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Abstract: A new model of gravity is presented here similar to the earlier work of Verlinde on Emergent Gravity but without the use of Verlinde's main assumption on the nature of gravity as an entropic force using the First Law of Thermodynamics. Instead, the model will be shown to be consistent with the Second Law of Thermodynamics as used in Black Hole Physics. Moreover, it does not use the Equipartition Theorem such that there is no need to define a thermal bath enclosed within a holographic screen. The theory uses $E = NE_p$, for the total energy of a massive object where E_p is the Planck Energy while N is the number of Planck Energy to represent the maximum possible density of information that can reside in matter. The theory uses also the Holographic Principle as the basis for an information-theoretic approach to the nature of gravity. It is shown here that gravity emerges whenever there is an updating of the information within a given volume of space by the presence of matter.

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1. Introduction

In the first paper of Verlinde's Emergent Gravity (EG) [1], it was conjectured that ordinary surfaces are holographic screens that obey the First Law of Thermodynamics similar to what had been conjectured in Black Hole Physics [2]. The theory primarily used the equation, $F\Delta r = T\Delta S$, where Δr is the distance of the test particle from the holographic screen, T is the temperature in the screen and ΔS is the change in entropy S. It was argued that as $\Delta r \to 0$, i.e., as the test particle touches the screen and increases the entropy, it induces gravitational force as a kind of entropic force. An analogy in Thermodynamics was used where a test particle that enters a gravitational field is likened to a polymer molecule that enters a region where it is immersed in a thermal bath. Such a condition is known to give rise to an entropic force at molecular and atomic levels. According to EG, a test particle that enters a gravitational field is also undergoing a similar condition where the entropic force is the gravitational force. This analogy was heavily criticized and seems to be experimentally proven to be flawed [3,4]. Wang et al. [5], argued that horizons are indeed thermodynamic in nature but general ordinary surfaces that are considered in the Emergent Gravity program are not.

In this paper, what is used as a starting guide in formulating a new theory of gravity is the fact that the changes in the strength of gravity given by the scalar potential ϕ are proportional to the changes in the density $\rho = \rho(r)$, as described by Poisson Equation $\nabla^2\phi = 4\pi G\rho$. In the context of the holographic screen, one can also use the Gauss Theorem of gravity, $A\Delta F = -4\pi GV\Delta\rho$, for a constant volume and area. It gives us,

$$\frac{dF}{d\rho} = -\frac{4}{3}\pi Gr \tag{1}$$

which shows that the change in the magnitude of gravity within the space enclosed by the screen defined by r, is proportional to the change of density or the total number of matter in a given volume of space. It is suggested here, that this classic description of gravity can be used in a fundamental way to show that gravity is an emergent phenomenon in Nature. Also, instead of using the First Law of Thermodynamics, what is primarily used here is the Second Law of Thermodynamics as used in the seminal work of Bekenstein in Black Hole Physics. Historically, it was conjectured by Bekenstein in the 1970s that the entropy of a black hole is proportional to its area [6], i.e., $S = \gamma A$, where S is the

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entropy within the black hole, $A = A(r_s)$ is the horizon area of the black hole with radius r_s and γ is a constant of proportionality. Subsequently, the value for γ was established to be related to the Planck length [7] such that,

$$S = A/4l_p^2 = A/A_p = N_a \tag{2}$$

where $l_p = \sqrt{\hbar G/c^3}$ is the Planck length and A_p is the Planck area. This result implies that the quantity N_a or the number of cells in the holographic screen is a measure of entropy (or density of information) within the volume of space enclosed by the holographic screen. The question that Verlinde wanted to answer in EG is how can this result be applicable to describe gravity in a non-black hole setting. However, the relation $A = N_a A_p$ seems to be the only central argument in Verlinde's work and missed other important results in Black Hole Thermodynamics. Here, instead of just using the quantity N_a, the quantity

$$N_d = \frac{\rho}{\rho_{pl}} = \frac{M/r}{M_p/L_p} = \frac{M/M_p}{r/L_p} = \frac{N}{N_l} \tag{3}$$

will be used here where $\rho_{pl} = M_p/L_p = \sqrt{c^4/G^2}$ is the Planck (linear) density, $M_p = \sqrt{\hbar c/G}$ is the Planck Mass and L_p is the Planck length. Notice that N_d is also written in terms of quantities $N_l = r/L_p$ and $N = M/M_p = Mc^2/M_pc^2 = E/E_p$ which can also be used to measure the density of information for a given length r and total mass M, respectively. The use of N_d is consistent with what had been mentioned above that the magnitude of gravity is proportional to the density and N_d is related to N_a since $N_l^2 = N_a$. Moreover, the mass M of any gravitating object is represented in units of Planck mass, M_p, i.e., $M = NM_p$, and will be shown here to have a key role in describing gravity in a fundamental way. Hence, the main difference of our work with Verlinde's, is that the Energy Equipartition Principle, $E = \frac{1}{2}k_b NT$, will not be used here but to be replaced by the expression, $E = NE_p$, as a quantized representation of energy in terms of the Planck energy $E_p = M_pc^2$. Moreover, the consequence of the Area Theorem which shows the relation of entropy to the mass, given by the equation, $S = 4\pi M^2$, will also be used here to justify the theory. It was derived from the fact that a Schwarzschild black hole, has a horizon area $A = 4\pi r_s^2$, where $r_s = 2GM/c^2$. It is a key result that relates mass to how much information resides inside a black hole although quantities such as the electric charge and the spin also increase the entropy of a black hole. If only the mass-entropy relation is to be considered, can we still use this result in a non-black hole setting? Can the Area Theorem in Black Hole Physics be applied to a galactic setting in which a very large magnitude of gravity is also involved? Lastly, the main objective of this paper is not to derive Newtonian Gravity as Verlinde had done in his original paper on EG, but to derive a new model of gravity similar to the Modified Newtonian Dynamics (MOND) theory of Milgrom as a possible alternative to the Dark Matter hypothesis.

2. Modified Newtonian Gravity

The dimensionless form of Newton's Law of Gravity in terms of Planck scale units can be expressed as follows,

$$N_F = \frac{F_N}{F_p} = \frac{\left(\frac{M_1}{M_p}\right)\left(\frac{M_2}{M_p}\right)}{r^2/l_p^2} = \frac{N_1 N_2}{N_a} \tag{4}$$

where, $F_p = c^4/G$, is the Planck Force. The expression above must be modified in cases when the magnitude of gravity is very large. However, the modification must be done such that the expression above will simply become a special case when gravity is relatively weak. One such particular case where modification is needed for a large magnitude of gravity is the case of a black hole. Another one is the case of the magnitude of gravity by a large number of objects that collectively generate a gravitational effect like in the case of a galaxy or a galaxy cluster. The modification of Equation (4) for both cases mentioned above was attempted, historically, by General Relativity theory and by MOND theory respectively. Here, as mentioned in the previous section, the proposed modification will be similar to Verlinde's theory of emergent gravity where he primarily used N_a that represents the

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number of bits of information within the volume enclosed by the holographic screen. However, it is argued here that N_a gives only the entropy of virtual particles generated out of empty space inside the black hole and even those outside the event horizon that enter into the black hole. One must also consider the information associated with the real particles within the black hole which are either free particles or bounded particles within the atoms of gravitating matters inside the black hole. It is posited here that this consideration must also be true for non-black hole settings. For example, the maximum density of information given by all real particles in a 2-body system can be represented by the quantity $N = \frac{M_1}{M_p} + \frac{M_2}{M_p} = N_1 + N_2$, where M_1, M_2 are the masses of the two gravitating objects that are composed of real particles while $N_1 = M_1/M_p$, and $N_2 = M_2/M_p$ represent the maximum possible density of information that can be stored for each real particles inside each of the gravitating matter. Now, a purely information-theoretic approach to gravity would be that the magnitude of gravity is dependent solely on the amount of information that resides in space and matter within a gravitational system. If gravity is strong enough such that it can generate virtual particles in empty space within the vicinity of the objects, that must be included in the general description of entropy within the system. Hence, the magnitude of gravity F should only be dependent on the value of N and N_a. The former represents the amount of information that resides in a gravitating matter and the latter, by the Holographic Principle, represents the amount of information within a given volume of space represented by the amount of energy generated due to gravity within the system. Gravity, therefore, in essence, would only be proportional to the information density. To quantify this, consider the square of N_d such that the ratio of the magnitude of the gravity and Planck Force is proportional to it, that is, $\frac{F}{F_p} = \epsilon N_d^2$, for some unitless constant of proportionality ϵ. This will yield us,

$$F = \frac{c^4}{G}\frac{N_1 N_2}{N_l^2}\epsilon + \frac{c^4}{G}\left(\frac{N_1^2 + N_2^2}{N_l^2}\right)\epsilon = F_{NG} + F_{HG} \tag{5}$$

where, it can be shown that,

$$F_{NG} = \frac{c^4}{G}\frac{N_1 N_2}{r^2/L_p^2}\epsilon = \hbar c\frac{M_1 M_2}{r^2 M_p^2}\epsilon = G\frac{M_1 M_2}{r^2}\epsilon \tag{6}$$

which is the usual expression for the magnitude of gravity in Newtonian Gravity (NG) except for the addition of the unitless quantity ϵ. Hence, it can be said that the quantity, $F = \frac{c^4}{G}\left(\frac{N}{N_l}\right)^2\epsilon$, must also be an expression that measures the magnitude of gravity that is expressed in terms of the density or amount of information that resides within a gravitational system. Meanwhile, the quantity, $F_{HG} = \frac{c^4}{G}\left(\frac{N_1^2+N_2^2}{N_l^2}\right)\epsilon$, is a magnitude of an excess gravity i.e., a "Hidden Gravity" (HG), in addition to the magnitude of gravity that is already given by the Newtonian equation of gravity. In terms of masses, M and m, for a two-body system, we can rewrite Equation (5) as follows,

$$F = G\frac{Mm}{r^2}\epsilon + G\left(\frac{M^2 + m^2}{r^2}\right)\epsilon \tag{7}$$

which can be simplified further as follows,

$$F = G\frac{Mm}{r^2}f\epsilon \tag{8}$$

where $f = f\left(\frac{M}{m}\right) = 1 + \left(\frac{M}{m} + \frac{m}{M}\right)$. This result is similar in a black hole setting where the addition of mass does not necessarily imply the addition of individual entropy. The increase in mass M in a black hole by introducing a test object with mass m, i.e., $M \to M + m$, increases the entropy within the volume occupied by M but with total entropy greater than the sum of the individual entropy S_M and S_m of the masses, respectively. Thus, $S = 4\pi M^2 \to 4\pi(M+m)^2 > S_M + S_m$, since the total entropy would become,

$$S = 4\pi(M^2 + 2Mm + m^2) > 4\pi(M^2 + m^2) \tag{9}$$

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This similarity in a black hole scenario should not be surprising since when a particle is added to a gravitational system it not only increases the entropy of the system, but the addition of its mass also increases the magnitude of the gravity generated by the whole system. However, since it is posited here that the magnitude of gravity is proportional to the entropy, thus, the excess entropy, $S_e = 8\pi Mm$, would then correspond to an excess gravity that is usually unaccounted for in calculating the magnitude of gravity if one is to use the classical equation of gravity. In Sections 3 and 4, this new approach to gravity that relates gravity with the entropy of the system will be applied to larger systems that involve a large number of gravitating objects. This approach is different from the most commonly used approach in introducing a new theory of gravity where one is to generalize the Einstein-Hilbert action, $S = \int \sqrt{-g} R d^4 x$, by imposing additional parameters into the action, such as scalar, vector, tensor, and spinor fields, and then making the action conformally invariant in order to produce a new field equation for gravity. One of the well-known examples of this approach is the Tensor-Vector-Scalar (TeVeS) gravity theory by Bekenstein [8] as a relativistic generalization of MOND paradigm of Milgrom [9]. This Lagrangian method will not be used here since the model presented here will focus more on the relation of gravity with the information density within a gravitational system rather than on its energy density. In addition, the main difference between the proposed theory here with MOND theories is that the so-called "interpolating function", $\mu(x)$, that is added to Newton's laws in MOND theories [10] was identified and derived here to be a product of the function f and the unitless constant ϵ. It has a specific physical interpretation and is not just added, arbitrarily. To show this, note that the correction term $f = f(M/m)$ is a function in terms of mass ratio. In Observational Astronomy, one can never calculate the mass ratio by getting the individual masses of the gravitating objects since the mass of one celestial object can never be known separately from the other mass of a celestial object that is gravitationally bound to it. However, one can express the function f in other terms instead of mass ratio. One of which is in terms of the acceleration of the two gravitating objects toward each other. For simplicity, let $M >> m$ which gives as, $f = 1 + \frac{M}{m}$. Using Newton's second and third law, $f = 1 + \frac{a}{a_0}$ which then gives the familiar modification of MOND theories to Newton's gravity equation,

$$F = G\frac{Mm}{\mu(a/a_0)r^2} \tag{10}$$

where $\mu(a/a_0) = (\epsilon f)^{-1} = \frac{1}{\epsilon\left[1+\frac{a}{a_0}\right]}$. In MOND theories, it was originally suggested by Milgrom that $a_0 \approx 1.2 \text{ x}10^{-10}\ m/s^2$ would be the optimal value based on his analysis to rotation curve data. Here, the quantity, a_0, is not arbitrarily added without any physical meaning or justification. It is not a new fundamental constant that marks the transition between the Newtonian and deep-MOND regimes. It is interpreted here as the acceleration of the gravitating matter towards the test object orbiting it. However, there are complications in knowing the exact value of a_0, especially if the gravitating matter is a collection of smaller bodies with spaces in between like in the case of galaxies where there are different distributions of stars or density distribution within the galaxy. Such variation leads to various shapes and configurations of galaxies making the corresponding range of the gravitational influence around a galaxy unique for every galaxy. The mathematical details of this will be further discussed in Section 4. This scenario for a galaxy is very different in the case of a black hole in which the are no pockets of different density distribution within the black hole since all its mass is only concentrated at that center called the singularity. However, the magnitude of the gravitational effect should be relatively the same with galaxy, hence describing the magnitude of gravity must also be the same for both cases.

3. Tully-Fisher Relation

In this section, it will be shown that the correction term $f(M/m)$ can also be used to derive the Tully-Fisher Relation just like in MOND theories. In Observational Astronomy, there are already known methods that can be used to measure the mass ratio of two bodies in a two-body system.

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For non-luminous objects in a 2-body system, the distances R_1 and R_2 from the barycenter can be used. Since each force felt by both bodies acts only along the line joining the centers of the masses and both bodies must complete one orbit in the same period, the centripetal forces can be equated using Newton's 3rd law, such that we can have the relation $\frac{M}{m} = \frac{R_1}{R_2}$. On the other hand, to get the mass ratio of distant luminous objects in a two-body system like in a binary star, one can use an approximation [11] via the mass-luminosity relationship, $\frac{M}{m} \approx \left(\frac{L_M}{L_m}\right)^{\gamma}$, where $1 < \gamma < 6$. The value, $\gamma = 3.5$, is commonly used for main-sequence stars. For a galaxy with halo mass M and a star with mass, $m_{\odot}$, equal to one solar mass and revolves along the halo mass, we can use the work of Vale et al. [12] that relates the observed luminosity of the galaxy L and the halo mass of the galaxy via a double power law equation, i.e., $\frac{M}{m_{\odot}} \approx \left(\frac{L}{L_{\odot}}\right)^{\frac{1}{b}}$ where the range $0.28 \leq b \leq 4$ for exponent b, is for galaxies with galactic halo mass that ranges from high-mass to low-mass. The mass-luminosity relation above will yield us,

$$v^2 = \frac{\epsilon GM}{r}\left(1 + \frac{M}{m_{\odot}}\right) \approx \frac{\epsilon GM}{r}\left[1 + \left(\frac{L}{L_{\odot}}\right)^{\frac{1}{b}}\right] \tag{11}$$

by equating F with the magnitude of the centripetal force mv^2/r. This implies that $L \sim v^{2b}$ where for the average, $b = 2$, the equation give us the Tully-Fisher relation $L \sim v^4$.

4. External Field Effect

In most MOND theories [13], it was suggested that any local measurement of the magnitude of gravity is not absolute. It will always depend on the external gravity of other masses. This is known as the External Field Effect (EFE) which according to [14] implies that "the internal dynamics of a system are affected by the presence of external gravitational fields". Note, however, that the effect or magnitude of gravity from a source varies depending on the density or distribution of matter in its vicinity. In the galactic scale in particular, one must consider the variation of density from its bulge, to the galactic disk, and up to the galactic halo. Although MOND theories were the first to suggest the existence of EFE, as far as we know, it was never translated into concrete mathematical terms as will be shown in this section. For large-scale gravity which involves a larger group of gravitational sources we now have, $N = N_1 + N_2 + \ldots + N_k$, for k number of gravitating objects. Squaring N, we have, $N^2 = (N_1)N_1 + (N_1 + N_2)N_2 + \ldots (N_1 + N_2 + \ldots + N_k)N_k$. Distributing and rearranging terms, we can have a more compact expression using the summation symbol, i.e.,

$$N^2 = \sum_{i<j}^{k} N_i N_j + \sum_{i}^{k} N_i^2 = \left(\sum_{i<j}^{k-1} N_i N_j + N_k \sum_{i}^{k-1} N_i\right) + \left(\sum_{i}^{k-1} N_i^2 + N_k^2\right)$$

By using Equation (5) and the convention $G = \hbar = c = 1$, the magnitude of the gravity of a galaxy, F_G, acting on kth star, would be,

$$F_G = \epsilon\left(\frac{N_k \sum_{i}^{k-1} N_i}{r_{cg}^2}\right) + \epsilon\left(\frac{\sum_{i<j}^{k-1} N_i N_j}{r_{cg}^2}\right) + \epsilon\left(\frac{\sum_{i}^{k-1} N_i^2}{r_{cg}^2}\right) + \epsilon\left(\frac{N_k^2}{r_{cg}^2}\right) = F_{NG} + F_{HG} \tag{12}$$

where r_{cg} is the distance of separation of the kth object from the center of gravity of all other stars within the galaxy. The number of stars with gravity acting on the kth star is given by $k - 1$. The "stars" mentioned here include the supermassive black holes at the center of the galaxy which usually have the greatest contribution to the overall gravity of the galaxy. The first term in Equation (12) can be associated with a magnitude of gravity, F_{NG}, that is similar to Newtonian Gravity (NG) while the last three terms constitute the magnitude, F_{HG}, of a "Hidden Gravity" (HG). The additional gravity given by F_{HG} will make the gravity of the galaxy extend not just on stars at the edge of the visible part of the

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galaxy but even beyond it, up to the edge of the galactic halo that surrounds the visible part of the galaxy. Hence, the observed flat rotation curve of galaxies is explained here not by an unobservable additional matter within the galaxy halo but by the excess gravity that was unaccounted for when one is using the classical theory of gravity. The value of F_{HG}, however, varies from one galaxy to another since it depends not only on the k number of objects that can contribute to the magnitude of gravity but also on the distribution of the objects in a given volume of space or the density of matter defined by the quantity N and the distance r_{cg}.

5. Conclusions

A new theory of gravity is presented here with a basis that is partially similar to Verlinde's emergent theory of gravity but with a result that is similar to a MOND theory. Gravity here is not described by the amount of curvature of spacetime (*à la* Einstein) or described as an entropic force that emerges in a thermal bath (*à la* Verlinde), but described by the density of information that can be contained within a gravitational system. Also, the theory neither introduced a new baryonic particle as suggested by the Dark Matter hypothesis nor introduced a new field as suggested by MOND theories. It modifies Newtonian gravity by using the fundamental role of information and entropy in the description of gravity. It was also shown that the so-called "interpolating function" in MOND theories is a function defined by the accelerations toward each other of gravitating matter at the center of gravity and the test object orbiting it. The similarity of the description of gravity for black hole and non-black hole setting like in the case of the galaxy was also established.

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