# Exact solutions for a novel $H_2$ optimal design of electromagnetic tuned mass damper

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**Abstract:** To realize structural vibration control,a two parameters $H_2$ optimization design was proposed to optimize the tuning ratio and damping ratio for electromagnetic tuned mass damper (EMTMD). The control effect of this two parameters optimization design is better than that of classical tuned mass damper (TMD).For this two parameters optimization,the most important thing is that the inductance of the coil can be set very small and the external load resistance can be positive ,which can avoid the use of complex negative impedance circuit. If Ref.[6] were designed according to the $H_2$ optimization of two parameters, the EMTMD can be used for multi-modal vibration control of structures without connecting negative inductance and negative resistance spontaneously.

**Key words:** electromagnetic tuned mass damper; $H_2$ optimization; structural vibration control; negative inductance ; negative resistance.

## 1 Introduction

With the development of structural vibration control technology, a large number of vibration control dampers have been proposed and applied to the fields of machinery, automobile and aerospace. Among the many dampers controlling vibration, the Tuned Mass Damper (TMD) is the most widely used.However, the general damping element of TMD is viscous damping, which is sensitive to temperature and difficult to adjust the damping coefficient.Electromagnetic damper is a research hotspot in recent years. Compared with traditional viscous damper, electromagnetic damper has the advantages of low noise, easy maintenance and long service life. It is widely used in vehicle suspension system and train braking system. Therefore, it can be considered to replace viscous damper with electromagnetic shunt damper to form a new type of electromagnetic tuned mass damper (Electromagnetic tuned mass damper，EMTMD).Zuo et al. [1] verified the feasibility of electromagnetic transducer replacing the viscous damping element in classical TMD through experiments. Liu et al. [2] proposed an EMTMD with L-R-C shunt circuit . In his study, the four designed parameters of EMTMD, including circuit tuning ratio, damping ratio, structural tuning ratio and electromechanical coupling coefficient, were optimized by $H_2$ optimization method. And the dual functions of vibration control and vibration energy collection of EMTMD were verified by experiments.

Luo[3] also studied an EMTMD with stiffness coupling to control the structural vibration of single degree of freedom system, and obtained the analytical solutions of structural frequency ratio, electromagnetic damping ratio and electromechanical coupling coefficient of EMTMD based on the optimization method.

However, although L-R-C resonant shunt circuit can effectively suppress structural vibration, there is a problem of frequency detunes. Once the resonant frequency of the circuit can not match the natural frequency of the structure, which will lead to the sharp decline or even failure of the control effects of the resonant shunt circuit. This high sensitivity to frequency seriously restricts the application and popularization of damping vibration technology of electromagnetic shunt circuit.

Yan [5] designed an EMTMD with L-R shunt circuit for multi-modal structural vibration control of cantilever beam, but the designed coil inductance and coil DC resistance are too large. Therefore, the negative inductive negative resistance circuit is connected to the shunt circuit to improve the vibration control effect. However, for his EMTMD designed,after the shunt circuit was connected to these analog circuits, the amplifier needs to be connected with additional power supply. Moreover, spontaneously the control form of EMTMD changes from passive control to semi-active control, and its control system will become complex and expensive.

In fact, for the optimal design of EMTMD, the real pursuit should be a flexible design method. For Liu, Luo s mentioned above, in their research, all other design parameters except mass ratio are considered, and the analytical solution of the optimal design parameters is obtained. However, some of these parameters are insensitive to the control effect, such as damping ratio and electromechanical coupling coefficient. Among them, it is difficult to make the electromechanical coupling coefficient reach the optimal design parameters.

According to the conclusions of scholars such as Zheng[6] and Inoue[7], the electromechanical coupling coefficient is related to the stiffness of the main system , electromagnetic coefficient and coil inductance ,while the electromagnetic coefficient and coil inductance coefficient are related to the number of turns n, width D and length of the coil. Therefore, the optimization of EMTMD could not consider the parameter electromechanical coupling coefficient, but only consider the optimization of two parameters including tuning ratio and damping ratio.The electromechanical coupling coefficient can be given a reasonable value. Thus, the advantage of this two parameters optimization method is that the coil inductance can be set relatively small, so the coil DC resistance will be small, and the connection of analog circuits such as negative inductance and resistance as in Ref.[6] can be avoided, also the complexity of the control system being reduced spontaneously .

Therefore, based on the above discussion, aiming at the vibration control of single degree of freedom structure with EMTMD (as shown in Fig.1), this paper uses $H_2$ optimization to obtain the analytical solution when only two parameters including tuning ratio and damping ratio are considered, also the analytical solution when three parameters such as tuning ratio, damping ratio and electromechanical coupling coefficient are considered.Then using numerical calculation and concrete data to show the superiority of two parameters $H_2$ optimization design than that of the three parameters.

This paper is organized as follows: Section 2 is a mathematical model for the EMTMD,also the displacement amplified factor being deduced .In Sec.3 the exact solutions and the optimal H2 performance for the two parameters H2 optimization was deduced.In Sec.4, the numerical calculation of the EMTMD was presented ,in comparision to classic TMDs and that of three parameters H2 optimization for EMTMD. Finally ,this paper was concluded in Sec.5.

## 2 Electromechanical coupling model

The model of electromagnetic tuned mass damper (EMTMD) is shown in Fig.1, where $m_1$ is the mass of the main system, $m_2$ is the mass of the dynamic vibration absorber, $k_1$ is the stiffness coefficient of the main system, $k_2$ is the stiffness coefficient of the absorber of the EMTMD. $e_M$ is the induced electromotive force. $\phi$ is the electromagnetic coefficient, which is related to the number of turns N and length $l$ of the coil and the magnetic induction intensity B of the permanent magnet [7]. L is the coil inductance. $R_0$ is the DC resistance of the coil. $R_{sh}$ is the external resistance of the coil,which can be positive resistance or negative resistance. If $R_{sh}$ is negative resistance,the system is semi-active control. On the contrast ,if $R_{sh}$ is positive resistance, the system is passive control.

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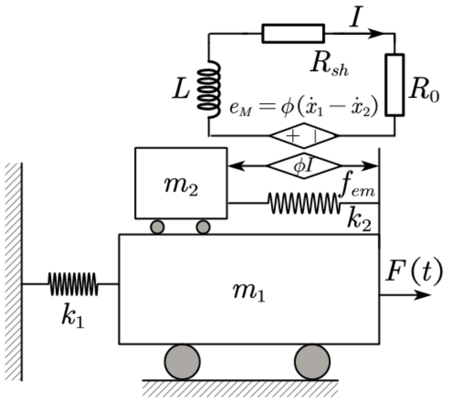


Fig.1 The Electromagnetic tuned mass damper

Assuming that resistance R is the equivalent resistance of shunt circuit, the electromechanical coupling equation of EMTMD shown in Fig.1 is as follows:

$$\begin{cases} m_1\ddot{x}_1 = -k_1x_1 - k_2(x_1 - x_2) - \phi I + F(t) \\ m_2\ddot{x}_2 = k_2(x_1 - x_2) + \phi I \\ L\dot{I} + RI = \phi(\dot{x}_1 - \dot{x}_2) \end{cases} \tag{1}$$

According to Eq.(1), the displacement amplification factor of the main system is:

$$X_n = \frac{X_1}{F/k_1} = \frac{k_1(-(k_2 - m_2\omega^2)(R + iL\omega) - i\omega\phi^2)}{(m_2\omega^2(k_2(R + iL\omega) + i\omega\phi^2) - (-(k_1 - m_1\omega^2))(-(k_2 - m_2\omega^2)(R + iL\omega) - i\omega\phi^2))} \tag{2}$$

The following parameters are defined: $\mu = m_2/m_1$ is the ratio of the mass of the vibration absorber to the mass of the main system; $\omega_a = \sqrt{k_2/m_2}$ ,is the natural circular frequency of the EMTMD; $\omega_0 = \sqrt{k_1/m_1}$ ,is the natural circular frequency of the main system; $\lambda = \omega/\omega_0$, is the ratio of the excitation force frequency to the frequency of the main system; $\gamma = \omega_a/\omega_0$ ,is the tuning ratio; $\zeta = R/(2L\omega_0)$ ,is the damping ratio; Ignoring the electromagnetic loss of the coil,the mechanical electrical coupling coefficient is $\psi = \phi^2/(Lk_1)$, which is a dimensionless coefficient. After dividing the numerator and denominator of Eq.(2) by $k_1^2 L\omega_0$ at the same time,we can obtain the dimensionless form of Eq.(2):

$$X_n(j\lambda) = X_1/(F/k_1)$$

$$= \frac{\gamma^2\mu(2\zeta + j\lambda) - \lambda^2\mu(2\zeta + j\lambda) + j\lambda\psi}{(\lambda^2\mu(-(2\zeta + j\lambda)(\gamma^2\mu - \lambda^2 + 1) - j\lambda\psi) + (1 - \lambda^2)(\gamma^2\mu(2\zeta + j\lambda) + j\lambda\psi))} \tag{3}$$

Let $X_N = |X_n(\lambda)|$, after calculating the absolute value of the Eq.(3), we can get:

$$X_N = \sqrt{\frac{(2\gamma^2\zeta\mu - 2\zeta\lambda^2\mu)^2 + (\gamma^2\lambda\mu - \mu\lambda^3 + \lambda\psi)^2}{(2\gamma^2\zeta(1 - \lambda^2)\mu - 2\zeta\lambda^2\mu(\gamma^2\mu - \lambda^2 + 1))^2 + ((1 - \lambda^2)(\gamma^2\lambda\mu + \lambda\psi) + \lambda^2\mu(\lambda^3 - \gamma^2\lambda\mu - \lambda\psi - \lambda))^2}} \tag{4}$$

## 3. $H_2$ optimization design of EMTMD

### 3.1 $H_2$ peformance index PI

According to the $H_2$ optimization principle, assuming that the electromechanical coupling system is excited by random white noise [1-8-9], the $H_2$ optimization criterion is to minimize the value of the RMS[8]. Assuming that the power spectral density of the external excitation force in the whole frequency band is $S_f$, a function PI of $H_2$ optimization performance index can be defined [8-10]. Through the optimization of PI, the vibration energy of the main system in the

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whole frequency band can be minimized[11]. The mathematical meaning of PI is the generalized integral of the square root of the displacement amplification factor of the main system to the frequency ratio $\lambda$.

$$PI = \frac{E[X_1^2]}{2\pi S_f \omega_0 / k_1^2} \tag{5}$$

Where:

$$E[X_1{}^2] = \int_{-\infty}^{+\infty} \left|\frac{X_1}{F}\right|^2 S_f d\omega = \frac{S_f}{k_1^2}\int_{-\infty}^{+\infty}\left|\frac{X_1}{F/k_1}\right|^2 d\lambda = \frac{S_f\omega_0}{k_1^2}\int_{-\infty}^{+\infty}|X_n(j\lambda)|^2 d\lambda \tag{6}$$

According to the conclusion deduced by Ref.[12] and Ref.[13], combined with Eq.(4) and Eq.(5), we can get:

$$\begin{gathered} PI = \frac{M_5}{2a_0\Delta_5} = \frac{1}{2\pi}\int_{-\infty}^{+\infty}|X_n(j\lambda)|^2 d\lambda = \frac{1}{2\pi j}\int_{-\infty}^{+\infty}\frac{|Num(j\lambda)|^2}{Den(j\lambda)Den(-j\lambda)}d(j\lambda) = \\ \frac{1}{2\pi}\int_{-\infty}^{+\infty}\frac{b_0(j\lambda)^{2n-2}+b_1(j\lambda)^{2n-4}+...+b_{n-1}}{((a_0(j\lambda)^n+a_1(j)^{n-1}+...+a_n)\cdot(a_0(-j\lambda)^n+a_1(-j\lambda)^{n-1}+...+a_n))}d(\lambda) \end{gathered} \tag{7}$$

The coefficient $b_0 \sim b_4$ of the numerator can be calculated according to Eq.(7), and the coefficient $a_0 \sim a_5$ of the denominator can be calculated according to Eq. (3),whose expression are as follows:

$$\begin{cases} a_0 = \mu \\ a_1 = 2\zeta\mu \\ a_2 = (\mu+\gamma^2\mu+\gamma^2\mu^2+\psi+\mu\psi) \\ a_3 = (2\zeta\mu+2\gamma^2\zeta\mu+2\gamma^2\zeta\mu^2) \\ a_4 = (\gamma^2\mu+\psi) \\ a_5 = 2\gamma^2\zeta\mu \end{cases} \tag{8}$$

$$\begin{cases} b_0 = 0 \\ b_1 = -\mu^2 \\ b_2 = 4\zeta^2\mu^2 - 2\gamma^2\mu^2 - 2\mu\psi \\ b_3 = 8\gamma^2\zeta^2\mu^2 - \gamma^4\mu^2 - 2\gamma^2\mu\psi - \psi^2 \\ b_4 = 4\gamma^4\zeta^2\mu^2 \end{cases} \tag{9}$$

Replace the coefficients in Eq.(8) and Eq.(9) into Eq.(7), according to Ref.[12-13] and the appendix, the PI's expression can be getted as follows:

$$\begin{gathered} PI = \frac{1}{4\zeta\mu^2\psi}[4\gamma^4\zeta^2\mu^4 + \mu\psi(2\gamma^2+\psi-2) + \mu^2(\gamma^4(4\zeta^2+1)+\gamma^2(-8\zeta^2+2\psi-2)+4\zeta^2+1) \\ +\psi^2+\mu^3(\gamma^4(8\zeta^2+1)-4\gamma^2\zeta^2)] \end{gathered} \tag{10}$$

Calculate the partial derivatives of PI for tuning ratio $\gamma$, damping ratio $\zeta$ and electromechanical coupling coefficient $\psi$ respectively, and make $\partial PI/\partial\gamma = \partial PI/\partial\zeta = \partial PI/\partial\psi = 0$.After setting their molecules zero one by one ,we can obtain:

$$4\gamma^2\zeta^2\mu^3 + \mu^2(\gamma^2(8\zeta^2+1)-2\zeta^2) + \mu(\gamma^2(4\zeta^2+1)-4\zeta^2+\psi-1)+\psi = 0 \tag{11}$$

$$\begin{gathered} \mu^2(\gamma^4(-(\mu+1))(4\zeta^2(\mu+1)-1)+\gamma^2(4\zeta^2(\mu+2)-2)-4\zeta^2+1)+ \\ 2\mu\psi(\gamma^2(\mu+1)-1)+(\mu+1)\psi^2 = 0 \end{gathered} \tag{12}$$

$$4\gamma^4\zeta^2\mu^4 + (\gamma^2-1)^2(4\zeta^2+1)\mu^2 + \mu^3(\gamma^4(8\zeta^2+1)-4\gamma^2\zeta^2) - \mu\psi^2 - \psi^2 = 0 \tag{13}$$

## 3.2 Two parameters $H_2$ optimization including tuning ratio and damping ratio

When it comes to designing the coil, it is the most feasible method to set the inductance L of the coil to a small value, and then reasonably set a suitable value for the electromechanical coupling coefficient $\psi$. Under this design idea,the exact solution for optimal design parameters only including tuning ratio $\gamma$ and damping ratio $\zeta$ can be obtained by solving the Eq.(11) and

Eq.12,whose expressions of $\gamma_{opt1}$ and $\zeta_{opt1}$ are obtained by solving Eq.(11) and Eq. (12):

$$\begin{cases} \gamma_{opt1} = \sqrt{-\dfrac{\sqrt[3]{2}(3c_0c_2 - c_1^2)}{3\sqrt[3]{A_1}c_0} + \dfrac{\sqrt[3]{A_1}}{3\sqrt[3]{2}c_0} - \dfrac{c_1}{3c_0}} \\ \zeta_{opt1} = \sqrt{\dfrac{\mu - \mu\psi - \mu^2\gamma_{opt1}^2 - \mu\gamma_{opt1}^2 - \psi}{2\mu(-\mu + 2\mu^2\gamma_{opt1}^2 + 4\mu\gamma_{opt1}^2 + 2\gamma_{opt1}^2 - 2)}} \end{cases} \tag{14}$$

Where: $A_1 = A_2 + \sqrt{4(3c_0c_2 - c_1^2)^3 + (A_2)^2}$ , $A_2 = -2c_1^3 + 9c_0c_2c_1 - 27c_0^2c_3$

$$\begin{cases} c_0 = 4\mu^2(\mu+1)^3 \\ c_1 = 3\mu(\mu+1)\left((2\psi-3)\mu^2 + 4(\psi-1)\mu + 2\psi\right) \\ c_2 = 2\begin{pmatrix} \mu^4 + (\psi^2 - 4\psi + 5)\mu^3 + (3\psi^2 - 10\psi + 6)\mu^2 \\ + \psi^2 + 3(\mu(\psi-2))\psi \end{pmatrix} \\ c_3 = 6\psi\mu - \mu^3 - \psi^2\mu^2 + 4\psi\mu^2 - 4\mu^2 - 3\psi^2\mu - 2\psi^2 \end{cases} \tag{15}$$

From the dimensionless parameters defined in Section.2, we can obtain : $R_{opt1} = 2L\omega_0\zeta_{opt1}$, $k_2 = \mu\gamma_{opt1}^2 k_1$. Therefore, the optimal performance index PI of $H_2$ optimization is:

$$PI_{opt1} = PI[\gamma_{opt1}(\psi,\mu), \zeta_{opt1}(\psi,\mu), \psi, \mu] \tag{16}$$

### 3.3 Three parameters $H_2$ optimization for $\gamma$,$\zeta$,and $\psi$

The optimization of the three parameters is the most ideal optimal design for the EMTMD. The exact solutions for optimal design parameters can be obtained from Eq. (11) (12) (13) as follows:

$$\begin{cases} \gamma_{opt2} = \sqrt{1 - \sqrt{\dfrac{\mu}{\mu+1}}} \\ \psi_{opt2} = \dfrac{2\mu^2\left(1 + \mu - \sqrt{\mu(\mu+1)}\right)}{(\mu+1)\left(\sqrt{\mu(\mu+1)} - \mu\right)} \\ \zeta_{opt2} = \sqrt{\dfrac{1}{8\mu^2 + \left(14 - 8\sqrt{\mu(\mu+1)}\right)\mu - 10\sqrt{\mu(\mu+1)} + 4}} \end{cases} \tag{17}$$

By substituting Eq.(17) into Eq.(10), we can get the optimal performance index PI:

$$PI_{opt2} = \sqrt{\frac{2 + \mu - \sqrt{\mu(\mu+1)}}{2\mu}} \tag{18}$$

Thus, the optimal inductance $L_{opt2} = \phi^2/(k_1\psi_{opt2})$ ; The optimal equivalent resistance in shunt circuit is $R_{opt2} = 2L_{opt2}\omega_0\zeta_{opt2}$ ; The optimal stiffness coefficient of spring in EMTMD is $k_{2opt2} = \mu\gamma_{opt2}^2 k_1$.

### 3.4 The two parameters optimization design can avoid the use of negative resistance circuit

When considering the optimization of two parameters, if $R_{01}$ is the coil DC resistance corresponding to the coil inductance coefficient taken as L, the shunt resistance $R_{sh1}$ connected outside the coil is: $R_{sh1} = 2L\omega_0\zeta_{opt1} - R_{01}$

Generally speaking the coil inductance L is proportional to the coil DC resistance. Assuming that the proportional coefficient is K, we can get

$$R_{sh1} = 2L\omega_0\zeta_{opt1} - R_{01} = 2KR_{01}\omega_0\zeta_{opt1} - R_{01} = R_{01}(2K\omega_0\zeta_{opt1} - 1) \tag{19}$$

Let the resistance value corresponding to the coil inductance coefficient $L_{opt2}$ be $R_{02}$.So the shunt resistance $R_{sh2}$ external to the coil optimized by the three parameters is:

$$R_{sh2} = 2L_{opt2}\omega_0\zeta_{opt1} - R_{02} = R_{02}(2K\omega_0\zeta_{opt2} - 1) \tag{20}$$

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According to Eq.(17), when $0 < \mu < 1$, we can get $0.5 < \zeta_{opt2} < 1.3566$. For the parameter $\zeta_{opt1}$, as long as the electromechanical coupling coefficient $\psi$ is large enough, $\zeta_{opt1} \gg \zeta_{opt2}$ hold true(see Table 2 and Fig.3) is , which means that $R_{sh1}$ can be a positive resistance, while $R_{sh2}$ can be a positive resistance only under the assumption of that the mass ratio $\mu$ is very large.Therefore, $R_{sh2}$ is a negative resistance generally .

## 4 Numerical calculation

### 4.1 $H_2$ optimal performance index for $PI_{opt1}$ and $PI_{opt2}$

If the mass ratio $\mu$ is constant, the $H_2$ optimal performance index $PI_{opt2}$ of the three parameters is a value, while the performance index $PI_{opt1}$ of the two parameters optimization is a curve about the parameter $\psi$, where the point $PI_{opt2}$ is on as shown in Fig.2:

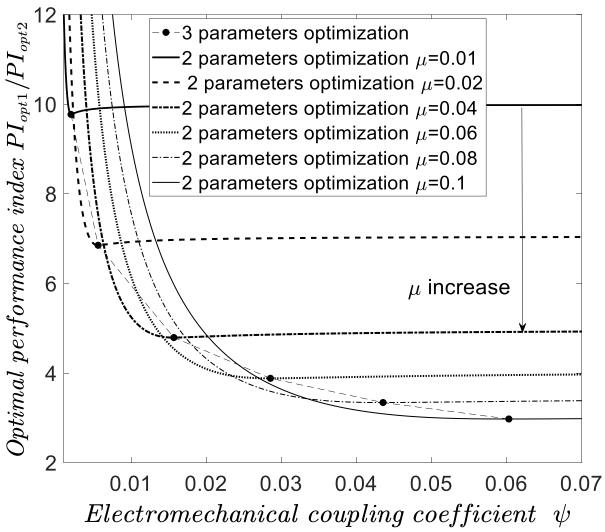
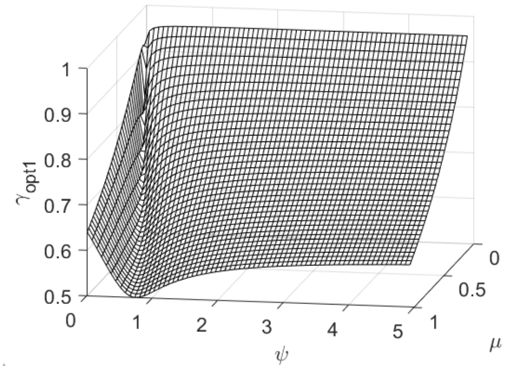



Fig.2 Under the two $H_2$ optimization strategies, the optimal performance index PI changes with the electromechanical coupling coefficient.

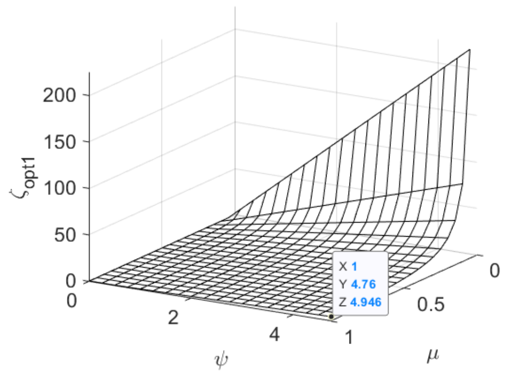
It can be seen from Fig.2 that with the increase of mass ratio $\mu$, the curve of optimal performance index PI will move down,which means that the effect of reducing vibration is better . For the two parameters $H_2$ optimization, if the parameter $\psi$ is close to zero, the optimal performance index $PI_{opt1}(\mu_i)$ will increase greatly which means that the control effect will be reduced and spontaneously far less effective than the optimizing the three parameters。If the parameter $\psi$ is larger than $\psi_{opt2}(\mu_i)$, the curve of $PI_{opt1}(\mu_i,\psi)$ will tend to be stable and very close to $PI_{opt2}(\mu_i)$, which means that the $H_2$ optimization effect of the two parameters is very close to the three parameters optimization design under the assumption that the $\psi$ is large enough to ensure optimal control force is almost equal between the two parameters optimal design and that of the three parameters.

### 4.2 Numerical calculation for the optimal parameters

The three-dimensional curved surface diagram of the optimal tuning ratio $\gamma_{opt1}$ and the optimal damping ratio $\zeta_{opt1}$ with respect to the changes of mass ratio $\mu$ and electromechanical coupling coefficient $\psi$ are shown in Fig.3:




(a) Optimal tuning ratio $\gamma_{opt1}$




(b) Optimal damping ratio $\zeta_{opt1}$

Fig.3 The changes of the $H_2$ optimal parameters with the respect to the mass ratio and electromechanical coupling coefficient

It can be seen from Fig. 3 that if the parameters tuning ratio and damping ratio are only optimized, when the mass ratio remains unchanged, the optimal tuning ratio $\gamma_{opt1}$ will first decrease and then increase with the increase of electromechanical coupling coefficient $\psi$. Assuming that the mass ratio $\mu$ is constant, the optimal damping ratio $\gamma_{opt1}$ will increase with the increase of electromechanical coupling coefficient $\psi$.

### 4.3 Frequency domain numerical analysis

Tab.1 shows the data of the example, in which the electromagnetic coefficient $\phi$, coil inductance L, coil DC resistance $R_0$ are from the linear voice coil motor of the model VCAR0032-0050-00A of Supt Motion Company.

Tab.1 Parameters of EMTMD

| Description of the parameter | Value |
|---|---|
| Mass of the main system $m_1$ | 13.6 Kg |
| Stiffness of the main system $k_1$ | 59193.94N/m |
| Mass ratio $\mu$ | 0.05 |
| Electromagnetic coefficient $\phi$ | 7.1N/A |
| Coil inductance L | 0.0033H |
| Coil DC resistance $R_0$ | 6.5 Ω |
| Stiffness of EMTMD $k_2$ | To be optimized |
| Equivalent resistance of circuit | To be optimized |

When calculating the frequency response function of the main system corresponding to the two parameters optimization, it is assumed that the electromagnetic coefficient $\phi$=10N/A is greater than the value of $\phi$ in Tab.1; When calculating the three parameter optimization, the electromagnetic coefficient $\phi = 7.1N/A$ remains unchanged. Fig.5 shows the main system's

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frequency response function of EMTMD with two parameters optimization and three parameters optimization (The optimization of classical TMD can be found in Ref.[14]).

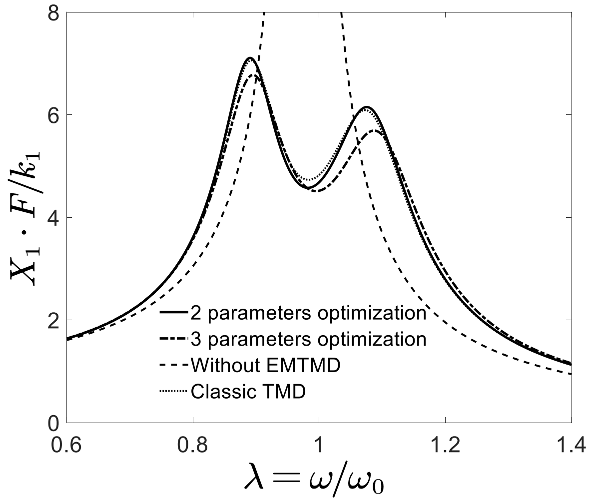


Fig.4 Frequency response curve of displacement amplification factor

It can be seen from Fig.4 that the control effects of the two parameters optimization is better than that of the classical TMD. It can be calculated that the frequency response function corresponding to the two parameter optimization decreases by 0.17% compared with the area surrounded by the coordinate axis of frequency ratio $\lambda$ , while that of the three parameters are 4.1% from the aspect of the reduced area .

Tab.2 The values of optimal design parameters

| Description of the parameter | Value |
|---|---|
| Two parameters optimal resistance $R_{opt1}$ | 10.5024$\Omega$ |
| Two parameter optimal stiffness $k_{2opt1}$ | 2739.27N/m |
| Three parameter optimal resistance $R_{opt2}$ | 3.368$\Omega$ |
| Optimal inductance $L_{opt2}$ | 0.0390 H |
| Optimal stiffness $k_{2opt2}$ | 2313.84N/m |
| Optimal damping ratio $\zeta_{opt1}$ | 24.1188 |
| Optimal tuning ratio $\gamma_{opt1}$ | 0.9620 |
| Optimal damping ratio $\zeta_{opt2}$ | 0.65413 |
| Optimal tuning ratio $\gamma_{opt2}$ | 0.88418 |
| Optimal electromechanical coupling coefficient $\psi_{opt2}$ | 0.02182 |
| Two parameter optimization design—electromechanical coupling coefficient $\psi$ | 0.5119 |

Since the optimal equivalent resistance $R_{opt1} > R_0 > R_{opt2}$ and $L_{opt2} \gg L$ , so compared with the $H_2$ optimal design of three parameters, the $H_2$ optimal design of two parameters can set the coil inductance L to be small and the electromagnetic coefficient $\phi$ to be large, and then make the optimal resistance $R_{opt1}$ greater than the DC resistance $R_0$ of the coil. Thus, it is not necessary to connect a negative resistance in the shunt circuit to neutralize the coil DC resistance

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$R_0$, which means Ref.[6] can be used for multi-modal control of the structure without connecting a negative inductive and negative resistance circuit according to the $H_2$ optimization design of two parameters.

It is worth noting that the optimal resistance $R_{opt1}$ of the two parameter optimization is larger than that of the three parameter optimization , so the current of the shunt circuit corresponding to the two parameter optimization is smaller than that of the three parameter optimization, but the electromagnetic coefficient $\phi$ of the two parameters optimization is larger than that of the three parameter optimization, so the optimal control force corresponding to the two optimization is very close, The correctness of this interpretation can be reflected in Fig.2.

The optimal curve of the frequency response function of the main system is a three-dimensional surface diagram about the electromechanical coupling coefficient $\psi$, as shown in Fig. 5

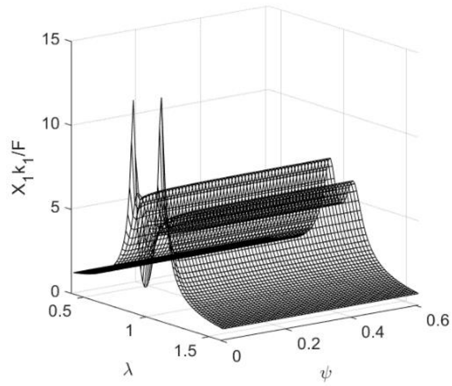


Fig.5 The influence of electromechanical coupling coefficient on the optimal control effect under the same mass ratio when tuning ratio and damping ratio are optimized.

As can be seen from Fig. 5, in an ideal case of that the electromechanical coupling coefficient is equal to 0, EMTMD is transformed into an undamped dynamic vibration absorber; When the electromechanical coupling coefficient $\psi$ is large enough, the optimal control effect tends to be stable. No matter how the electromechanical coupling coefficient is increased, the frequency response curve of the optimal displacement amplification factor of the main system hardly changes, which is the same as the reason in Fig.2, that is, the optimal control force sent by the coil hardly changes.

**4.4 Robustness analysis of two parameters optimization**

In order to analyze the influence of the deviation of design parameters from the optimal values on the frequency response function of the main system and the $H_2$ optimal performance index $PI_{opt1}$, the curves shown in Fig. 6 was drawn.

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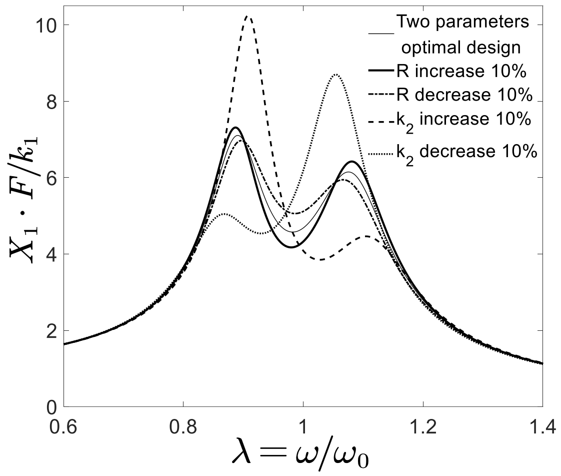


Fig. 6 (a) Sensitivity analysis of tuning ratio and damping ratio to the frequency response function of the main system.

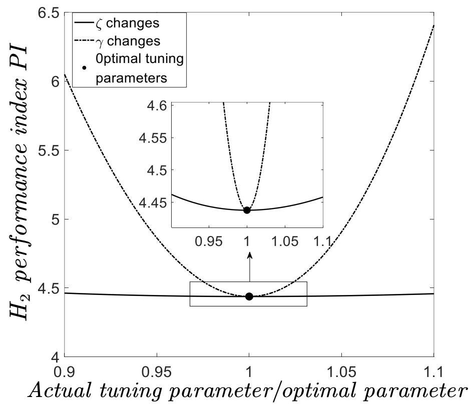


Fig. 6 (b) Sensitivity analysis of tuning ratio and damping ratio to the $H_2$ optimal performance index $PI_{opt1}$.

Fig.6 Parameters sensitivity analysis

It can be seen from Fig.6 that the parameter determining the damping ratio is the equivalent resistance R, which has little influence on the frequency response curve of the main system, so does control effect as shown in Fig.6(a). The parameter $k_2$, which determines the tuning ratio, has a great influence on the displacement amplification factor curve of the main system, so does the control effect.

### 4.5 Time domain numerical analysis

Under the stationary random excitation of Gaussian white noise, the variance of structural displacement response of under the action of Gaussian white noise can be calculated combined with the data in Tab.1 and Tab.2 by using the virtual excitation method ,under the assumption of the power spectral density $S_f$=1, , which as is shown in Fig.7.

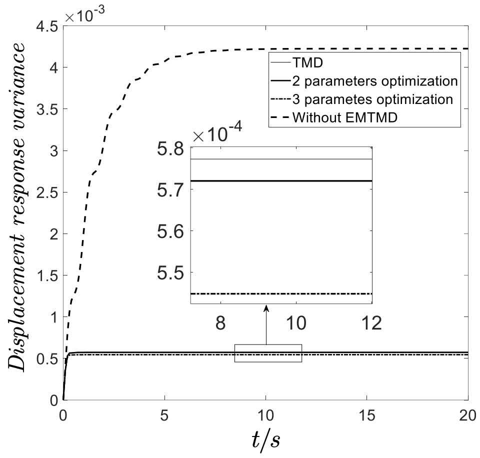


Fig.7 response variance of displacement $X_1$ of main system

From the variance of displacement $x_1$ of the main system structure on the Fig.7, it can be seen that the control effect of two parameters optimization is slightly lower than that of three parameters optimization, and the structural displacement response variance of both is slightly lower than that of classical TMD, indicating that the both $H_2$ optimal design of the EMTMD is better than that of classical TMD，under the assumption of that electromechanical coupling coefficient is large enough. The time domain analysis shows that by sacrificing the slight effect of control, the two parameters $H_2$ optimization can avoid the use of negative impedance circuit, so that the form of control system is complete passive control, which is completely feasible for the Ref.[6].

## 5 Conclusion

In this paper, the $H_2$ optimization of two parameters for EMTMD was proposed, and the exact solutions of the optimal parameters was obtained. Through numerical calculation, the following conclusions are obtained:

(1)The optimal control effect of two parameters optimization, is better than that of classical TMD by 0.17%.

(2) The optimal design of two parameters $H_2$ optimization can set the coil's inductance L very small; For the two parameters $H_2$ optimization ,by increasing the electromagnetic coefficient $\phi$ and electromechanical coupling coefficient $\psi$, the optimal resistance will be greater than the DC resistance of the coil spontaneously , so that the external resistance of the coil is a positive resistance and there is no need to connect to the complex negative resistance circuit spontaneously . Therefore, according to the $H_2$ optimization design of two parameters, the EMTMD in Ref.[6] can be used for multi-modal control of structures without connecting the negative inductance and negative resistance circuit.

(3) Conclusion (2) shows that the optimal design of two parameters can make the shunt circuit of EMTMD avoid connecting to the complex impedance circuit and achieve the optimal control effect, which means that according to the optimal design of two parameters, EMTMD is completely in the form of passive control, and its engineering feasibility is completely more higher than the semi-active control form of EMTMD connected to the complex impedance circuit as in Ref.[6].

## Attention :

For the author had already graduated from the campus ,so there is no conditions to make an experiment. If the reader want to design this EMTMD,the author will provide help do his best.

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## Appendix:

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$$PI = \frac{1}{2\pi}\int_{-\infty}^{+\infty} |X_n(i\lambda)|^2 d\lambda = \frac{1}{2\pi}\int_{-\infty}^{+\infty} \frac{|Num(i\lambda)|^2}{Den(i\lambda)Den(-i\lambda)} d(\lambda)$$

$$= \frac{1}{2\pi i}\int_{-\infty}^{+\infty} \frac{|Num(i\lambda)|^2}{Den(i\lambda)Den(-i\lambda)} d(i\lambda)$$

$$= \frac{1}{2\pi}\int_{-\infty}^{+\infty} \frac{b_0(i\lambda)^{2n-2} + b_1(i\lambda)^{2n-4} + \ldots + b_{n-1}}{\begin{pmatrix}(a_0(i\lambda)^n + a_1(i\lambda)^{n-1} + \ldots + a_n) \\ (a_0(-i\lambda)^n + a_1(-i\lambda)^{n-1} + \ldots + a_n)\end{pmatrix}} d(\lambda) =$$

$$\frac{M_n}{2a_0\Delta_n} = \begin{vmatrix} b_0 & b_1 & b_2 & \cdots & b_{n-1} \\ a_0 & a_2 & a_4 & \cdots & 0 \\ 0 & a_1 & a_3 & \cdots & 0 \\ \vdots & \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & 0 & \cdots & a_n \end{vmatrix} \Bigg/ \left( 2a_0 \begin{vmatrix} a_1 & a_3 & a_5 & \cdots & 0 \\ a_0 & a_2 & a_4 & \cdots & 0 \\ 0 & a_1 & a_3 & \cdots & 0 \\ \vdots & \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & 0 & \cdots & a_n \end{vmatrix} \right) = I_n$$

When n is equal to 5 ,we can obtain:

$$\Delta_5 = a_0{}^2 a_5{}^2 - 2a_0a_1a_4a_5 - a_0a_2a_3a_5 + a_0a_3{}^2a_4 + a_1{}^2a_4{}^2 + a_1a_2{}^2a_5 - a_1a_2a_3a_4$$

$$\mathrm{M}_5 = \frac{(a_0b_4)(-a_0a_1a_5 + a_0a_3^2 + a_1^2a_4 - a_1a_2a_3)}{a_5} + b_0(-a_0a_4a_5 + a_1a_4^2 + a_2^2a_5 - a_2a_3a_4) +$$

$$a_0b_3(a_1a_2 - a_0a_3) + a_0b_2(a_0a_5 - a_1a_4) + a_0b_1(a_3a_4 - a_2a_5)$$