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The Dark Sector and the Standard Model from a Local Generalisation of Extra Dimensions

David J. Jackson

Independent Researcher, UK

david.jackson.th@gmail.com

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Abstract

Many models with structures of matter associated with a structure of extra spatial dimensions have been proposed in recent decades. On employing a further generalisation from the local 4-dimensional spacetime form to a general form for proper time, we describe how matter fields resembling the Standard Model of particle physics can be accommodated far more directly than with a higher-dimensional spacetime theory. The successful identification of key features of visible matter in this *non-spatial* sector of extra dimensions in turn motivates seeking a candidate for dark matter residing in the original extra *spatial* dimension sector, and provides a close guide for the explicit form this invisible matter might take. We describe how such Standard Model and dark matter sectors in different extra-dimensional branches of generalised proper time are gravitationally connected through their common root in the local 4-dimensional spacetime and consider further possible mutual interactions and implications in comparison with existing dark matter models. A yet further possible branch of generalised proper time can be connected with dark energy models, hence in principle accounting for all three major components of cosmological structure within this framework.

Keywords: Standard Model; dark matter; dark energy; unification; extra dimensions

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1 Introduction and Outline

Observations in cosmology from galactic scales and above, from the rotation curves, clustering and lensing effects of galaxies to studies of the cosmic microwave background, distant supernovae and the large scale structure of the universe, collectively indicate the presence of a significant amount of 'dark matter' (see for example [1] section 2.1, [2] section 3.1, [3] section 27.1). This compelling conclusion assumes the gravitational field equation of general relativity to hold on these scales and implies the existence of a novel form of matter that gravitates but does not interact electromagnetically. Such an effectively charge-neutral entity might also have been termed 'invisible matter' to highlight its transparent nature.

Fluctuations in the dark matter distribution in the early universe, as modelled in simulations, are expected to have seeded the formation of visible galaxies. While tending to be accumulated more densely towards the centre of galaxies, at later times and at the present epoch, the dark matter spreads out some distance beyond the boundaries of visible stars forming extended dark haloes around galaxies and underlying a vast cosmic web interlinking galactic structures. To account for the overall gravitational impact the total amount of dark matter in the universe is required to be around five times that of ordinary visible matter. Hence while it is generally known *where* the dark matter is and *how much* of it there is, it is not known *what* it is.

A good fit to the data, again assuming the standard theory of gravity, is provided by the hypothesis of a new stable particle or class of particles. Such a source for dark matter, in principle also subject to non-gravitational 'dark forces' of its own, could have been created in the Big Bang as a new form of elementary particles alongside, and distinct from, the visible states of the Standard Model of particle physics. Empirical observations of cosmological structure are consistent with a non-relativistic form of *cold* dark matter (CDM) – that is heavy, slow moving particles – that can interact no more than *weakly* with the Standard Model states of visible matter to account for the present lack of any laboratory detection. The CDM candidates of one generic class of theoretical models are termed WIMPs, 'weakly interacting massive particles', where the 'weak' nature of the interaction with visible matter is typically associated with the familiar $SU(2)_L$ 'weak interaction'. More generally WIMPs are linked with the electroweak scale with a presumed particle mass of order 100 GeV, although the proposed mass can vary by several orders of magnitude ([4] section 1). The interaction between WIMP and Standard Model states is sufficient for thermal equilibrium in the high temperatures of the early universe, with the model parameters determining the appropriate thermal decoupling and 'freeze-out' relic density of dark matter states as the universe expands ([4] section 2).

Dark matter with non-gravitational interactions could in principle be observable in the laboratory through rare impacts with microscopic structures of visible matter or as signalled through missing energy in events of high energy physics collider experiments, and also through astrophysical detection such as via gamma-ray sources. The failure to date to detect dark matter through any of these means (see for example [3] sections 27.6 and 27.7, [4] section 3, [5]) places limits on the parameter space of models for dark matter and puts pressure on minimal scenarios for the WIMP hypothesis (see for example [6]). Observations of the aftermath of galactic interactions, with

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clouds of dark matter having seemingly passed through each other, also suggest that the degree of non-gravitational self-interaction exhibited by the stable structures of dark matter must be limited in strength and/or range (for example via analysis of the 'Bullet Cluster' [7]).

The true nature of dark matter, given its dominant gravitational effects on the galactic scale together with its elusiveness in the laboratory, is one of the biggest mysteries in modern-day physics. The pressing need for an underlying theoretical explanation of dark matter has stimulated a large number of models, the properties of several of which we shall focus upon in this paper. In the present theory we propose an explicit source for dark matter as directly associated with structures of extra spatial dimensions that are in turn generated within the context of a generalisation of the local form for proper time, and describe how this dark matter relates to the models reviewed. Significantly, rather than constructing a dark matter model tailored to empirical observations, here we begin with an underlying conceptual motivation that will lead to an account of *what dark matter is* and how it can interact with the Standard Model, as deriving from the first principles and elementary basis of the theory. The paper is organised as outlined below.

In the three subsections of section 2 we briefly review the proposals of existing models for dark matter, pertinent for the theory to be developed here, involving in turn a new non-Abelian gauge interaction, the production of black holes in the very early universe and the geometric structures of extra spatial dimensions. The latter case, with 4-dimensional spacetime augmented by a small number of extra spatial dimensions to accommodate new particle states as the dark matter candidate, contrasts in particular with the foundations of the approach to be taken in the present theory. A natural further augmentation from the local structure of 4-dimensional spacetime to generalised proper time will be introduced and motivated in section 3. The successes in accounting for the Standard Model through this generalisation of proper time beyond the restricted case of extra spatial dimensions will then be reviewed in section 4.

This leaves free an alternative sector of generalised proper time that can be interpreted as the underlying generator of apparent extra spatial dimensions and in turn as the source of an explicit physical form for dark matter as we describe in section 5, where comparison with the models reviewed in section 2 will be made. The nature of the gravitational impact of both sectors of generalised proper time leads to a direct gravity-mediated link between the corresponding visible and dark sectors of matter, as will be explained in section 6. There we also speculate on the nature of possible non-gravitational interactions between these two sectors of extra dimensions that might in principle be detectable in the laboratory. The exploration of different branches of generalised proper time also opens up the possibility of accommodating dark energy, and hence accounting for the full composition of the 'cosmological pie chart', within this unifying theory as we describe in section 7. We conclude with further discussion and comments in the final section.

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2 Review of Selected Dark Matter Models

2.1 New Confining Gauge Interaction

On considering that most of the mass of the visible matter in the universe is associated with baryonic states and quantum chromodynamics (QCD), in the form of protons and neutrons, a class of models propose a new hidden strong sector, or a 'dark QCD', as a basis for dark matter. A suitable new confining non-Abelian gauge force would result in bound composite states of dark matter analogous to the familiar hadrons. The novel 'dark hadron' states, typically composed of a new set of 'dark fermion' or 'dark quark' elementary states, might then have a similar origin, mass and density as standard hadronic matter in the universe [8, 9, 10, 11, 12, 13]. While in some cases such dark states could be considered a WIMP candidate [12], these are generally a distinct class of models and in turn typically not ruled out by laboratory limits. The hidden gauge group may be an additional SU(3), similar to the Standard Model colour gauge group $\mathrm{SU}(3)_c$ of ordinary QCD, or more generally SU(n) or even an SO(m) gauge group for arbitrary $n, m \geq 3$ [9, 11].

Such gauge groups have been studied in analogy with, and to help elucidate the properties of, standard QCD itself [14, 15]. Connections have been established between the gauge theories of the closely related gauge groups SO(6) and SU(4) ([15] equation 4.4) as well as for SO(m) and SU(n) for $m = 2n$ in the large n limit for which some simplifications apply while keeping essential features ([15] equations 1.1 and 4.6; the links in particular involve the deconfining temperatures of the respective theories). The coupling of the gauge group SO(m) to a real scalar field has also been studied ([14] with the Lagrangian of equation 1 therein), although with such a model there considered non-physical since there is no known empirical correlate of such a theory.

In the context of establishing candidates for dark matter there are also models with a new hidden gauge sector but no matter field equivalent of quark states transforming under the gauge symmetry. This describes the minimal case in which the dark matter candidate consists of the lightest stable 'dark glueball' states [11, 16]. However, more typically new fermions are also included to open up further possibilities in terms of the lightest dark baryon or dark pion states as dark matter candidates.

In a further proposal the formation of much heavier dark composite states has been considered. It was originally suggested that the excess of standard quarks over antiquarks in the early universe could be concentrated in dense invisible nuggets of quarks as a basis for dark matter derived from within the Standard Model alone [17]. However, the need for the appropriate properties, such as a first order phase transition in the early universe, again motivates the introduction of a new confining gauge theory with the requisite features to permit the formation of heavy 'dark nuclei' or 'dark quark nuggets' [18, 19, 20]. With no electromagnetic interaction and hence no Coulomb repulsion such stable dark nuclei, studied using a simple 'liquid drop' type model, with nucleon number $N \geq 10^8$ or even much larger can form through Big Bang 'dark nucleosynthesis' [18, 21, 22].

The dark nucleosynthesis, based on a new non-Abelian gauge force with the dark colour group acting on new fermion states transforming under the fundamental representation, is analogous to the formation of the lightest nuclei, isotopes of hydrogen, helium and lithium, through the Standard Model processes of ordinary nucleosynthesis in the early universe. However, the dark quark nuggets can have a macroscopic mass in the range of 10^{-10}–10^{20} kg and a physical radius of up to 10^3 km (around the size of Australia); and hence theoretically populate a large part of dark matter parameter space [19]. The Fermi degeneracy pressure supports the dark quark nugget against collapse ([19] figure 2 and section 4.1). In other models new fermions may be bound by a Yukawa interaction, rather than a gauge force, again balanced by the Fermi pressure to form stable massive composite states [23, 24].

It is conceivable that if a dark quark nugget grew too massive a collapse into a black hole could occur ([23] section IV, [25] sections II and III). However, while lacking the potential for detecting any weak non-gravitational interactions with these states, apart from at very close range the gravitational impact of such black holes would be identical to that of stable dark quark nuggets of the same mass and number and hence still constitute a candidate for dark matter with a similar large scale impact. Indeed the properties of black holes make them a suitable candidate for dark matter in their own right, as we review in the following subsection.

2.2 Primordial Black Holes

While standard QCD processes are not feasible as a source of dark matter quark nuggets as noted in the previous subsection, the possibility remains of a purely Standard Model basis for dark matter through the formation of primordial black holes (PBHs). These may be produced through various possible mechanisms in the very early universe, for example from the collapse of large density fluctuations generated by an inflationary era ([26] section 2, [27], [28] section II, [29] section II). Such PBHs with a mass greater than 10^{12} kg (or around $10^{-18}\,M_\odot$ in units of solar masses) would be expected to survive Hawking evaporation through to at least the present epoch and can be considered a dark matter candidate [30].

In recent years the negative result from the lack of any direct detection of WIMP dark matter particles in the laboratory, as noted in section 1, together with the positive result in the observation of black hole merger events [31] have increased the motivation for the PBH dark matter proposal. Consistent with the characteristics of these gravity wave detection events the impact of black holes of around $30\,M_\odot$ as a dark matter candidate can be analysed [32]. However, the constraints of the lower bound from evaporation and upper bounds from gravitational lensing and other means suggest that only the mass window of approximately 10^{-16}–$10^{-12}\,M_\odot$ (equivalent to the mass of an asteroid around 5–100 km across) is still available for PBHs with a monochromatic mass to account for all of the dark matter ([29] figure 3).

The limits naturally depend upon this assumption of a monochromatic mass, and are more flexible if a linear combination [33] or extended mass function [28] is considered. The PBH background could also be considered a partial contribution to dark matter alongside for example a WIMP candidate ([28] section VI), with both components consistent with the respective empirical limits. While the analysis of a variety of observations and the development of models is ongoing there is still a suggestion that PBHs in the mass range of around 1–100 $M_\odot$ could make a significant contribution to,

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or even the total composition of, dark matter [32, 34, 35]. Approximately solar mass PBHs can also be considered MACHO, 'massive compact halo object', candidates [36].

Further support for the PBH hypothesis would be obtained if gravity waves were to be observed from a black hole merger event with the mass of one partner less than $1.4\,M_\odot$ (with $2.6\,M_\odot$ seemingly the current lightest black hole observed as the secondary object in event GW190814 [37] section 1, point 5). Such a black hole, below the Chandrasekhar limit, would imply a non-stellar and presumably primordial origin [34]. Another means of testing the PBH dark matter proposal remains through the prediction of gravitational microlensing effects [30]. Further if black holes of a much lower mass of up to around 10^{12} kg ($10^{-18}\,M_\odot$) were also produced in the Big Bang they might still be observed as a gamma-ray source coming from the Hawking radiation and evaporative explosion at the end of a black hole's lifetime ([26] section 3.3). While the ultimate nature of a complete 'quantum gravity' theory remains an open question if the physics in the vicinity of a black hole singularity is such that black holes do not completely evaporate but rather leave a Planck mass relic then in principle such relics, if charged, could even be observed in the laboratory [38].

While PBHs provide a potential means of accounting for dark matter without having to propose new elementary states of matter beyond the Standard Model, all of the above empirical and possible observational effects of black holes apply regardless of the type of matter from which they originate. This would include the collapse of massive dark quark nuggets constructed from a new physics sector as considered at the end of the previous subsection. As well as novel forms of matter, such as dark quark nuggets, and extreme structures of 4-dimensional spacetime, in the form of black holes, candidates for dark matter have also been proposed in the framework of novel forms for spacetime itself – that is through the addition of extra dimensions of space as we review in the following subsection.

2.3 Extra Spatial Dimension Framework

Models with extra dimensions of space involve a direct generalisation from the basic 4-dimensional spacetime structure of general relativity. In relativity a local infinitesimal proper time interval δs in 4-dimensional spacetime can be expressed as:

$$(\delta s)^2 = \eta_{ab}\delta x^a \delta x^b \qquad (1)$$

where $\eta = \text{diag}(+1, -1, -1, -1)$ is the local Lorentz metric and $a, b = 0, 1, 2, 3$ with the summation convention over repeated indices implied. The interval δs is invariant under Lorentz transformations acting upon the spacetime components $\{\delta x^a\} \in \mathbb{R}^4$. Increasing the number of components, while maintaining a quadratic form under an extended metric of the appropriate signature, corresponds to appending extra spatial dimensions. This provides a means of opening up new degrees of freedom and a potential arena for accommodating matter fields with the desired empirical properties from the perspective of the original 4-dimensional spacetime.

In particular the deployment of extra spatial dimensions can provide a framework in which to identify a dark matter candidate, as proposed by a number of models. The Standard Model particles are typically confined to our 4-dimensional spacetime

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brane embedded within a higher-dimensional spacetime *bulk*. A field propagating in the bulk giving rise to a Kaluza-Klein tower of states in the effective 4-dimensional theory can generate a stable dark matter candidate as the lightest Kaluza-Klein particle [39]. Alternatively the dark matter candidate may itself be identified with a field confined on a 4-dimensional brane within the bulk (see for example [40] and figure 1 therein). The construction of such models typically involves a dark matter candidate exhibiting a minimal coupling with the Standard Model that may be *weak*, on including a small non-gravitational interaction as consistent with a WIMP, or *feeble*, if this interaction is *extremely weak* or potentially mediated through gravity alone as correspondingly termed FIMP dark matter [41].

The above models generally involve an extension from the local form of equation 1 to for example the form ([40] equation 1):

$$(\delta s)^2 = f(y)^2\,\eta_{ab}\delta x^a \delta x^b - (\delta y)^2 \qquad (2)$$

with y a new spacelike coordinate in a 5-dimensional bulk spacetime and $f(y)$ a simple function. In general there may be further spacelike components and the geometry of the extra dimensions may or may not be compactified. Such a form for proper time intervals in equation 2 is seen for example in models in which the extra dimensions are considered 'flat' – with $f(y) = 1$ for a constant metric in equation 2 ([40] section 3), 'spherical' – with two extra space dimensions compactified on a sphere ([42] equation 3), 'warped' – with a y-dependent 4-dimensional metric via a 'warp factor' of the form $f(y) = e^{-k|y|}$ in equation 2 with k a curvature scale ([40] section 4, [43] equation 2.1), or 'universal' – with all fields including those of the Standard Model propagating in the bulk [44, 45]. Further developments, and tension with regions of parameter space ruled out by the lack of any dark matter particle detection (as noted in section 1 with reference to [3] sections 27.6 and 27.7, [4] section 3, [5]; for searches at the Large Hadron Collider see also [46, 47]) have led to models generalising and combining elements of the above approaches (such as [48, 49]).

In this paper we shall be led to an explicit and direct relation between a structure of extra spatial dimensions and a candidate for dark matter of a different nature to the models reviewed in this subsection. In particular, rather than deriving from a new field postulated to *reside in* extra-dimensional structures, as for the above models, in the present theory the dark matter candidate will be more literally *identified with* the local structures of the extra dimensions *themselves*, as will be described in section 5. There are of course a great deal of models and theories postulating a role for structures of extra spatial dimensions without any motivation in connection with dark matter (see for example [50, 51]). There are also a large number of dark matter candidates proposed that make no use of extra-dimensional structures (see for example [1, 2], [3] section 27, as well as the two previous subsections of this paper).

While increasingly constrained by observations dark matter remains empirically relatively simple, compared to the structures of visible matter, in the sense that the properties have been inferred through gravitational detection alone. It is then perhaps not surprising that the appropriate dark matter phenomenology might be accommodated through a variety of theoretical means via structures of extra spatial dimensions. This is in contrast with the long-standing programme of attempts to accommodate the elementary properties of visible matter through extra dimensions of

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space. These attempts date from the 5-dimensional spacetime theory of Kaluza one hundred years ago [52] to the modern-day quest to associate structures deriving from additional spatial dimensions over 4-dimensional spacetime with the matter fields of the Standard Model itself (see for example [53, 54]). However, it has proven difficult to establish such a connection without the need for a somewhat contrived approach.

In the following section we motivate and introduce the basic idea of the present theory for which the possibility of a further sector of 'extra dimensions' will be identified on making a further generalisation from the proper time interval of equation 1. Compared with the *restricted* case of extra *spatial* dimensions we then describe in section 4 how the matter fields deriving from the new sector of *generalised proper time* exhibit a far more direct connection with features of the Standard Model of particle physics. This will provide a firm grounding for the essentially parallel identification of a dark matter candidate in the original extra spatial dimension sector. As we describe in section 5 this dark matter candidate takes a physical form closely connecting with the models reviewed in subsection 2.1 and potentially also those of subsection 2.2, hence incorporating elements of all three classes of models summarised in this section.

3 Extra Dimensions and Generalisation of Proper Time

In equation 1 the physically directly measurable quantity is proper time on the left-hand side, or strictly an integrated $\int \delta s$ finite interval. While proper time is expressed as a real number $\delta s \in \mathbb{R}$, with the familiar simple additive property associated with intervals of time, it is the arithmetic substructure of the real line that permits the expression of such infinitesimal intervals of time in a *quadratic* form, as for the case of equation 1 as well as in the extension to the $f(y) = 1$ case of equation 2. Such a quadratic form naturally describes 'spatial' dimensions, given an appropriate metric signature, through conformity with the Pythagorean theorem and the algebraic properties of a local Euclidean geometry associated with the corresponding local coordinate components, such as $\{\delta x^1, \delta x^2, \delta x^3\}$ in equation 1.

However, in considering generalisations beyond the familiar 4-dimensional external spacetime there is no compelling reason for any *additional* structure to be of a *spatial* form, since we do not perceive extra dimensions in such a geometrical manner. Rather the arithmetic generalisation from the real proper time interval in equation 1 can take the n-dimensional homogeneous p^{th}-order polynomial form:

$$(\delta s)^p = \alpha_{abc\ldots} \delta x^a \delta x^b \delta x^c \ldots \tag{3}$$

Here in general $p > 2$, corresponding to the possibility of forms of higher than quadratic order, with each coefficient $\alpha_{abc\ldots} = -1, 0$ or $+1$ as a generalisation from the components of the Lorentz metric η_{ab} in equation 1. The indices $a, b, c, \ldots$ each now run over $0, \ldots, n-1$ and a full symmetry group $\hat{G}$ acts upon the n components $\{\delta x^a\} \in \mathbb{R}^n$, as a generalisation from the Lorentz group, leaving the generalised proper time interval δs in equation 3 invariant.

This equation can be considered a very conservative starting point for a theory. While there is no empirical evidence for *extra spatial* dimensions here we are describing

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an inherent arithmetic generalisation for the local form for proper time, with *time itself* an elementary component of the physical world with which we are very familiar. As well as providing a simpler and more unique starting point for a theory, compared with the large array of possible initial constructions based on posited extra dimensions of space, time also has the intrinsic property of pervading all phenomena in the universe. These are all characteristic features that might be desired in a fundamental unifying theory ([55] section 1).

For the rigorous analysis of the implications of this approach the expression for infinitesimal intervals in equation 3 can first be rearranged in a more convenient form. On dividing both sides by $(\delta s)^p = \delta s\, \delta s\, \delta s \ldots$, taking the limit $\delta s \to 0$, and defining the generally finite components $v^a := \frac{\delta x^a}{\delta s}\big|_{\delta s \to 0}$, for $a = 0 \ldots n-1$, of an n-dimensional vector $\boldsymbol{v}_n = (v^0, v^1, v^2, \ldots, v^{n-1}) \in \mathbb{R}^n$, equation 3 can be rewritten as:

$$L_p(\boldsymbol{v}_n)_{\hat{G}} \;:=\; \alpha_{abc\ldots} \frac{\delta x^a \delta x^b \delta x^c \ldots}{\delta s \;\; \delta s \;\; \delta s \;\ldots}\Big|_{\delta s \to 0} \;=\; \alpha_{abc\ldots}\, v^a\, v^b\, v^c \ldots \;=\; 1 \qquad (4)$$

Here p is the homogeneous polynomial power, n the dimension, and $\hat{G}$ the full symmetry of this generalised form for proper time ([55] equation 4 and references therein). The possible mathematical realisations for this expression, and corresponding values for $(p, n, \hat{G})$ as extensions from the original local 4-dimensional spacetime values of $(p = 2, n = 4, \hat{G} = \text{Lorentz})$, will determine the physical properties of the theory.

The physical content of the theory derives from the necessary breaking of the full symmetry $\hat{G}$ of equation 4 in extracting the substructure required to *identify* the local geometric structure of the external 4-dimensional spacetime manifold M_4 itself, entailing the local choice of $\boldsymbol{v}_4 = (v^0, v^1, v^2, v^3) \in TM_4$ as a specific subset of four components projected onto the local tangent space of the external spacetime. The absolute distinction between these external spacetime components and the residual internal components of equation 4 implies an *absolute* breaking of the full mathematical symmetry $\hat{G}$. As the starting point for all physics this symmetry $\hat{G}$ is hence in general reduced to the direct product form:

$$\text{Lorentz} \times G \;\subset\; \hat{G} \qquad (5)$$

(which is consistent with the Coleman-Mandula theorem, [56] equation 92 discussion). The *external* Lorentz symmetry, acting upon the external $\boldsymbol{v}_4 \in TM_4$ components, will be expressed by the group $\text{SO}^+(1,3)$ or its double cover $\text{SL}(2,\mathbb{C})$, as for equation 6 below, while the residual group factor G describes an *internal* symmetry. The residual components of $\boldsymbol{v}_n \in \mathbb{R}^n$ from equation 4 over and above $\boldsymbol{v}_4 \in TM_4$, together with their transformation properties under the broken symmetry of equation 5, will provide a basis for physical matter fields in the external 4-dimensional spacetime.

In the following section we describe a mathematically unique series of expressions for proper time in the form of equation 4 augmenting the 4-dimensional spacetime form. The corresponding physical consequences of the symmetry breaking of equation 5 will then be described explicitly and connections established with the Standard Model of particle physics.

4 Sector with Standard Model Structure

We first note that equation 1 for a local proper time interval δs in 4-dimensional spacetime can itself be rearranged in the form of equation 4 and expressed in a standard way as the determinant of a 2×2 Hermitian complex matrix $\boldsymbol{h} \in \mathrm{h}_2\mathbb{C}$, within which the components of $\boldsymbol{v}_4 = (v^0, v^1, v^2, v^3) \in \mathbb{R}^4$ are embedded as:

$$L_2(\boldsymbol{v}_4)_{\mathrm{SL}(2,\mathbb{C})} = \eta_{ab} v^a v^b = |\boldsymbol{v}_4|^2 = \det(\boldsymbol{h}) = \det \begin{pmatrix} v^0 + v^3 & v^1 - v^2 i \\ v^1 + v^2 i & v^0 - v^3 \end{pmatrix} = 1 \quad (6)$$

This determinant is invariant under the matrix transformations $\boldsymbol{h} \to \boldsymbol{S}\boldsymbol{h}\boldsymbol{S}^\dagger$ for any element $S \in \mathrm{SL}(2,\mathbb{C})$ of the symmetry group $\hat{G} = \mathrm{SL}(2,\mathbb{C})$ as the double cover of the Lorentz group $\mathrm{SO}^+(1,3)$.

Explicitly exploiting the potential for $p > 2$ in the general form for proper time in equation 4 a direct extension from the 4-dimensional *quadratic* form of equation 6 can be constructed as a *cubic* form defined as the determinant of a 3×3 Hermitian complex matrix $\boldsymbol{v}_9 \in \mathrm{h}_3\mathbb{C}$ with a full $\hat{G} = \mathrm{SL}(3,\mathbb{C})$ symmetry:

$$L_3(\boldsymbol{v}_9)_{\mathrm{SL}(3,\mathbb{C})} = \det(\boldsymbol{v}_9) = \det \left(\begin{array}{cc|c} v^0+v^3 & v^1-v^2 i & v^4+v^5 i \\ v^1+v^2 i & v^0-v^3 & v^6+v^7 i \\ \hline v^4-v^5 i & v^6-v^7 i & v^8 \end{array}\right) = \det \left(\begin{array}{c|c} \boldsymbol{h} & \psi \\ \hline \psi^\dagger & v^8 \end{array}\right) = 1 \quad (7)$$

Here the four components of $\boldsymbol{h} \in \mathrm{h}_2\mathbb{C}$, as projected onto the external spacetime tangent space with $\boldsymbol{h} \equiv \boldsymbol{v}_4 = (v^0, v^1, v^2, v^3) \in TM_4$, are now embedded within a subspace of $\boldsymbol{v}_9 \in \mathrm{h}_3\mathbb{C}$. The five residual components of $\psi \in \mathbb{C}^2$ and $v^8 \in \mathbb{R}$ can then be associated with 'matter fields' in the extended 4-dimensional spacetime as deriving from this 9-dimensional case for generalised proper time. With an external Lorentz symmetry $\mathrm{SL}(2,\mathbb{C}) \subset \mathrm{SL}(3,\mathbb{C})$ acting on the projected components $\boldsymbol{h} \in \mathrm{h}_2\mathbb{C} \subset \mathrm{h}_3\mathbb{C}$ as extracted from the higher-dimensional space in equation 7 the breaking of the full symmetry $\hat{G} = \mathrm{SL}(3,\mathbb{C})$ as described for equation 5 leaves a residual internal $G = \mathrm{U}(1)$ symmetry acting upon the matter field components $\psi(x), v^8(x)$ (where $x \in M_4$ represents location in the extended 4-dimensional spacetime manifold, see also [57] equations 48–51 and figure 2 with the notation $n = v^8 \in \mathbb{R}$ therein). In a similar manner the physical properties of the resulting matter fields can be analysed for further extensions to higher-dimensional forms for proper time and for the general case.

In considering higher-dimensional forms for equation 4 with a high degree of symmetry a natural extension from the quadratic Lorentzian form of equation 6 via the cubic form of equation 7 is obtained on generalising from elements of the complex algebra $\mathbb{C}$ to the octonions $\mathbb{O}$, as the largest normed division algebra [58]. In a consistent manner this leads to the structure of a 27-dimensional cubic form for proper time with a full $\hat{G} = \mathrm{E}_6$ symmetry that can be written as:

$$L_3(\boldsymbol{v}_{27})_{\mathrm{E}_6} = \det(\boldsymbol{v}_{27}) = 1 \qquad \text{with} \qquad \boldsymbol{v}_{27} \in \mathrm{h}_3\mathbb{O} \quad (8)$$

The elements of $\mathrm{h}_3\mathbb{O}$ are Hermitian 3×3 matrices over the octonions $\mathbb{O}$ ([57] subsection 4.2, employing an $\mathrm{E}_6 \equiv \mathrm{SL}(3,\mathbb{O})$ symmetry action constructed as described for example in [59, 60]). In turn the symmetry and structure of equation 8 can itself be embedded directly within an E_7 symmetry acting upon a 56-dimensional *quartic* form:

$$L_4(\boldsymbol{v}_{56})_{E_7} = q(\boldsymbol{v}_{56}) = 1 \qquad \text{with} \qquad \boldsymbol{v}_{56} \in F(\mathrm{h}_3\mathbb{O}) \tag{9}$$

The 4^{th}-order form $q(\boldsymbol{v}_{56})$ is defined in terms of the algebraic properties of elements of the Freudenthal triple system $F(\mathrm{h}_3\mathbb{O})$ ([57] subsection 4.3, employing the relevant mathematical structures described for example in [61] section 9, [62]).

In the case of equation 9 the symmetry breaking projection of the subcomponents $\boldsymbol{v}_4 \in TM_4$ out of the full set of components $\boldsymbol{v}_{56} \in F(\mathrm{h}_3\mathbb{O})$ onto the local tangent space of the external spacetime M_4 breaks the full $\hat{G} = E_7$ symmetry. This directly yields the transformation properties of the reduced 56-dimensional representation space under the resulting external Lorentz $\mathrm{SL}(2,\mathbb{C})$ and a residual internal gauge symmetry identified as $G = \mathrm{SU}(3) \times \mathrm{U}(1)$ that describe matter fields with the properties summarised in table 1. These structures bear a close resemblance to properties of one generation of leptons and quarks in the Standard Model. In particular Dirac spinor structures under the external Lorentz symmetry are identified as well as internal $\mathrm{SU}(3)_c$ colour singlets **1** and triplets **3** with the appropriate relative fractional charge magnitudes $1 : \frac{2}{3} : \frac{1}{3}$ under an internal $\mathrm{U}(1)_Q$ associated with electromagnetism.

56 \ $E_7 \supset$	Lorentz	$\times$ $\mathrm{SU}(3)_c$	$\times$ $\mathrm{U}(1)_Q$	matter
4	$\underline{\text{vector}}$	**1**	0	'ν_L'
8	Dirac	**1**	1	$\begin{pmatrix} e_L \\ e_R \end{pmatrix}$
12	$\underline{\text{scalar}}$	**3**	$\frac{2}{3}$	$\begin{pmatrix} \text{`}u_L\text{'} \\ \text{`}u_R\text{'} \end{pmatrix}$
24	Dirac	**3**	$\frac{1}{3}$	$\begin{pmatrix} d_L \\ d_R \end{pmatrix}$
4	$\underline{\text{vector}}$	**1**	0	'Higgs' ($\boldsymbol{v}_4$)
4	scalar	**1**	0	Yukawa

Table 1: The symmetry breaking of the E_7 quartic form for proper time of equation 9 and partitioning of the 56 components of $\boldsymbol{v}_{56} \in F(\mathrm{h}_3\mathbb{O})$ through the extraction of a local Lorentz symmetry acting upon the external $\boldsymbol{v}_4 \in TM_4$ subcomponents. The final column lists the matter field interpretation ([57] figure 4). While incomplete, and with discrepancies indicated by the underlined entries, in identifying this correspondence between the fragmented components of $\boldsymbol{v}_{56}$ and one generation of Standard Model states there is very little redundancy.

The four components of $\boldsymbol{v}_4 \in TM_4$ are projected from the 56 components of $\boldsymbol{v}_{56} \in F(\mathrm{h}_3\mathbb{O})$, as subsumed from $\boldsymbol{v}_4 \equiv \boldsymbol{h} \in \mathrm{h}_2\mathbb{C}$ for the original 4-dimensional case of equation 6. This necessary extraction to identify the local structure of the external spacetime M_4 is intimately associated with the symmetry breaking for the higher-dimensional forms of proper time as described for equation 5, with the components of the 4-vector $\boldsymbol{v}_4 \in TM_4$ in turn associated with a non-standard 'Higgs' as indicated in table 1 (and also discussed in [57] after figure 4; this clearly implies new physics since in the Standard Model the Higgs is an elementary scalar field). Further, Yukawa coupling factors derive from the scalar invariant components listed at the bottom of table 1 as proposed to account for the spectrum of individual particle masses, as will be discussed for equation 17 in section 6.

While a complete correspondence with Standard Model electroweak theory is not identified at this level there are structures closely analogous to elements of standard electroweak symmetry breaking $\mathrm{SU}(2)_L \times \mathrm{U}(1)_Y \to \mathrm{U}(1)_Q$ (with L for left-handed and Y hypercharge) found for the above $\hat{G} = \mathrm{E}_6$ and $\hat{G} = \mathrm{E}_7$ stages, compatible with the $\binom{\nu}{e}$-lepton and $\binom{u}{d}$-quark assignments in table 1. These connections further motivate the association of the projected $\boldsymbol{v}_4 \in TM_4$ components with the Higgs ([63] section 8.3). An intrinsic left-right asymmetry, a key feature of weak interactions, is also distinctly identified at the E_7 level ([64] section 6, as also discussed in [65] towards the end of subsection 3.1). At this 'one generation' level the neutrino components, identified through the internal symmetry properties alone, can *only* be accommodated in *either* the left *or* right-handed sector of the fragmented components of $\boldsymbol{v}_{56} \in F(\mathrm{h}_3\mathbb{O})$, unlike the case for the other lepton and quark states identified in table 1. On choosing the left-handed sector this neutrino state is hence denoted 'ν_L' in table 1 and is complementary to the projected $\boldsymbol{v}_4 \in TM_4$ components associated with the 'Higgs', with the quote marks indicating the need for further developments to uncover an appropriate external Lorentz symmetry structure for both the neutrino and the Higgs (see also discussion in [57] before figure 4).

The main structures of the Standard Model that remain to be identified are then the appropriate Lorentz spinor representations for the neutrino and u-quark states, as underlined in table 1, together with a full electroweak and Higgs sector and a full three generations of leptons and quarks. These features are all correlated through the need to fully identify a weak $\mathrm{SU}(2)_L$ symmetry component acting upon doublets of left-handed spinor states while also mixing across three generations of Standard Model particles. The complete set of properties is predicted to arise through one further augmentation beyond the E_7 stage of equation 9 and table 1 to a full $\hat{G} = \mathrm{E}_8$ symmetry of a potential 248-dimensional *octic* form for proper time, as consistent with equation 4, that can be written provisionally as ([57], [65] equation 37):

$$L_8(\boldsymbol{v}_{248})_{\mathrm{E}_8} = 1 \tag{10}$$

As for the E_6 and E_7 levels of equations 8 and 9 this realisation of E_8 is anticipated to be rich in octonion elements, with for example the 'triality' property of the octonion algebra closely linked with the construction and mutual relations of three generations of Standard Model states. An intrinsic and compact octonion-rich account of mixing effects could provide an underlying explanation for the properties of both the neutrino and quark sectors, in a deeper manner than can be achieved by a direct modelling of empirical phenomena, and in turn prove predictive. More generally, as well as potentially completing the Standard Model particle multiplet structure through the breaking pattern of an E_8 symmetry over a $\boldsymbol{v}_4 \in TM_4$ subcomponent projection,

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the possible existence of such a form for proper time in equation 10 can also be provisionally connected with prospects for new physics beyond the Standard Model and the corresponding empirical consequences. These new features include a means of accommodating two right-handed neutrinos ν_R alongside three generations of ν_L states, consistent with the observations of solar and atmospheric neutrino oscillations, and in a manner related to the possible implication of a composite Higgs structure (as described in detail in [65] subsection 4.1).

Owing to an element of flexibility in the standard construction of Lagrangian terms, within a few consistency requirements, it is possible to extract a self-contained 'one generation' description from the usual full Standard Model Lagrangian. For the present theory the aim of accounting for neutrino oscillation and quark mixing structures interlinking three generations through a unique and tightly interconnected mathematical expression utilising octonion triality and the breaking of a unifying E_8 symmetry implies that from such a complete picture it may well not be possible to cleanly extract a mathematical substructure describing the properties of 'one generation' on its own. The potential existence of such a compact and complete octonion-rich description of a full three generation structure through the proposed E_8 level of equation 10 is hence considered a main reason why the *approximately* 'one generation' picture at the intermediate level of the E_7 symmetry breaking in table 1 provides an incomplete description of the corresponding lepton and quark states. However, the concept of a *single isolated* pristine generation of leptons and quarks without any mixing is *not* something physically encountered in nature and does not necessarily provide an appropriate intermediate target on the way to a full account of the Standard Model within a unified theory.

The sequence of generalised forms for proper time with exceptional Lie group symmetries based on equations 8 and 9 then potentially terminates in the ultimate form for proper time described for equation 10, with a realisation of E_8 as uniquely the largest exceptional Lie group constructed as a symmetry action utilising the composition of octonions as uniquely the largest division algebra. This sequence of forms for proper time can be classified generically through the branch of equation 4 denoted:

$$L_{p>2}(\boldsymbol{v}_n)_{\hat{E}} = 1 \tag{11}$$

with $(p, n, \hat{E}) = (3, 27, \mathrm{E}_6), (4, 56, \mathrm{E}_7)$ and provisionally $(8, 248, \mathrm{E}_8)$ for equations 8, 9 and 10 respectively.

Mathematical properties of both the exceptional Lie groups and the octonion algebra are well known to have links with the physical structures of the Standard Model (see [66], [67] and [68] for examples involving E_6, E_7 and E_8 respectively; [69] section 1, [70] and [71] for cases with the octonions). However, here we *begin* with a well-defined *conceptual* motivation through equation 4 as the pivotal expression for generalised proper time and an account of *why* the explicit actions of these *mathematical* symmetry structures in equations 8–10 and corresponding breaking patterns should apply directly in the *physical* world. In particular here the notion of a continuous symmetry of forms for generalised proper time in equation 4 is *more fundamental* than the standard classification of symmetry groups and analysis of their representations based on the properties of complex Lie algebras (for example [72]). The employment of the octonion algebra in describing symmetry transformations, with a non-associative composition,

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hence opens up room for new results including some deviation from a standard analysis of the 248-dimensional representation of E_8 that may be sufficient to close the gap with the Standard Model observed in [68] ([57] section 5, [69] section 3).

While there is a high degree of uniqueness for the mathematical structures discussed above in relation to equation 11 there remains the possibility of an *entirely different branch* for generalised proper time fully consistent with equation 4. In particular the original generalisation from the 4-dimensional spacetime form of equation 1 to the quadratic form of equation 2 with $f(y) = 1$, and as extended further for an arbitrary number of additional components, is also a consistent expression for generalised proper time in equations 3 and 4 for a restricted $p = 2$ case. While we have reviewed above how the visible matter of the Standard Model can be accommodated through the $p > 2$ sector of equation 11 this alternative $p = 2$ sector, as might then also be associated with *extra spatial dimensions*, has *not* been utilised. This latter branch then leads directly to the possibility of also accommodating a dark matter candidate.

One motivation for reviewing in detail the means of uncovering the Standard Model through the branch of equation 11 in this section has been to illustrate how the explicit physical form for dark matter might be constructed under this approach through such an alternative extra spatial dimensions branch of generalised proper time, as we describe in the following section. The Standard Model and dark matter 'branches' of this theory, having a common 'root' in the local 4-dimensional spacetime form of equation 1, will interact through their mutual gravitational impact (as originally suggested in [65] subsection 4.2 in the discussion of equation 42 therein). This link and other potential interactions between the sectors will be described in section 6, where further connections with the models reviewed in section 2 and possible empirical consequences will also be considered.

5 Sector with Dark Matter Candidate

The development of this theory has been mainly motivated through the desire to account for the Standard Model within the structures of extra dimensions, as reviewed in the previous section. However, as well as directly connecting with the elementary states of visible matter through the $p > 2$ sector of generalised proper time of equation 11, in principle culminating in the proposed $(p = 8, n = 248, \hat{E} = \mathrm{E}_8)$ case of equation 10, a further feature of this approach is that there are other possible sectors that remain completely free. In particular, there remains a possible $p = 2$ sector consistent with the generalised form for proper time in equations 3 and 4, as noted at the end of the previous section, in principle with the potential to accommodate a dark matter candidate. Interpreted as locally appending extra spatial dimensions this $p = 2$ sector has some connection with the dark matter models reviewed in subsection 2.3. However, as we shall explain, the physical form the proposed dark matter takes here is more closely related to the models of subsection 2.1 and even of subsection 2.2.

The simplest extension from the local 4-dimensional form for proper time of equation 1, via the $p = 2$ extra spatial dimension case for equation 3, is to the

n'-dimensional spacetime form:

$$(\delta s)^2 \;=\; \hat{\eta}_{ab}\delta x^a \delta x^b \tag{12}$$

with $\hat{\eta} = \mathrm{diag}(+1, -1, \ldots, -1)$ here the extended local $n' \times n'$ Lorentz metric, now with indices $a, b = 0, \ldots, n'-1$. The interval δs is invariant under the augmented Lorentz transformations of the group $\mathrm{SO}^+(1, n'-1)$ acting upon the components $\{\delta x^a\} \in \mathbb{R}^{n'}$.

Equation 12 can be rearranged in the form of equation 4 as:

$$L_2(\boldsymbol{v}_{n'})_{\hat{L}} \;=\; \hat{\eta}_{ab}v^a v^b \;=\; 1 \tag{13}$$

for this quadratic $p = 2$, n'-dimensional, spacetime form with a full Lorentz group symmetry $\hat{G} \equiv \hat{L} = \mathrm{SO}^+(1, n'-1)$. (This expression is distinguished from the $p > 2$, n-dimensional, extension via equations 6 and 7 to the $\hat{G} \equiv \hat{E}$ exceptional Lie group symmetry branch of equation 11). The full $\hat{L}$ symmetry of equation 13 is again broken on extracting the $\boldsymbol{v}_4 = (v^0, v^1, v^2, v^3) \in TM_4$ subcomponents in identifying the local external 4-dimensional spacetime as described for equation 5. In this case, with an external Lorentz $\mathrm{SO}^+(1,3)$ symmetry acting on $\boldsymbol{v}_4 \in TM_4$, a residual internal symmetry $G = \mathrm{SO}(n'-4)$ is obtained. The corresponding symmetry breaking pattern for $\hat{L} = \mathrm{SO}^+(1, n'-1)$ in equation 13 is summarised in table 2 (as may be contrasted with table 1 as determined specifically for equation 9 for the $\hat{E} = \mathrm{E}_7$ level of equation 11).

n' \ $\mathrm{SO}^+(1, n'-1) \supset$	Lorentz	$\times$ SO(m)	matter
m $(= n'-4)$	scalar	m-vector	'dark quarks' ($\boldsymbol{v}_m$)
4	4-vector	invariant	'Higgs' ($\boldsymbol{v}_4$)

Table 2: The partitioning of the $\boldsymbol{v}_{n'} \in \mathbb{R}^{n'}$ components of the n'-dimensional spacetime form for proper time of equation 13 under the symmetry breaking projection of the subcomponents $\boldsymbol{v}_4 \in \mathbb{R}^4 \equiv TM_4$ onto the local external 4-dimensional spacetime M_4. The residual matter field $\boldsymbol{v}_m(x) \in \mathbb{R}^m$, with $m = n'-4$, is a scalar under the external Lorentz symmetry $\mathrm{SO}^+(1,3)$ transforming as an m-vector under the residual internal SO(m) gauge symmetry (see also [65] equations 6–9 and figure 1(b)).

The manner in which structures resembling the Standard Model have been directly identified through to the $\hat{E} = \mathrm{E}_7$ level for equation 11, as residing in the $\boldsymbol{v}_{56}(x) \in F(\mathrm{h}_3\mathbb{O})$ components of equation 9 and reviewed for table 1, provides a guide for the determination of the empirical properties of any dark matter candidate deriving from the fragmentation of the $\boldsymbol{v}_{n'} \in \mathbb{R}^{n'}$ components in table 2. The common components $\boldsymbol{v}_4(x) \in TM_4$ are already associated with the Higgs as discussed for table 1 while here the residual components $\boldsymbol{v}_m(x) \in \mathbb{R}^m$ of table 2 provide the potential basis for dark matter. The elementary particle states associated with the multiplet $\boldsymbol{v}_m \in \mathbb{R}^m$ are scalars under the external $\mathrm{SO}^+(1,3)$ Lorentz symmetry that transform under the fundamental representation of the internal non-Abelian gauge group SO(m), acting as simple 'rotations' in this space. With n', and $m = n'-4$, arbitrarily large these elementary states are then likely to be highly mutually interacting via the SO(m) gauge force.

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While in the visible sector $SU(3)_c$ acts upon states of coloured quarks that are confined in hadrons, by analogy here the states associated with the components of $\boldsymbol{v}_m(x)$, termed 'dark quarks' in table 2, under $SO(m)$ gauge interactions would be expected to be bound into distinct physical particle states that can be termed 'dark hadrons'. Here we are using the name 'quark' in 'dark quark' as a generic term for a fundamental bound constituent, by analogy with QCD, although in this case these are not fermions but rather the scalar states associated with $\boldsymbol{v}_m(x)$ in table 2. Similarly we are employing the term 'hadron' in 'dark hadron' in a generic sense bearing in mind the translation of the original Greek *hadros* as 'bulky'. The analogy between $SO(m)$ gauge theory and QCD was noted in subsection 2.1 with reference to [14, 15]. This structure with $SO(m)$ acting upon elementary scalar states, while considered an unphysical model in [14], is here physically realised through table 2 as a natural basis for a dark matter candidate.

In the Standard Model the up and down quarks are very light compared with pions, composed of two quarks, and nucleons, composed of three quarks, with such hadronic states gaining a large part of their mass from the strength of the binding $SU(3)_c$ interaction. By comparison, assuming a similar elementary coupling strength for the dark gauge sector, even if the dark quark states are very light or massless (in the sense of lacking any equivalent of a Lagrangian mass term) the physical dark hadrons, composed of a number of elementary dark quarks bound together by the 'dark gluons' of $SO(m)$ gauge interactions for large m, could be very massive. Indeed such dark hadrons might be *far more* 'bulky' than their visible hadronic and nuclear counterparts.

While the masses of ordinary stable heavy nuclei, for example from iron to lead, range from around 50–200 GeV, the massive dark hadrons, in principle composed of a large number of dark quarks, with no weak force decays and with no destabilising Coulomb repulsion, could be far heavier than the nuclear states of visible matter. In lacking any electromagnetic coupling, and subject to no gauge interactions other than that of the internal $SO(m)$ symmetry, the dark hadrons would also indeed be invisible. The massive dark hadrons would then be analogous to the 'dark quark nugget' models reviewed in subsection 2.1 with reference to [18, 19, 20, 21, 22]. The stability of massive dark hadrons might be compared with that of ordinary heavy nuclei and modelled for example in an analogous manner to a 'liquid drop' model (such as employed by some of the models reviewed in subsection 2.1, although presumably with different volume and surface terms as modified for scalar constituents).

As alluded to in the opening of subsection 2.1 the level of mass density of visible matter is related to the degree of interaction of standard QCD. Hence the greater propensity for interaction, given a large number of dark quark states and $SO(m)$ dark gluon states, could be related to the higher overall mass density of dark matter by the observed factor of five. With the elementary states of the sector of equation 13 and table 2 being initially created in the Big Bang in parallel with the Standard Model states of the sector of equation 11 and table 1, the higher degree of self-interaction and corresponding energy density of dark QCD could also have led to the forging of heavy dark hadrons in the immediate aftermath of the Big Bang.

That is, around the epoch of the standard QCD phase transition from a quark-gluon plasma to standard hadronic states, the massive dark hadrons may also have initially formed. In the visible QCD sector there remains a key question over the nature of baryogenesis and the origin of the asymmetry between matter and antimatter as required to account for the net excess of matter states. This may be a further significant difference with the proposed dark sector here, with the dark quarks being charge-neutral scalar states transforming under a real representation of SO(m) with no distinction between matter and antimatter particles.

Even if the dark quarks do not interact with Standard Model states other than through gravity the dark hadrons could transmute through collisions with each other or through the breakup of unstable massive states, and have their own thermal history originating in the Big Bang. In particular in the early universe this further evolution of dark states might progress in parallel with the period of standard primordial nucleosynthesis, that is alongside the production of the light elements of ordinary matter through to ^{7}Li. Heavy unstable dark hadrons or dark glueballs formed in that early era might eventually decay to lighter stable states. If there are corresponding lifetimes on the order of cosmic timescales (by analogy with some ordinary heavy radioactive isotopes forged much later in stellar and supernova processes) that would generate a further time development history for such a dark matter component of the universe and its thermal properties.

As noted towards the end of subsection 2.1 for the case of fermionic constituents the Fermi degeneracy pressure supports a massive dark quark nugget against gravitational collapse. However, here such fermion states have been replaced by the scalar dark quarks of table 2. Hence as well as lacking a Coulomb repulsion that would destabilise the formation of very massive dark hadrons here the lack of any Fermi pressure presumably also makes gravitational collapse into a black hole far more likely, and at a potentially much lower mass scale. The formation of primordial black holes from such a dark hadron source may then also not require the underlying large density fluctuations from an initial inflationary epoch, unlike the case for some PBH models reviewed in subsection 2.2 as formed with Standard Model matter states only. That is, the massive dark hadron states deriving from the dark quark constituents of table 2 may be inherently more prone to dense clumping and collapse to PBHs.

The PBHs forming through inflationary perturbations and subsequent Standard Model physics can have an enormous mass range of 10^{-8} kg–10^{35} kg ($10^5\,M_\odot$) depending on a formation time ranging from the Planck time to one second after the instant of the Big Bang, with for example a mass of order $1\,M_\odot$ associated with the QCD epoch at around 10^{-5} seconds as a potential PBH dark matter candidate [28, 36]. However, as noted in the opening of subsection 2.2, there are a number of different known mechanisms for the formation of PBHs, including via new superheavy metastable particles that might dominate the early universe ([26] section 2.1). In the present approach, while aided by some level of initial density fluctuations, PBH formation might be primarily seeded through the new physics of the growth of massive dark hadrons composed of many scalar dark quarks in the very early universe. The potential timing and mass range for this PBH production is an open question. Importantly, here the new confining gauge force and scalar constituent part of this mechanism has a natural underlying physical source, through the extra spatial dimension branch of generalised proper time of equation 13, providing a well-motivated underlying conceptual and theoretical basis for this dark matter candidate.

As noted at the end of subsection 2.1, assuming a degree of stability, from a gravitational point of view the general impact of massive dark hadrons would be very similar to that of PBHs of the same mass and number, although the former might also exhibit weak non-gravitational interactions with ordinary matter as we consider in the following section. The potential for light PBHs to evaporate and for stable PBHs to merge as the universe evolves implies that this form of dark matter candidate can also have cosmic time-dependent phenomenological properties to some degree.

While sketching the possibilities above for the consequences of table 2, as deriving from the scalar dark quark states associated with the components of the field $\boldsymbol{v}_m(x)$ and initially bound by SO(m) gauge interactions, the scope of this paper is not however aimed at calculating the detailed physics of the resulting massive dark hadrons or primordial black holes. Indeed from the SU$(3)_c$ gauge symmetry and triplet states in table 1 (or even at a full Standard Model multiplet level deriving from an E_8 symmetry breaking as predicted for the equation 11 branch of generalised proper time) significant further advances would be needed (including an account of quantum mechanical behaviour) to calculate the empirical properties of the physical hadron and nuclear states of visible matter generated.

Rather here the purpose has been to show how a dark matter candidate *can naturally* arise in this theory based on generalised proper time through equation 4. Employing the sector of generalised proper time of equation 13, with a direct interpretation as a source for extra spatial dimensions, comparisons with the framework of the dark matter models reviewed in subsection 2.3 can be made. However, through the resulting confining gauge interaction with the internal non-Abelian symmetry SO(m) in table 2 and the potential for high density collections of scalar dark quark constituents to gravitationally collapse, the practical physical form this dark matter candidate takes more closely resembles the models reviewed in subsections 2.1 and 2.2. Elements of all three classes of models considered in section 2, and others besides, might then be utilised in developing a full understanding of the present theory of dark matter.

As alluded to above the identification of this dark matter candidate is made possible through the availability of more than one sector of extra dimensions that is opened up this way through forms of generalised proper time. Similarly as the branch of equation 11 and table 1 directly resembles features of the Standard Model, as reviewed in the previous section, in this section we have described how physical structures deriving from the branch of equation 13 and table 2 exhibit appropriate features desired for a particle or macroscopic dark matter candidate. In the following section we continue this theme and describe in more detail how this proposed dark matter exhibits suitable features in terms of the nature and degree of possible interactions with visible Standard Model matter.

6 Interaction Between the Two Sectors

Any sector of generalised proper time of physical interest will involve an augmentation from the 4-dimensional spacetime form of equation 1, which is necessarily extracted to identify the local geometry of the external spacetime M_4 itself. This is the case for the generalisation to equation 11, and the resulting Standard Model branch, as well as for the generalisation to equation 13, as the proposed dark matter branch of extra dimensions associated with generalised proper time. In extracting the local 4-dimensional Lorentz metric structure of the external spacetime of equation 1, such as expressed in equation 6, from the full form of proper time in equation 4, such as via the explicit embedding in equation 7, the magnitude $h(x) = |\boldsymbol{v}_4(x)|$ of the projected components $\boldsymbol{v}_4(x) = (v^0, v^1, v^2, v^3) \in TM_4$ is now free to vary. In place of the normalised value of unity in equation 6 in general we then have:

$$L\!\!\!/_2(\boldsymbol{v}_4)_{\text{Lorentz fragment}} = \eta_{ab} v^a v^b = |\boldsymbol{v}_4|^2 = h^2 \neq 1 \tag{14}$$

for the projected set of four components $\{v^a = \delta x^a/\delta s; a = 0, 1, 2, 3\}$ of the external 4-vector $\boldsymbol{v}_4(x) \in TM_4$. The notation $L\!\!\!/$ in equation 14 indicates that this expression represents the external Lorentz *fragment* of a higher-dimensional form for proper time in equation 4 that is broken upon extracting this 4-dimensional substructure, with the complete expression for this broken form $L\!\!\!/_p(\boldsymbol{v}_n) = 1$ for $n > 4$ presented in equation 17 below.

In being projected out of the full form for proper time variations of $h(x)$ in equation 14 for the external spacetime fragment imply local time dilation effects in the extended 4-dimensional spacetime manifold M_4. These in turn are directly associated with distortions from an otherwise flat Minkowski spacetime with a geometric warping described by an extended metric $g_{\mu\nu}(x)$ of the conformal form (as explained for [63] figure 13.1 and equation 13.2; with $\eta_{\mu\nu}$ a global Lorentz metric):

$$g_{\mu\nu}(x) = \frac{1}{h^2(x)} \eta_{\mu\nu} \tag{15}$$

In turn the Einstein tensor $G^{\mu\nu}(x)$, as a standard function of the metric tensor $g_{\mu\nu}(x)$ as constructed in general relativity (see [63] sections 3.3 and 3.4 for the conventions of Riemannian geometry employed), takes the explicit form ([65] equation 24; with $\rho, \mu, \nu = 0, 1, 2, 3$ spacetime indices):

$$G^{\mu\nu} \;=\; -3h^{-2} \partial_\rho h \, \partial^\rho h \, g^{\mu\nu} - 2h^{-1} \partial^\mu \partial^\nu h + 2h^{-1} \Box h \, g^{\mu\nu} \;=:\; -\kappa T^{\mu\nu} \tag{16}$$

The energy-momentum tensor $T^{\mu\nu}(x)$ is here *defined* via Einstein's field equation $G^{\mu\nu} = -\kappa T^{\mu\nu}$ with κ a normalisation constant ([63] equation 3.75). For the present theory the chain of relations in equations 14–16 then describes a direct underlying geometric *source* of the physical property of mass.

In the context of the symmetry breaking structures arising from the $\hat{G} = \mathrm{E}_7$ form for proper time of equation 9 the four components $\boldsymbol{v}_4(x) \in TM_4$ (subsumed from $\boldsymbol{v}_4 \equiv \boldsymbol{h} \in \mathrm{h}_2\mathbb{C}$ in equations 6 and 7), as central to the symmetry breaking and now taking a 'vacuum value' for the corresponding scalar magnitude $h = |\boldsymbol{v}_4| = \sqrt{\det(\boldsymbol{h})}$, are associated with a non-standard Higgs in this theory, as reviewed in connection with electroweak symmetry breaking after table 1 in section 4. The corresponding geometric deformation of the spacetime geometry via equation 14 in equations 15 and 16 then connects the mutual relation between spacetime curvature and energy-momentum as conceived in the framework of general relativity with the 'origin of mass' and the Higgs mechanism of the Standard Model (as also noted for [65] equations 22–24).

The standard scalar Higgs in the *flat* spacetime background of special relativity introduces a *dynamical mass* for particle states in equations of motion via mass terms in the Standard Model Lagrangian. This contrasts with the concept of *gravitational mass* in general relativity which is intrinsically and irreducibly associated with *deviations from flatness* in 4-dimensional spacetime. In the context of the present theory the projected components $\boldsymbol{v}_4(x) \in TM_4$ (equivalent to $\boldsymbol{h}(x) \in \mathrm{h}_2\mathbb{C}$ through to the E_7 level for the $p > 2$ branch of generalised proper time of equation 11) hence play a pivotal role in relating the Standard Model of particle physics and the general relativistic theory of gravitation, through the corresponding conceptions of mass in these two frameworks.

The residual components of $\boldsymbol{v}_n(x)$, which already resemble leptons and quarks in the symmetry breaking for this branch at the $(p = 4, n = 56, \hat{E} = \mathrm{E}_7)$ level of equation 9 as summarised for table 1, gain mass by interacting with these 'Higgs' $\boldsymbol{h}(x) \equiv \boldsymbol{v}_4(x) \in TM_4$ components. This interaction takes place through the complete expression for equation 14, that is via the fragmented terms of equation 4 as partitioned into a sum of parts individually invariant under the broken symmetry of equation 5 (with the Lorentz symmetry $\mathrm{SL}(2, \mathbb{C})$ employed for the $p > 2$ branch considered here; [56] equation 43):

$$L_p(\boldsymbol{v}_n)_{\mathrm{Lorentz}\times G} = \sum(\text{invariant parts}) = 1 \tag{17}$$

As alluded to in the discussion after table 1 some of the terms in this expression can be interpreted as containing factors in the lepton or quark components from the main part of table 1 as well as factors in the 'Higgs' field $\boldsymbol{h}(x) \equiv \boldsymbol{v}_4(x)$ components and a Yukawa coupling, the latter associated with the 'vacuum values' of the scalar invariants listed in the bottom part of table 1. With the structure of such terms closely analogous to the construction of the Standard Model Lagrangian mass terms they are hence expected to describe the mass of the lepton and quark states, with a more complete connection anticipated for the predicted full E_8 level of equation 10 (see discussion of [56] equations 37 and 43, and equations 36–45 therein for the connection with Lagrangian terms more generally). Here terms of equation 17 physically *generate* mass for elementary particle states through an impingement on the external spacetime geometry as described above for equations 14–16 as consistent with the general theory of relativity.

In the case of the quadratic $p = 2$ branch of generalised proper time in the n'-dimensional spacetime form of equation 13 the broken form corresponding to equation 17 can be readily written out explicitly, incorporating the external 4-dimensional Lorentz fragment of equation 14, as simply:

$$L_2(\boldsymbol{v}_{n'})_{\mathrm{Lorentz}\times\mathrm{SO}(m)} = |\boldsymbol{v}_4|^2 + |\boldsymbol{v}_m|^2 = 1 \tag{18}$$

where here the external Lorentz symmetry is $\mathrm{SO}^+(1,3)$ rather than the double cover. The residual matter field component $\boldsymbol{v}_m(x)$, with $m = n' - 4$, is a Lorentz scalar that transforms under the fundamental representation of the internal $G = \mathrm{SO}(m)$ gauge symmetry, as described for table 2.

Here again the external fragment $\boldsymbol{v}_4 \in TM_4$ of equation 14 can have variable magnitude $h(x) = |\boldsymbol{v}_4(x)|$ as now projected out of the form for proper time of equation 13 and balanced by the magnitude $|\boldsymbol{v}_m(x)|$ under the normalised expression of equation 18 for this case of breaking a full $\hat{L} = \mathrm{SO}^+(1, n'-1)$ symmetry. Hence a dark matter candidate associated with the extra components $\boldsymbol{v}_m(x)$ can again impinge upon $h(x) = |\boldsymbol{v}_4(x)|$ generating a gravitational interaction through the time dilation and corresponding geometric warping effects in the external 4-dimensional spacetime manifold M_4 as described for equations 14–16 above, now through the mutual constraint of equation 18. While Standard Model and dark matter states may hence reside in different sectors of extra dimensions, as associated with the different generalisations of proper time of equations 11 (table 1 for the E_7 level) and 13 (table 2) respectively, they are able to interact through the necessary intersection pictured in figure 1 and their shared impact on the external spacetime geometry via this 'gravitational portal' of equations 14–16.

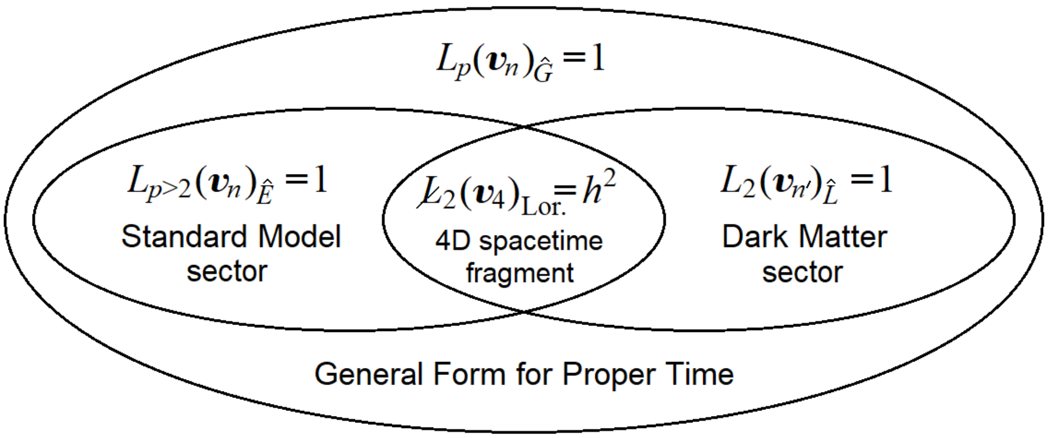

Figure 1: The relations between the generalised form for proper time of equation 4 and the particular sectors of equations 11 and 13, with exceptional Lie group $\hat{E}$ and higher-dimensional Lorentz group $\hat{L}$ symmetries respectively, together with their overlap in the local 4-dimensional spacetime geometry through the fragment of equation 14.

The common extended external 4-dimensional spacetime geometry is also directly warped by the internal gauge fields in a manner analogous to the construction of non-Abelian Kaluza-Klein theories. The Einstein tensor $G^{\mu\nu}(x)$ is related to the gauge field strength ${F_\alpha}^{\mu\nu}(x)$ as ([73] equation 93, [56] equation 40):

$$G^{\mu\nu} \;=\; 2\chi\left(-F^{\alpha\mu}{}_{\rho}{F_\alpha}^{\rho\nu} - \frac{1}{4}g^{\mu\nu}\,F^{\alpha}{}_{\rho\sigma}{F_\alpha}^{\rho\sigma}\right) \;=:\; -\kappa T^{\mu\nu} \tag{19}$$

where $\{\mu, \nu, \rho, \sigma\}$ are spacetime indices, α is a Lie algebra index and χ is a further normalisation constant. This implies a further, and potentially dominant, source of gravitational interaction between visible baryonic and dark hadron states of matter.

We note here the contrast between the basic geometric construction underlying equation 19 and that typically employed in non-Abelian Kaluza-Klein theories (for example [74]). In the latter case 4-dimensional spacetime is initially augmented to an extended $(n > 4)$-dimensional spacetime manifold, with the components of the augmented metric accommodating the degrees of freedom of a non-Abelian gauge field ([73] equation 61, [74] equation 12) and with no further 'matter fields' identified. In

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the present theory the internal non-Abelian symmetry G is identified directly with the residual symmetry from the breaking of $\hat{G}$ in equation 4 over the local 4-dimensional spacetime Lorentz subgroup symmetry (consistent with the Coleman-Mandula theorem as noted for equation 5) and acts on the residual 'matter field' components of $\boldsymbol{v}_n(x)$ from the generalised local form for proper time. For the sector of equation 13 the residual $G = \mathrm{SO}(m)$ symmetry acts directly on the extra spatial dimension components $\boldsymbol{v}_m(x)$ themselves identified as the matter field described for table 2.

Similarly as multiplets of the Standard Model are *directly* accounted for in the symmetry breaking structures of equation 11 as analysed in detail through to the level of table 1, the dark matter candidate described for table 2 derives *directly* from the local components of extra spatial dimensions that originate from the second possible sector for generalised proper time of equation 13. This dark quark and corresponding dark hadron construction, as a potential source of primordial black holes, described in the previous section is very different to the models in which a dark matter candidate is *introduced* either as a lightest Kaluza-Klein particle or a new field confined to a brane as reviewed in subsection 2.3. However, with extra spatial dimensions involved in all cases there are possible connections as we consider next.

In particular if the interaction between the dark and visible sectors is through gravity alone as described above for figure 1 and equation 19 the dark hadrons identified here could be considered a form of 'feebly interacting massive particle' or FIMP (by analogy with [43, 49]) and might be compared with other models for gravity-mediated dark matter in extra dimensions. The time dilation and 4-dimensional spacetime geometric warping induced by variations of $|\boldsymbol{v}_m(x)|$ for the dark matter components in equation 18 through equations 14–16 is also analogous to dark matter models with a 'dilaton field' associated with the 'warp factor' of the 4-dimensional brane embedding in the bulk space of a framework with warped extra dimensions [49], alluded to following equation 2, or a 'radion field' associated with variations in the parameters of the compactified geometry of extra dimensions [45, 49].

In the present theory gravitational mediation can take place through the common external spacetime components $\boldsymbol{v}_4 \in TM_4$ of equation 14 as listed in tables 1 and 2 which, as described after table 1 and for equations 16 and 17, are also associated with the Standard Model Higgs. Hence this construction might also involve a non-gravitational mediation with particle physics interactions through these Higgs components that might be compared to models of a 'Higgs portal' connection with dark matter (see for example [75, 76]). Such a non-gravitational interaction, again mediated through the common central overlapping $\boldsymbol{v}_4 \in TM_4$ components in figure 1, might then be expected to be at the electroweak scale similarly as characteristic of WIMP models (as is the case in [76]). If there are dark hadrons in the appropriate mass range then, similarly as for the models in [75, 76], in principle these weak interactions with ordinary visible matter might be detectable in the laboratory through impacts on atomic nuclei or as a missing energy signal in a particle physics collider experiment, as consistent with existing limits ([3] sections 27.6 and 27.7, [4] section 3 and [5] as cited in section 1; see also [77]).

We also note that the $(n' = 10)$-dimensional quadratic spacetime case of equation 13 for the dark sector of generalised proper time is effectively already subsumed in the visible sector of equation 11 through the quadratic determinant structure of a $h_2\mathbb{O}$ subspace embedded within $h_3\mathbb{O}$ and $F(h_3\mathbb{O})$ at the E_6 and E_7 levels of equations 8 and 9 respectively ([64] table 5, [57] equations 25, 26, 72 and 73). Similarly as the overlapping components $\boldsymbol{v}_4 = (v^0, v^1, v^2, v^3) \in TM_4$ of equation 14 imply a gravitational interaction and a potential Higgs interaction, the six further overlapping components denoted $\boldsymbol{v}_6 = (v^5, \ldots, v^{10}) \in \mathbb{R}^6$ might then in principle generate a further non-gravitational interaction between the visible and dark sectors, through this augmentation of the intersection region in figure 1.

In subsection 2.1 (with reference to [15]) we alluded to the close relationship between the gauge group SO(6) and its double cover SU(4), which share the same Lie algebra structure. In the present theory the internal $G = \mathrm{SU}(3)_c \times \mathrm{U}(1)_Q$ symmetry in table 1, identified at the E_6 and E_7 levels for equation 11, forms a subgroup of an $\mathrm{SU}(4) \subset E_6 \subset E_7$ that is equivalent to an SO(6) action on the six real components underlying the set of scalar 'u_L'-quark states (and similarly for the scalar 'u_R'-quarks) in table 1 (see discussion in [57] from equation 62 to the end of subsection 4.2). In particular the provisional 'u_R' states, as embedded in the components of $F(h_3\mathbb{O})$ at the E_7 level (as pictured in [64] equation 66), consist of six real components that can be identified directly with the above $\boldsymbol{v}_6 = (v^5, \ldots, v^{10}) \in \mathbb{R}^6$ components and upon which this SO(6) acts as simple rotations.

Such an SO(6) can also be interpreted as part of the internal symmetry identified in the symmetry breaking of the equation 13 sector, that is as a subgroup of $\mathrm{SO}(m)$ for $m > 6$ acting upon a 6-dimensional subset $\boldsymbol{v}_6$ of the $\boldsymbol{v}_m \in \mathbb{R}^m$ dark quark components of table 2. A form of interaction can in principle take place between 'visible' and 'dark' quarks through these overlapping components that possess both Standard Model and dark gauge symmetries. Such an interaction could be possible through gauge field components common to both the visible and dark sector in terms of equation 19. This would be analogous to the 'kinetic mixing' generated by fields that have both visible and dark gauge charges, through the corresponding Lagrangian gauge field terms, that results in a very weak interaction between the Standard Model and dark matter in some existing models ([40] equation 2).

However, the full E_8 level of equation 10 for the branch of equation 11, as predicted to complete the Standard Model as reviewed towards the end of section 4, will be needed to properly assess this possibility. The sequence of neatly aligned direct embeddings of equations 6, 7, 8 and 9, with $h_2\mathbb{C} \subset h_3\mathbb{C} \subset h_3\mathbb{O} \subset F(h_3\mathbb{O})$, for this branch of generalised proper time is anticipated to be completed by a final E_8 'Russian doll' level that is somewhat 'skewed' with respect to the others ([57] section 5). Under this further embedding, with a full Standard Model internal symmetry action on full spinor u-quark states identified at the level of breaking the full E_8 symmetry of equation 10, a $h_2\mathbb{O}$ subspace and role for an intermediate $\mathrm{SU}(4) \equiv \mathrm{SO}(6)$ subgroup action may not be so distinctly identified (see also [57] subsection 4.2 final paragraph) and the potential 'kinetic mixing' with the dark sector diminished or lost.

To avoid any internal space overlap between the visible and dark sectors it is also possible that the extra spatial dimensions for the dark sector may in fact begin for $n' > 10$ rather than $n' > 4$ as assumed in table 2 and for equation 18, and hence instead associated with $\boldsymbol{v}_m(x)$ components with $m = n' - 10$. Since n' (and m) is still arbitrarily large this may have little or no effect on the physics of dark quarks and dark hadrons described in the previous section. For the case in which massive

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dark hadrons collapse into primordial black holes, as also considered in section 5, only gravitational interactions would remain and the above potential for any (very) weak non-gravitational coupling with the Standard Model would become redundant, apart from for the surviving lower mass dark hadrons. Before concluding in section 8 with further comments on the suitability of this proposal for a dark matter sector, in the following section we speculate on the possibility of also accommodating a source for dark energy within the framework of generalised proper time.

7 Potential Phantom Dark Energy Sector

A thorough theoretical understanding of the origin and nature of dark matter would aid the analysis of cosmological observations more generally, based on fitting the parameters of cosmological models to the data, and help clarify our understanding of the dark sector as a whole. Indeed the cosmological observations alluded to in the opening of section 1 demonstrate the need for a dark sector beyond a dark matter component alone. In particular the cold dark matter scenario can be augmented to the ΛCDM cosmological model with a cosmological constant Λ parametrising the apparent accelerating expansion of the universe. The source of 'dark energy' driving such an acceleration is if anything an even greater mystery in physics than the nature of dark matter ([1] section 3, [3] section 28).

The 'cosmological constant problem' involves the enormous discrepancy between the apparent empirical value of Λ and the far larger computed value of the vacuum energy using standard quantum theory. More generally, to understand the nature of dark energy it may first be necessary to understand how gravitation and quantum phenomena can be coherently combined in a unified theory of 'quantum gravity' (see for example [56] subsections 2.3 and 7.1). While the dark matter branch of equation 13 seems too simple to also account for dark energy, in the context of the present theory the origin of the cosmological constant Λ, or of the cosmic acceleration in an alternative model, might also relate to the vacuum value for a scalar field component identified in the symmetry breaking of the branch of equation 11 (as an additional empirical consequence over the origin of Yukawa couplings described for the E_7 level of table 1 and after equation 17). The impact of such a scalar field might be to generate the appropriate large-scale geometric effect of an overall accelerating expansion through the projection of the local 4-dimensional spacetime fragment of equation 14 ([63] chapter 13, in particular discussion on pages 384–386 and 417–418 therein). Such a possibility, with very different scalar field vacuum values possible in the very early universe, as permitted by dilation symmetries, might also connect with an inflationary scenario in the Big Bang ([63] section 13.2, [57] discussion following equation 90).

While the Standard Model resides in the branch of equation 11 and dark matter in the branch of equation 13 the elementary field or mechanism driving such accelerating expansions might again derive from another branch of equation 4 for generalised proper time altogether. Indeed there remains the theoretical consideration of further mathematically permitted branches of generalised proper time, that is forms for equation 4 other than the branch of equation 11 or 13, that might generate physical structures including further contributions to dark matter and the dark sector gener-

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ally (see also discussion of [65] equation 42). Further progress may then be needed, to establish a complete account of quantum gravity (based on [56]) as well as of the Standard Model from the symmetry breaking structure of a full E_8 realisation for generalised proper time in equation 10 (based on [57]) and to address the mathematical possibility and implications of further branches for equation 4, to fully assess the origin of dark energy in the context of the present theory. Here we consider in particular the properties of a further possible branch of equation 4.

The branch of equation 11 resulted from the extension of the 4-dimensional spacetime form of equation 1, rewritten as described for equation 4 and as expressed in equation 6, via the determinant of 3×3 Hermitian complex matrices in equation 7. However, from that stage, in addition to the possible extension with octonion elements to equations 8 and 9, an alternative chain of extensions from equation 7, consistent with the general homogeneous polynomial form for proper time of equations 3 and 4, can be identified simply as:

$$L_{p>2}(\boldsymbol{v}_{p^2})_{\mathrm{SL}(p,\mathbb{C})} = \det(\boldsymbol{v}_{p^2}) = 1 \qquad \text{with} \qquad \boldsymbol{v}_{p^2} \in \mathrm{h}_p\mathbb{C} \tag{20}$$

The action of a full $\hat{G} = \mathrm{SL}(p, \mathbb{C})$ symmetry on the $p \times p$ Hermitian complex matrices $\boldsymbol{v}_{p^2} \in \mathrm{h}_p\mathbb{C}$, with p^2 independent real components, then leaves invariant intervals of proper time as expressed in this p^{th}-order determinant form, as a direct matrix generalisation from the 4-dimensional form of equation 6.

For $p \geq 4$ the symmetry breaking extraction of the original local external 4-dimensional spacetime form, as associated with an $\mathrm{h}_2\mathbb{C}$ subspace, results in a group product structure, as described for equation 5, of the form:

$$\mathrm{SL}(2, \mathbb{C}) \; \times \; \mathrm{SL}(p-2, \mathbb{C}) \times \mathrm{U}(1) \; \subset \; \mathrm{SL}(p, \mathbb{C}) \tag{21}$$

Here, alongside the external Lorentz $\mathrm{SL}(2, \mathbb{C})$ symmetry, the internal symmetry G includes a factor of $\mathrm{U}(1)$ and a *non-compact* gauge group $\mathrm{SL}(p-2, \mathbb{C})$ acting on the residual components of $\boldsymbol{v}_{p^2} \in \mathrm{h}_p\mathbb{C}$.

A non-compact internal symmetry, such as $\mathrm{SL}(p-2, \mathbb{C})$ above for $p \geq 4$, implies an indefinite sign for the kinetic energy in the corresponding gauge field strength Lagrangian term (see also discussion of [78] equation 1.1):

$$\mathcal{L} \sim F^{\alpha}{}_{\mu\nu} F_{\alpha}{}^{\mu\nu} \tag{22}$$

Such terms are closely related to the structure of equation 19 ([73] equations 91–93). This inevitable consequence of negative kinetic energy, associated with such non-compact gauge groups, is also described for ([57] equations 86–88, [56] equation 93) where such branches for generalised proper time were hence considered non-physical on the grounds of consistency.

However, a source of negative kinetic energy is *precisely* what is required for models of 'phantom dark energy' [79, 80]. In such dark energy models the equation of state relating the negative pressure p and positive energy density ρ is of the form $p = w\rho$ with the parameter $w < -1$ [81]. In this case the energy density actually increases as the universe expands resulting in an accelerating expansion of the universe that increases at a faster rate than the case for a cosmological constant, for which $w = -1$.

Phantom dark energy models typically introduce a non-standard kinetic energy term through a new scalar field, as motivated by the simplicity of such a field. The negative kinetic energy in such toy models results in the appropriate dynamical properties with $w < -1$ (see also [82]).

In the present theory a direct source of negative kinetic energy is identified in the form of equation 22 for the non-compact gauge symmetry from the mathematically permitted sector of generalised proper time described above for equations 20 and 21. To understand the full picture in this sector the properties of matter fields deriving from the partitioning of the set of $\boldsymbol{v}_{p^2} \in \mathbb{R}^{(p^2)}$ components in equation 20, as transforming under the broken symmetry of equation 21, should also be taken into account. These matter fields consist of a set of $(p-2)$ 2-component complex spinors, charged under the internal U(1), and a set of $(p-2)^2$ neutral real scalars, in each case with corresponding transformations under the non-compact internal $\mathrm{SL}(p-2, \mathbb{C})$. This full structure, while more elaborate than the above toy models, incorporates a negative kinetic energy contribution from this latter gauge symmetry. Such a phantom dark energy sector would gravitationally interact with the Standard Model and dark matter sectors similarly as described for equations 14–19 in the previous section.

This sector of equation 20 is then also to be included in figure 1, as a third overlapping expression for generalised proper time with a common root in the local 4-dimensional spacetime. A highly unifying picture emerges with the three primary sectors of the 'cosmological pie chart' (consisting of around 5% ordinary matter, 26% dark matter and 69% dark energy [3]) deriving in parallel from three possible branches of forms for infinitesimal proper time intervals consistent with equation 4. The exceptional Lie group branch of equation 11 leads to structures of the Standard Model and beyond, as underlying ordinary visible matter as described in section 4. The 'extra spatial dimension' branch of equation 13, with an SO(m) internal symmetry, together with any further branches with hidden compact gauge groups, results in a natural dark matter candidate as described in sections 5 and 6. Finally, as proposed in this section, the branch of equation 20, together with further cases involving non-compact internal symmetries, can be associated with models of phantom dark energy. There is also the potential for some degree of non-gravitational interaction between all three sectors, extending the analysis discussed towards the end of section 6.

The dramatic increase in the expansion rate for phantom dark energy models implies a finite lifetime for the universe culminating in a 'Big Rip' [83]. Within this framework the dark energy density may be within an order of magnitude of the combined dark and visible matter density for a significant fraction of the lifetime of the universe, in principle accounting for the 'coincidence problem' regarding the present day observed values (see also [82] section 4 and references therein). While we described in section 5 how the present theory might explain the empirically observed dark matter to visible matter average density ratio, with both sectors dominated by a confining gauge interaction, there is hence also an argument in the phantom dark energy sector that may account for the present observed rate of cosmic acceleration. Hence this unified theory, based on forms for generalised proper time, may lead to not only the three main sectors of the large scale cosmological composition but also the appropriate relative proportions between them. However, more work is needed to explore this proposed connection between the present theory and the dark energy sector.

In the meantime, more generally, the nature and extent of the interconnection between dark matter and dark energy is still open to developments in theoretical understanding and empirical observations ([1], [84] for example figure 9). The 'Hubble tension', that is the 4σ or more disagreement between 'early time' and 'late time' determinations of the present universe expansion rate, places some pressure on the ΛCDM cosmological model ([85] for example figure 1). There is also the question of interpreting the data itself and even an element of debate over whether the large-scale expansion of the universe is actually accelerating and whether dark energy is needed at all [86, 87]. However, fits to the cosmological data allowing for a free, yet constant, equation of state parameter yield a value of $w = -1.028 \pm 0.031$, consistent with a cosmological constant ([88] section 7.4.1). A more recent analysis within the context of extended dark energy models yields a measurement of $w = -1.187^{+\,0.038}_{-\,0.030}$ and 'more than 4.9σ evidence for a phantom dark energy equation of state' in a manner that considerably alleviates the Hubble tension [89].

Hence while the branch of generalised proper time of equation 20 provides a direct source of negative kinetic energy via equations 21 and 22 as a key feature of phantom dark energy models, such models also receive a degree of empirical support from the analysis of cosmological observations. This branch might then complete the cosmological pie chart with all three sectors, while having radically different empirical properties, deriving in parallel from a common unifying origin through the permitted explicit expressions for equation 4. In turn all mathematically possible expressions for generalised proper time, augmenting the common 4-dimensional spacetime root, might make some contribution to the full cosmological picture.

8 Discussion and Outlook

A number of models have attempted to associate the properties of visible matter, as described by the Standard Model, or dark matter, as inferred from cosmological observations, with the structures of extra spatial dimensions. Even if *both* forms of matter could be accommodated together in extra dimensions of space the challenge would remain to account for the very limited nature of the interaction between the visible and dark sectors. Alternatively if only *either* visible *or* dark matter were to reside in the structures of extra spatial dimensions that would introduce something of an uncomfortable asymmetry, with the *other* sector of matter having an apparently different origin and nature. In that case, with the visible and dark matter sectors produced by different mechanisms, they could also easily have had corresponding average mass densities in the universe differing by many orders of magnitude.

In the present theory a symmetric and mutually balanced picture is achieved. Here visible and dark matter are generated in parallel as associated with *different sectors* of extra dimensions originating in possible mathematical forms for generalised proper time. This identification and employment of a common basis in the generalisation of local proper time intervals implies weak or vanishing non-gravitational interactions between the Standard Model and dark matter sectors while also being consistent with the empirical observation of their similar overall mass density.

While the empirical properties of dark matter, other than determined through its gravitational impact and inferred from the lack of any other detection, are presently unknown and may be fundamentally unobservable, the Standard Model in the visible sector has been established and analysed in great detail in the laboratory. On employing generalised proper time, as introduced for equations 3 and 4, the sector with $p > 2$ described for equation 11 provides an efficient means of *directly* identifying non-trivial connections with the esoteric properties of the Standard Model as we have reviewed in section 4. These links have been established through to the E_7 level of equation 9, as explicitly determined and summarised for table 1, and lead to the prediction of an octonion-rich E_8 realisation for the ultimate form of proper time of equation 10 to fully accommodate the elementary structure of visible matter. Given the explicit progress that has been made through to the E_7 level mathematical constraints on the potential form that the E_8 level could take already strongly hint at new physics beyond the Standard Model with possible predictive power in the Higgs and neutrino sectors [65].

In this paper we have focussed upon the new physics that can be identified in an alternative branch of generalised proper time. While the mathematical structures with octonion-based realisations of exceptional Lie group symmetries have a high degree of uniqueness for the $p > 2$ forms of equation 11, this augmentation is not itself entirely unique as *different branches* of generalised proper time are mathematically permitted. In particular there is a possible alternative $p = 2$ sector for equation 4 as described for equation 13. While the fundamental basis here is in terms of generalised proper *time* this possible *quadratic* augmentation in equations 12 and 13, having the structure of a higher-dimensional Lorentzian spacetime form, can here be *interpreted* as the *origin* of a local structure of extra dimensions of *space*.

This potential mathematical loose end of the theory together with the physical loose end in the need to account for dark matter can be mutually connected and resolved as we have described in this paper. Indeed, the success in accounting for the Standard Model through the branch of equation 11 for generalised proper time itself provides motivation for seeking a source for dark matter through the extra spatial dimension branch of equation 13. Further, the directness of the symmetry breaking mechanism for identifying Standard Model features through the exceptional Lie group branch of generalised proper time as described for table 1 elucidates the means of identifying the explicit physical form of the dark matter deriving from the extended Lorentzian sector as described following table 2 in section 5. Hence, while empirically gravity provides a rather blunt instrument for deducing the potentially richer features of dark matter, such specific properties may be theoretically calculable in far more detail in the context of this framework.

As described for table 2 in section 5 the basis for this dark matter sector is the non-Abelian internal symmetry group $\mathrm{SO}(m)$ acting on the set of scalar fields $\boldsymbol{v}_m(x) \in \mathbb{R}^m$ for arbitrarily large m. By analogy with QCD this would be expected to result in the formation of massive 'dark hadrons', or 'dark quark nuggets', with a large number of constituent scalar 'dark quarks', associated with the components of $\boldsymbol{v}_m(x)$, as bound by the confining gauge symmetry $\mathrm{SO}(m)$. With neither a Coulomb repulsion limiting their size nor a Fermi pressure to support them collapse of such massive dark hadrons into black holes would be likely and even inevitable. Formed in the early universe such primordial black holes would then become a further, or even dominant, component of this dark matter sector.

Having a basis in a confining internal SO(m) gauge symmetry, as a 'dark QCD' counterpart of the standard QCD that dominates the mass density of visible matter as noted in the opening of subsection 2.1, this framework is again consistent with the generation of a similar quantity of dark and visible matter in the very early universe. As noted in section 5 the balance in overall energy density may be tilted in favour of a greater contribution from dark matter, by the observed factor of five, owing in part to a larger number of 'dark gluon' states and corresponding higher degree of interaction for the SO(m) gauge sector compared with the familiar gluons and interactions of the visible $SU(3)_c$ gauge sector. The relation of the strong coupling for elementary QCD interactions to that for dark QCD, with a presumed similar or identical value, remains to be more fully understood.

As a dark matter candidate this theory can be compared with the existing models of dark matter briefly reviewed in section 2. In terms of empirical structures the dark matter identified here has a physical basis similar to models employing a new confining gauge interaction [8, 9, 10, 11, 12, 13, 16] and may be closest to the models of massive 'dark nuclei' or 'dark quark nuggets' [18, 19, 20], also reviewed in subsection 2.1, albeit here with scalar dark quark constituents from table 2. For the case of the heavier dark hadrons gravitationally collapsing a connection with the primordial black hole models for dark matter [26, 27, 28, 29, 30, 32, 33, 34, 35, 36] discussed in subsection 2.2 also follows. The present approach can also be contrasted with the models using extra spatial dimensions [39, 40, 42, 43, 44, 45, 48, 49] described in subsection 2.3. However, rather than directly constructing a model to fit the observations, here we first motivate the underlying *source* of extra dimensions through generalised proper time and present an account of *why* dark matter exists at all, *where* it comes from and *what* it physically is, before seeking connections with the data.

Despite the underlying simplicity of the elementary components of dark matter in table 2 the physical form this takes can still exhibit a rich variety of empirical properties. By comparison with the models reviewed in subsection 2.1 dark glueballs as well as massive dark hadrons could form, all with a distribution of masses and able to mutually interact via the close-range dark QCD forces, in principle transmuting through collisions or the breakup of unstable states. Any primordial black holes produced could also have a range of masses, with the lighter black holes potentially evaporating and the heavier ones potentially merging through to the present epoch. Through this variety of dark matter structures there is then the possibility of both a multi-component distribution in space and a significant cosmic time-dependent evolution in the physical impact of this sector.

Compared with the matter fields deriving from the symmetry breaking of the Standard Model sector of equation 11, as explicitly determined through to the E_7 stage in table 1, the underlying physical basis of the matter fields deriving from the symmetry breaking of equation 13 in table 2, as representing a dark matter candidate, is nevertheless somewhat simpler at the level of elementary particle states. We hence naturally find *ourselves* 'residing' in the exceptional Lie group sector of equation 11, which exhibits a more complex microscopic physics apparently conducive to supporting biological life, by a trivial 'anthropic' argument. We then still feel the gravitational influence and possible further tenuous interaction effects of the Lorentzian extra spatial dimension sector of equation 13 as considered in section 6.

The common origin of the two sectors of generalised proper time in the local 4-dimensional spacetime substructure of equations 1 and 14 as pictured in figure 1 and utilised in equations 15–18, together with the direct relationship between external spacetime curvature and internal gauge fields of equation 19, implies a mutual gravitational interaction between Standard Model states and this dark matter candidate. However, weak non-gravitational interactions between the Standard Model and dark sectors might still in principle arise, either through an apparent 'Higgs portal' via the common external $\boldsymbol{v}_4(x) \in TM_4$ components or through a 'kinetic mixing portal' via potential overlapping internal components and common gauge symmetry subgroups, as described towards the end of section 6. As we noted there these properties could in principle permit the empirical detection of dark matter via such non-gravitational interactions in the laboratory ([3] sections 27.6 and 27.7, [4] section 3, [5], [77]). We note here that they could also result in specific features that might be observed indirectly through gravitation. For example these possible weak non-gravitational interactions between the two sectors would link the thermal history of dark hadrons with visible matter in the high temperatures of the early universe with potential predictive consequences observable to us in cosmological structure (in a manner analogous to WIMP, or as might be contrasted with some FIMP, models).

While in this paper we have identified and described a direct and simple source for dark matter through a basis in generalised proper time, in the previous section we have also considered the further branch of equation 20, with a non-compact internal symmetry component and the implication of negative kinetic energy, hence identifying a possible connection with phantom dark energy models. In principle an explanation of the full cosmological pie chart, with all three major sectors deriving from branches of generalised proper time and without any redundancy in this description, might then be achievable within the framework of this theory. This potential for a deeper and robust theoretical understanding of *all* components of the dark sector alongside visible matter might then significantly aid in the clarification of the overall picture accounting for cosmological observations.

In summary, with emphasis on a fundamental basis in the very familiar entity of *time*, with the innate arithmetic substructure expressed through equations 3 and 4, a theory can be constructed by contrast with models built upon posited extra dimensions of *space*. This new approach exhibits a number of properties such as simplicity and uniqueness desired for an underlying unified theory, as we reviewed in section 3. Complementary to the Standard Model branch of equation 11 described in section 4 the possible form for proper time of equation 13 provides a natural source *for* 'extra spatial dimensions'. However, here we do not interpret these extra components in a literal geometric manner but rather they provide a direct source of matter field structures identified as a dark matter candidate in 4-dimensional spacetime as described in sections 5 and 6. Without having to *postulate* either the existence of extra spatial dimensions or a new field the physical form of dark matter generated by the structure of this theory is found to exhibit suitable properties as we have discussed further in this section.

As described for table 2 this dark matter takes the form of scalar 'dark quark' states bound by an internal SO(m) gauge interaction into massive 'dark hadrons', with the potential to collapse into black holes, in a sector largely independent of

Standard Model particle states. This sector of dark matter in extra dimensions is then open to further theoretical development in itself and through comparison with existing models for dark matter such as reviewed in section 2. It remains to assess more fully the resulting physical features of this dark matter in connection with the empirical observations it is required to account for and to deduce potential predictive consequences both for the laboratory and in cosmology. In this paper we have, however, made the significant step of describing from first principles a fundamental underlying mechanism through which dark matter, and also potentially dark energy, can originate.

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