
Article

Application of Mode-Adaptive Bidirectional Pushover Analysis to an Irregular Reinforced Concrete Building Retrofitted via Base Isolation

Kenji Fujii ^{1*}, Takumi Masuda ²

¹ Department of Architecture, Faculty of Creative Engineering, Chiba Institute of Technology; Chiba 275-0016, Japan; kenji.fujii@p.chibakoudai.jp

² Graduate School of Creative Engineering, Chiba Institute of Technology; Chiba 275-0016, Japan; s16b1147bq@s.chibakoudai.jp

* Correspondence: kenji.fujii@p.chibakoudai.jp

Featured Application: Seismic response evaluation of a base-isolated building; seismic rehabilitation design for Reinforced Concrete buildings using the base-isolation technique.

Abstract: In this article, the main building of the former Uto City Hall, which was severely damaged in the 2016 Kumamoto earthquake, is investigated as a case study for the retrofitting of an irregular Reinforced Concrete building using the base-isolation technique. Its peak response is predicted via mode-adaptive bidirectional pushover analysis (MABPA), which was originally proposed by the authors. In the prediction step of MABPA, the peaks of the first and second modal responses are predicted considering the energy balance during a half cycle of the structural response. The numerical analysis results show that the peak relative displacement can be properly predicted by MABPA. The results also show that the performance of the retrofitted building models is satisfactory for the ground motion considered in this study, including the recorded motions in the 2016 Kumamoto earthquake.

Keywords: seismic isolation; asymmetric building; mode-adaptive bidirectional pushover analysis (MABPA); seismic retrofit; momentary energy input

1. Introduction

1.1. Background

Seismic isolation is widely applied to buildings for earthquake protection in earthquake-prone countries [1]. Unlike in the case of traditional (non-isolated) earthquake-resistant structures, seismic isolation ensures that the behaviors of building structures are within the elastic range and reduces acceleration in buildings during large earthquakes [2]. Therefore, this technique is applied to not only new buildings but also existing reinforced concrete and masonry buildings, including historical structures [3–11].

In general, there is some degree of irregularity in building structures, whether they are newly designed or preexisting. Therefore, the problem of torsional response due to plan irregularities needs to be studied for base-isolated structures as well as non-isolated structures. Several researchers have investigated the seismic behavior of base-isolated buildings with plan irregularities [12–25]; these include fundamental parametric studies using idealized models [12–16], as well as studies using more realistic frame building models [17–25]. Most of these studies are based on nonlinear time-history analyses [12–24]. However, there are a few investigations that examine the applicability of nonlinear static procedures to base-isolated buildings with asymmetry. Kilar and Koren [25] had examined the applicability of the extended N2 method [26, 27], which is one of the variants

of the nonlinear static procedures, to the base-isolated asymmetric buildings. They found the nonlinear peak response of base-isolated asymmetric building can be predicted by the extend N2 method. However, they have examined only those whose superstructure is regular in elevation. Therefore, further investigations are needed for the applicability of the nonlinear static procedures, especially for base-isolated building with plan and elevation irregularities.

1.2. Motivation

There are two main motivations for this study. The first is to extend nonlinear static analyses to base-isolated buildings with irregularities, and the second is to predict the nonlinear peak responses of base-isolated buildings according to the concept of energy input.

With respect to the first motivation, the authors have proposed mode-adaptive bi-directional pushover analysis (MABPA) [28–33]. This is a variant of nonlinear static analysis and was originally proposed for the seismic analysis of non-isolated asymmetric buildings subjected to horizontal bidirectional excitation. The first version of MABPA was proposed for non-isolated asymmetric buildings with regular elevation [28]; it was then updated following the development of displacement-based mode-adaptive pushover (DB-MAP) analysis [29]. This version has been applied to reinforced concrete asymmetric frame buildings with buckling-restrained braces [30] and to building models with bidirectional setbacks [31]. The applicability of MABPA has been discussed and evaluated based on the effective modal mass ratio of the first two modes [32]. The seismic capacity of an existing irregular building severely damaged in the 2016 Kumamoto earthquake (the former Uto City Hall) has been evaluated using MABPA [33]. Looking back on the development of MABPA, the logical next step should be to extend the method to base-isolated buildings with irregularities. Considering the case in which the seismic isolation period is well separated from the natural period of a superstructure, the behavior of the superstructure may be considered to be that of a rigid body, as discussed by several researchers, e.g., [16]. In such cases, the effective modal mass ratio of the first and second modes is close to unity, provided that the torsional resistance at the isolation layer is sufficient. It is expected that the seismic response of such a base-isolated building with plan irregularities will be accurately predicted by MABPA.

In the nonlinear static analysis shown in the American Society of Civil Engineers ASCE/SEI 41-17 document [3], the reduced spectrum considering the effective damping is used to predict the target displacement for a nonlinear static analysis. Similarly, as shown in Notification No. 2009 of the Ministry of Construction of Japan [34], the equivalent linearization technique can be used for target displacement evaluations of base-isolated buildings. However, several researchers have examined the responses of long-period building structures subjected to pulse-like ground motions, e.g., Mazza examined a base-isolated building [18, 20] and Güneş and Ulucan studied a tall reinforced concrete building [35]. Because the effective damping is calculated based on a steady response having the same displacement amplitude in the positive and negative directions, the use of effective damping for the prediction of the peak responses of base-isolated structures subjected to pulse-like ground motion is questionable. From this viewpoint, an alternative concept for predicting the peak response is required. Accordingly, the second motivation of this study is to investigate the use of the energy concept.

The concept of energy input was introduced by Akiyama in the 1980s [36] and is implemented in the design recommendation for seismically isolated buildings presented by the Architectural Institute of Japan [2]. In Akiyama's theory, the *total input energy* is considered to be a suitable seismic intensity parameter to access the cumulative response of a structure. Instead of the total input energy, Inoue et al. proposed the *maximum momentary input energy* [37–39] as an intensity parameter related to the peak displacement; the nonlinear peak displacement can be predicted by equating the maximum momentary input energy and the cumulative hysteresis energy during a half cycle of the structural

response. Following the work of Inoue et al., the authors investigated the relationship between the maximum momentary input energy and the total input energy of an elastic single-degree-of-freedom (SDOF) model [40]. In addition, the concept of the momentary input energy was extended to consider bidirectional horizontal excitation [41, 42]. Specifically, the time-varying function of the momentary energy input has been formulated for unidirectional [40] and bidirectional [41] excitation: this function can be calculated from the transfer function of the model and the complex Fourier coefficients of the ground acceleration. Using the time-varying function, both energy parameters, the total and the maximum momentary input energy, can be accurately calculated. In a previous study [42], it was shown that the nonlinear peak displacement and the cumulative energy of the isotropic two-degree-of-freedom model representing a reinforced concrete building can be properly predicted using a time-varying function. Therefore, the use of a time-varying function of the momentary energy input is promising for seismic response predictions for base-isolated buildings.

1.3. Objectives

Based on the above discussion, the following questions are addressed in this paper.

- Is MABPA capable of predicting the peak response of irregular base-isolated buildings?
- The prediction of the peak equivalent displacement of the first two modal responses is an essential step in MABPA. For this, the relationship between the maximum momentary input energy and the peak displacement needs to be properly evaluated. How can this relationship be evaluated from the pushover analysis results?
- In the prediction of the maximum momentary input energy of the first two modal responses, the effect of simultaneous bidirectional excitation needs to be considered. Can the upper bound of the peak equivalent displacement of the first two modal responses be predicted using the bidirectional maximum momentary input energy spectrum [41]?

In this study, the main building of the former Uto City Hall [33] is investigated as a case study of an irregular reinforced concrete building retrofitted using the base-isolation technique. The nonlinear peak responses of two retrofitted building models subjected to bidirectional excitation are investigated via a time-history analysis using artificial and recorded ground motion datasets. Then, their peak responses are predicted by MABPA to evaluate its accuracy.

The rest of paper is organized as follows. Section 2 briefly presents information concerning the original building. Then, two retrofitted building models using the base-isolation technique are presented, as well as the ground motion data. Section 3 presents an outline of MABPA. Then, the prediction procedure of the maximum equivalent displacement is conducted using the maximum momentary input energy. The validation of the prediction of the peak response is discussed in Section 4. Discussions focused on (i) the relationship between the maximum equivalent displacement and the maximum momentary input energy, (ii) the predictability of the largest maximum momentary input energy from the bidirectional momentary input energy spectrum, and (iii) the accuracy of the upper bound of the maximum equivalent displacement of the first two modes are presented in Section 5. Conclusions and future directions of study are discussed in Section 6.

2. Building and Ground Motion Data

2.1. Original Building

Figure 1 shows a simplified structural plan and the elevation of the main building of the former Uto City Hall [33]. This five-story reinforced concrete irregular building was constructed in 1965. Details concerning this building can be found in the literature [33].

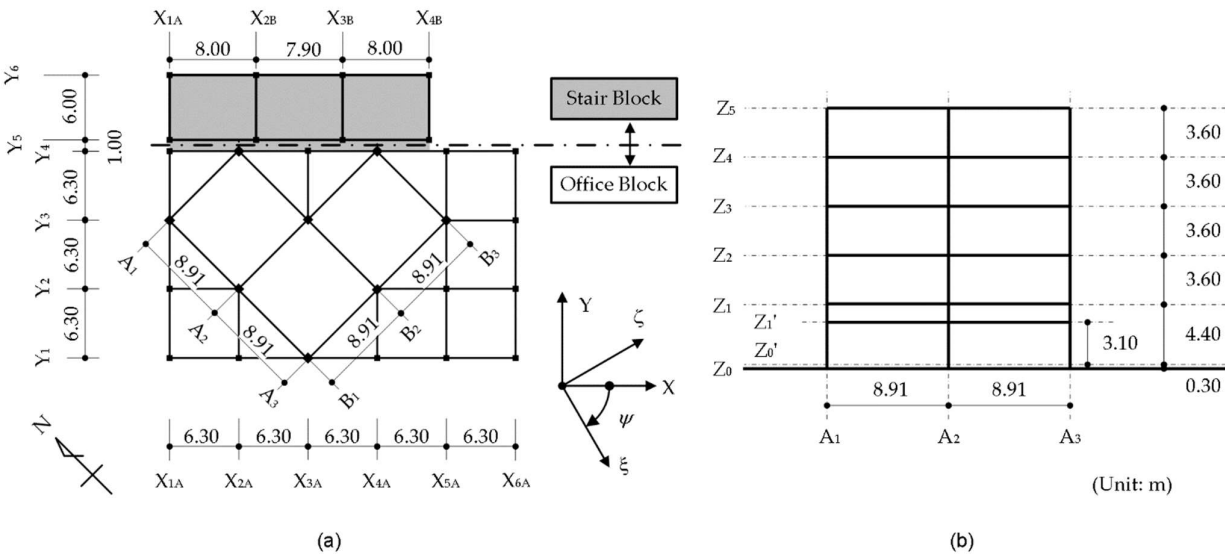


Figure 1. Simplified structural plan and elevation of the main building of the former Uto City Hall [33]: (a) structural plan (Level Z₀) and (b) simplified plan elevation (frame B₁).

2.2. Properties of the Isolation Layer

Two retrofitted building models are examined in this study to consider different seismic isolation periods (T_f). The properties of the isolation layer were determined as follows based on the energy-balanced design method introduced in the design recommendation [2].

First, the mass and the moment of inertia of each floor were calculated. Table 1 shows the floor mass (m_j), moment of inertia (I_j), and the radius of gyration of the floor mass (r_j) for the j^{th} floor. Here, the values of the floor mass and the moment of inertia above level 1 are taken from a previous study [33], while the weight per unit area at level 0 is assumed to be 24 kN/m² for the calculation of the floor mass and the moment of inertia of each floor. Therefore, the calculated total mass (M) is 5412 t.

Table 1. Floor mass, moment of inertia, and radius of gyration of the floor mass of each floor level.

Floor level	Floor mass	Moment of Inertia	Radius of gyration of
j	m_j (t)	I_j ($\times 10^3$ tm ²)	floor mass r_j (m)
5	677.8	78.37	10.75
4	548.5	62.85	10.70
3	543.0	62.50	10.73
2	581.0	67.12	10.75
1	1208.1	199.0	12.83
0	1853.7	274.9	12.18

Figure 2 shows the horizontal response spectra of the recorded ground motions of the first (April 14, 2016: UTO0414) and second (April 16, 2016: UTO0416) earthquakes observed at the K-NET Uto station. Note that the spectral acceleration (S_A) and velocity (S_V) shown in Figures 2(a) and 2(b) are calculated as the absolute (vector) value of the two horizontal components. The total input energy spectrum (the V_I spectrum) is also calculated considering the simultaneous input of the two horizontal components. The viscous damping ratio is set to 0.05 for the calculations of S_A and S_V , while it is set to 0.10 for the calculation of V_I following the work done by Akiyama [36].

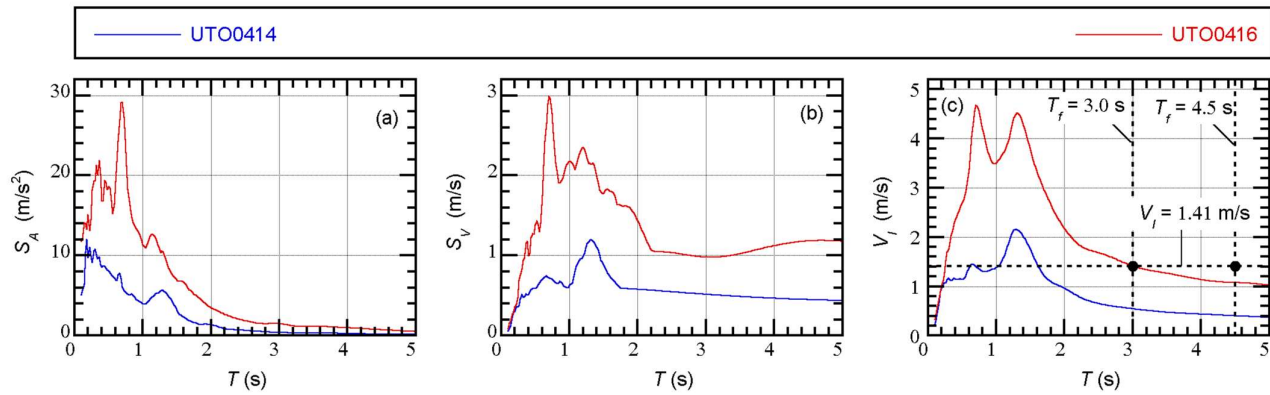


Figure 2. Horizontal response spectra of the recorded ground motions at the K-NET Uto station: (a) elastic acceleration response spectrum (damping ratio: 0.05); (b) elastic velocity response spectrum (damping ratio: 0.05); and (c) elastic total input energy spectrum (damping ratio: 0.10).

In this study, the range of the seismic isolation period (T_f) is considered to be from 3 s to 4.5 s when determining the properties of the isolation layer. Based on Figure 2(c), the equivalent velocity of the total input energy (V_I) is set to 1.41 m/s to determine the properties of the isolation layer.

Assuming that the superstructure is a rigid body, the equation of the energy balance is expressed as

$$\frac{1}{2}K_f\delta_{\max}^2 + 4{}_s n_s Q_y \delta_{\max} = \frac{1}{2}MV_I^2. \quad (1)$$

In Eq. (1), δ_{\max} is the maximum horizontal displacement, ${}_s n$ is the equivalent number of repetitions, K_f is the total horizontal stiffness of the flexible element, and ${}_s Q_y$ is the total yield strength of the rigid-plastic element. Equation (1) can be rewritten as

$$\left(\frac{2\pi}{T_f}\right)^2 \delta_{\max}^2 + 8{}_s n_s \alpha_y g \delta_{\max} = V_I^2, \quad (2)$$

$$\text{where } K_f = M \left(\frac{2\pi}{T_f}\right)^2, {}_s \alpha_y = \frac{{}_s Q_y}{Mg}. \quad (3)$$

In Eq. (3), g is the acceleration due to gravity, assumed to be 9.8 m/s², and ${}_s \alpha_y$ is the yielding shear strength coefficient. In this study, the design-allowable horizontal displacement (δ_a) is set to 0.40 m, while the value of ${}_s n$ is set to 2 following the design recommendation [2]. Then, the two parameters of the isolation layer, T_f and ${}_s \alpha_y$, are adjusted such that the following condition is satisfied:

$$\left(\frac{2\pi}{T_f}\right)^2 \delta_a^2 + 16{}_s \alpha_y g \delta_a \geq V_I^2. \quad (4)$$

The isolation layer below level 0 comprises natural rubber bearings (NRBs), elastic sliding bearings (ESBs), and steel dampers. Figure 3 shows the layout of the isolators and dampers in the isolation layer of the two retrofitted building models: Model-Tf34, shown in Figure 3(a), and Model-Tf44, shown in Figure 3(b). As shown in the figure, the steel dampers are placed at the perimeter frames to provide torsional resistance. The point G shown in this figure is the center of mass of the superstructure, the point S_0 is the center

of stiffness of the isolation layer calculated according to the initial stiffness of the isolators and dampers, while the point S_1 is the center of stiffness of the isolation layer calculated according to the secant stiffness of the isolators and dampers considering their displacement (δ) of 0.40 m.

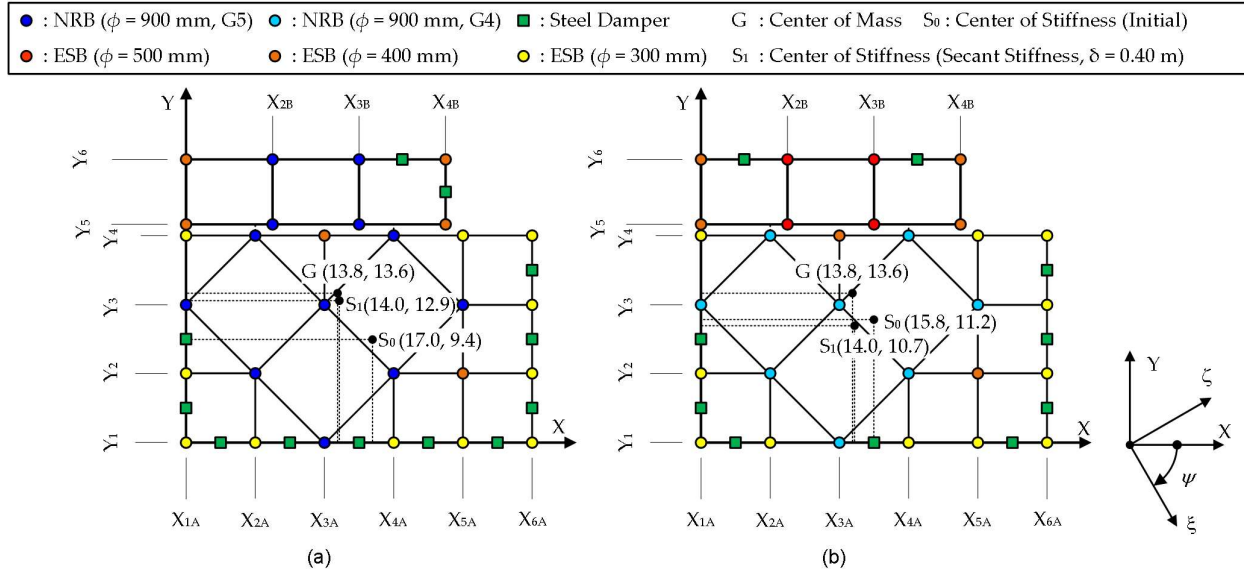


Figure 3. Layout of the isolators and dampers in the isolation layer: (a) Model-Tf34 and (b) Model-Tf44.

For Model-Tf34, $T_f = 3.38$ s and ${}_s\alpha_y = 0.0448$, while for Model-Tf44, $T_f = 4.40$ s and ${}_s\alpha_y = 0.0396$. Note that T_f is calculated according to the horizontal stiffness of the NRBS, while ${}_s\alpha_y$ is calculated according to the yield shear strength of the ESBs and steel dampers.

As shown Figure 3, the eccentricity at the isolation layer is non-negligible in both models. The eccentricity indices [34] of Model-Tf34 calculated according to the initial stiffnesses in the X- and Y-directions are $R_{eX} = 0.285$ and $R_{eY} = 0.216$, respectively, while those calculated according to the secant stiffnesses in the X- and Y-directions are $R_{eX} = 0.014$ and $R_{eY} = 0.054$, respectively. Similarly, the eccentricity indices of Model-Tf44 calculated according to initial stiffnesses in the X- and Y-directions are $R_{eX} = 0.155$ and $R_{eY} = 0.126$, respectively, while those calculated according to the secant stiffnesses in the X- and Y-directions are $R_{eX} = 0.233$ and $R_{eY} = 0.015$, respectively. In this study, the perimeter frames X_{1A} and Y_6 are referred as the “flexible-side frames,” while the frames X_{6A} and Y_1 are referred to as the “stiff-side frames.” Note that no optimization to minimize the torsional response has been made to choose the dampers in this study because such optimization is beyond the scope of this study.

Figure 4 shows envelopes of the force-deformation relationship for the isolators and dampers. The behavior of the NRBS is assumed to be linear elastic, while that of the ESBs and dampers is assumed to be bilinear.

All of the isolators were chosen from a catalog provided by the Bridgestone Corporation [43,44], considering that the ultimate horizontal deformation is larger than 150% of δ_a (1.5×0.40 m = 0.60 m). Meanwhile, the steel dampers were chosen from a catalog provided by Nippon Steel Corporation Engineering Co. Ltd. [45]. Tables 2–4 list the properties of the isolators and dampers used in the two models.

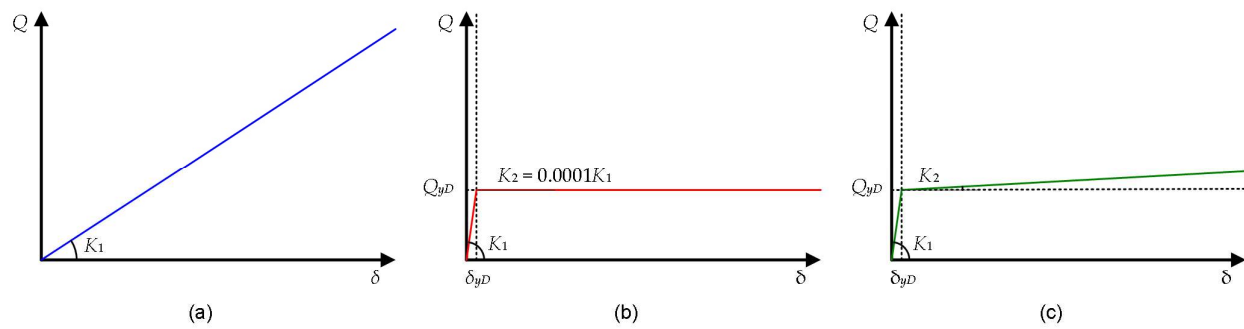


Figure 4. Envelope of the force-deformation relationship for the isolators and dampers: (a) natural rubber bearings (NRBs); (b) elastic sliding bearings (ESBs); (c) and steel dampers.

Table 2. Properties of the selected natural rubber bearings (NRBs).

Type	Outer Diameter (mm)	Total rubber thickness (mm)	Shear modulus (MPa)	Horizontal stiffness K_1 (MN/m)	Vertical stiffness K_V (MN/m)
NRB ($\phi = 900$ mm, G5)	900	180	0.441	1.56	3730
NRB ($\phi = 900$ mm, G4)	900	180	0.392	1.38	3420

Table 3. Properties of the selected elastic sliding bearings (ESBs).

Type	Outer Diameter (mm)	Shear modulus (MPa)	Friction coefficient μ	Initial horizontal stiffness K_1 (MN/m)	Vertical stiffness K_V (MN/m)
ESB ($\phi = 300$ mm)	300	0.392	0.010	0.884	1380
ESB ($\phi = 400$ mm)	400	0.392	0.010	1.48	2270
ESB ($\phi = 500$ mm)	500	0.392	0.010	2.40	3710

Table 4. Properties of the selected steel dampers.

Initial stiffness K_1 (MN/m)	Yield Strength Q_{yd} (kN)	Post yield stiffness K_2 (MN/m)
7.60	184	0.128

The yield strength of an ESB is calculated as

$$Q_{yd} = \mu P_V. \quad (5)$$

In Eq. (5), P_V is the vertical load of the ESB due to gravity.

2.3. Structural Modeling

The two building models are modeled as three-dimensional spatial frames, wherein the floor diaphragms are assumed to be rigid in their own planes without an out-of-plane stiffness. Figure 5 shows the structural modeling. For the numerical analyses, a nonlinear analysis program for spatial frames developed by the authors in a previous study [46] was used. The structural models are based on Model-RuW4-100 from a previous study [33]. As shown in Figures 5(b) and 5(c), all the vertical and rotational springs in the basement of the original model were replaced by isolators and dampers were installed in the isolation layer. The shear behavior of the isolators is modeled using the multi-shear-spring model proposed by Wada and Hirose [47], while their axial behavior is modeled using a linear elastic spring, as shown in Figure 5(d). The shear behavior of the steel dampers is

also modeled using the multi-shear-spring model, as shown in Figure 5(e). No bending stiffness is considered in either the isolators or the dampers. The hysteresis rules of the ESBs and steel dampers are modeled following the normal bilinear rule for simplicity of analysis. Details of the structural modeling of the superstructure can be found in the literature [33]. The damping matrix of the superstructure was then assumed to be proportional to the tangent stiffness matrix of the superstructure, with 2% of the critical damping of the first mode. The damping of the isolators and dampers was not considered, assuming that their energy absorption effects are already included in the hysteresis rules. Note that the force-displacement relationships of all members, including those in the superstructures and in the isolation layers, are assumed to be symmetric in the positive and negative loading directions, as in previous studies [33, 46].

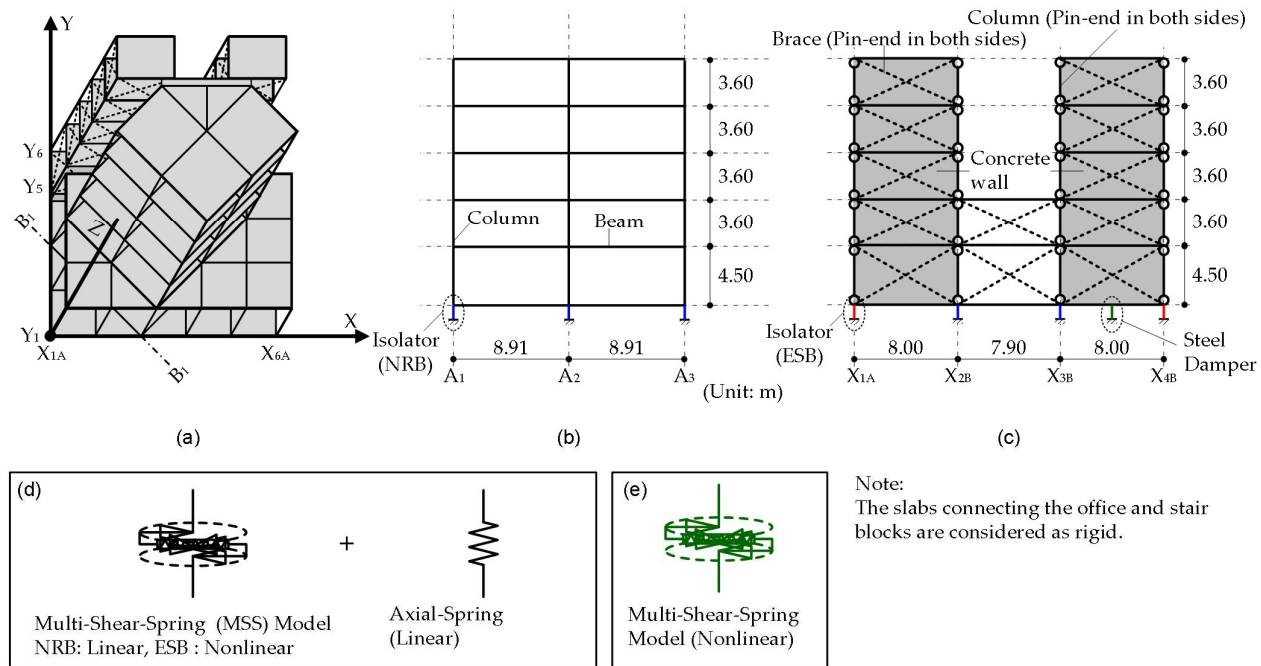


Figure 5. Structural modeling: (a) overview of the structural model; (b) frame B1; (c) frame Y6; (d) modeling of the isolators (NRBs and ESBs); and (e) modeling of the steel dampers.

Figures 6 and 7 show the first three natural modes of Model-T34 and Model-T44 in the elastic range. Here, T_{ie} is the natural period of the i^{th} mode in the elastic range ($i = 1-3$), m_{ie}^* is the effective modal mass ratio of the i^{th} mode with respect to its principal direction in the elastic range, ψ_{ie} is the incidence of the principal direction of the i^{th} modal response in the elastic range, and R_{pie} is the torsional index of the i^{th} mode in the elastic range. The values of m_{ie}^* , ψ_{ie} , and R_{pie} are calculated according to the i^{th} elastic mode vector (ϕ_{ie}) as follows:

$$m_{ie}^* = \frac{1}{M} \frac{\left(\sum_{j=0}^5 m_j \phi_{Xjie} \right)^2 + \left(\sum_{j=0}^5 m_j \phi_{Yjie} \right)^2}{\sum_{j=0}^5 m_j \phi_{Xjie}^2 + \sum_{j=0}^5 m_j \phi_{Yjie}^2 + \sum_{j=0}^5 I_j \phi_{\Theta jie}^2}, \quad (6)$$

$$\tan \psi_{ie} = - \frac{\sum_{j=0}^5 m_j \phi_{Yjie}}{\sum_{j=0}^5 m_j \phi_{Xjie}}, \quad (7)$$

$$R_{\rho ie} = \sqrt{\frac{\sum_{j=0}^5 I_j \phi_{\Theta jie}^2}{\left(\sum_{j=0}^5 m_j \phi_{Xjie}^2 + \sum_{j=0}^5 m_j \phi_{Yjie}^2 \right)}}, \quad (8)$$

$$\boldsymbol{\varphi}_{ie} = \{ \phi_{X0ie} \quad \cdots \quad \phi_{X5ie} \quad \phi_{Y0ie} \quad \cdots \quad \phi_{Y5ie} \quad \phi_{\Theta 0ie} \quad \cdots \quad \phi_{\Theta 5ie} \}^T. \quad (9)$$

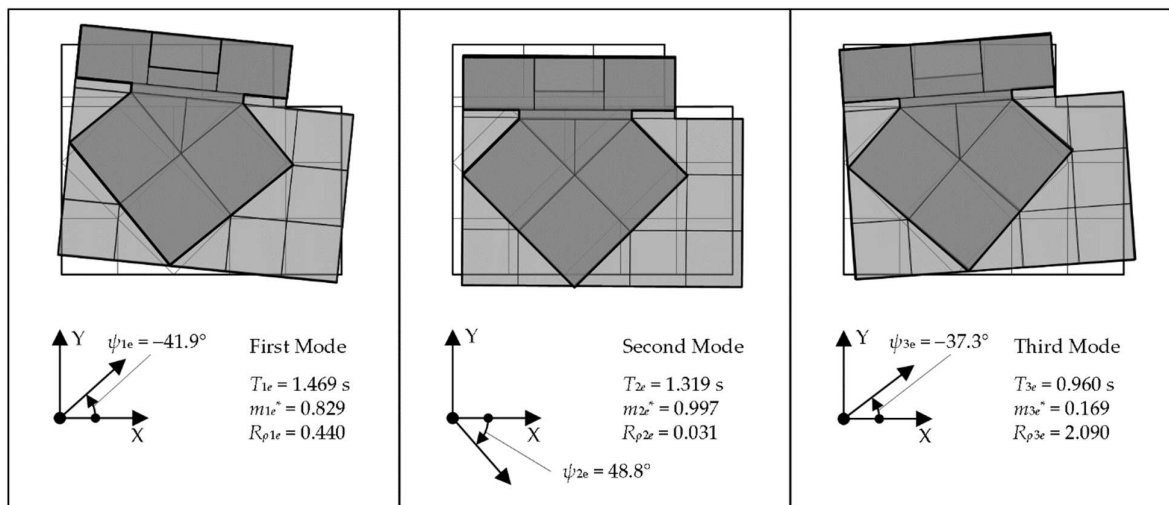


Figure 6. Shape of the first three natural modes of Model-Tf34 in the elastic range.

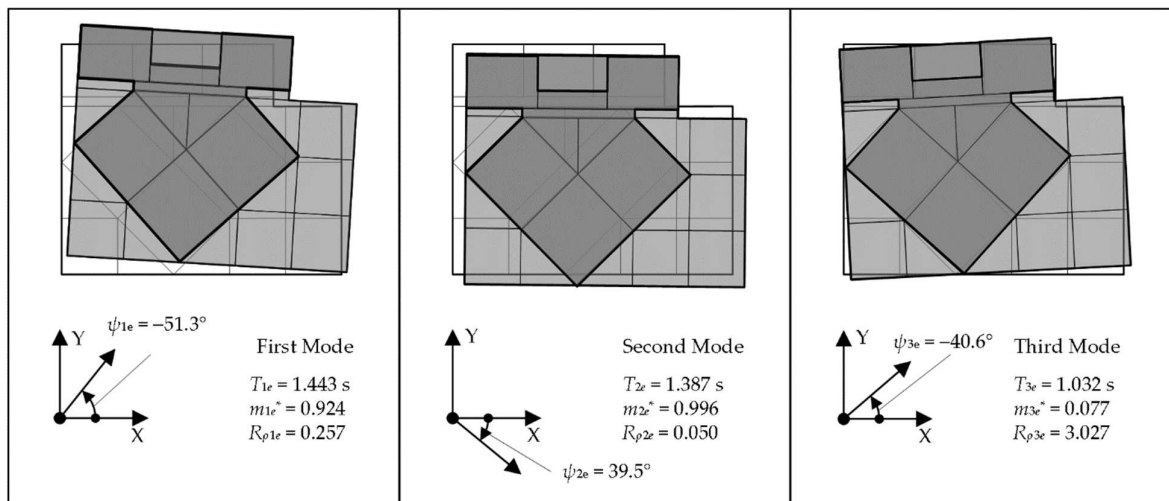


Figure 7. Shape of the first three natural modes of Model-Tf44 in the elastic range.

The behavior of the superstructure can be considered as a rigid body in the first three modes in the elastic range, as observed from Figures 6 and 7. In both building models, the effective modal mass ratios of the first two modes are close to 1. In addition, the first mode is predominantly translational ($R_{\rho 1e} < 1$), the second mode is nearly purely translational ($R_{\rho 2e} \ll 1$), and the third mode is predominantly torsional ($R_{\rho 3e} > 1$). In addition, the angles between the principal directions of the first two modes are close to 90° (90.7° for Model-

Tf34 and 90.8° for Model-Tf44). Therefore, both building models are classified as torsionally stiff buildings: in a previous study [32], the conditions for classification as a torsionally stiff building are $R_{\rho1}, R_{\rho2} < 1$, and $R_{\rho3} > 1$.

The natural periods of the superstructure in the elastic range, assuming that the entire basement is supported by pins, are 0.373 s (first mode), 0.342 s (second mode), and 0.192 s (third mode). Therefore, the natural period of the superstructure is well separated from the seismic isolation period in both models.

2.4. Ground motion data

In this study, the seismic excitation is bidirectional in the X–Y plane and both artificial ground motions and recorded ground motions are considered.

2.4.1. Artificial ground motions

In this study, two series of artificial ground motion datasets (the Art-1 and Art-2 series) were generated. As noted in Section 2.2, the equivalent velocity of the total input energy (V_I) was set to 1.41 m/s to determine the properties of the isolation layer. Therefore, the artificial ground motion datasets were generated such that their bidirectional V_I spectrum fit the predetermined target V_I spectrum as follows.

The target bidirectional V_I spectrum with a damping ratio of 0.10 was determined such that

$$V_I(T) = \begin{cases} 1.41(T/0.75) & \text{m/s} : T \leq 0.75\text{s} \\ 1.41 & : T > 0.75\text{s} \end{cases} \quad (10)$$

To generate each horizontal component, it was assumed that the intensity of V_I in both orthogonal components (ξ - and ζ - components) was identical. The target equivalent velocities of the total input energy of the ξ - and ζ -components, $V_{I\xi}(T)$ and $V_{I\zeta}(T)$, respectively, were calculated as

$$V_{I\xi}(T) = V_{I\zeta}(T) = V_I(T)/\sqrt{2} \quad (11)$$

The phase angle is given by a uniform random value, and to consider the time-dependent amplitude of the ground motions, a Jennings-type envelope function ($e(t)$) is assumed. In this study, two envelope functions were considered. In Art-1-00, the envelope function was set as in Eq. (12), while in Art-2-00 the envelope function was set as in Eq. (13).

$$e(t) = \begin{cases} (t/5)^2 & : 0\text{s} \leq t \leq 5\text{s} \\ 1 & : 5\text{s} \leq t \leq 25\text{s} \\ \exp\{-0.066(t-25)\} & : 25\text{s} \leq t \leq 60\text{s} \end{cases} \quad (12)$$

$$e(t) = \begin{cases} (t/2.5)^2 & : 0\text{s} \leq t \leq 2.5\text{s} \\ 1 & : 2.5\text{s} \leq t \leq 12.5\text{s} \\ \exp\{-0.132(t-12.5)\} & : 12.5\text{s} \leq t \leq 30\text{s} \end{cases} \quad (13)$$

Figures 8 and 9 show the time-histories of the two components and the orbits of Art-1-00 and Art-2-00, respectively. Because the same target V_I spectrum is applied while the duration of Art-2-00 is the half that of Art-1-00, the peak ground acceleration of Art-2-00 is larger than that of Art-1-00.

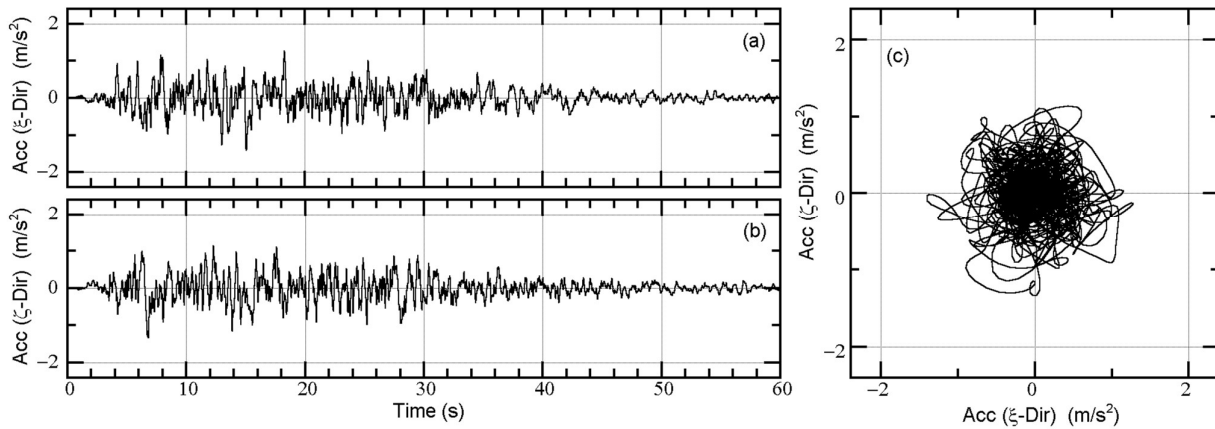


Figure 8. Two components of the generated artificial ground motion (Art-1-00): (a) ξ -direction; (b) ζ -direction; and (c) orbit.

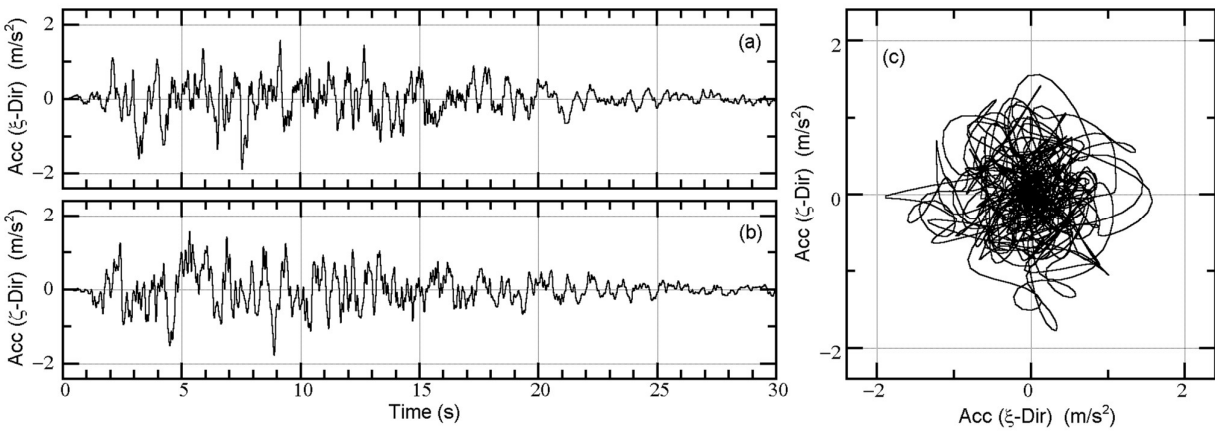


Figure 9. Two components of the generated artificial ground motion (Art-2-00): (a) ξ -direction; (b) ζ -direction; and (c) orbit.

Note that the artificial ground motions used in this study were generated independently. The correlation coefficients of Art-1-00 and Art-2-00 are close to zero, even though the envelope functions of the two components are the same; therefore, the two components can be considered independently of each other.

Next, 11 artificial ground motion datasets were generated from Art-1-00 and Art-2-00 by shifting the phasing angle. The generated artificial ground motion vector ($\mathbf{a}_g(t, \Delta\phi_0)$) is expressed as

$$\mathbf{a}_g(t, \Delta\phi_0) = \begin{Bmatrix} a_{g\xi}(t, \Delta\phi_0) \\ a_{g\zeta}(t, \Delta\phi_0) \end{Bmatrix} = \sum_{n=-N}^N \begin{Bmatrix} c_{\xi,n} \\ c_{\zeta,n} \end{Bmatrix} \exp[i\{\omega_n t - \text{sgn}(\omega_n) \Delta\phi_0\}]. \quad (14)$$

In Eq. (14), $c_{\xi,n}$ and $c_{\zeta,n}$ are the complex Fourier coefficients of the n^{th} harmonics of $a_{g\xi}(t)$ and $a_{g\zeta}(t)$, respectively, ω_n is the circular frequency of the n^{th} harmonic, and $\Delta\phi_0$ is the constant used to shift the phase angle of all the harmonics. As in previous studies [40, 42], the constant $\Delta\phi_0$ was set from $\pi/12$ to $11\pi/12$ with an interval of $\pi/12$. Notably, the phase difference of each ground motion component does not change when shifting the phase angle. In addition, the total input energy spectrum (the V_i spectrum) is independent of the shifting phase angle ($\Delta\phi_0$). This is because the shifting phase angle does not affect the absolute value of the complex Fourier coefficient, and the total input energy

per unit mass can be calculated from the Fourier amplitude spectrum without the phase characteristics of the ground motion, as shown by Ordaz [48]. The generated artificial ground motion datasets are numbered from 01 to 11 depending on $\Delta\phi_0$, with a total of $2 \times 12 = 24$ artificial ground motion datasets generated and used in this study.

Figures 10 and 11 show the horizontal response spectra of the generated ground motion datasets. As shown in these figures, the spectral acceleration (S_A) and the spectral velocity (S_V) differs slightly because of the shift in the phase angle. A comparison of Figures 10 and 11 reveals that S_A and S_V of the Art-2 series are larger than those of the Art-1 series, even though the target V_I spectrum is identical.

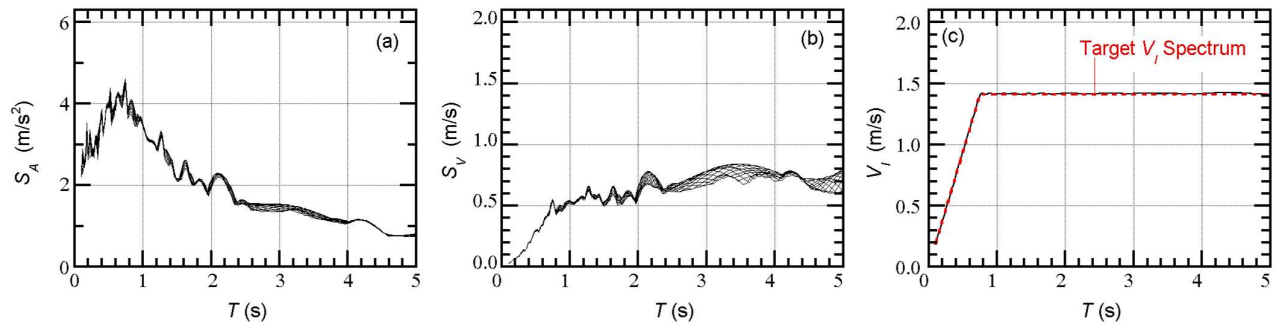


Figure 10. Horizontal response spectra of the artificial ground motion (Art-1 series): (a) elastic acceleration response spectrum (damping ratio: 0.05); (b) elastic velocity response spectrum (damping ratio: 0.05); and (c) elastic total input energy spectrum (damping ratio: 0.10).

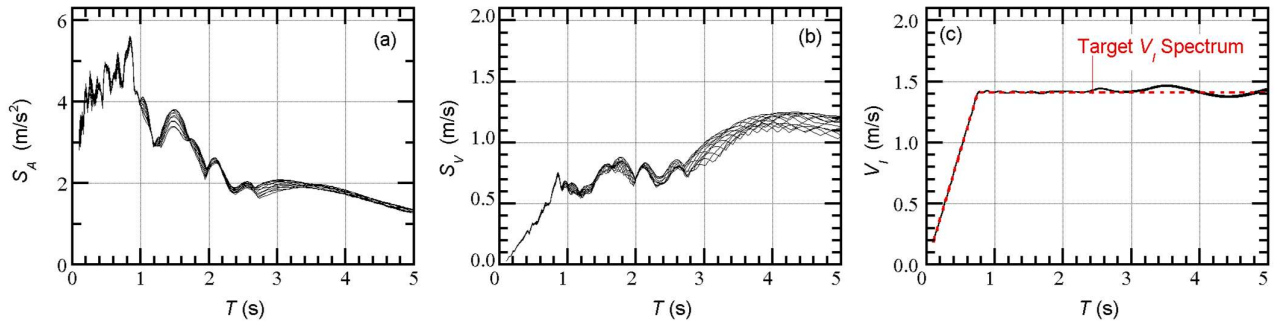


Figure 11. Horizontal response spectra of the artificial ground motion (Art-2 series): (a) elastic acceleration response spectrum (damping ratio: 0.05); (b) elastic velocity response spectrum (damping ratio: 0.05); and (c) elastic total input energy spectrum (damping ratio: 0.10).

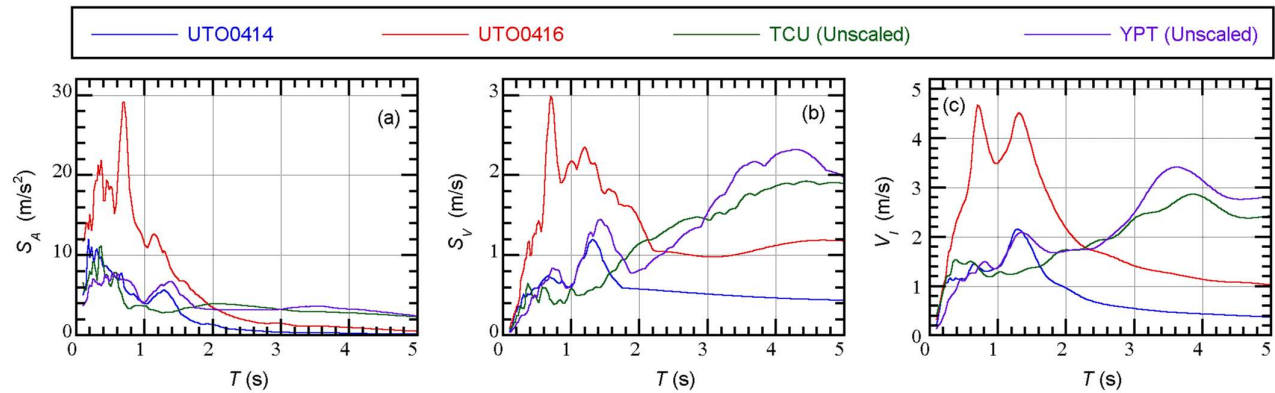
2.4.2. Recorded ground motions

Table 5 lists the four datasets of recorded ground motions used in this study. Details concerning the original ground motions can be found in the Appendix. Figure 12 shows the horizontal response spectra of the unscaled recorded ground motion datasets.

One of the motivations of this analysis is to evaluate the seismic performance of the two retrofitted models under the ground motions recorded during the 2016 Kumamoto earthquake. Therefore, the two recorded ground motion datasets (UTO0414 and UTO0416) were used without scaling. A further two datasets of ground motions, TCU and YPT, were chosen as examples of long-period pulse-like ground motions, as investigated by Güneş and Ulucan [35]. These two ground motion datasets were scaled such that the equivalent velocity of the total energy input (V_I) at the isolation period (T_f) equals 1.41 m/s, which is consistent with the intensity considered in the determination of the properties of the isolation layer, as discussed in Section 2.2.

Table 5. List of the recorded ground motion datasets used in this study.

Earthquake of the Original Record	Ground Motion ID	Scale Factor	
		Model-Tf34	Model-Tf44
Kumamoto, 2016/04/14	UTO0414	1.000	1.000
Kumamoto, 2016/04/16	UTO0416	1.000	1.000
Chichi, 1999	TCU	0.5540	0.5718
Kocaeli, 1999	YPT	0.4293	0.5057

**Figure 12.** Horizontal response spectra of the recorded ground motions: (a) elastic acceleration response spectrum (damping ratio: 0.05); (b) elastic velocity response spectrum (damping ratio: 0.05); and (c) elastic total input energy spectrum (damping ratio: 0.10).

3. Description of MABPA

3.1. Outline of MABPA

Figure 13 shows the flow of the updated MABPA. Here, the U-axis is the principal axis of the first modal response, while the V-axis is the axis perpendicular to the U-axis, following previous studies [28, 30]. In this study, the predictions of the peak responses of the first and second modes (steps 2 and 4) were updated as follows.

- The bidirectional momentary input energy proposed in the previous study is applied as the seismic intensity parameter.
- The peak response of each mode is predicted from the energy balance in a half cycle of the structural response.

It is assumed that the upper bound of the unidirectional maximum momentary input energy considering the various directions of the seismic input is approximated by the bidirectional maximum momentary input energy. For the prediction of the peak equivalent displacements of the first and second modal responses, the bidirectional maximum momentary input energy spectrum (bidirectional $V_{\Delta E}$ spectrum) is used. This is because the influences of the simultaneous input of two horizontal components are included in the bidirectional $V_{\Delta E}$ spectrum, as discussed in previous studies [41, 42]. Note that the maximum momentary input energy of each modal response varies depending on the angle of incidence of the horizontal ground motion. The assumption that the maximum momentary input energy of each modal response can be predicted from the bidirectional $V_{\Delta E}$ spectrum can lead to conservative predictions. The validation of this assumption is discussed in Section 5.

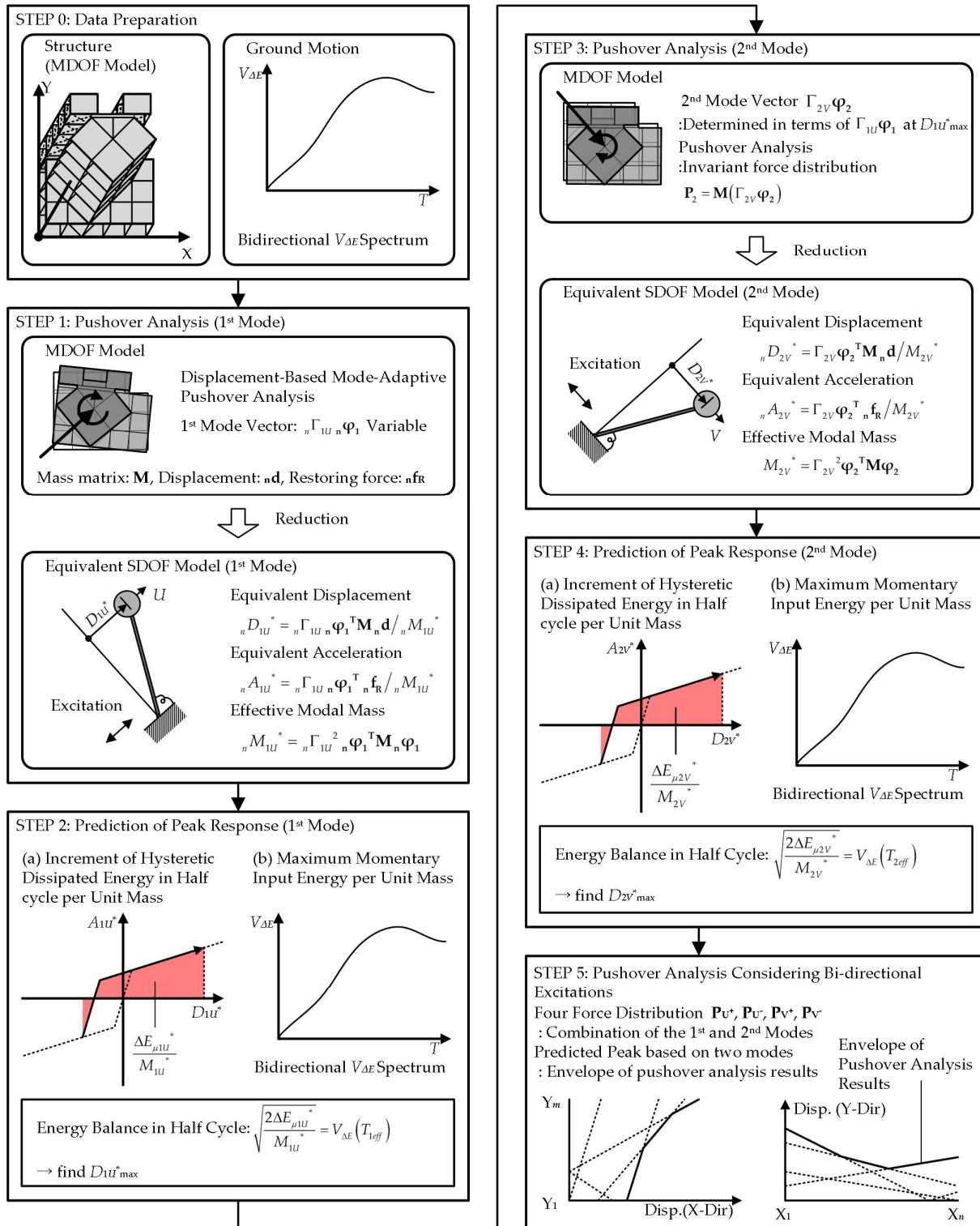


Figure 13. Flow of the updated mode-adaptive bi-directional pushover analysis (MABPA).

3.2. Prediction of the peak response using the momentary energy input

3.2.1. Calculation of the bidirectional momentary input energy spectrum

The bidirectional momentary input energy spectrum, which was proposed in previous studies [41, 42], is calculated as follows.

First, the complex Fourier coefficient of the two horizontal components, $c_{\xi,n}$ and $c_{\zeta,n}$, can be calculated via the discrete Fourier transform of the two horizontal components $a_{g\xi}(t)$ and $a_{g\zeta}(t)$, respectively.

The displacement and velocity transfer function of the linear system with viscous and complex damping can be calculated as follows:

$$H_{CVD}(i\omega_n) = \frac{1}{\omega_0^2 - \omega_n^2 + 2\omega_0\{h\omega_n + \beta\omega_0 \operatorname{sgn}(\omega_n)\}i}, H_{CVV}(i\omega_n) = i\omega_n H_{CVD}(i\omega_n). \quad (15)$$

In Eq. (15), $\omega_0 (= 2\pi/T)$ is the natural circular frequency and h and β are the viscous and complex damping of the linear system, respectively. From the complex Fourier coefficients ($c_{\xi,n}$ and $c_{\zeta,n}$) and the transfer functions of the linear system ($H_{CVD}(i\omega_n)$ and $H_{CVV}(i\omega_n)$), the duration of a half cycle of response (Δt) can be calculated as

$$\Delta t = \pi \sqrt{\sum_{n=1}^N |H_{CVD}(i\omega_n)|^2 \left\{ |c_{\xi,n}|^2 + |c_{\zeta,n}|^2 \right\}} / \sum_{n=1}^N |H_{CVV}(i\omega_n)|^2 \left\{ |c_{\xi,n}|^2 + |c_{\zeta,n}|^2 \right\}. \quad (16)$$

Then, the Fourier coefficient of the time-varying function of the momentary energy input can be calculated as follows:

$$E_{\Delta BI,n}^* = \begin{cases} \frac{\sin(\omega_n \Delta t / 2)}{\omega_n \Delta t / 2} \sum_{n_1=n+1}^N \left\{ H_V(i\omega_{n_1}) + H_V(-i\omega_{n_1-n}) \right\} \left\{ c_{\xi,n_1} c_{\xi,-(n_1-n)} + c_{\zeta,n_1} c_{\zeta,-(n_1-n)} \right\} & : n > 0 \\ 2 \sum_{n_1=1}^N \operatorname{Re} \left\{ H_V(i\omega_{n_1}) \right\} \left\{ |c_{\xi,n_1}|^2 + |c_{\zeta,n_1}|^2 \right\} & : n = 0 \\ E_{\Delta BI,n}^* & : n < 0 \end{cases} \quad (17)$$

The momentary input energy per unit mass at time t can be calculated as follows:

$$\frac{\Delta E_{BI}(t)}{m} = \int_{t-\Delta t/2}^{t+\Delta t/2} \sum_{n=-N+1}^{N-1} E_{\Delta BI,n}^* \exp(i\omega_n t) dt. \quad (18)$$

The maximum momentary input energy per unit mass ($\Delta E_{BI,\max}/m$) can be evaluated as the maximum value calculated by Eq. (18) over the course of the seismic event. The bidirectional maximum momentary input energy spectrum (the bidirectional $V_{\Delta E}$ spectrum) is calculated as

$$V_{\Delta E} = \sqrt{2\Delta E_{BI,\max}/m}. \quad (19)$$

The bidirectional $V_{\Delta E}$ spectrum is prepared at the beginning of the MABPA (step 0). The range of the natural period (T) needs to be properly considered. In this study, a range of T from 0.1 s to 6.0 s (longer than the seismic isolation period) is considered. The assumption of damping is also important. Because only a hysteresis (displacement-dependent) damper is installed in the isolation layer, complex damping was chosen in this study. The complex damping ratios (β) were set to 0.10, 0.20, and 0.30.

3.2.2. Formulation of the effective period and the hysteretic dissipated energy in a half cycle

To predict the peak responses of the first two modes, bilinear idealization of the equivalent acceleration-equivalent displacement relationship was made as follows. In the following discussion, the formulations are made for the first modal response. Figure 14 shows the bilinear idealization procedure used in this study.

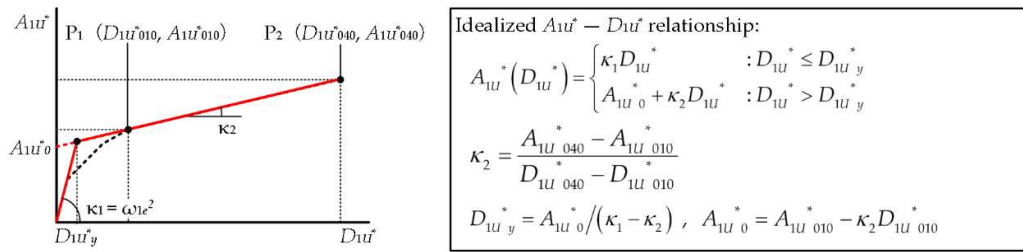


Figure 14. Bilinear idealization of the equivalent acceleration–equivalent displacement relationship.

Then, the effective period (T_{1eff}) and the hysteretic dissipated energy in the half cycle per unit mass ($\Delta E_{\mu 1u}^* / M_{1u}^*$) are formulated. Figure 15 shows the modeling of the structural response in a half cycle used to predict the peak response. In Figure 15, the parameter η is the displacement ratio in the positive and negative directions and is assumed to be in the range from 0 to 1. In addition, the horizontal deformation of the isolation layer is assumed to be predominant in the first modal response.

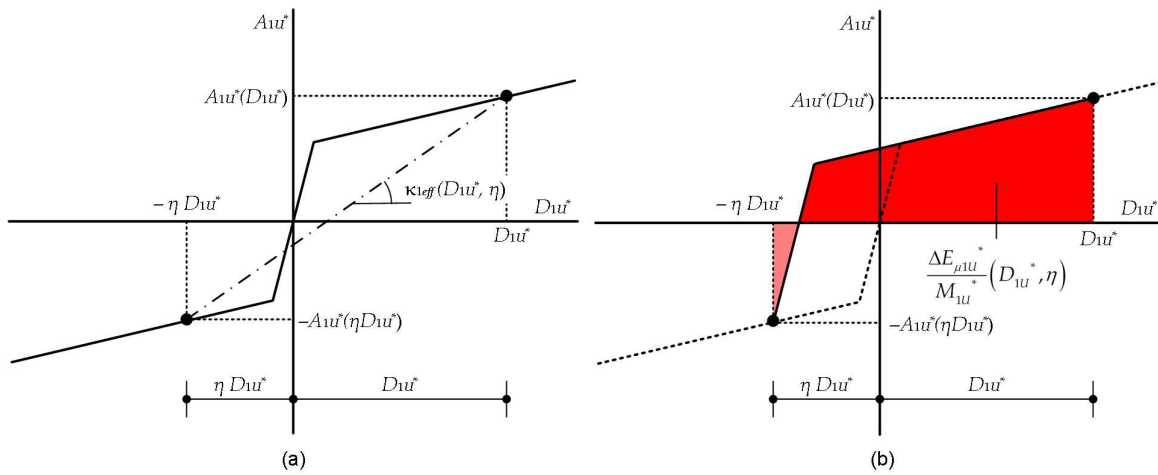


Figure 15. Modeling of the structural response in a half cycle used to predict the peak response: (a) the effective slope (κ_{1eff}) and (b) the hysteretic dissipated energy in a half cycle per unit mass ($\Delta E_{\mu 1u}^* / M_{1u}^*$).

In this study, the effective period of the first mode corresponding to D_{1u}^* is calculated as

$$T_{1eff}(D_{1u}^*) = 2\pi / \sqrt{\kappa_{1eff}(D_{1u}^*)}. \quad (20)$$

In addition, the equivalent velocity of the hysteretic dissipated energy in a half cycle corresponding to D_{1u}^* is calculated as

$$V_{\Delta E_{\mu 1u}^*}(D_{1u}^*) = \sqrt{\frac{2\Delta E_{\mu 1u}^*}{M_{1u}^*}(D_{1u}^*)}. \quad (21)$$

The effective slope corresponding to D_{1u}^* is calculated as

$$\kappa_{1eff}(D_{1U}^*) = \int_0^1 \kappa_{1eff}(D_{1U}^*, \eta) d\eta$$

$$= \begin{cases} \kappa_1 & : D_{1U}^* \leq D_{1U}^{*y} \\ \left(2\ln 2\right) \frac{A_{1U0}^*}{D_{1U}^*} + \kappa_2 - \left\{ \ln \left(1 + \frac{D_{1U}^{*y}}{D_{1U}^*}\right) \right\} \left(\frac{A_{1U0}^*}{D_{1U}^*} + \kappa_1 - \kappa_2 \right) + \frac{D_{1U}^{*y}}{D_{1U}^*} (\kappa_1 - \kappa_2) & : D_{1U}^* > D_{1U}^{*y} \end{cases} \quad (22)$$

The hysteretic dissipated energy in a half cycle per unit mass corresponding to D_{1U}^* is calculated as

$$\frac{\Delta E_{\mu 1U}}{M_{1U}^*}(D_{1U}^*) = \int_0^1 \frac{\Delta E_{\mu 1U}}{M_{1U}^*}(D_{1U}^*, \eta) d\eta$$

$$= \begin{cases} \frac{1}{3} \kappa_1 D_{1U}^{*2} & : D_{1U}^* \leq D_{1U}^{*y} \\ \frac{1}{3} \kappa_2 D_{1U}^{*2} + \frac{3}{2} A_{1U0}^* D_{1U}^* \left\{ \left(1 - \frac{D_{1U}^{*y}}{D_{1U}^*}\right)^2 + \frac{2}{3} \frac{D_{1U}^{*y}}{D_{1U}^*} \left(1 - \frac{2}{3} \frac{D_{1U}^{*y}}{D_{1U}^*}\right) \right\} & : D_{1U}^* > D_{1U}^{*y} \end{cases} \quad (23)$$

The predicted peak response D_{1U}^{*max} is obtained from the following equation:

$$V_{\Delta E \mu 1U}^* \{T_{1eff}^*(D_{1U}^{*max})\} = V_{\Delta E} \{T_{1eff}^*(D_{1U}^{*max})\}. \quad (24)$$

Note that no viscous damping is considered because the isolation layer is assumed to have no viscous damping.

4. Analysis Results

The analysis results are split into two subsections. In the first subsection, an example of the prediction of the peak equivalent displacement using the updated MABPA is shown. In the second subsection, comparisons are made between the nonlinear time-history analysis results and the predicted results.

4.1. Example of a prediction of the peak equivalent displacement

The structural model shown here as an example is Model-Tf44. Figure 16 shows the nonlinear properties of the equivalent SDOF model calculated according to Section 3.2.2.

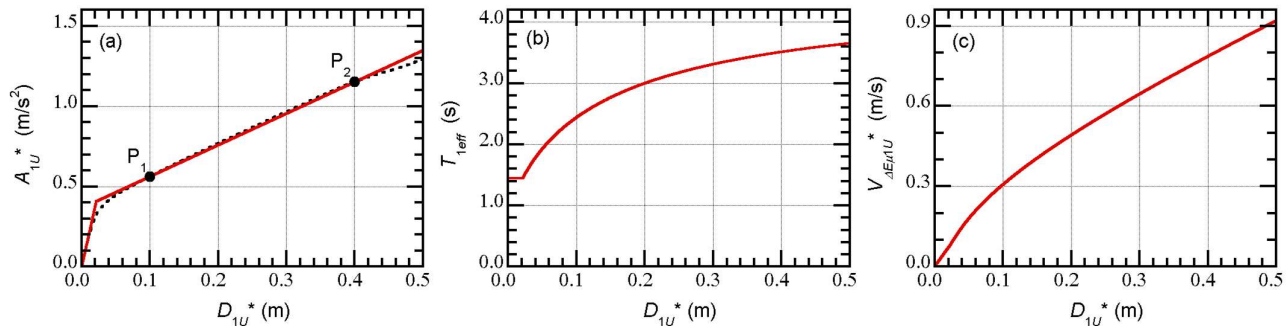


Figure 16. Nonlinear properties of the equivalent single-degree-of-freedom (SDOF) model representing the first modal response (Model-Tf44): (a) the $A_{1U}^*-D_{1U}^*$ relationship; (b) the $T_{1eff}^*-D_{1U}^*$ relationship; and (c) the $V_{\Delta E \mu 1U}^*-D_{1U}^*$ relationship.

In the results shown in Figures 16(b) and 16(c), the $V_{\Delta E \mu 1U}^* - T_{1eff}$ curve is constructed and overlaid with the bidirectional $V_{\Delta E}$ spectrum. Figure 17 shows the prediction of the peak equivalent displacements of the first two modes.

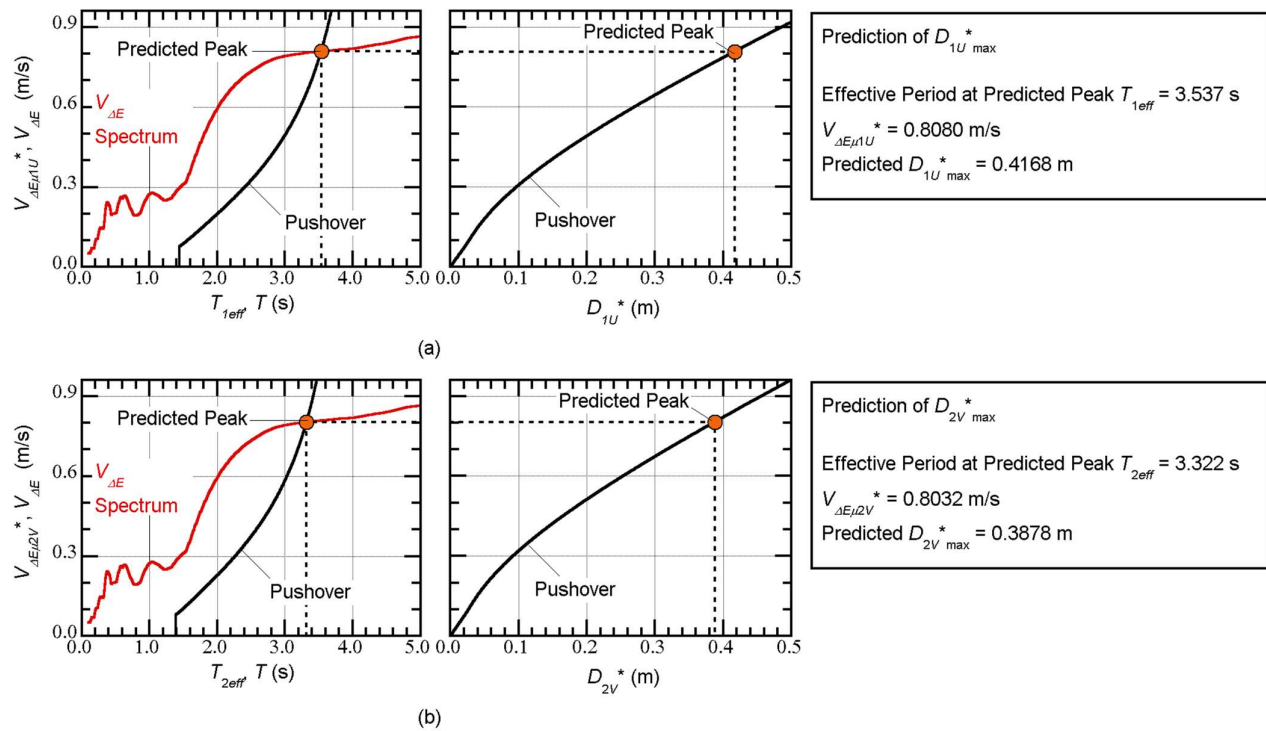


Figure 17. Prediction of the peak equivalent displacements (model: Model-Tf44, ground motion: TCU): (a) prediction of the peak of the first modal response and (b) prediction of the peak of the second modal response.

The predicted peak was obtained as the intersection point of the $V_{\Delta E \mu 1U}^* - T_{1eff}$ curve according to the pushover analysis results and the bidirectional $V_{\Delta E}$ spectrum, as shown on the left side of Figure 17(a). Then, the peak equivalent displacement $D_{1U}^*_{max}$ was predicted from the $V_{\Delta E \mu 1U}^* - D_{1U}^*_{max}$ curve. The prediction of the peak equivalent displacement of the second mode, $D_{2V}^*_{max}$, can be made using the same procedure as that of $D_{1U}^*_{max}$, as shown in Figure 17(b). Note that, for the predictions of $D_{1U}^*_{max}$ and $D_{2V}^*_{max}$, the same $V_{\Delta E}$ spectrum (bidirectional $V_{\Delta E}$ spectrum) is used.

4.2. Comparisons with the nonlinear time-history analysis

In Section 4.2.1, the nonlinear time-history analysis results using 24 artificial ground motion datasets are compared with the predicted results. In this analysis, $24 \times 4 = 96$ cases are analyzed for each model: the angle of incidence of the horizontal ground motion (ψ) in Figure 3 is set to -45° , 0° , 45° , and 90° considering the symmetry of the force-deformation relationship assumed in the structural model. Because the characteristics of the two horizontal components are similar, the discussion focuses on comparisons between the envelope of the time-history analysis results and the predicted results.

In Section 4.2.2, the nonlinear time-history analysis results using the four recorded ground motion datasets are compared with the predicted results. In this analysis, $4 \times 12 = 48$ cases are analyzed for each model: the angle of incidence of the horizontal ground motion (ψ) is set to values from -75° to 90° with an interval of 15° .

4.2.1. Artificial ground motion

Figures 18 and 19 show the peak relative displacement at $X_3A_3Y_3$, which is the closest point to the center of the floor mass at each level. In these figures, the nonlinear time-history analysis results are compared with the results predicted by MABPA (for complex

damping ratios (β) of 0.10, 0.20, and 0.30). As shown here, the predicted peaks can approximate the envelope of the time-history analysis results, except in the case of Model-Tf34 subjected to the Art-2 series shown in Figure 18(b). In addition, the predicted peak with $\beta = 0.10$ is larger than that with $\beta = 0.30$. When the models are subjected to the Art-1 series, the predicted peak closest to the envelope of the time-history analysis results is found with $\beta = 0.10$ for Model-Tf34 (Figure 18(a)) and with $\beta = 0.30$ for Model-Tf44 (Figure 18(b)). Differences in the predicted peaks resulting from the value of the complex damping are small in the case of Model-Tf34 but relatively noticeable in the case of Model-Tf44.

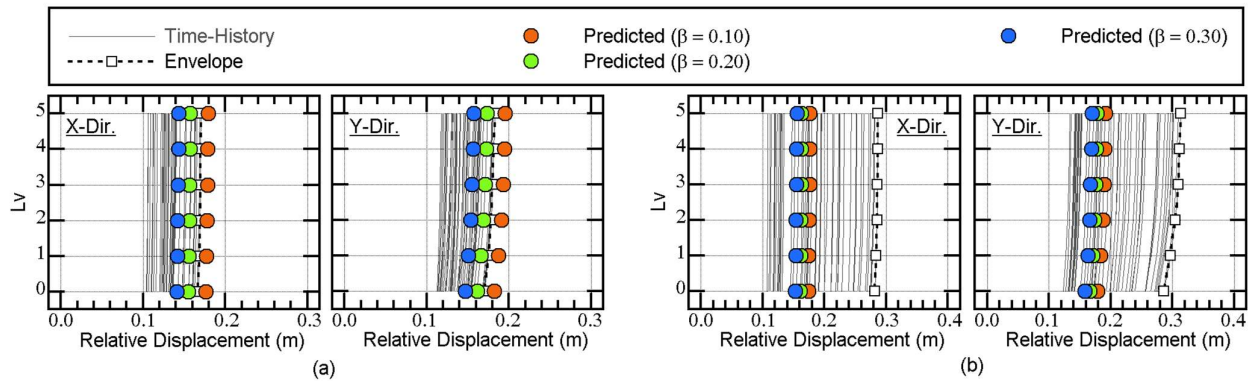


Figure 18. Comparison of the peak relative displacements at $X_{3A}Y_3$ (Model-Tf34) for the (a) Art-1 and (b) Art-2 series.

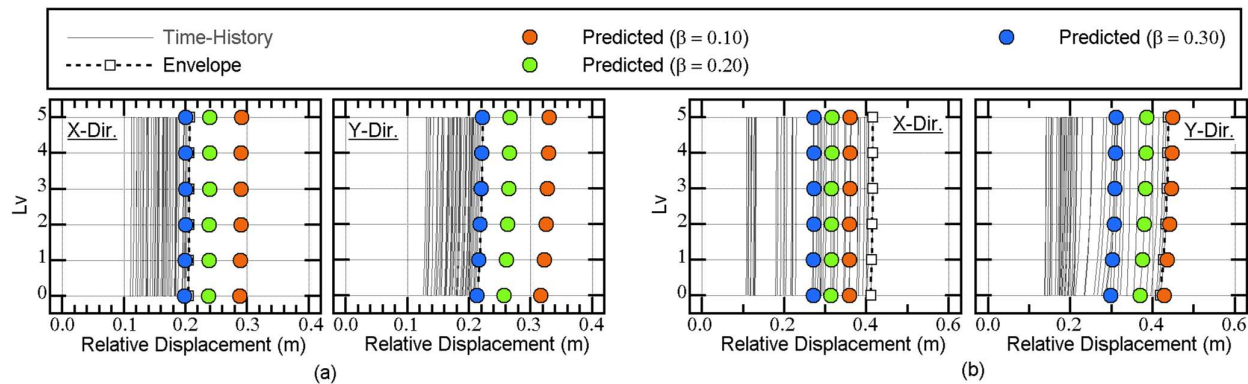


Figure 19. Comparison of the peak relative displacements at $X_{3A}Y_3$ (Model-Tf44) for the (a) Art-1 and (b) Art-2 series.

Figures 20 and 21 show comparisons of the horizontal distributions of the peak displacement at level 0, which is just above the isolation layer. In Figure 20, the results of Model-Tf34 subjected to the Art-1 series are shown. The predicted peak agrees with the envelope of the time-history analysis results, even though the predicted peak is not conservative at Y_1 and Y_2 (the stiff side in the X-direction). A similar observation can be made in the case of Model-Tf44 subjected to the Art-1 series, as shown in Figure 21, where the predicted peak with $\beta = 0.30$ agrees very well with the envelope of the time-history analysis results, except at Y_1 and Y_2 .

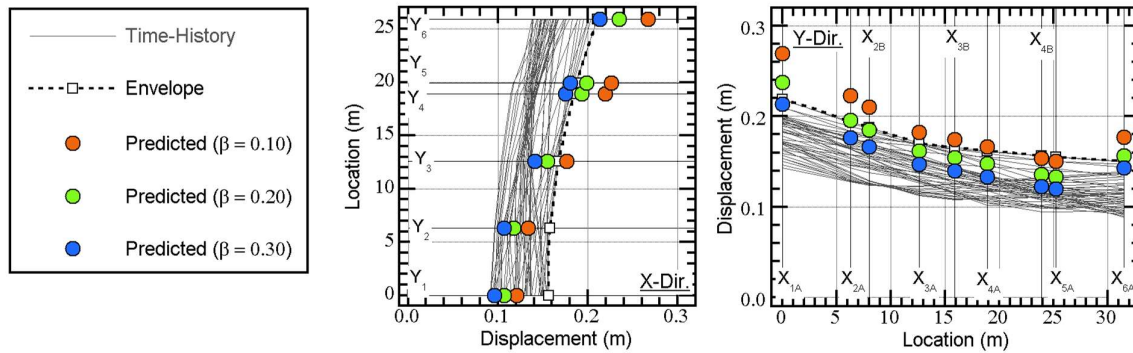


Figure 20. Comparison of the horizontal distributions of the peak displacement at level 0 (model: Model-Tf34, ground motion: Art-1 series).

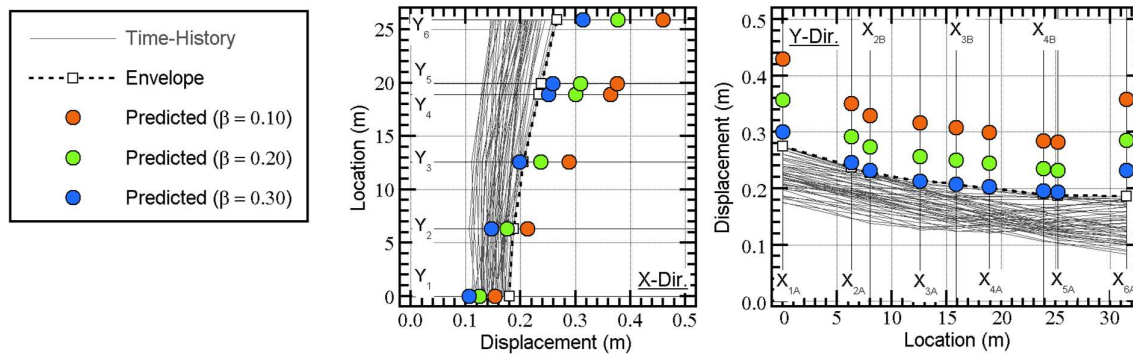


Figure 21. Comparison of the horizontal distributions of the peak displacement at level 0 (model: Model-Tf44, ground motion: Art-1 series).

Based on the above results, the updated MABPA can predict the peak relative displacement at $X_{3A}Y_3$ and the horizontal distribution of the peak displacement at level 0 for both models according to the artificial ground motion datasets.

4.2.2. Recorded ground motion

Figures 22 and 23 show the peak relative displacement at $X_{3A}Y_3$. The accuracy of the predicted peak displacement is satisfactory: the predicted peak approximates the envelope of the nonlinear time-history analysis results, even though some cases are overestimated (e.g., model: Model-Tf44, ground motion: UTO0416, Figure 23(b)). The variation of the predicted peak due to the assumed complex damping ratio (β) depends on the ground motion. In some cases, the largest peak is obtained when β is 0.30 (e.g., model: Model-Tf34, ground motion: UTO0416, Figure 22(b)), while in other cases, the largest peak is obtained when β is 0.10 (e.g., model: Model-Tf34, ground motion: TCU, Figure 22(c)).

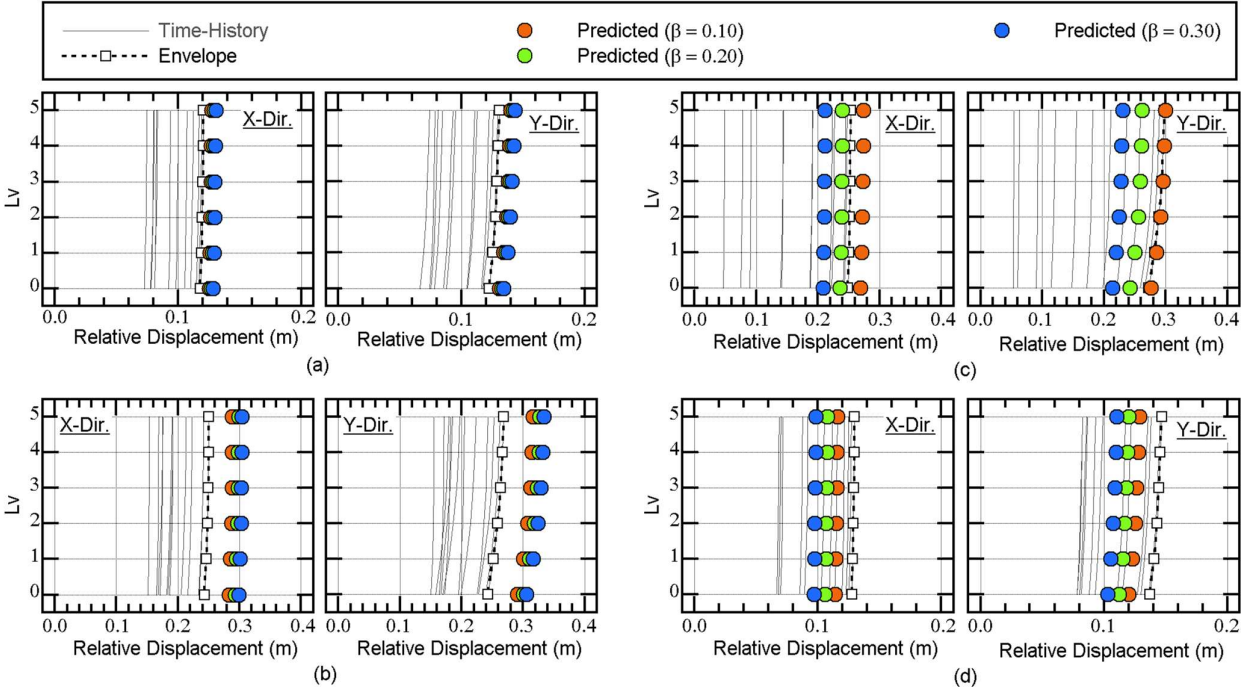


Figure 22. Comparison of the peak relative displacements at X_{3A}Y₃ (Model-Tf34): (a) UTO0414; (b) UTO0416; (c) TCU; and (d) YPT.

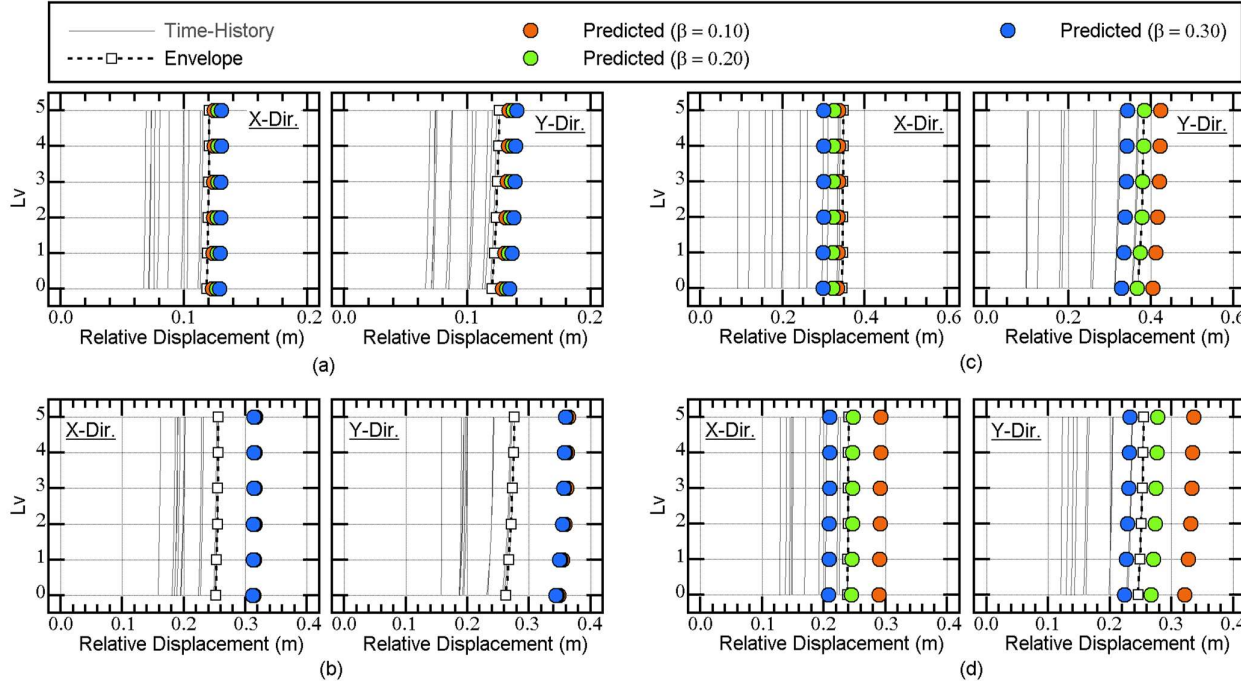


Figure 23. Comparison of the peak relative displacements at X_{3A}Y₃ (Model-Tf44): (a) UTO0414; (b) UTO0416; (c) TCU; and (d) YPT.

Figure 24 shows comparisons of the horizontal distribution of the peak displacement at level 0. In this figure, the Model-Tf34 results are shown, with the results for UTO0414 shown in Figure 24(a) and those for TCU shown in Figure 24(b).

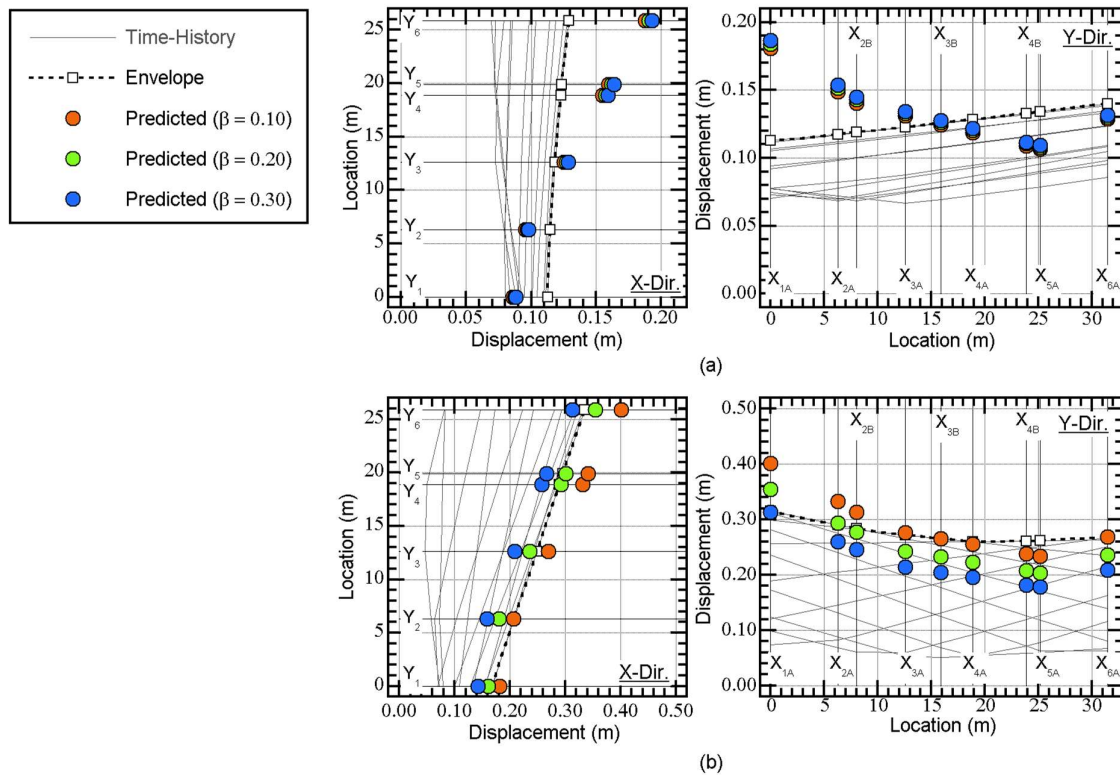


Figure 24. Comparison of the horizontal distributions of the peak displacement at level 0 (model: Model-Tf34) for (a) UTO0414 and (b) TCU.

The predicted horizontal distribution of the peak displacement in the Y-direction is notably different from the envelope of the time-history analysis in the case of UTO0414, as shown in Figure 24(a). In the predicted distribution, the largest displacement occurs at X_{1A} (the flexible side in the Y-direction); meanwhile, in the envelope of the time-history analysis results, the displacement at X_{1A} is the smallest. Conversely, the predicted horizontal distribution of the peak displacement in the Y-direction fits the envelope of the time-history analysis in the case of TCU very well, as shown in Figure 24(b). In the predicted distribution, the largest displacement occurs at X_{1A} , which is consistent with the envelope of the time-history analysis results. The modal displacement responses at X_{1A} and X_{6A} (the stiff side in the Y-direction) are compared and discussed further in Section 5.5.

One of the motivations of this analysis is to evaluate the seismic performance of the retrofitted building models under the recorded motions from the 2016 Kumamoto earthquake. It has already been reported in a previous study [33] that the seismic capacity evaluation of the main building of the former Uto City Hall indicated that the structure was insufficient to withstand the earthquake that occurred on April 14, 2016. Therefore, the drift of the superstructure was examined as follows.

Figure 25 shows comparisons of the peak drift for the three columns in Model-Tf34 in cases with UTO0414 and UTO0416. The peak drifts for the three columns obtained from the nonlinear time-history analysis results are smaller than 0.4%. The largest peak drift occurs for column A_3B_3 on the second story, with a value of 0.12% in the case of UTO0414 (Figure 25(a)) and a value of 0.30% in the case of UTO0416 (Figure 25(b)). Therefore, the seismic performance of Model-Tf34 is excellent for the motions recorded during the 2016 Kumamoto earthquake. Figure 26 shows comparisons of the peak drift of Model-Tf44. Similar observations can be made for Model-Tf44. Comparisons of Figures 25 and 26 indicate that the peak drift of Model-Tf44 is smaller than that of Model-Tf34. These figures

also illustrate that the accuracy of the predicted peak depends on the column, with the accuracy at column A_1B_1 being satisfactory and that at column A_3B_3 being less satisfactory.

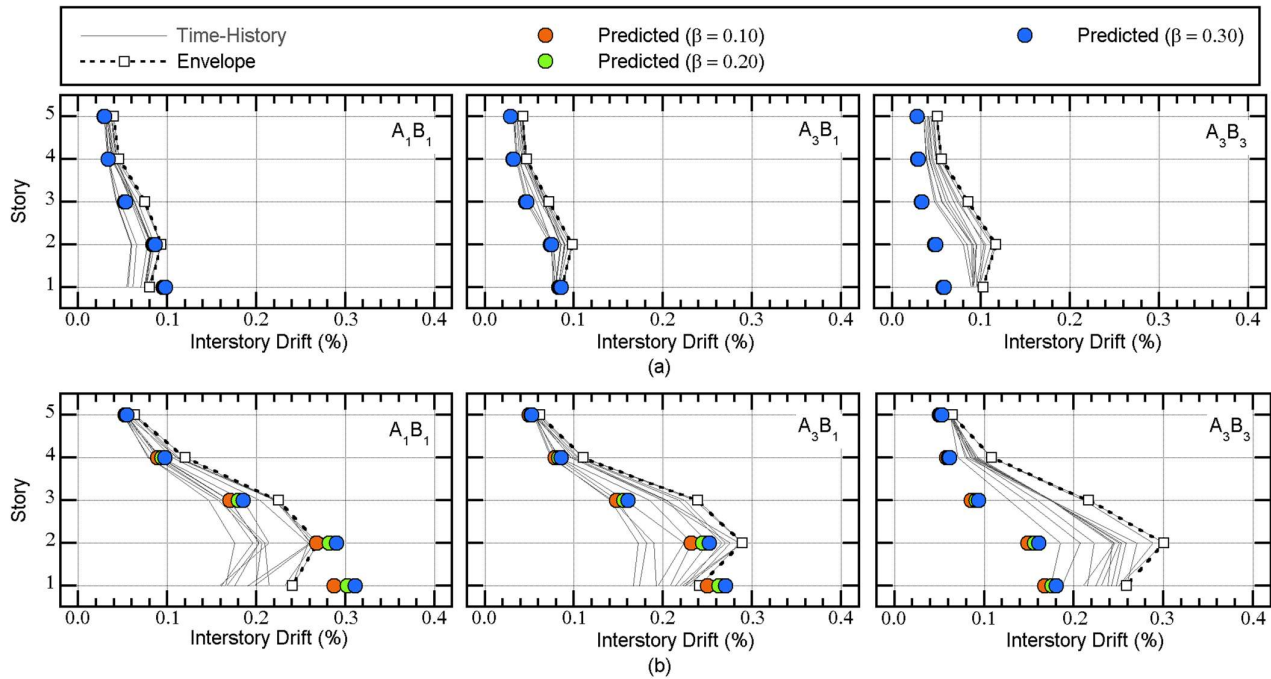


Figure 25. Comparison of the peak drift for the columns (Model-Tf34) with (a) UTO0414 and (b) UTO0416.

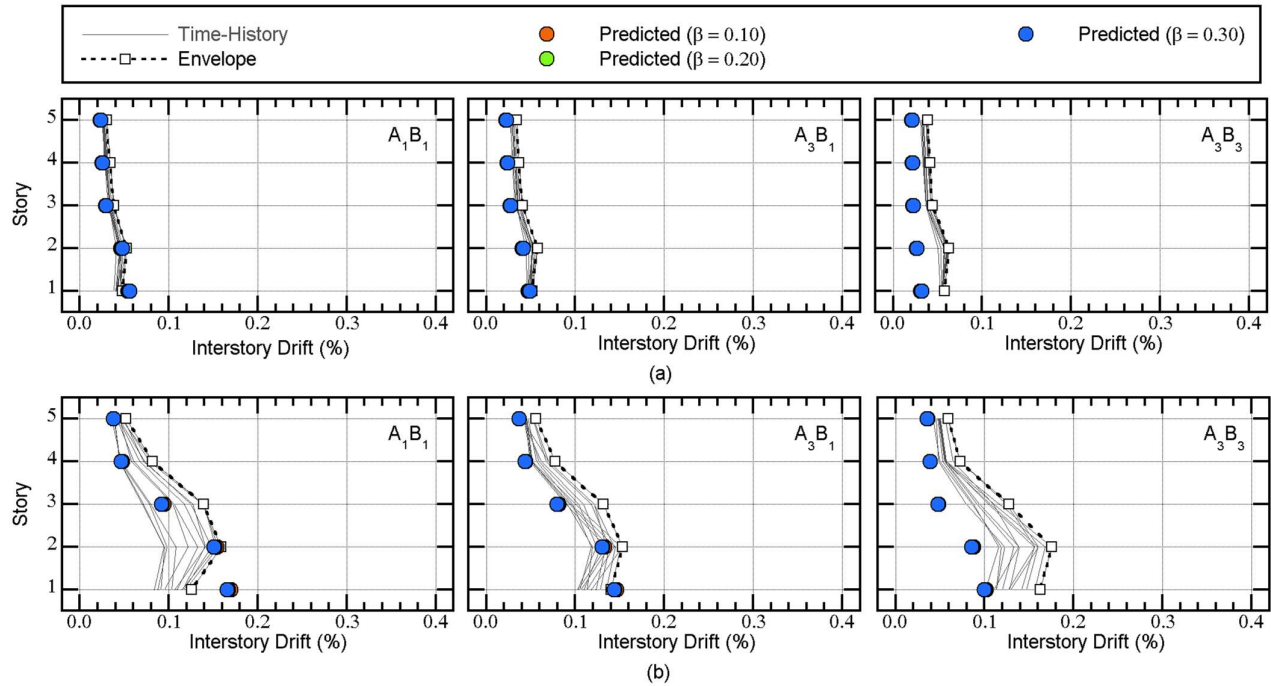


Figure 26. Comparison of the peak drift for the columns (Model-Tf44) with (a) UTO0414 and (b) UTO0416.

Figures 27 and 28 show the peak displacements at two isolators ($X_{1A}Y_6$ and $X_{6A}Y_1$) for various angles of incidence of seismic input (ψ). In these figures, the displacement of each isolator is calculated as the absolute (vector) value of the two horizontal directions and

the predicted peaks are shown by the colored lines. As shown in these figures, the upper bounds of the peak displacement of the isolators can be satisfactorily evaluated using the updated MABPA presented in this study.

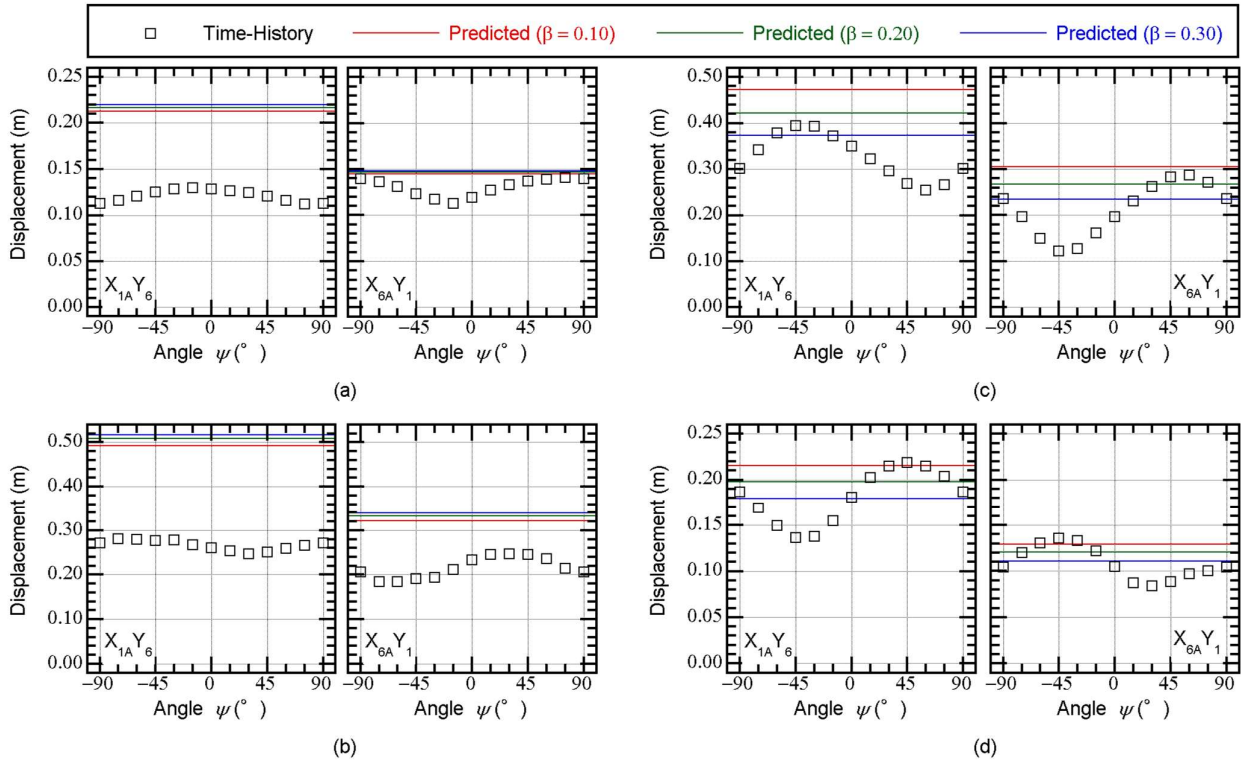


Figure 27. Peak displacement at isolators for various angles of incidence of seismic input (Model-Tf34): (a) UTO0414; (b) UTO0416; (c) TCU; and (d) YPT.

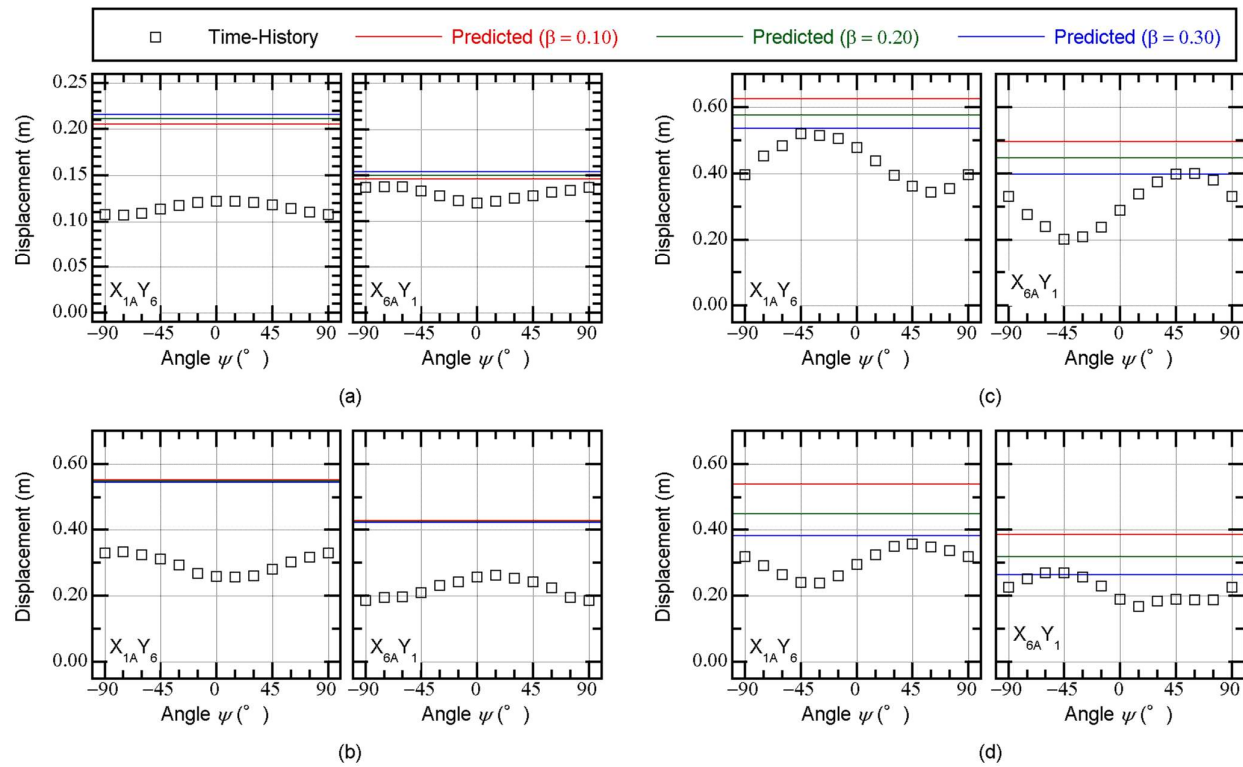


Figure 28. Peak displacement at isolators for various angles of incidence of seismic input (Model-Tf44): (a) UTO0414; (b) UTO0416; (c) TCU; and (d) YPT.

These figures also indicate that the variation in the peak displacement of the isolators due to the angle of incidence of the seismic input is noticeable, e.g., in the cases of TCU and YPT, for both models. This point is discussed in Section 5.

5. Discussion

Here, there are four points of focus: (i) the relationship between the maximum momentary input energy and the peak equivalent displacement of the first modal response and its predictability from the pushover analysis results, (ii) the predictability of the maximum momentary input energy of the first modal response from the bidirectional $V_{\Delta E}$ spectrum, (iii) the accuracy of the predicted peak equivalent displacements of the first and second modal responses, and (iv) the contribution of the higher mode to the displacement response at the edge of level 0.

5.1. Calculation of the modal responses

Figure 29 shows the calculation flow for the first and second modal responses from the nonlinear time-history analysis results. This procedure is based on the procedure originally proposed by Kuramoto [49] for analyzing the nonlinear modal response of a non-isolated planar frame structure extended to analyze that of three-dimensional base-isolated structures.

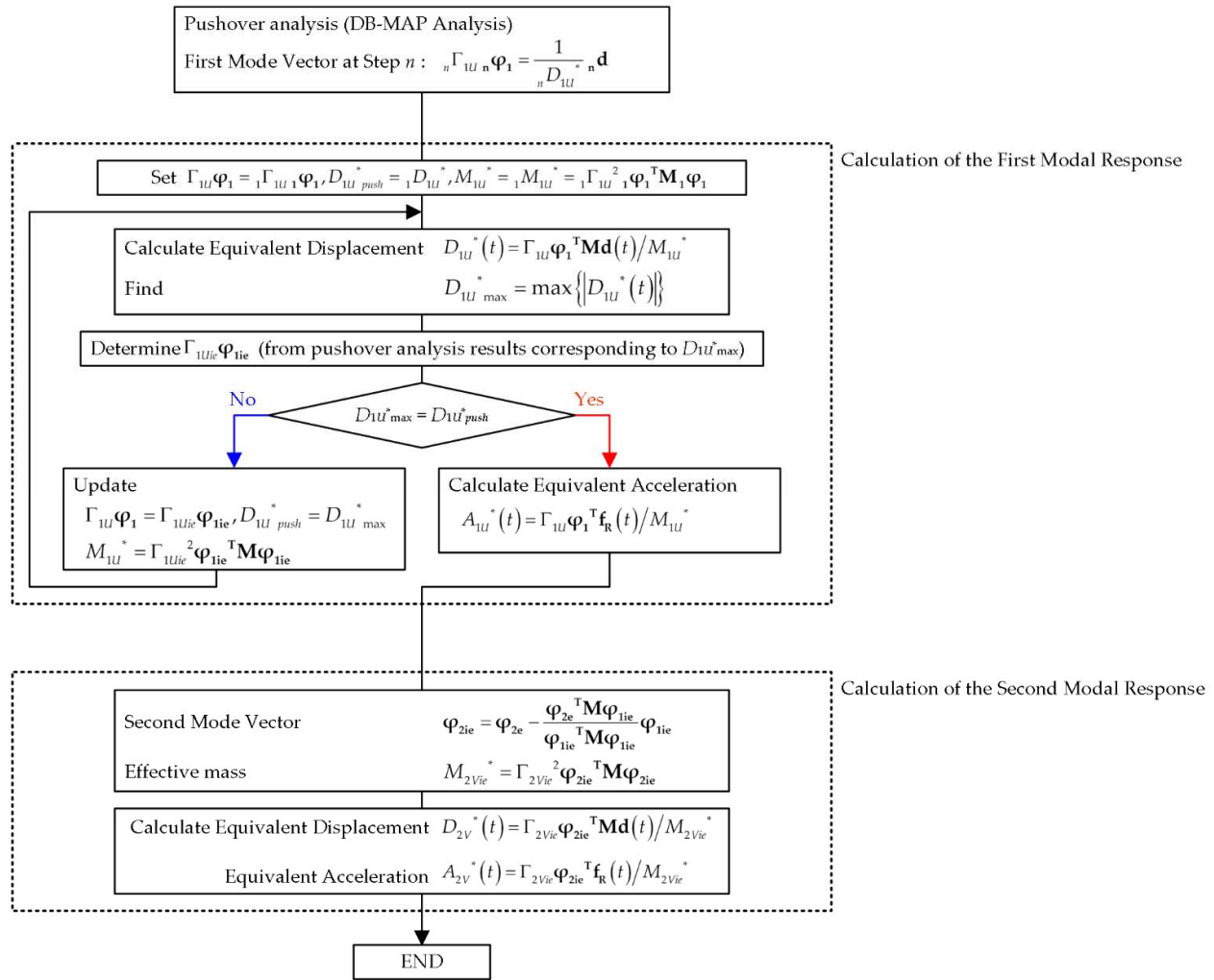


Figure 29. Calculation flow for the first and second modal responses.

Next, the momentary input energy of the first modal response per unit mass was calculated as follows. From the time-history of the equivalent displacement of the first modal response ($D_{1U}^*(t)$), the momentary input energy of the first modal response per unit mass is calculated as

$$\frac{\Delta E_{1U}^*}{M_{1U}^*} = - \int_t^{t+\Delta t} \dot{D}_{1U}^*(t) a_{gU}(t) dt. \quad (25)$$

In Eq. (25), $\dot{D}_{1U}^*(t)$ is the derivative of $D_{1U}^*(t)$ with respect to t , $a_{gU}(t)$ is the ground acceleration component of the U-axis (the principal axis of the first modal response corresponds to $D_{1U}^*_{max} = \max\{|D_{1U}^*(t)|\}$), and Δt is the duration of a half cycle of the first modal response. The maximum momentary input energy of the first modal response per unit mass ($\Delta E_{1U}^*_{max} / M_{1U}^*$) is the maximum value of $\Delta E_{1U}^* / M_{1U}^*$ calculated by Eq. (25) over the course of the seismic event. Figure 30 shows the definition of the maximum momentary input energy of the first modal response per unit mass. As shown in this figure, the half cycle when the maximum equivalent displacement ($D_{1U}^*_{max}$) occurs corresponds to the half cycle when the maximum momentary energy input occurs.

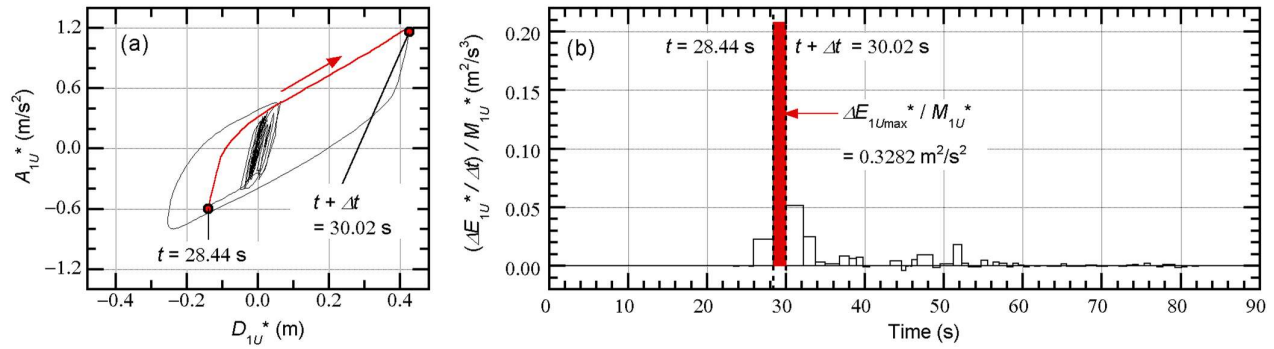


Figure 30. Definition of the maximum momentary input energy of the first modal response per unit mass (structural model: Model-Tf44, ground motion: TCU, angle of incidence of seismic input: $\psi = -30^\circ$): (a) hysteresis of the first modal response and (b) time-history of the momentary energy input.

The equivalent velocity of the maximum momentary input energy of the first modal response ($V_{\Delta E_{1U}}^*$) and the response period of the first modal response (T_1') are calculated such that

$$V_{\Delta E_{1U}}^* = \sqrt{2\Delta E_{1U \max}^* / M_{1U}^*}, \quad (26)$$

$$T_1' = 2\Delta t. \quad (27)$$

5.2. Relationship between the peak equivalent displacement and the maximum momentary input energy of the first modal response

Figures 31 and 32 show comparisons between the $V_{\Delta E_{1U}}^* - D_{1U}^*$ curve and the $V_{\Delta E_{1U}}^* - D_{1U \max}^*$ relationship obtained from the nonlinear time-history analysis, with the results of Models Tf34 and Tf44 shown in Figures 31 and 32, respectively. These figures confirm that the $V_{\Delta E_{1U}}^* - D_{1U}^*$ curve fits the plots obtained from the time-history analysis results very well. Therefore, the accuracy of the predicted $D_{1U \max}^*$ relies on the accuracy of the predicted $V_{\Delta E_{1U}}^*$, which is discussed in the next subsection.

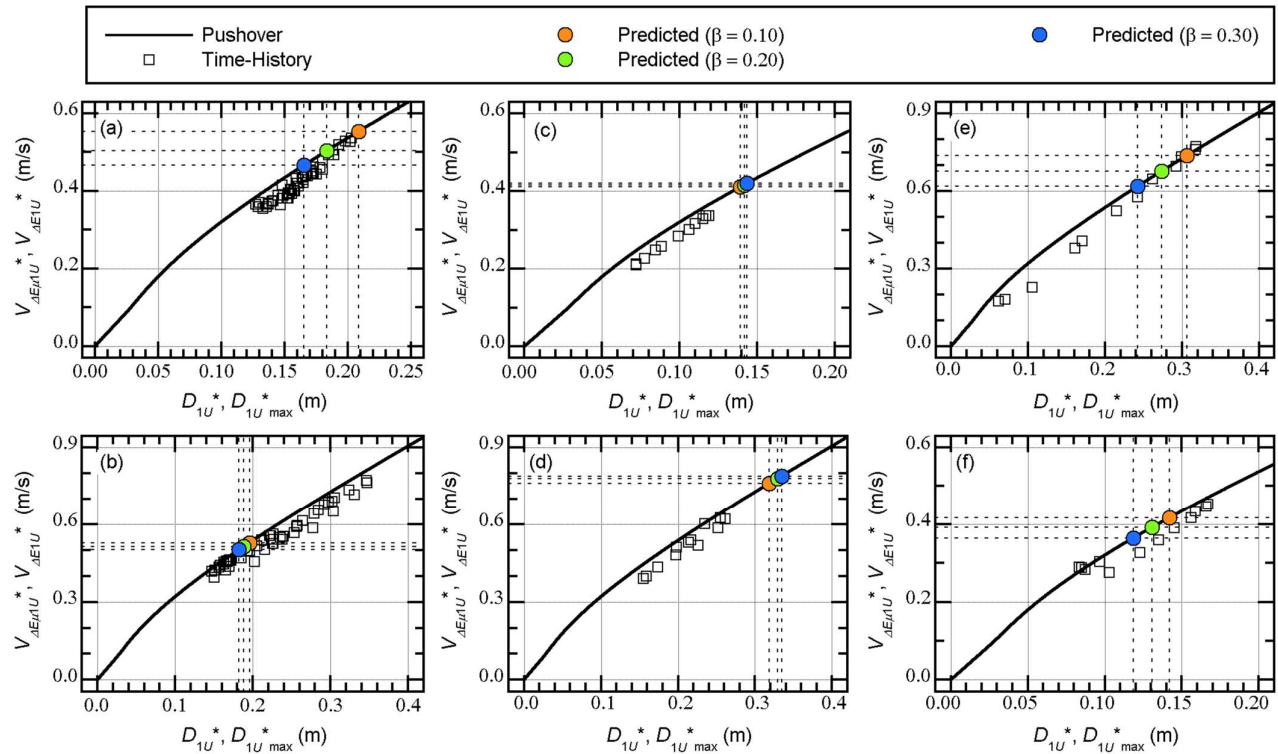


Figure 31. Comparison between the $V_{\Delta E_{1U}}^* - D_{1U}^*$ curve and the $V_{\Delta E_{1U}}^* - D_{1U}^*_{max}$ relationship obtained from the time-history analysis (Model-Tf34): (a) Art-1 series; (b) Art-2 series; (c) UTO0414; (d) UTO0416; (e) TCU; (f) and YPT.

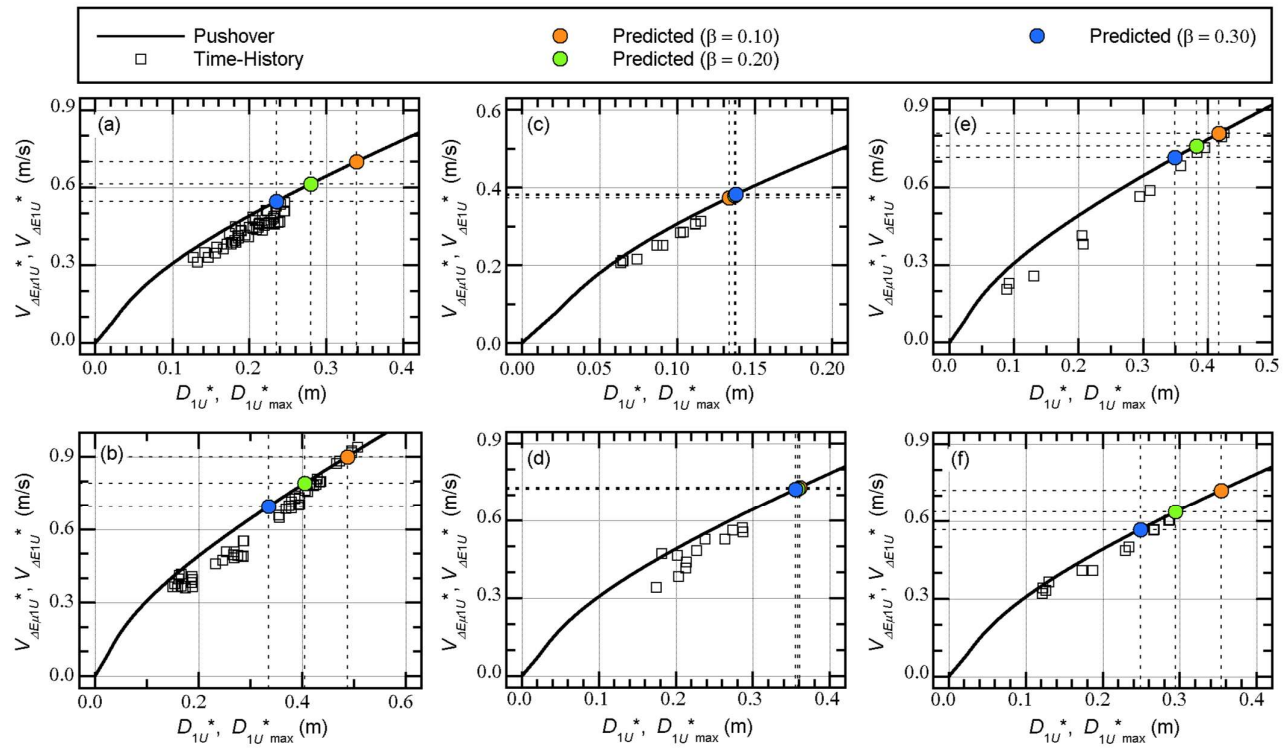


Figure 32. Comparison between the $V_{\Delta E_{1U}}^* - D_{1U}^*$ curve and the $V_{\Delta E_{1U}}^* - D_{1U}^*_{max}$ relationship obtained from the time-history analysis (Model-Tf44): (a) Art-1 series; (b) Art-2 series; (c) UTO0414; (d) UTO0416; (e) TCU; (f) and YPT.

5.3. Comparison of the maximum momentary input energy and the bidirectional momentary input energy spectrum

Figure 33 shows the prediction of $V_{\Delta E1U}^*$ from the bidirectional $V_{\Delta E}$ spectrum for Model-Tf34. The plots shown in this figure indicate the nonlinear time-history analysis results.

In most cases, the bidirectional $V_{\Delta E}$ spectrum with complex damping $\beta = 0.10$ approximates the upper bound of the plot of the time-history analysis results, as shown in Figures 33(a), 33(b), 33(e), and 33(f). However, in the other cases shown in Figures 33(c) and 33(d), the plots of the time-history analysis results are below those of the bidirectional $V_{\Delta E}$ spectrum.

From the comparisons between the predicted $V_{\Delta E1U}^*$ and that obtained from the time-history analysis results, the predicted $V_{\Delta E1U}^*$ provides a conservative estimation, except in the case of the Art-2 series shown in Figure 33(b). This is because the predicted response points correspond to the “valley” of the bidirectional $V_{\Delta E}$ spectrum, therefore making the predicted $V_{\Delta E1U}^*$ smaller.

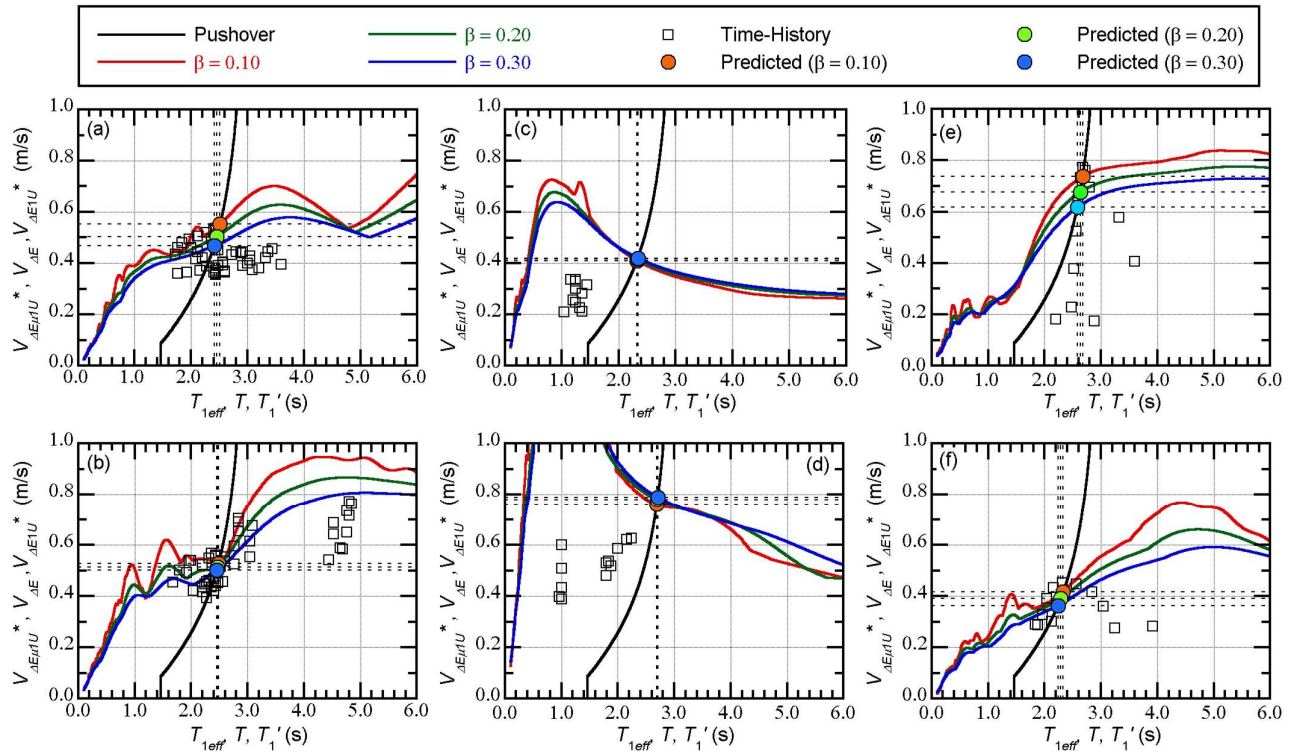


Figure 33. Prediction of $V_{\Delta E1U}^*$ from the bidirectional $V_{\Delta E}$ spectrum and its accuracy (Model-Tf34): (a) Art-1 series; (b) Art-2 series; (c) UTO0414; (d) UTO0416; (e) TCU; and (f) YPT.

Figure 34 shows the prediction of $V_{\Delta E1U}^*$ from the bidirectional $V_{\Delta E}$ spectrum for Model-Tf44. Similar observations to those made for Model-Tf34 can be made for Model-Tf44.

Based on the above results, it can be concluded that that bidirectional $V_{\Delta E}$ spectrum can approximate the upper bound of the equivalent velocity of the maximum momentary input energy of the first mode $V_{\Delta E1U}^*$. Therefore, $V_{\Delta E1U}^*$ can be properly predicted using the bidirectional $V_{\Delta E}$ spectrum.

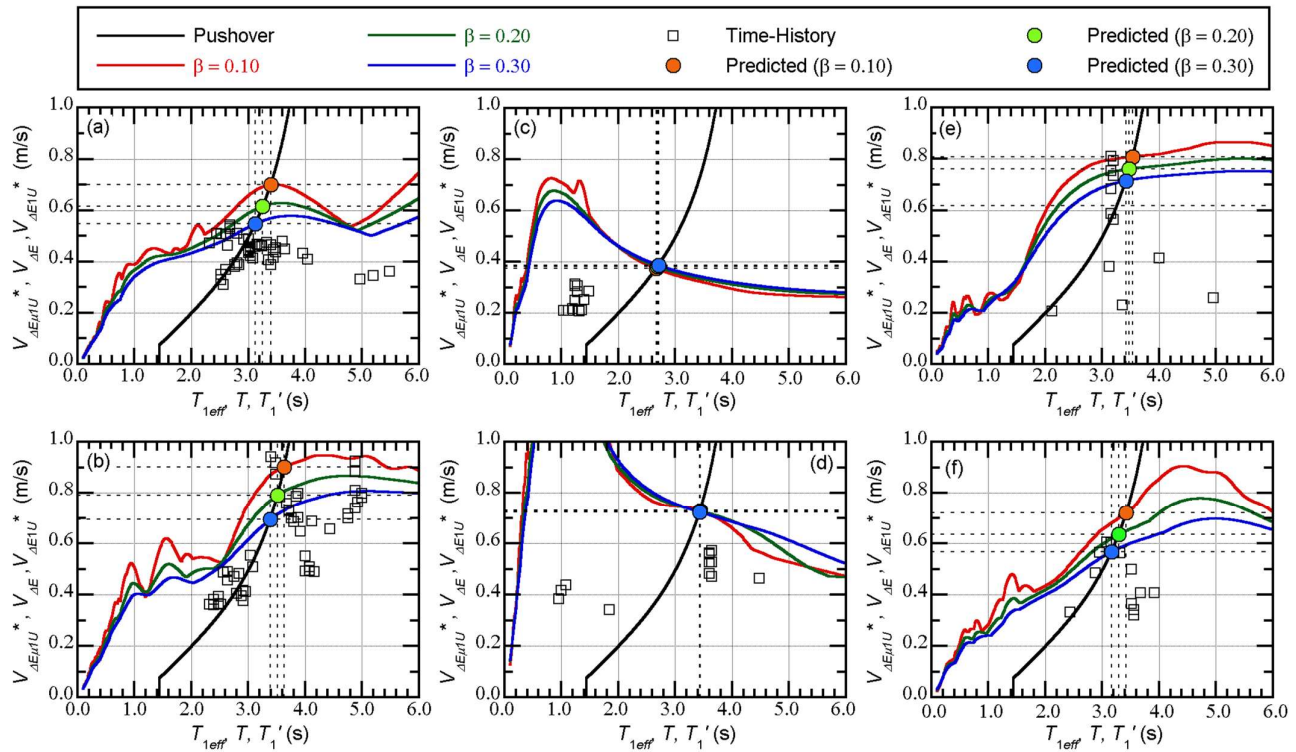


Figure 34. Prediction of $V_{\Delta EIU}^*$ from the bidirectional $V_{\Delta E}$ spectrum and its accuracy (Model-Tf44): (a) Art-1 series; (b) Art-2 series; (c) UTO0414; (d) UTO0416; (e) TCU; and (f) YPT.

5.4. Accuracy of the predicted peak equivalent displacements of the first and second modal responses

Figure 35 shows the accuracy of the predicted peak equivalent displacement of the first and second modal responses, D_{1U}^* and D_{2V}^* , respectively, for Model-Tf34. The predicted peak in the case of complex damping $\beta = 0.10$ approximates the upper bound of the time-history analysis results, except when the ground motion dataset is the Art-2 series (Figure 35(b)) or YPT (Figure 35(f)).

Figure 36 shows the accuracy of the predicted D_{1U}^* and D_{2V}^* for Model-Tf44. The predicted peak in the case of $\beta = 0.10$ approximates the upper bound of the time-history analysis results in all cases.

It is also observed from Figures 35(c)–35(f) and 36(c)–36(f) that the largest D_{1U}^* and D_{2V}^* do not occur simultaneously. To understand this phenomena, Figures 37 and 38 show D_{1U}^* and D_{2V}^* for varying angles of incidence of the seismic input (ψ) for both models. As shown here, the angle where the largest D_{2V}^* occurs is different from the angle where the largest D_{1U}^* occurs, with the difference between the two angles being approximately 90° . This is clearly observed in the cases of TCU and YPT for both models.

Based on the above discussion, the upper bound of the peak equivalent displacements of the first and second modal responses, D_{1U}^* and D_{2V}^* , can be predicted by the updated MABPA.

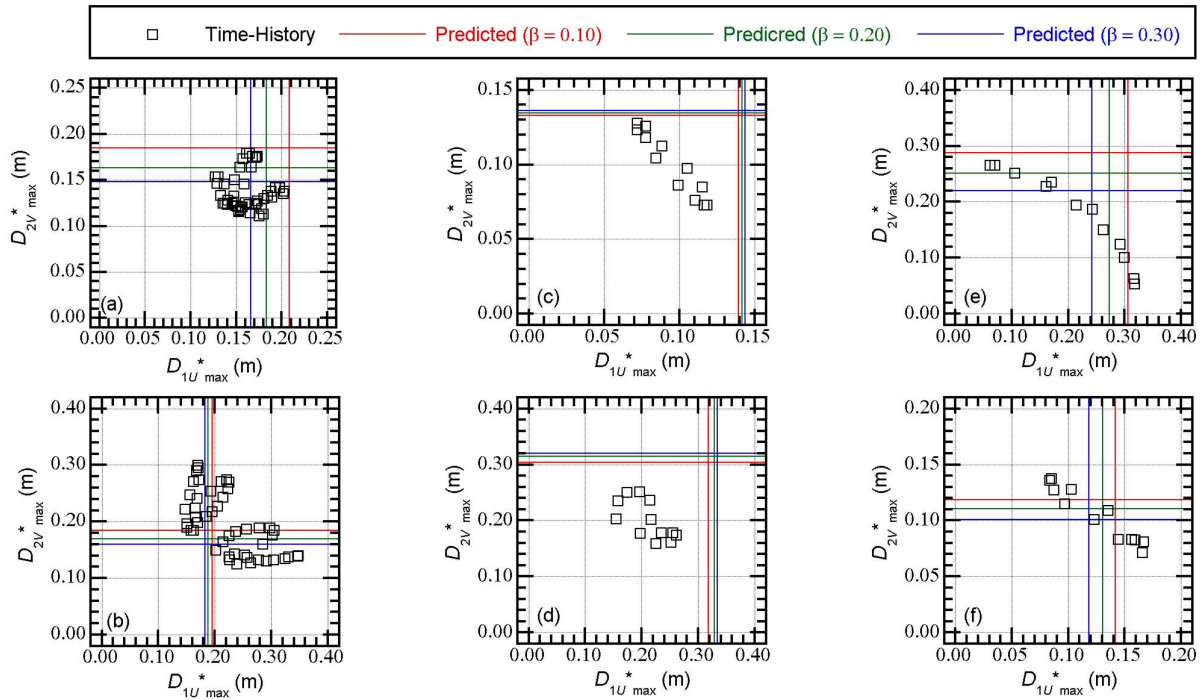


Figure 35. Accuracy of the predicted peak equivalent displacements of the first two modes (Model-Tf34): (a) Art-1 series; (b) Art-2 series; (c) UTO0414; (d) UTO0416; (e) TCU; and (f) YPT.

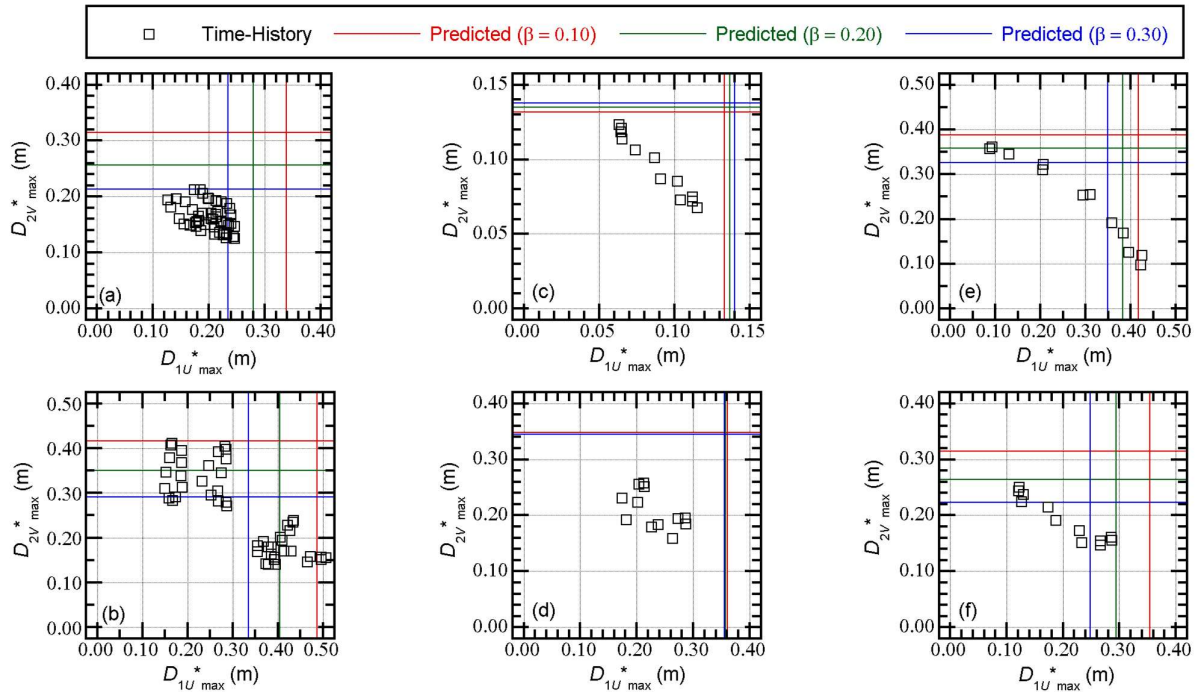


Figure 36. Accuracy of the predicted peak equivalent displacements of the first two modes (Model-Tf44): (a) Art-1 series; (b) Art-2 series; (c) UTO0414; (d) UTO0416; (e) TCU; and (f) YPT.

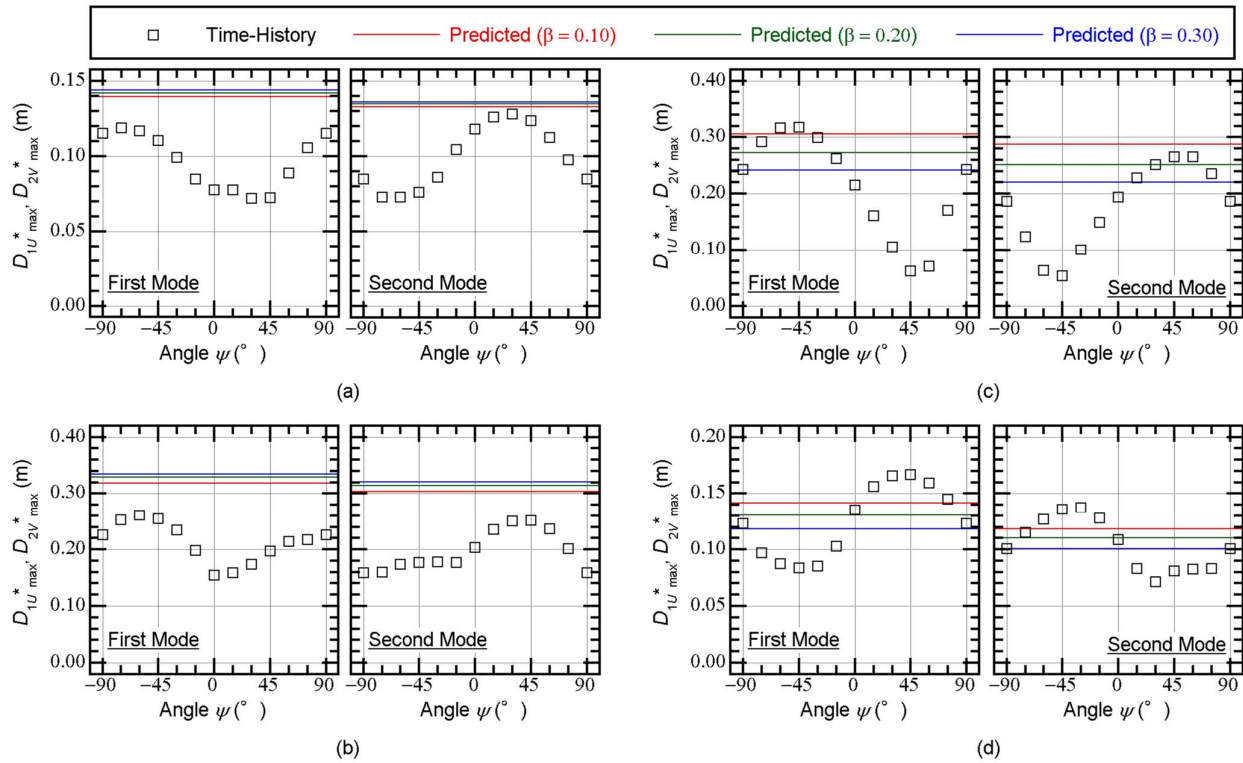


Figure 37. Peak equivalent displacements of the first two modes for various directions of seismic input (Model-Tf34): (a) UTO0414; (b) UTO0416; (c) TCU; and (d) YPT.

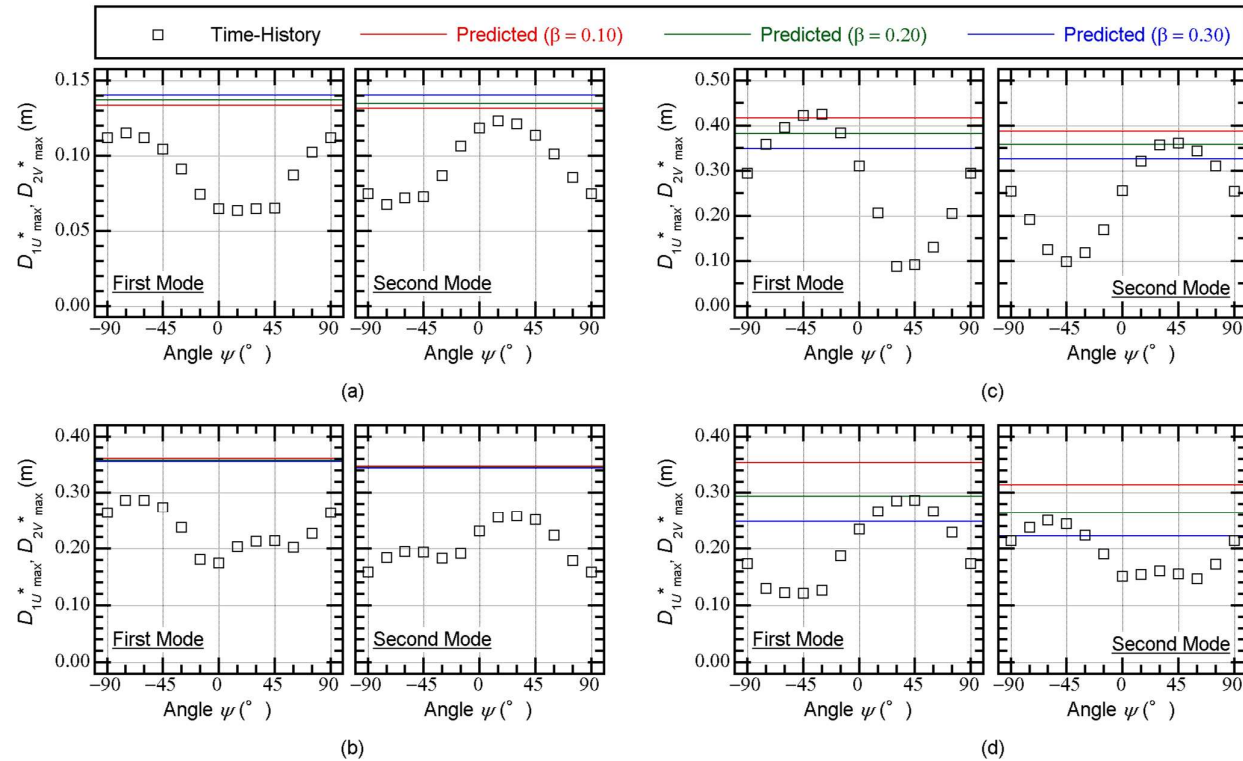


Figure 38. Peak equivalent displacements of the first two modes for various directions of seismic input (Model-Tf44): (a) UTO0414; (b) UTO0416; (c) TCU; and (d) YPT.

5.5. Contribution of the higher mode to the displacement response at the edge of level 0

As discussed in Section 4.2.2, the accuracy of the predicted horizontal distribution of the peak response at level 0 depends on the ground motion dataset. According to Model-Tf34, the predicted largest peak displacement in the Y-direction occurs at X_{1A} (the flexible side in the Y-direction) while the envelope of the nonlinear time-history analysis results indicates that the largest peak occurs at X_{6A} (the stiff side in the Y-direction), the opposite side to X_{1A} , in the case of UTO0414, as shown in Figure 24(a). Conversely, in the case of TCU shown in Figure 24(b), the envelope of the time-history analysis results indicates that the largest peak occurs at X_{1A} , which is consistent with the predicted results. In this subsection, the modal response at level 0 is calculated and discussed.

From the time-history of the displacement at the center of mass of level 0, the horizontal displacement in the Y-direction at point j on level 0 can be calculated as

$$d_{Y0j}(t) = y_0(t) - L_{X0j}\theta_0(t). \quad (28)$$

In Eq. (28), L_{X0j} is the location of point j in the X-direction from the center of mass of level 0. Therefore, the modal response of the horizontal displacement at point j can be calculated as

$$d_{Y0j1}(t) = \Gamma_{1U}(\phi_{Y01} - L_{X0j}\phi_{\theta01})D_{1U}^*(t), \quad (29)$$

$$d_{Y0j2}(t) = \Gamma_{2V}(\phi_{Y02} - L_{X0j}\phi_{\theta02})D_{2V}^*(t), \quad (30)$$

$$d_{Y0jh}(t) = d_{Y0j}(t) - \{d_{Y0j1}(t) + d_{Y0j2}(t)\}. \quad (31)$$

Figure 39 shows comparisons of the modal responses at the edge of level 0. The structural model shown in this figure is Model-Tf34, the input ground motion dataset is UTO0414, and the angle of incidence of the seismic input (ψ) is -75° , the angle at which the largest peak response at X_{1A} occurs. Note that “All modes” is the response originally obtained from the time-history analysis results ($d_{Y0j}(t)$), “First mode” and “Second mode” are the first and second modal responses ($d_{Y0j1}(t)$ and $d_{Y0j2}(t)$), respectively, and “Higher mode” is the higher (residual) modal response calculated from Eq. (31) ($d_{Y0jh}(t)$).

In the response of X_{1A} shown in Figure 39(a), the contribution of the higher modal response is non-negligible, even though the contribution of the first modal response is predominant. In addition, the sign of the higher modal response at the time the peak response occurs at X_{1A} is opposite to that of the “All mode” response, with the contribution of the higher mode reducing the peak response at X_{1A} .

In the response of X_{6A} shown in Figure 39(b), the contribution of the first modal response is negligibly small and that of the second and higher modal responses are noticeable. In addition, the sign of the higher modal response at the time the peak response occurs at X_{6A} is the same as that of the “All mode” response, with the contribution of the higher mode increasing the peak response at X_{6A} .

This indicates that the reason why the largest peak response in the envelope of the time-history analysis results occurs at X_{6A} (not at X_{1A}) in the case of UTO0414 can be explained by the contributions of the higher modal response. In the case of UTO0414, the contribution of the higher modal response is non-negligibly large.

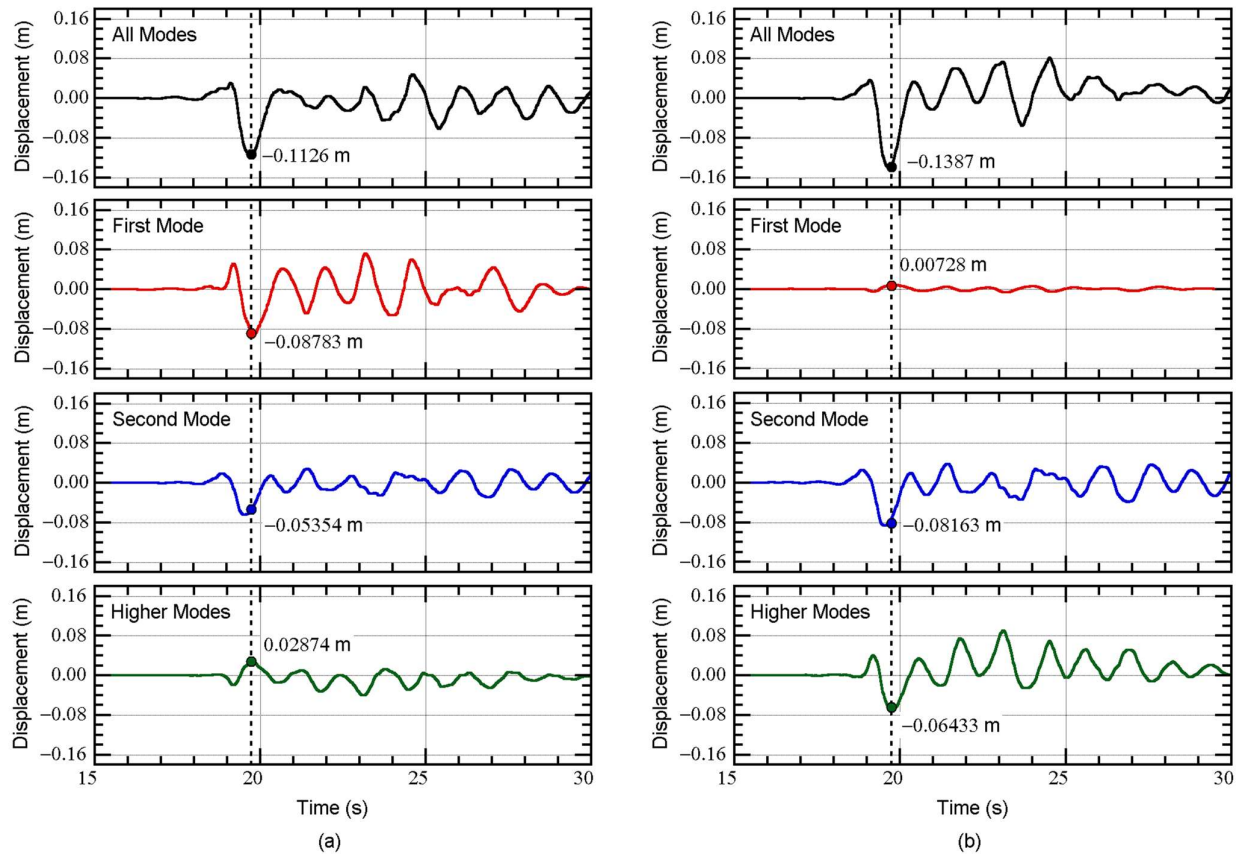


Figure 39. Comparisons of the modal responses at the edge of level 0 (structural model: Model-Tf34, ground motion: UTO0414, angle of incidence of seismic input: $\psi = -75^\circ$): (a) X_{1A} and (b) X_{6A} .

Another comparison is made in Figure 40 for the structural model Model-Tf34, the input ground motion dataset TCU, and an angle of incidence of the seismic input (ψ) of 60° , where the angle of the largest peak response at X_{1A} occurs. In the response of X_{1A} shown in Figure 40(a), the contribution of the first modal response is predominant while that of the higher modal response is small. Meanwhile, in the response of X_{6A} shown in Figure 40(b), the contribution of the first modal response is negligibly small and that of the second and higher modal responses are noticeable.

Therefore, the accuracy of the predicted horizontal distribution of the peak response at level 0 relies on the contribution of the higher modal response. The envelope of the time-history analysis results is notably different from the predicted results in the case of UTO0414 because the contribution of the higher modal response is significant. Meanwhile, the predicted results are close to the envelope of the time-history analysis results in the case of TCU because the contribution of the higher modal response is small. To confirm this, Table 6 lists the equivalent velocities of the maximum momentary input energy predicted from the bidirectional $V_{\Delta E}$ spectrum (complex damping ratio (β) of 0.10) for the two cases. In this table, the values of the first and second modal responses are those predicted using MABPA, while the value of the third mode is predicted assuming that the effective period (T_{3eff}) equals the natural period in the elastic range (T_{3e}). This table confirms that the contribution of the third mode may be noticeable in the case of UTO0414, while it may be small in the case of TCU.

In the MABPA prediction, only the contributions of the first and second modal responses are considered. Therefore, a discrepancy of the predicted results from the time-

history analysis may occur because of the lack of a contribution from the higher modal responses.

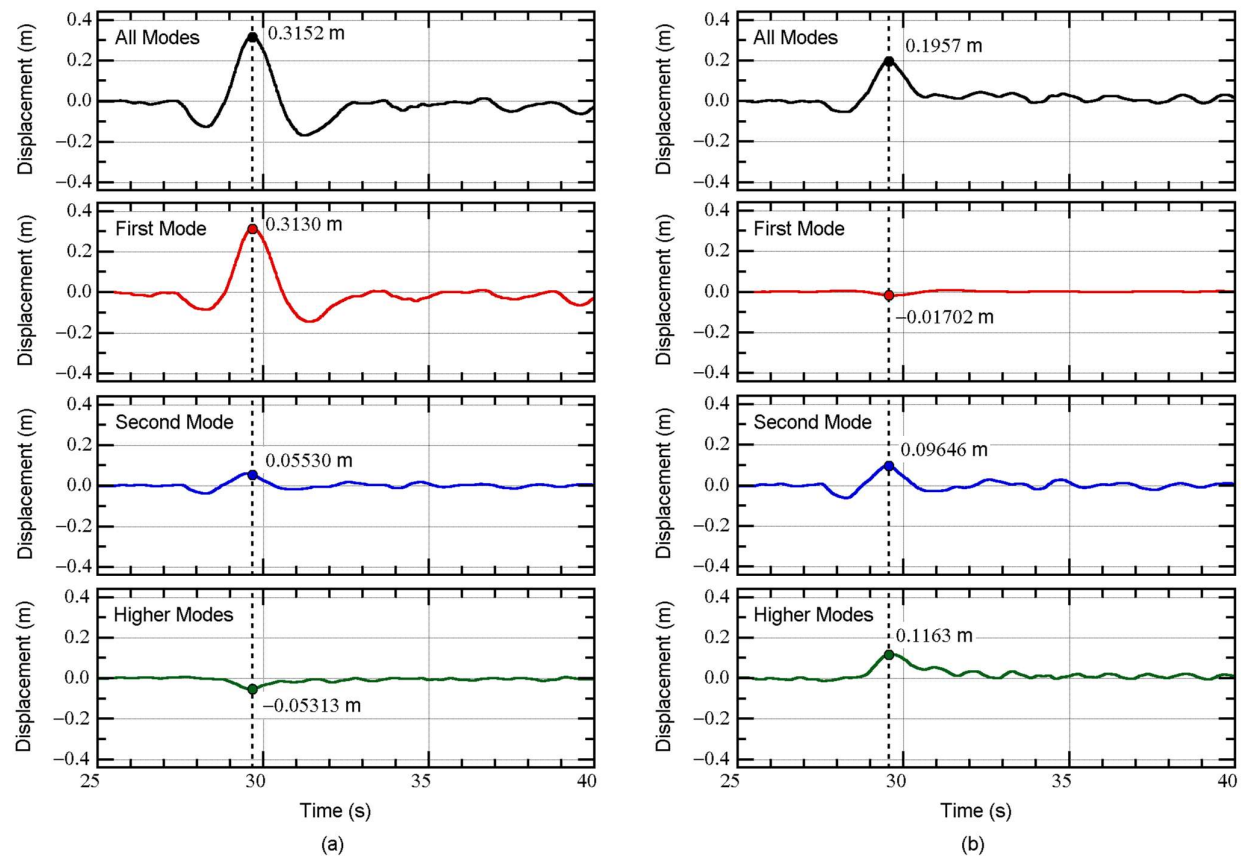


Figure 40. Comparisons of the modal responses at the edge of level 0 (structural model: Model-Tf34, ground motion: TCU, angle of incidence of seismic input: $\psi = 60^\circ$): (a) X_{1A} and (b) X_{6A} .

Table 6. Equivalent velocities of the maximum momentary input energy predicted from the bidirectional $V_{\Delta E}$ spectrum ($\beta = 0.100$).

Ground motion set	First Mode $V_{\Delta E}(T_{1eff})$ (m/s)	Second Mode $V_{\Delta E}(T_{2eff})$ (m/s)	Third Mode $V_{\Delta E}(T_{3e})$ (m/s)	Ratio (2nd / 1st)	Ratio (3rd / 1st)
UTO0414	0.4108	0.4232	0.7151	1.030	1.741
TCU	0.7575	0.7319	0.2519	0.9662	0.3325

6. Conclusions

In this study, the main building of the former Uto City Hall was investigated as a case study of the retrofitting of an irregular reinforced concrete building using the base-isolation technique. The nonlinear peak responses of two retrofitted building models subjected to horizontal bidirectional ground motions were predicted by MABPA, and the accuracy of the method was evaluated. The main conclusions and results are as follows.

- The predicted peak response according to the updated MABPA agrees satisfactorily with the envelope of the time-history analysis results. The peak relative displacement at $X_{3A}Y_3$ at each floor can be satisfactorily predicted. The predicted distribution of the peak displacement at level 0 (just above the isolation layer) approximates the envelope of the nonlinear time-history analysis results, even though in some cases the

predicted distributions differ from the envelope of the nonlinear time-history analysis.

- The relationship between the equivalent velocity of the maximum momentary input energy of the first modal response (V_{AE1U}^*) and the peak equivalent displacement of the first modal response (D_{1U}^*) can be properly evaluated from the pushover analysis results. The plots obtained from the nonlinear time-history analysis results fit the evaluated curve from the pushover analysis results well.
- The upper bound of the peak equivalent displacements of the first two modal responses can be predicted using the bidirectional V_{AE} spectrum [41]. Comparisons between the predicted peak equivalent displacements and those calculated from the nonlinear time-history analysis results show that the predicted peak approximates the upper bound of the nonlinear time-history analysis results. The upper bound of V_{AE1U}^* can be approximated by the bidirectional V_{AE} spectrum.

Based on the above findings, the updated MABPA appears to predict the peak responses of irregular base-isolated buildings with accuracy. However, MABPA still has two shortcomings. The first shortcoming is the limitation of the applicability of MABPA. As discussed in a previous study [32], the application of MABPA is limited to buildings classified as torsionally stiff buildings. The current (updated) MABPA has the same restriction. This limitation can be avoided if the torsional resistance of the isolation layer is sufficiently provided, as shown in this study. The second shortcoming involves the contributions of the higher modal responses. In the original MABPA for non-isolated buildings, only the first two modes were considered for the prediction. Therefore, the prediction was less accurate for cases when the response in the stiff-side perimeter was larger than that in the flexible-side perimeter. The contributions of the third and higher modal responses need to be investigated.

Another aspect of this study to be emphasized is the application of the bidirectional V_{AE} spectrum for the prediction of the peak response of a base-isolated building. The results shown in this study imply that the bidirectional V_{AE} spectrum [41] is a promising candidate for a seismic intensity parameter for the peak response. As discussed in a previous study [42], one of the biggest advantages of the bidirectional momentary input energy is that it can be directly calculated from the Fourier amplitude and phase angle of the ground motion components using a time-varying function of the momentary energy input, without knowing the time-history of the ground motion. This means that researchers can eliminate otherwise unavoidable fluctuations from the nonlinear time-history analysis results. Therefore, the pushover analysis and the bidirectional V_{AE} spectrum are an optimal combination to understand the fundamental characteristics of both base-isolated and non-isolated asymmetric buildings.

The optimal distribution of the hysteresis dampers according to the design of the isolation layer needed to minimize the torsional response is not discussed in this study. However, the updated MABPA presented here can help in the optimization of the damper distribution. By carrying out the pushover analysis and calculating the torsional index of the first three modes in each step, as shown in previous studies [31, 32], the contribution of the torsional response in each modal response can be monitored. Based on this, the damper distributions can be optimized. This is one of the issues that will be investigated in subsequent studies.

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Writing – review & editing, Kenji Fujii. All authors have read and agreed to the published version of the manuscript.

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Conflicts of Interest: The authors declare no conflict of interest.

Appendix: Time-histories of the recorded ground motions used in this study

Table A1 shows the date of event, magnitude (Meteorological Agency Magnitude M_J , or moment magnitude M_W), location of epicenter, distance, and station name of each record. Figures A1–A4 show the time-histories and orbits of the original ground motion records.

Table A1. Event date, magnitude, location of epicenter, distance, and station name of each record.

ID	Event date	Magnitude	Distance	Station name	Direction of components	
					ξ -Dir	ζ -Dir
UTO0414	2016/04/14	$M_J = 6.5$	15 km	K-Net UTO (KMM008)	EW	NS
UTO0416	2016/04/16	$M_J = 7.3$	12 km	K-Net UTO (KMM008)	EW	NS
TCU	1999/09/20	$M_W = 7.6$	0.89 km*	TCU075	major**	minor**
YPT	1999/08/17	$M_W = 7.5$	4.83 km*	Yarimca	major**	minor**

* This distance is the closest distance from the rupture plane defined in the Pacific Earthquake Engineering Research Center (PEER) database, while those from the Japanese database are the epicentral distances.

** Horizontal major and minor axes are determined following the work done by Arias [50] and Penzien and Watabe [51].

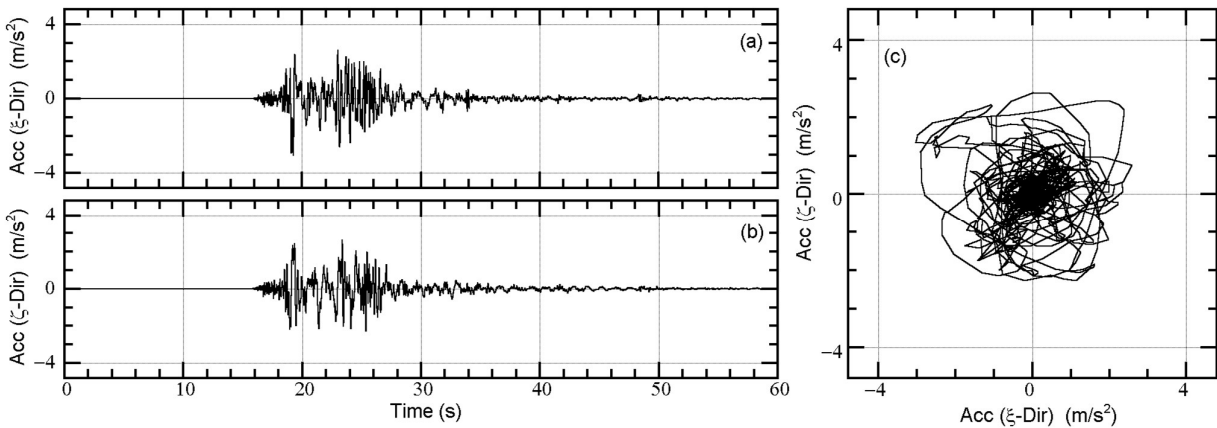


Figure A1. Two components of the generated artificial ground motion (UTO0414): (a) ξ -direction; (b) ζ -direction; and (c) orbit.

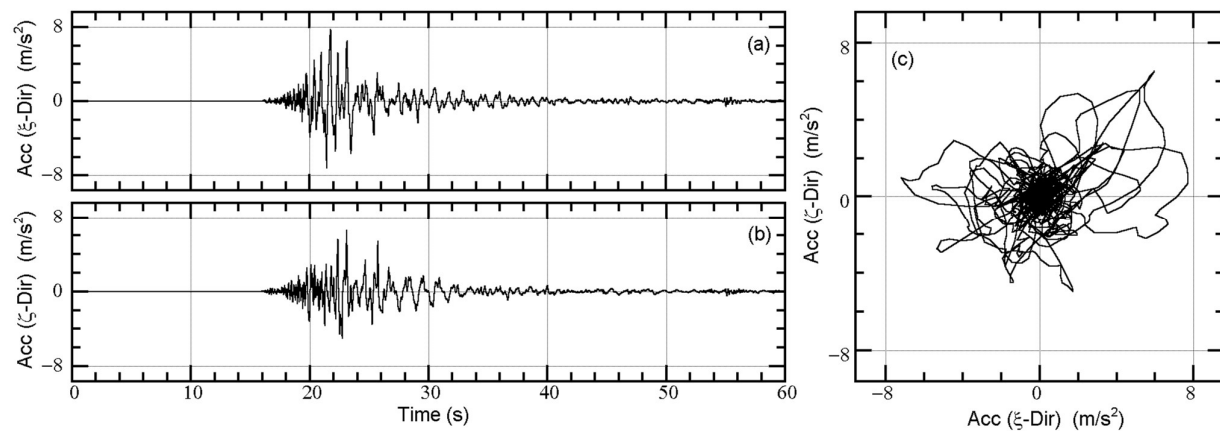


Figure A2. Two components of the generated artificial ground motion (UTO0416): (a) ξ -direction; (b) ζ -direction; and (c) orbit.

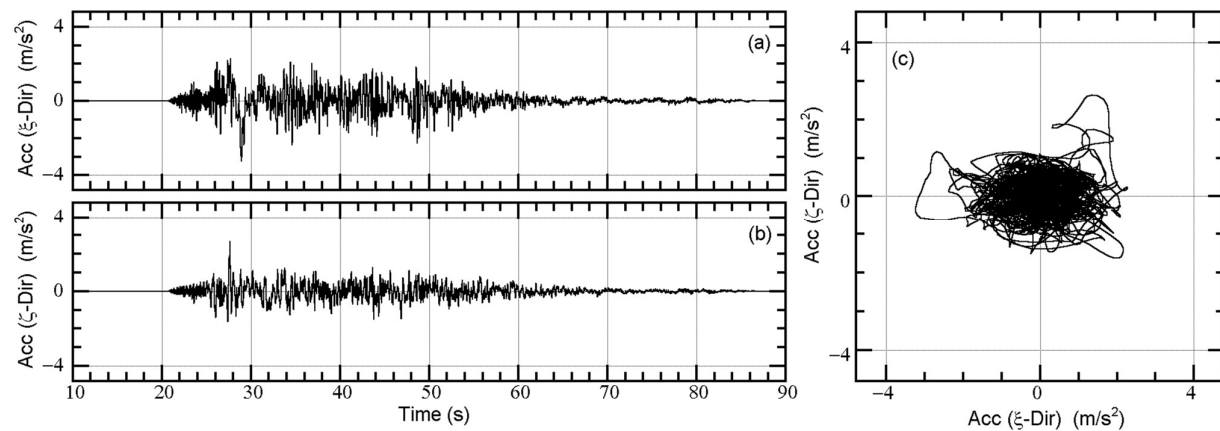


Figure A3. Two components of the generated artificial ground motion (TCU): (a) ξ -direction; (b) ζ -direction; and (c) orbit.

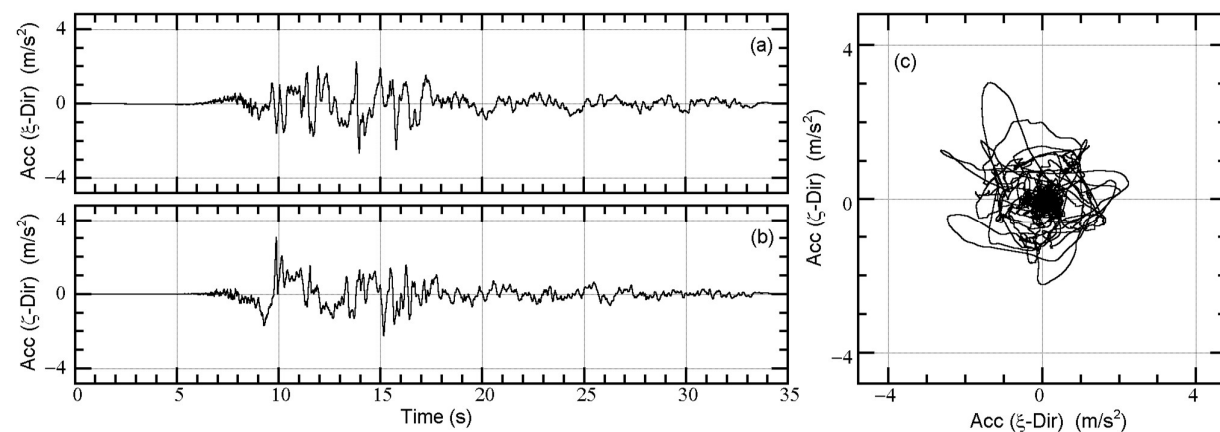


Figure A4. Two components of the generated artificial ground motion (YPT): (a) ξ -direction; (b) ζ -direction; and (c) orbit.

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