Class 1 heating cycles: A new class of thermodynamic cycles

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Supplementary Information

Figure S1 shows three typical examples of class 1 heating cycles (HC-1s), which have been presented in the main text. Without loss of generality, they are all in basic form A (HC-1A). Here we derive their coefficients of performance (COPs, the ratio of the cycle’s output to input) for medium-temperature heating or low-temperature cooling when the cycle’s net work output $W_{\text{net}} = 0$. The former two cycles are regarded as internally reversible [¹].

1. The HC-1A with isothermal heat transfer processes (Figure S1a)

According to the law of conservation of energy, we have

$$W_{\text{net}} = Q_{\text{in}, \text{H}} + Q_{\text{in}, \text{L}} - Q_{\text{out}} = 0$$

(S1)

where $Q_{\text{in}, \text{H}}$ is the amount of heat absorbed from the high-temperature heat source by the cycle, $Q_{\text{in}, \text{L}}$ is the amount of heat absorbed from the low-temperature heat source
by the cycle, and $Q_{out}$ is the amount of heat rejected to the medium-temperature heat sink by the cycle. The cycle’s net entropy change should be zero. Thus,

$$\int dS = \frac{Q_{in,H}}{T_H} + \frac{Q_{in,L}}{T_L} - \frac{Q_{out}}{T_M} = 0$$ (S2)

Combining Eq. (S1) and Eq. (S2), we can express the COPs as

$$\text{COP}_H = \frac{Q_{out}}{Q_{in,H}} = \frac{T_M (T_H - T_L)}{T_H (T_M - T_L)}$$ (S3)

$$\text{COP}_C = \frac{Q_{in,L}}{Q_{in,H}} = \frac{T_L (T_H - T_M)}{T_H (T_M - T_L)}$$ (S4)

where COP$_H$ and COP$_C$ are the cycle’s COPs for medium-temperature heating and low-temperature cooling respectively.

2. The HC-1A with isobaric heat transfer processes, employing an ideal gas as its working medium (Figure S1b)

Notice that state point 7 follows the rules of both heat rejection and pressurization. Dividing the medium-temperature heat $Q_{out}$ into two parts, and regarding the ideal gas’s isobaric specific heat $c_p$ as constant [1], we obtain

$$W_{net} = Q_{in,H} + Q_{in,L} - Q_{out} = Q_{in,H} + Q_{in,L} - (Q_{out}^{(1)} + Q_{out}^{(2)}) = 0$$ (S5)

$$Q_{in,H} = H_3 - H_2 = mc_p (T_3 - T_2)$$

$$Q_{in,L} = H_1 - H_6 = mc_p (T_1 - T_6)$$

$$Q_{out}^{(1)} = H_4 - H_7 = mc_p (T_4 - T_7)$$

$$Q_{out}^{(2)} = H_7 - H_5 = mc_p (T_7 - T_5)$$ (S6)

where $H$ is the working medium’s enthalpy at each state point, $m$ is the working
medium’s mass, and \( T \) is the working medium’s thermodynamic temperature at each state point.

According to the behavior of the ideal gas \(^1\) and Eq. (S6), we have

\[
\left( \frac{p_{\text{H}}}{p_{\text{M}}} \right)^{\frac{k-1}{k}} = \frac{T_{\text{in}}}{T_{\text{out}}} = \frac{T_{\text{in}} - T_{\text{out}}}{T_{\text{out}} - T_{\text{in}}} = \frac{Q_{\text{in}, \text{H}}}{Q^{(1)}_{\text{out}}}
\]

(S7)

\[
\left( \frac{p_{\text{M}}}{p_{\text{L}}} \right)^{\frac{k-1}{k}} = \frac{T_{\text{in}}}{T_{\text{out}}} = \frac{T_{\text{in}} - T_{\text{out}}}{T_{\text{out}} - T_{\text{in}}} = \frac{Q_{\text{out}}^{(2)}}{Q_{\text{in}, \text{L}}}
\]

(S8)

where \( k \) is the ideal gas’s specific heat ratio.

Combining Eq. (S5), Eq. (S7) and Eq. (S8), we can express the COPs as

\[
\text{COP}_{\text{H}} = \frac{Q_{\text{out}}}{Q_{\text{in}, \text{H}}} = \frac{Q^{(1)}_{\text{out}} + Q^{(2)}_{\text{out}}}{Q_{\text{in}, \text{H}}} = \frac{\frac{k-1}{k} \left( \frac{p_{\text{H}}^{\frac{k-1}{k}} - p_{\text{L}}^{\frac{k-1}{k}}} {p_{\text{H}}^{\frac{k-1}{k}} - p_{\text{M}}^{\frac{k-1}{k}}} \right)}
\]

(S9)

\[
\text{COP}_{\text{C}} = \frac{Q_{\text{in}, \text{L}}}{Q_{\text{in}, \text{H}}} = \frac{\frac{k-1}{k} \left( \frac{p_{\text{L}}^{\frac{k-1}{k}} - p_{\text{M}}^{\frac{k-1}{k}}} {p_{\text{M}}^{\frac{k-1}{k}} - p_{\text{L}}^{\frac{k-1}{k}}} \right)}
\]

(S10)

or

\[
\text{COP}_{\text{H}} = \frac{Q_{\text{out}}}{Q_{\text{in}, \text{H}}} = \frac{T_{\text{out}} (T_{\text{in}} - T_{\text{out}})} {T_{\text{out}} (T_{\text{in}} - T_{\text{out}})} = \frac{T_{\text{out}} (T_{\text{in}} - T_{\text{out}})} {T_{\text{out}} (T_{\text{out}} - T_{\text{in}})}
\]

(S11)

\[
\text{COP}_{\text{C}} = \frac{Q_{\text{in}, \text{L}}}{Q_{\text{in}, \text{H}}} = \frac{T_{\text{in}} (T_{\text{out}} - T_{\text{in}})} {T_{\text{in}} (T_{\text{out}} - T_{\text{in}})} = \frac{T_{\text{out}} (T_{\text{out}} - T_{\text{in}})} {T_{\text{out}} (T_{\text{in}} - T_{\text{out}})}
\]

(S12)

3. The HC-1A with isobaric heat transfer processes, employing a phase-change working medium (Figure S1c)
\[
\text{COP}_{\text{h}} = \frac{Q_{\text{out}}}{Q_{\text{in}, \text{h}}} = \frac{H_1 - H_5}{H_3 - H_2} = \frac{h_1 - h_5}{h_3 - h_2} \tag{S13}
\]

\[
\text{COP}_{\text{c}} = \frac{Q_{\text{in}, \text{c}}}{Q_{\text{in}, \text{h}}} = \frac{H_1 - H_6}{H_3 - H_2} = \frac{h_1 - h_6}{h_3 - h_2} \tag{S14}
\]

where \(h\) is the working medium’s enthalpy per unit of mass at each state point. These two formulae cannot be further simplified because the behavior of the phase-change working medium is much more complex than that of the ideal gas.

When \(W_{\text{net}} \neq 0\), we can also obtain the cycles’ COPs in a similar way.

However, since heat and power differ in grade, the meanings of such formulae are not clear.
Figure S1. Three typical examples of HC-1s. (a) An HC-1A with isothermal heat transfer processes. $T_H$, $T_M$ and $T_L$ are the working medium’s thermodynamic temperatures during high-temperature heat absorption, medium-temperature heat rejection, and low-temperature heat absorption, respectively. (b) An HC-1A with isobaric heat transfer processes, employing an ideal gas as its working medium. State point 7 is the state passed through by both process 1-2 and process 4-5. (c) An HC-1A with isobaric heat transfer processes, employing a phase-change working medium. The depressurization (throttling) process 5-6 is internally irreversible and thus...
expressed as a dotted line. In (b) and (c), $p_h$, $p_m$ and $p_l$ are the working medium’s pressures during high-temperature heat absorption, medium-temperature heat rejection, and low-temperature heat absorption, respectively.
References