

Information Theory of Gravity

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Abstract

A new model of gravity is presented here similar to the earlier work of Verlinde on Emergent Gravity but without the use of thermodynamic assumptions. The theory does not use the main assumption of Verlinde on the nature of gravity as an entropic force using the First Law of Thermodynamics. Moreover, it does not use the Equipartition Theorem such that there is no need to define a thermal bath enclosed within a holographic screen. Instead of Equipartition Theorem, the theory uses $E = NE_p$, for the total energy of a massive object where E_p is the Planck Energy while N is the number of Planck Energy to represent the maximum possible density of information that can reside in matter. The theory uses also the Holographic Principle as the basis for an information-theoretic approach to the nature of gravity. It is shown here that gravity emerges whenever there is an updating of the information within a given volume of space by the presence of matter.

1 Introduction

The main criticism to Verlinde's Emergent Gravity(EG) theory is the fact that it uses thermodynamic assumptions. In his first paper on EG [1], he used an analogy in Thermodynamics where a test particle that enters in a gravitational field is liken to a polymer molecule that enters in a region where it is immerse in a thermal bath. Such condition is known to induce an entropic force at molecular and atomic level. Similarly, according to Verlinde, a test particle that enters a gravitational field is also undergoing similar condition where the entropic force is the gravitational force. This analogy was heavily criticized and experimentally proven to be flawed [2, 3]. Wang et.al., [4], argues that horizons are indeed thermodynamic in nature but general ordinary surfaces that are considered in Emergent Gravity program are not. In essence, Verlinde conjectured that ordinary surfaces are holographic screens that

obey the First Law of Thermodynamics similar to what had been conjectured on horizons in Black Hole Physics [5]. He used the equation, $F\Delta x = T\Delta S$, where Δx is the distance of the test particle from the holographic screen, T is the temperature in the screen and ΔS is the change in entropy S . He argues that as $\Delta x \rightarrow 0$, i.e., as the test particle touches the screen and increases the entropy, it will induce a gravitational force as a kind of entropic force. In this paper, instead of using the First Law of Thermodynamics, we are guided by the fact that the changes in the strength of gravity given by the scalar potential ϕ , as describe by Poisson Equation $\nabla^2\phi = 4\pi G\rho$, is proportional to the changes in the density ρ , i.e., $\nabla\phi = 4\pi G \int \rho dr$ where $\rho = \rho(r)$. In the context of holographic screen, one can use Gauss Theorem of gravity which can be written as $A\Delta F = -4\pi GV\Delta\rho$ for a constant volume and area. It gives us, $\frac{dF}{d\rho} = -\frac{4}{3}\pi Gr$, which shows that the change in magnitude of gravity within the space enclosed by the screen, is always proportional to the change of density or the total number of matter in a given volume of space. Hence, at the very least, one can assume that the change in density is what induces gravity to change its magnitude and to emerge in empty space without necessarily involving macroscopic conditions like changes in the temperature or the amount of heat in the region. It is suggested here, that this classic description of gravity can be used in a fundamental way to show that gravity is an emergent phenomenon in Nature. Another assumption that was used by Verlinde is the Holographic Principle which considers the fundamental role of the quantity $N_a = A/A_p$, or the number of cells in the holographic screen in units of Planck Area A_p , to represent the density of information within the volume of space enclosed by the holographic screen. Here, instead of just using the quantity N_a , we will be using the quantity

$$N_d = \frac{\rho}{\rho_p} = \frac{M/r}{M_p/L_p} = \frac{M/M_p}{r/L_p} = \frac{N}{N_l} \quad (1)$$

where $\rho_p = M_p/L_p$ is the Planck (linear) density which is about $10^{27}kg/m$, $M_p = \sqrt{\hbar c/G}$ is the Planck Mass which is about $10^{-8}kg$ and L_p is the Planck length which is about

$10^{-35}m$. Notice that N_d is also written in terms of quantities $N_l = r/L_p$ and $N = M/M_p = Mc^2/M_p c^2 = E/E_p$ which can also be used to measure the density of information for a given length r and total mass M , respectively. The use of the quantity N_d is consistent with what we had mentioned above that gravity must be proportional to the density ρ . It is also related to N_a since $N_l^2 = N_a$. Thus, one of the main differences of our work with Verlinde's, is that the Energy Equipartition Principle, $E = \frac{1}{2}k_bNT$, which is known to be applicable only at microscopic and macroscopic scale, will not be used here but to be replaced by a new representation of energy, $E = NE_p$, as a quantized representation of energy in terms of the Planck energy E_p . Such approach, we think, is more compatible in a theory that uses the quantity N_a which is a quantity defined in Planck scale units. Furthermore, the main objective of this paper is not to derive Newtonian Gravity as what Verlinde had done in his original paper on EG, but to derive a modified form of Newtonian Gravity that can be used to explain the problem of flat rotation curve of galaxies as a possible alternative to the Dark Matter hypothesis and Modified Newtonian Dynamics (MOND).

2 Modified Newtonian Gravity

As mentioned in the previous section, in Verlinde's theory, he primarily used N_a or the number of bits that can be occupied within the holographic screen. This, however, is only the number of fundamental units of space where the energy associated with gravity can occupy. One must also consider the information that resides within the gravitating matter which can be represented by the quantity $N = \frac{M_1}{M_p} + \frac{M_2}{M_p} = N_1 + N_2$, where M_1, M_2 are the masses of two bodies gravitating towards each other, while $N_1 = M_1/M_p$, and $N_2 = M_2/M_p$ represent the maximum possible density of information that can be stored for each gravitating matter. At this point, we aim to achieve here a purely information-theoretic approach to gravity where its magnitude will not be dependent on the amount of heat or curvature of spacetime within the gravitational field but solely on the amount of information that resides in space and matter within a gravitational system. Hence, the magnitude of gravity F

should only be dependent on the value of N and N_a . The former represents the amount of information on matter and the latter, by Holographic Principle, represents the amount of information within a given volume of space that is occupied by the gravitational system. Gravity therefore would only be proportional to the information density. To quantify this idea, we consider the square of N_d and multiplying it with constant $c^4/2G$, which yield us,

$$F = \frac{c^4}{2G} \left(\frac{N}{N_l} \right)^2 = \hbar c \frac{N_1 N_2}{r^2} + \frac{c^4}{2G} \left(\frac{N_1^2 + N_2^2}{N_l^2} \right) = F_{NG} + F_{HG} \quad (2)$$

where

$$F_{NG} = \hbar c \frac{N_1 N_2}{r^2} = \hbar c \frac{M_1 M_2}{r^2 M_p^2} = G \frac{M_1 M_2}{r^2} \quad (3)$$

is the usual expression for the magnitude of gravity in Newtonian Gravity (NG) that describes it as a force. Hence, the quantity

$$F = \frac{c^4}{2G} \left(\frac{N}{N_l} \right)^2 = \frac{c^4}{2G} N_d^2 \quad (4)$$

must also be an expression that we can relate to the magnitude of gravity which is not necessarily a force as it can be purely expressed in terms of the number of bits or amount of information that resides in a gravitational system. Meanwhile, the quantity

$$F_{HG} = \frac{c^4}{2G} \left(\frac{N_1^2 + N_2^2}{N_l^2} \right) \quad (5)$$

is a magnitude of an excess gravity i.e., a ‘‘Hidden Gravity’’(HG), in addition to the magnitude of gravity given by the Newtonian Gravity. In terms of masses, M and m , for a two-body system, we can rewrite Eqn. (2) as follows,

$$F = G \frac{Mm}{r^2} + \frac{k}{2} \left(\frac{M^2 + m^2}{r^2} \right) \quad (6)$$

where $k = \frac{c^4}{G\rho_p^2} \approx 10^{-11}m^4/Ns^4$. For a terrestrial experiment that uses two 1-kg objects that is 1-meter apart, like in Cavendish experiment, the second term will be negligible and Newton's law of gravity will be observed since the value of k is very small. By unit analysis, $\frac{[m]^4}{[s]^4} = \left([m]\frac{[m]}{[s]^2}\right)^2 = \left([m]\frac{[N]}{[kg]}\right)^2 = [m]^2\frac{[N]^2}{[kg]^2}$, which gives us $k \approx 10^{-11}Nm^2/kg^2 \approx G$ such that Eqn.(6) can be simplified further as follows,

$$F \approx G \frac{Mm}{r^2} f \quad (7)$$

where $f = f\left(\frac{M}{m}\right) = 1 + \alpha$ and $\alpha = \frac{1}{2}\left(\frac{M}{m} + \frac{m}{M}\right)$. It is surprising that the value of the constant k is about the same value of the gravitational constant G which allow us to have the simplified equation above. The modified Newton's law of gravity given by Eqn. (7) is considered to be applicable for larger masses and will be used later for systems in galactic scale. Although this modification of Newtonian Law of Gravity is simple, it must be realized that the derivation of it is not without a theoretical basis. Other similar models that also try to modify Newton's Law of Gravity by adding additional term are mostly done arbitrarily with the aim of fitting the model to the observed data and even reconciling it with dark matter hypothesis. See [6] for different types of such models as examples. These non-Newtonian law of gravity approaches, according to [6], "Although... an old idea that could appear rudimentary...and it is mostly abandoned in modern literature, we think that a reconsideration of this approach could motivate further research in the area of modified gravity theories." On the other hand, the most commonly used approach in introducing a new theory of gravity nowadays is to generalize the Einstein-Hilbert action, $S = \int \sqrt{-g}Rd^4x$, by imposing additional parameters into the action, such as scalar, vector, tensor and spinor fields for the purpose of making the action conformally invariant and to produce field equations that might explain the dark energy and dark matter problems. One of the well-known examples of this, is the Tensor-Vector-Scalar (TeVeS) gravity theory by Bekenstein [7] as a relativistic generalization of MOND paradigm of Milgrom[8]. This Lagrangian method will

not be used here since the model presented here will focus more on the relation of gravity with information density rather than with the energy density within a gravitational system.

3 Mass Correction and Tully-Fisher Relation

The new model of gravity presented here would lead to a small correction to the mass of a test object with motion that is under the influence of gravity. The derivaton of the correction term will be used later to derive the Tully-fisher Relation. For simplicity, we consider a test object with mass m , that has a circular orbit around the center of the source of gravity with mass M . We note of the fact that any object that is under the influence of gravity will react by accelerating and experiencing a force which is a fictitious one, known as the centrifugal force. It has a magnitude that is equal to the magnitude of the centripetal force F_c which is usually associated with the source of gravity. The magnitude is given by $F_c = mv^2/r$, where r is the distance from the source of gravity and v is the rotational velocity of the test object. Equating this to Eqn.(7) we yield, $M \approx \frac{rv^2}{G}\gamma$, where $\gamma = f^{-1}$. For the case of a binary system where $M \approx m$, $\gamma \approx \frac{1}{2}$ and $M \approx \frac{1}{2}\frac{rv^2}{G}$ while for $M \gg m$, $\gamma \approx (1 + \frac{M}{2m})^{-1} = (\frac{2m+M}{2m})^{-1} \approx (\frac{1}{2}\frac{M}{m})^{-1}$ which gives us $M \approx \frac{rv^2}{G}\frac{2m}{M}$. For both cases, the correction term is too small that the model approximates the results of known classic theories of gravity. The correction term γ can be computed by getting the mass ratio without necessarily measuring the individual masses and dividing it. There are already known methods that can be used to measure the mass ratio in a two-body system. For non-luminous objects in a 2-body system, the distances R_1 and R_2 from the barycenter can be used. Since each force felt by both bodies acts only along the line joining the centers of the masses and both bodies must complete one orbit in the same period, the centripetal forces can be equated using Newton's 3rd law, such that we can have the relation $\frac{M}{m} = \frac{R_1}{R_2}$. On the other hand, to get the mass ratio of distant luminous objects in a two-body system like in a binary star, one can use an approximation via the mass-luminosity relationship, $\frac{M}{m} \approx \left(\frac{L_M}{L_m}\right)^{\frac{1}{3.9}}$, which applies for main sequence stars. For galaxy with mass M and a star that revolves around it with mass m_\odot equal to one solar mass, we can use the work of Vale et.al.[10] that relates the luminosity and mass of the galaxy via a double power law equation. It will give us the relation $\frac{M}{m_\odot} \approx \left(\frac{L}{L_\odot}\right)^{\frac{1}{b}}$, where L is the observed luminosity of the galaxy and the range $0.28 \leq b \leq 4$, is for galaxies with galactic halo mass that ranges form high-mass end to low-mass end. If one is to get the square of the rotational velocity v of the revolving star, the mass-luminosity relation will yield us, $v^2 = \frac{GM}{2r}\frac{M}{m_\odot} \approx \frac{GM}{2r}\left(\frac{L}{L_\odot}\right)^{\frac{1}{b}}$, since $f \approx \frac{1}{2}\frac{M}{m}$, for $M \gg m$. This implies that $L \sim v^{2b}$ which for the average, $b \approx 2$, the equation give us the Tully-Fisher relation $L \sim v^4$. Comparing

this result with Relativity, although the notion of mass in Newton's theory was given its relativistic correction in Special Relativity, however, nowhere in Einstein's theory of gravity that it was able to derive the Tully-Fisher relation.

4 External Field Effect and Mach's Principle

It should be realized that any local measurement of the magnitude of gravity is not absolute. It will always depend on external gravity of other masses. This is known in MOND theories as the External Field Effect (EFE)[9]. To illustrate, if an apple is acted by Earth's gravity, and Earth is acted by the Sun's gravity, all other gravity acting on the apple should be accounted for in the calculation. These include not only the gravity generated by the Milky Way acting on the Solar System, but also the gravity of the Local Group and supercluster where the Milky Way belongs which in turn is acted upon by the gravity of all matter in the Universe. The gravity in each sources varies depending on the density or distribution of matter in its vicinity. In galactic scale in particular, one must consider the variation of density from the nucleus of the galaxy, to its bulge, to the galactic disk, up to the galactic halo. Quantitatively, as the distance of a test object from the source of gravity increases or the area of the holographic screen becomes larger such that it encloses more matter, it will increase the density and therefore increases also the magnitude of gravity acting on the test object located at the holographic screen. In addition, it will also result for the direction of the center of gravity to shift from one point to another depending on the distribution of enclosed matter. Although MOND theories were the first to suggest the existence of EFE, as far as we know, it was never translated in concrete mathematical terms. In this section we wanted to express EFE, mathematically, based on the results from the previous section. For large scale gravity which involves a larger group of gravitational sources we now have, $N = N_1 + N_2 + \dots + N_k$, for k number of gravitating objects. Squaring N , we have, $N^2 = (N_1)N_1 + (N_1 + N_2)N_2 + \dots (N_1 + N_2 + \dots + N_k)N_k$. Distributing and rearranging terms, we can have a more compact expression using the summation symbol,

i.e., $N^2 = \sum_{i<j}^k N_i N_j + \sum_i^k N_i^2$, which can be expanded as follows:

$$N^2 = \left(\sum_{i<j}^{k-1} N_i N_j + N_k \sum_i^{k-1} N_i \right) + \left(\sum_i^{k-1} N_i^2 + N_k^2 \right) \quad (8)$$

By using Eqn. (2) and the convention $G = \hbar = c = 1$, the magnitude of the gravity of a galaxy, F_G , acting on k th star, would be

$$F_G = \left(\frac{N_k \sum_i^{k-1} N_i}{r_{cg}^2} \right) + \left(\frac{\sum_{i<j}^{k-1} N_i N_j}{r_{cg}^2} \right) + \frac{1}{2} \left(\frac{\sum_i^{k-1} N_i^2}{N_l^2} \right) + \frac{1}{2} \left(\frac{N_k^2}{N_l^2} \right) = F_{NG} + F_{HG} \quad (9)$$

where $N_l = r_{cg}/L_p$ and r_{cg} is the distance of separation of the k th object from the center of gravity of all other stars within the galaxy. The number of stars with gravity acting on the k th star is given by $k - 1$. The “stars” mentioned here include black holes and neutron stars which usually have greater contribution to the overall gravity of the galaxy than the usual stars. Contributions of planets, asteroids and other matter in the overall gravity of a galaxy would probably be just equal to the gravity of one or two black holes. The first term in Eqn. (9) can be associated with Newtonian Gravity (F_{NG}) while the last three terms are with the “Hidden Gravity” (F_{HG}). It should be noticed that F_{HG} is exceedingly larger than F_{NG} which means that the influence of the gravity of the galaxy extends not just on stars at the edge of the visible galaxy but beyond it, even up to the edge of the galactic halo that surrounds the visible part of the galaxy. If one is to understand this result in the perspective of Newtonian or Einsteinian theory of gravity, such amount of gravity would be associated to a mass greater than the visible mass within the galactic halo. Such is the very reason why the so-called Dark Matter was hypothesized by thinking within such paradigm. The observed flat rotation curve of galaxies is therefore explain here not by an unobservable additional matter within the galaxy halo but by the excess gravity that was not accounted for when one is using the classical theory of gravity of Newton or Einstein. In addition to this, Eqn. (9)

can also be used at cosmological scale not just at galactic scale. Instead of considering all the stars within a galaxy, we can consider all the galaxies within the Universe. The simplicity of the mathematical formalism of the new theory presented here should not be considered as its weakness but in fact its advantage. It removes any mathematical problem like the existence of singularities that may result to various interpretations. Furthermore, although it is not as sophisticated as the mathematical formalism of Einstein's theory, the most important aspect of the mathematical formalism of the theory is it naturally includes Mach's Principle which is not apparent in the mathematics of General Relativity (GR). Historically, according to Pais [13], Einstein strongly believed that to have a "satisfactory theory of gravity", Mach's Principle must be included along with the Principle of General Covariance and Principle of Equivalence. In GR, the latter two were incorporated mathematically but not Mach's Principle. In our theory, when $\sum N_i = 0$, i.e., the total density of all possible information that can be contained within all matter in the Universe is zero, gravity is totally non-existent. This must also be true for inertia if one is to uphold the Principle of Equivalence. In comparison with GR, its field equations allow for matter-free solutions which seems to suggest that it is incompatible with Mach's Principle. The theory presented here incorporates Mach's Principle where the interpretation of Mach's Principle that we used here is aligned with the interpretation of de Sitter [11]. Although de Sitter's interpretation is just one out of many possible interpretations of Mach's Principle according to Bondi [12], the fact remains that the theory of gravity presented here incorporates Mach's Principle and must be considered as its advantage over other theories of gravity. Lastly, the Principle of General Covariance will not be violated by the theory since one can use the Planck scale quantities in the formulation of physical laws where the measurements of those quantities by observers in different frames of reference can be correlated without ambiguity. The quantities like the Planck length and Planck mass can be considered as invariant quantities which are even more fundamental than other known constants in Nature, like the speed of light, since their values will not be varying at the Planck scale.

5 Conclusions and Recommendations

We presented an emergent theory of gravity without the use of thermodynamic assumptions and incorporates Mach's Principle. Gravity is not described by the amount of curvature of spacetime (*à la* Einstein) or as a force that emerges in a thermal bath (*à la* Verlinde), but by the density of information that can be contained within a gravitational system. Also, the theory neither introduced a new baryonic particle as suggested by Dark Matter hypothesis nor introduced a new field as suggested by MOND theories. It modifies Newtonian gravity by using the fundamental role of information in the description of gravity. If in Newton's theory, gravity is described by the gravitational potential ϕ , while in Einstein's theory it is described by the spacetime metric $g_{\mu\nu}$, here we have the unitless quantity N_d which is defined in Planck scale units. It is possible that the information-theoretic approach to gravity that we proposed here can be applied at Planck scale to unify gravity with Quantum Mechanics. All it takes is to reinterpret and to reformulate Quantum Mechanics, not only as a theory of entropy and information but also as an emergent theory that serve as a low-energy approximation of a more fundamental theory at the Planck scale. We also conjecture here that since the theory is possibly applicable at extreme scenario like at the Planck scale, it probably can be used to describe the information inside a black hole. Lastly, we suggested here that the theory presented here is not only applicable at galactic scale but also in cosmological scale. Particularly, we proposed here, that it has the potential to solve the problem on how the gravity generated by all the matter in the Universe, reaches its influence anywhere in the Universe and manifests itself as the local inertia. This Machian problem and all of the problems mentioned above, in our opinion, are interrelated and can be resolved via the information-theoretic approach that we have presented here.

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