

Information Theory of Gravity

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Abstract

A new model of gravity is presented here similar to the earlier work of Verlinde on Emergent Gravity but without the use of thermodynamic assumptions. The theory does not use the main assumption of Verlinde on the nature of gravity as an entropic force using the First Law of Thermodynamics. Moreover, it does not use the Equipartition Theorem such that there is no need to define a thermal bath enclosed within a holographic screen. Instead of Equipartition Theorem, the theory uses $E = NE_p$, for the total energy of a massive object where E_p is the Planck Energy while N is the number of Planck Energy to represent the maximum possible density of information that can reside in matter. The theory uses also the Holographic Principle as the basis for an information-theoretic approach to the nature of gravity. It is shown here that gravity emerges whenever there is an updating of the information within a given volume of space by the presence of matter.

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1 Introduction

The main criticism to Verlinde's Emergent Gravity(EG) theory is the fact that it uses thermodynamic assumptions. He conjectured in his first paper on EG [1] that ordinary surfaces are holographic screens that obey the First Law of Thermodynamics similar to what had been conjectured in Black Hole Physics [2]. He primarily used the equation, $F\Delta r = T\Delta S$, where Δr is the distance of the test particle from the holographic screen, T is the temperature in the screen and ΔS is the change in entropy S . He argued that as $\Delta r \rightarrow 0$, i.e., as the test particle touches the screen and increases the entropy, it induces gravitational force as a kind of entropic force. He used an analogy in Thermodynamics where a test particle that enters in a gravitational field is likened to a polymer molecule that enters in a region where it is immersed in a thermal bath. Such a condition is known to give rise to an entropic force at molecular and atomic levels. According to Verlinde, a test particle that enters a gravitational field is also undergoing a similar condition where the entropic force is the gravitational force. This analogy was heavily criticized and seems to be experimentally proven to be flawed [3, 4]. Wang et.al., [5], argued that horizons are indeed thermodynamic in nature

but general ordinary surfaces that are considered in the Emergent Gravity program are not.

In this paper, we are guided by the fact that the changes in the strength of gravity given by the scalar potential ϕ , as describe by Poisson Equation $\nabla^2\phi = 4\pi G\rho$, is proportional to the changes in the density ρ , i.e., $\nabla\phi = 4\pi G \int \rho dr$ where $\rho = \rho(r)$. In the context of holographic screen, one can use Gauss Theorem of gravity which can be written as $A\Delta F = -4\pi GV\Delta\rho$ for a constant volume and area. It gives us, $\frac{dF}{d\rho} = -\frac{4}{3}\pi Gr$, which shows that the change in magnitude of gravity within the space enclosed by the screen, is always proportional to the change of density or the total number of matter in a given volume of space. It is suggested here, that this classic description of gravity can be used in a fundamental way to show that gravity is an emergent phenomenon in Nature. Also, instead of using the First Law of Thermodynamics, we will be using the Second Law of Thermodynamics as used in the seminal work of Bekenstein and Hawking in Black Hole Physics. In retrospect, it was simply conjectured by Bekenstein in 1970s that the entropy of a black hole is proportional to its area[6], i.e., $S = \gamma A$, where S is the entropy within the black hole, $A = 2\pi r_s^2$ is the area of the black hole with radius r_s and γ is a constant of proportionality. Then Hawking[7] was able to confirm the area-entropy relation using quantum field theory and determined the value of γ . He came up with $S = A/4l_p^2 = A/A_p = N_a$ where $l_p = \sqrt{\hbar G/c^3}$ is the Planck length and A_p is the Planck area. This important result would later become the basis for the Holographic Principle. The question however that Verlinde wanted to answer is how can this result be applicable to describe gravity in a non-black hole setting. In doing so he was guided by the Holographic Principle as the physical interpretation of the Bekenstein-Hawking area theorem mentioned above. The principle states that the quantity N_a or the number of cells in the holographic screen is a measure of the density of information within the volume of space enclosed by the holographic screen. However, the relation $A = N_a A_p$ seems to be the only central argument in Verlinde's work and missed other important results in Black Hole Thermodynamics. Here, instead of just using the quantity N_a , we will be using the quantity

$$N_d = \frac{\rho}{\rho_p} = \frac{M/r}{M_p/L_p} = \frac{M/M_p}{r/L_p} = \frac{N}{N_l} \quad (1)$$

where $\rho_p = M_p/L_p$ is the Planck (linear) density which is about $10^{27}kg/m$, $M_p = \sqrt{\hbar c/G}$ is the Planck Mass which is about $10^{-8}kg$ and L_p is the Planck length which is about $10^{-35}m$. Notice that N_d is also written in terms of quantities $N_l = r/L_p$ and $N = M/M_p = Mc^2/M_p c^2 = E/E_p$ which can also be used to measure the density of information for a given length r and total mass M , respectively. The use of the quantity N_d is consistent with what we had mentioned above that gravity must be proportional to the density ρ . It is also related to N_a since $N_l^2 = N_a$. Hence, we are guided by the fact that the mass M of any gravitating

object can be represented in units of Planck mass, M_p , i.e., $M = NM_p$ and must also have a key role in describing gravity in a fundamental way. Hence, the main difference of our work with Verlinde's, is that the Energy Equipartition Principle, $E = \frac{1}{2}k_bNT$, will not be used here but to be replaced by the expression, $E = NE_p$, as a quantized representation of energy in terms of the Planck energy $E_p = M_pc^2$. Furthermore, besides the Area Theorem, the relation of entropy to the mass given by the equation below,

$$S = 4\pi M^2 \quad (2)$$

will be used also to justify the theory presented here. It is a significant result in relation to how much information can be known when matter enters a black hole. When a particle with mass m falls into a black hole, the entropy of the black hole increases where the increase of entropy is given by[8]:

$$\Delta S = 8\pi Mm + 4\pi m^2 \quad (3)$$

Again, the problem is how to relate this result to a non-black hole setting. Here, this result will be shown to be related to the galactic setting in which a very large magnitude of gravity is involved. Lastly, the main objective of this paper is not to derive Newtonian Gravity as Verlinde had done in his original paper on EG, but to derive a new model similar to Modified Newtonian Dynamics (MOND) as a possible alternative to the Dark Matter hypothesis.

2 Modified Newtonian Gravity

The dimensionless form of Newton's Law of Gravity in terms of Planck scale units can be expressed as follows,

$$N_F = \frac{F}{F_p} = \frac{\left(\frac{M_1}{M_p}\right)\left(\frac{M_2}{M_p}\right)}{r^2/l_p^2} = \frac{N_1N_2}{N_a} \quad (4)$$

This expression must be modified in cases when the magnitude of gravity is very large either due to the presence of a black hole or by a large number of gravitating bodies that collectively generate a gravitational effect like in the case of a galaxy. As mentioned in the previous section, the proposed modification will be similar to Verlinde's theory of emergent gravity where he primarily used N_a that represents the number of bits that can occupy within the holographic screen. However, it gives only the number of fundamental units of space that the energy associated with gravity can occupy. One must also consider the total information that resides within all matters in any gravitating system. For example, in a 2-body system, such quantity can be represented by the quantity $N = \frac{M_1}{M_p} + \frac{M_2}{M_p} = N_1 + N_2$, where M_1, M_2

are the masses while $N_1 = M_1/M_p$, and $N_2 = M_2/M_p$ represent the maximum possible density of information that can be stored for each gravitating matter where $M_p = \sqrt{\hbar c/G}$ is the Planck mass. By defining this, we aim to achieve here a purely information-theoretic approach to gravity where its magnitude will not be dependent on the amount of heat or curvature of spacetime within the gravitational field but solely on the amount of information that resides in space and matter within a gravitational system. Hence, the magnitude of gravity F should only be dependent on the value of N and N_a . The former represents the amount of information that resides in a gravitating matter and the latter, by Holographic Principle, represents the amount of information within a given volume of space that can be occupied by any amount of energy within the gravitational system. Gravity therefore would only be proportional to the information density. To quantify this idea, we consider the square of N_d such that the ratio of the magnitude of the gravity and Planck Force $F_p = c^4/2G$ is proportional to it, that is;

$$\frac{F}{F_p} = \epsilon N_d^2 = \epsilon \frac{(N_1 + N_2)^2}{N_l^2} \quad (5)$$

for some unitless constant of proportionality ϵ . This will yield us,

$$F = \frac{c^4}{2G} \left(\frac{N}{N_l} \right)^2 \epsilon = \hbar c \frac{N_1 N_2}{r^2} \epsilon + \frac{c^4}{2G} \left(\frac{N_1^2 + N_2^2}{N_l^2} \right) \epsilon = F_{NG} + F_{HG} \quad (6)$$

where

$$F_{NG} = \hbar c \frac{N_1 N_2}{r^2} \epsilon = \hbar c \frac{M_1 M_2}{r^2 M_p^2} \epsilon = G \frac{M_1 M_2}{r^2} \epsilon \quad (7)$$

is the usual expression for the magnitude of gravity in Newtonian Gravity (NG) that describes it as a force. Hence, the quantity

$$F = \frac{c^4}{2G} \left(\frac{N}{N_l} \right)^2 \epsilon = \frac{c^4}{2G} N_d^2 \epsilon \quad (8)$$

must also be an expression that we can relate to the magnitude of gravity which is not necessarily a force as it can be purely expressed in terms of the number of bits or amount of information that resides in a gravitational system. Meanwhile, the quantity

$$F_{HG} = \frac{c^4}{2G} \left(\frac{N_1^2 + N_2^2}{N_l^2} \right) \epsilon \quad (9)$$

is a magnitude of an excess gravity i.e., a ‘‘Hidden Gravity’’(HG), in addition to the magnitude of gravity given by the Newtonian Gravity. In terms of masses, M and m , for a

two-body system, we can rewrite Eqn. (6) as follows,

$$F = G \frac{Mm}{r^2} \epsilon + \frac{k}{2} \left(\frac{M^2 + m^2}{r^2} \right) \epsilon \quad (10)$$

where $k = \frac{c^4}{G\rho_p^2} \approx 10^{-11} m^4 / N s^4$. By unit analysis, $\frac{[m]^4}{[s]^4} = \left([m] \frac{[m]}{[s]^2} \right)^2 = \left([m] \frac{[N]}{[kg]} \right)^2 = [m]^2 \frac{[N]^2}{[kg]^2}$, which gives us $k \approx 10^{-11} N m^2 / k g^2 \approx G$ such that Eqn.(10) can be simplified further as follows,

$$F \approx G \frac{Mm}{r^2} f \epsilon \quad (11)$$

where $f = f\left(\frac{M}{m}\right) = 1 + \frac{1}{2} \left(\frac{M}{m} + \frac{m}{M} \right)$. It is surprising that the value of the constant k is about the same value of the gravitational constant G which allows us to have the simplified equation above. On the other hand, comparing this to the well-known estimate of the entropy S_1 for a black hole with mass M , its entropy changes upon the introduction of a test particle with mass m . Using Eq.(3), the changes can be expressed by the transformation below,

$$S_1 \rightarrow S_1 + \Delta S = 4\pi M_p^2 \frac{(M^2 + 2Mm + m^2)}{M_p^2} = 4\pi M_p^2 \left[\frac{Mm}{M_p^2} + \frac{M^2 + m^2}{M_p^2} \right] \quad (12)$$

The result is similar to Eqn.(10) since when particles are added in a gravitational system it not only increases the entropy of the system, but the addition of mass also increases the magnitude of the gravity generated by the system. Another way to derive this is by considering that the individual entropy of all gravitating bodies need not be multiplied but be added up where the sum is proportional to the square of the total mass $\mu = M_1 + M_2$, i.e., $S = S_1 + S_2 \propto \mu^2$ which is consistent with Eq. (2) for black holes. It can be conjectured therefore that the entropy associated with matter is not an exponential entropy but an entropy of information-bearing states that obeys an extended form of Landauer's Principle that shows the equivalence of mass and energy to information, similar to the recent work of Vopson[9]. The modified Newton's law of gravity given by Eqn. (11) is considered to be applicable for larger systems that involve a large number of gravitating objects just like in a galaxy. It should be noted that other similar models that also try to modify Newton's Law of Gravity by adding additional terms are mostly done arbitrarily with the aim of fitting the model to the observed data and even reconciling it with the dark matter hypothesis. See [10] for different types of such models as examples. This non-Newtonian law of gravity approaches, according to [10], "Although... an old idea that could appear rudimentary...and it is mostly abandoned in modern literature, we think that a reconsideration of this approach could motivate further research in the area of modified gravity theories." On the other hand, the most commonly used approach in introducing a new theory of gravity nowadays is to generalize the Einstein-

Hilbert action, $S = \int \sqrt{-g} R d^4x$, by imposing additional parameters into the action, such as scalar, vector, tensor and spinor fields for the purpose of making the action conformally invariant and to produce field equations that might explain the dark energy and dark matter problems. One of the well-known examples of this, is the Tensor–Vector–Scalar (TeVeS) gravity theory by Bekenstein [11] as a relativistic generalization of MOND paradigm of Milgrom[12]. This Lagrangian method will not be used here since the model presented here will focus more on the relation of gravity with the information density within a gravitational system rather than its energy density. Also, the main difference between the proposed theory here with MOND theories is that the so-called “interpolating function” added to Newton’s law in MOND theories was identified and derived here to be a product of the function f and the unitless constant ϵ rather than to be added, arbitrarily, to the equation.

3 Mass Correction and Tully-Fisher Relation

In stellar systems, the new model of gravity presented here would lead to a small correction on the calculated mass of the object that is generating the gravitational field. However, in a galactic scale, the corresponding correction term will be significant as implied in the previous section. In this section, it will be shown that the correction term can also be used to derive the Tully-Fisher Relation which relates the luminosity of the galaxy to the rotational velocity of its component stars. For simplicity, we consider first an object with mass m , that has a circular orbit around the center of the source of gravity with mass M . The test object under the influence of gravity will react by accelerating and experiencing a centrifugal force that has a magnitude that can be equated to the magnitude of the centripetal force F_c brought about by the effect of gravity. The magnitude is given by $F_c = mv^2/r$, where r is the distance from the source of gravity and v is the rotational velocity of the test object. This can be equated to Eqn.(11) which yield us,

$$M \approx \frac{rv^2}{\epsilon G} \gamma \quad (13)$$

where $\gamma = f^{-1}$. For the case of a binary system where $M \approx m$, $\gamma \approx \frac{1}{2}$ which gives us $M \approx \frac{1}{2} \frac{rv^2}{\epsilon G}$ while for $M \gg m$, $\gamma \approx (1 + \frac{M}{2m})^{-1} \approx (1 - \frac{M}{2m})$ which gives us $M \approx \frac{rv^2}{\epsilon G} (1 - \frac{m}{2M})$. For both cases, the correction term is too small within the stellar system scale, i.e., $(1 - \frac{m}{2M}) \lesssim \epsilon \lesssim 0.5$ such that the model will approximate the results of known classic theories of gravity in which the mass of the gravitating object is given by the equation, $M = rv^2/G$. For the case of the Earth-Sun system, we have $\epsilon \approx 0.999998$ using the Sun-Earth Mass ratio of 333,000. Note that the correcting term $f = f(m/M)$ is a function in terms of mass ratio.

In Observational Astronomy, there are already known methods that can be used to measure the mass ratio of two bodies in a two-body system. For non-luminous objects in a 2-body system, the distances R_1 and R_2 from the barycenter can be used. Since each force felt by both bodies acts only along the line joining the centers of the masses and both bodies must complete one orbit in the same period, the centripetal forces can be equated using Newton's 3rd law, such that we can have the relation $\frac{M}{m} = \frac{R_1}{R_2}$. On the other hand, to get the mass ratio of distant luminous objects in a two-body system like in a binary star, one can use an approximation via the mass-luminosity relationship[13],

$$\frac{M}{m} \approx \left(\frac{L_M}{L_m} \right)^a \quad (14)$$

where $1 < a < 6$. The value $a = 3.5$ is commonly used for main-sequence stars. For a galaxy with halo mass M and a star with mass $m_\odot$ equal to one solar mass and revolves along the halo mass, we can use the work of Vale et.al.[14] that relates the observed luminosity of the galaxy L and the halo mass of the galaxy via a double power law equation, i.e.,

$$\frac{M}{m_\odot} \approx \left(\frac{L}{L_\odot} \right)^{\frac{1}{b}} \quad (15)$$

where the range $0.28 \leq b \leq 4$ for exponent b , is for galaxies with galactic halo mass that ranges from high-mass to low-mass. If one is to get the square of the rotational velocity v of the revolving star in Eqn. (13), the mass-luminosity relation above will yield us,

$$v^2 = \frac{\epsilon GM}{r} \left(1 + \frac{M}{2m_\odot} \right) \approx \frac{\epsilon GM}{r} \left[1 - \left(\frac{L}{2L_\odot} \right)^{\frac{1}{b}} \right] \quad (16)$$

This implies that $L \sim v^{2b}$ where for the average, $b \approx 2$, the equation give us the Tully-Fisher relation $L \sim v^4$.

4 External Field Effect

In MOND theories[15], it was suggested that any local measurement of the magnitude of gravity is not absolute. It will always depend on the external gravity of other masses. This is known as the External Field Effect (EFE). To illustrate, if an apple is acted by Earth's gravity, and Earth is acted by the Sun's gravity, all other gravity acting on the apple should be accounted for in the calculation. These include not only the gravity generated by the Milky Way acting on the Solar System but also the gravity of the Local Group and supercluster

where the Milky Way belongs which in turn is acted upon by the gravity of all matter in the Universe. The gravity in each source varies depending on the density or distribution of matter in its vicinity. In the galactic scale in particular, one must consider the variation of density from the nucleus of the galaxy, to its bulge, to the galactic disk, up to the galactic halo. Quantitatively, as the distance of a test object from the source of gravity increases such that the corresponding area of the holographic screen becomes larger and encloses more matter, it will increase the density and therefore increases also the number of sources of gravity acting on the test object located at the holographic screen. Although MOND theories were the first to suggest the existence of EFE, as far as we know, it was never translated in concrete mathematical terms. In this section, we wanted to express EFE, mathematically, based on the results from the previous sections. For large-scale gravity which involves a larger group of gravitational sources we now have, $N = N_1 + N_2 + \dots + N_k$, for k number of gravitating objects. Squaring N , we have, $N^2 = (N_1)N_1 + (N_1 + N_2)N_2 + \dots (N_1 + N_2 + \dots + N_k)N_k$. Distributing and rearranging terms, we can have a more compact expression using the summation symbol, i.e., $N^2 = \sum_{i < j}^k N_i N_j + \sum_i^k N_i^2$, which can be expanded as follows:

$$N^2 = \left(\sum_{i < j}^{k-1} N_i N_j + N_k \sum_i^{k-1} N_i \right) + \left(\sum_i^{k-1} N_i^2 + N_k^2 \right) \quad (17)$$

By using Eqn. (6) and the convention $G = \hbar = c = 1$, the magnitude of the gravity of a galaxy, F_G , acting on k th star, would be

$$F_G = \epsilon \left(\frac{N_k \sum_i^{k-1} N_i}{r_{cg}^2} \right) + \epsilon \left(\frac{\sum_{i < j}^{k-1} N_i N_j}{r_{cg}^2} \right) + \epsilon \left(\frac{\sum_i^{k-1} N_i^2}{r_{cg}^2} \right) + \epsilon \left(\frac{N_k^2}{r_{cg}^2} \right) = F_{NG} + F_{HG} \quad (18)$$

where r_{cg} is the distance of separation of the k th object from the center of gravity of all other stars within the galaxy. The number of stars with gravity acting on the k th star is given by $k - 1$. The “stars” mentioned here include the supermassive black holes at the center of the galaxy which usually has the greatest contribution to the overall gravity of the galaxy. The first term in Eqn. (18) can be associated with Newtonian Gravity (F_{NG}) while the last three terms measures the magnitude, F_{HG} , of what we call as “Hidden Gravity”. Since F_{HG} is exceedingly larger than F_{NG} , it means that the influence of the gravity of the galaxy extends not just on stars at the edge of the visible part of the galaxy (i.e., at the bulge and the disk for example) but even beyond it, up to the edge of the galactic halo that surrounds the visible part of the galaxy. If one is to understand the effect of this from the

perspective of Newtonian or Einsteinian theory of gravity, it would be associated with a mass greater than the visible mass within the galactic halo. Such is the very reason why the Dark Matter was hypothesized by thinking within such a paradigm. The observed flat rotation curve of galaxies is therefore explained here not by an unobservable additional matter within the galaxy halo but by the excess gravity that was not accounted for when one is using the classical theory of gravity of Newton or Einstein. Lastly, the value of F_{HG} , however, varies from one galaxy to another since it depends not only on the number of objects k that can contribute to the magnitude of gravity but also to N (the amount of mass/energy of the gravity-contributing objects mentioned) and the distance r_{cg} .

5 Conclusions and Recommendations

We presented here a kind of emergent theory of gravity that is similar to a MOND theory. Gravity here is not described by the amount of curvature of spacetime (*à la* Einstein) or as a force that emerges in a thermal bath (*à la* Verlinde), but by the density of information that can be contained within a gravitational system. Also, the theory neither introduced a new baryonic particle as suggested by the Dark Matter hypothesis nor introduced a new field as suggested by MOND theories. It modifies Newtonian gravity by using the fundamental role of information in the description of gravity. It is possible that the information-theoretic approach to gravity that we proposed here can be applied at the Planck scale to unify gravity with Quantum Mechanics. Perhaps, all it takes is to reinterpret and reformulate Quantum Mechanics, not only as a theory of entropy and information but also as an emergent theory that serves as a low-energy approximation of a more fundamental theory at the Planck scale.

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