T_5) \\ Q_{\mathrm{out,\,L}} &= H_6 - H_1 = mc_p(T_6 - T_1) \end{aligned} \tag{S6}$$

where H is the working medium's enthalpy at each state point, m is the working medium's mass, and T is the working medium's thermodynamic temperature at each state point.

According to the behavior of the ideal gas [1] and Eq. (S6), we have

$$\left(\frac{p_{\mathrm{H}}}{p_{\mathrm{M}}}\right)^{\frac{k-1}{k}}=\frac{T_4}{T_3}=\frac{T_5}{T_7}=\frac{T_4-T_5}{T_3-T_7}=\frac{Q_{\mathrm{out,\,H}}}{Q_{\mathrm{in}}^{(2)}} \tag{S7}$$

$$\left(\frac{p_{\mathrm{M}}}{p_{\mathrm{L}}}\right)^{\frac{k-1}{k}}=\frac{T_7}{T_6}=\frac{T_2}{T_1}=\frac{T_7-T_2}{T_6-T_1}=\frac{Q_{\mathrm{in}}^{(1)}}{Q_{\mathrm{out,\,L}}} \tag{S8}$$

where k is the ideal gas's specific heat ratio.

Combining Eq. (S5), Eq. (S7) and Eq. (S8), we can express the COPs as

$$\mathrm{COP}_{\mathrm{HD}}=\frac{Q_{\mathrm{out,\,H}}}{Q_{\mathrm{in}}}=\frac{Q_{\mathrm{out,\,H}}}{Q_{\mathrm{in}}^{(1)}+Q_{\mathrm{in}}^{(2)}}=\frac{p_{\mathrm{H}}^{\frac{k-1}{k}}\left(p_{\mathrm{M}}^{\frac{k-1}{k}}-p_{\mathrm{L}}^{\frac{k-1}{k}}\right)}{p_{\mathrm{M}}^{\frac{k-1}{k}}\left(p_{\mathrm{H}}^{\frac{k-1}{k}}-p_{\mathrm{L}}^{\frac{k-1}{k}}\right)} \tag{S9}$$

$$\mathrm{COP}_{\mathrm{CD}}=\frac{Q_{\mathrm{out,\,H}}}{Q_{\mathrm{out,\,L}}}=\frac{p_{\mathrm{H}}^{\frac{k-1}{k}}\left(p_{\mathrm{M}}^{\frac{k-1}{k}}-p_{\mathrm{L}}^{\frac{k-1}{k}}\right)}{p_{\mathrm{L}}^{\frac{k-1}{k}}\left(p_{\mathrm{H}}^{\frac{k-1}{k}}-p_{\mathrm{M}}^{\frac{k-1}{k}}\right)} \tag{S10}$$

or

$$\mathrm{COP}_{\mathrm{HD}}=\frac{T_5\left(T_7-T_6\right)}{T_7\left(T_5-T_6\right)}=\frac{T_4\left(T_2-T_1\right)}{T_2T_4-T_1T_3} \tag{S11}$$

$$\mathrm{COP}_{\mathrm{CD}}=\frac{T_5\left(T_7-T_6\right)}{T_6\left(T_5-T_7\right)}=\frac{T_4\left(T_2-T_1\right)}{T_1\left(T_4-T_3\right)} \tag{S12}$$

3. The HC-2A with isobaric heat transfer processes, employing a phase-change working medium (Figure S1c)

$$\mathrm{COP}_{\mathrm{HD}}=\frac{Q_{\mathrm{out,\,H}}}{Q_{\mathrm{in}}}=\frac{H_4-H_5}{H_3-H_2}=\frac{h_4-h_5}{h_3-h_2} \tag{S13}$$

$$\mathrm{COP_{CD}} = \frac{Q_{\mathrm{out,\,H}}}{Q_{\mathrm{out,\,L}}} = \frac{H_4 - H_5}{H_6 - H_1} = \frac{h_4 - h_5}{h_6 - h_1} \tag{S14}$$

where h is the working medium's enthalpy per unit of mass at each state point. These two formulae cannot be further simplified because the behavior of the phase-change working medium is much more complex than that of the ideal gas.

When $W_{\mathrm{net}} \neq 0$, we can also obtain the cycles' COPs in a similar way. However, since heat and power differ in grade, the meanings of such formulae are not clear.

Figures

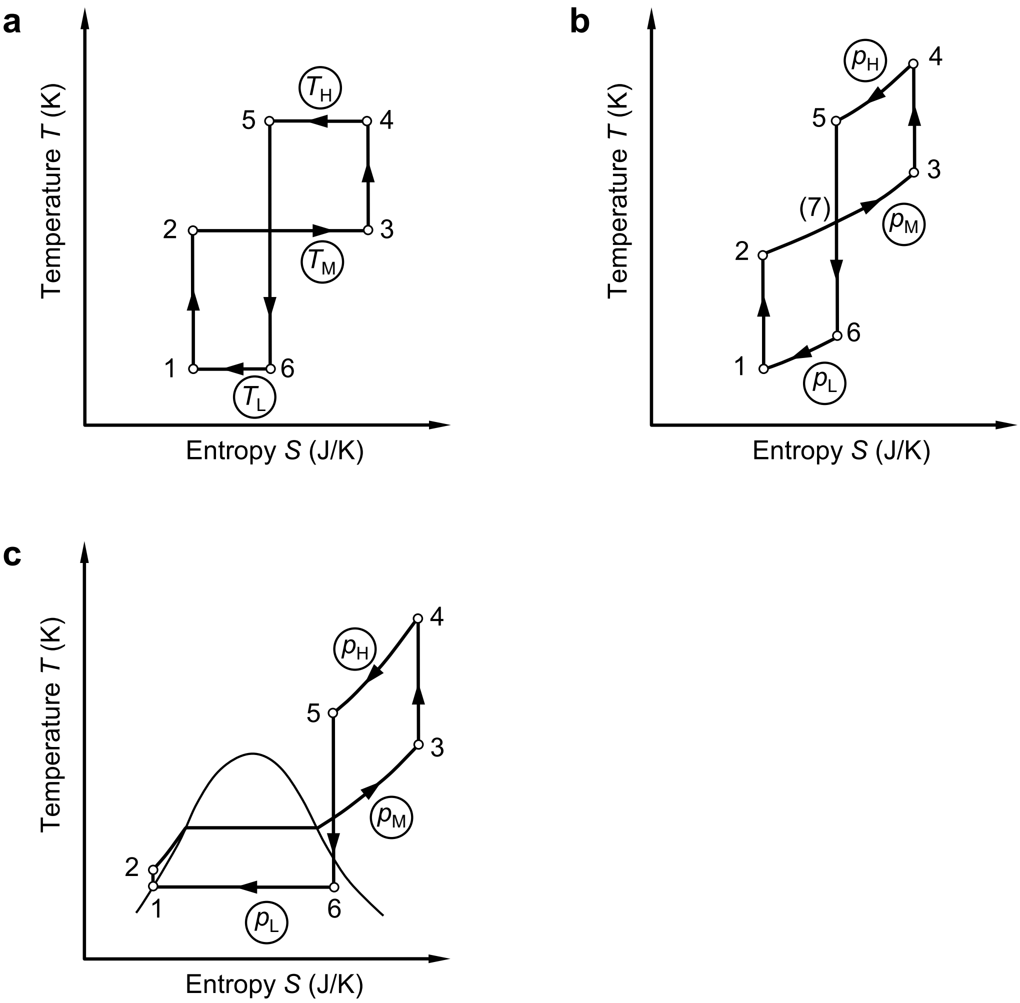

Figure S1. Three typical examples of HC-2s. (a) An HC-2A with isothermal heat transfer processes. T_H, T_M and T_L are the working medium's thermodynamic temperatures during high-temperature heat rejection, medium-temperature heat absorption, and low-temperature heat rejection, respectively. **(b)** An HC-2A with isobaric heat transfer processes, employing an ideal gas as its working medium. State point 7 is the state passed through by both process 2-3 and process 5-6. **(c)** An HC-2A with isobaric heat transfer processes, employing a phase-change working medium. In (b) and (c), p_H, p_M and p_L are the working medium's pressures during high-temperature heat rejection, medium-temperature heat absorption, and low-temperature heat rejection, respectively.

References

[1] Çengel YA, Boles MA, Kanoğlu M. Thermodynamics: An engineering approach. 9 ed. New York: McGraw-Hill Education, 2019.