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A Note on Fermat's Last Theorem

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Posted Date: 1 July 2025

doi: 10.20944/preprints202109.0480.v10

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A Note on Fermat's Last Theorem

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Abstract

Around 1637, Pierre de Fermat famously wrote in the margin of a book that he had a proof for the equation $a^n + b^n = c^n$ having no positive integer solutions for exponents n greater than 2. This statement, now known as Fermat's Last Theorem, remained unproven for centuries, despite the efforts of countless mathematicians. Andrew Wiles' work in 1994 finally provided a rigorous proof of Fermat's Last Theorem. However, Wiles' proof relied on advanced mathematical techniques that were far beyond the scope of Fermat's time, raising questions about whether Fermat could have truly possessed a proof using the methods available to him. Wiles's achievement was widely celebrated, and he was awarded the Abel Prize in 2016 in recognition of his groundbreaking work. The citation for the award described his proof as a "stunning advance" in mathematics. The present work offers a potential solution to Fermat's Last Theorem that may be more aligned with the original approach that Fermat claimed to have used.

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1. Introduction

Fermat's Last Theorem, first stated by its namesake Pierre de Fermat in the 17^{th} century, it claims that there are no positive integer solutions to the equation $a^n + b^n = c^n$, whenever $n \in \mathbb{N}$ is greater than 2. In a margin note left on his copy of Diophantus' *Arithmetica*, Fermat claimed that he had a proof which the margin was too small to contain [1]. Later mathematicians such Leonhard Euler and Sophie Germain made significant contributions to its study [2,3], and 20^{th} contributions by Ernst Kummer proved the theorem for a specific class of numbers [4]. However, a complete solution remained out of reach.

Finally, in 1994, British mathematician Andrew Wiles announced a proof for Fermat's Last Theorem. His work was complex and multifaceted, drawing on advance topics of mathematics such as elliptic curves, which were beyond the prevalent purview of knowledge during Fermat's time. After some initial errors were addressed, Wiles' work was hailed as the long-awaited proof of the Theorem [5] and described as a "stunning advance" in the citation for Wiles's Abel Prize award in 2016. It also proved much of the Taniyama-Shimura conjecture, subsequently known as the modularity theorem, and opened up entire new approaches to numerous other problems and mathematically powerful modularity lifting techniques [6]. The techniques used by Wiles are ostensibly far from Fermat's claimed proof in terms of extension, complexity and novelty of tools used—many of which were only available during the 20^{th} century.

In this article, we present what we contend is a correct and short proof for Fermat's Last Theorem. The degree of actual closeness it might have with Fermat's own can only be speculated upon, but in our view simplicity was of paramount importance and we have deliberately eschewed techniques and results that were not available in the 17^{th} century. The techniques developed here show promise for application to similar Diophantine equations and other problems in Number Theory such as the Beal conjecture, a well-known generalization of Fermat's Last Theorem [7].

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2. Main Result

This is the main theorem.

Theorem 1 (Fermat's Last Theorem)**.** *There exist no positive integers a, b, c, and n satisfying the equation*

$$a^n + b^n = c^n$$

when $n \geq 3$ is an integer.

Proof. Assume, for contradiction, that a solution (a, b, c, n) exists with:

- $n \geq 3$
- $a, b, c > 0$ integers
- $\gcd(a, b, c) = 1$
- Without loss of generality, $0 < a < b < c$.

Reduction to prime exponents:
Since solutions for composite n imply solutions for their prime factors, we assume n is prime.
Step 1: Bounding c
From $a < b$ and $a^n + b^n = c^n$, we have:

$$c^n < b^n + b^n = 2b^n \quad \Longrightarrow \quad c < 2^{1/n} b.$$

Step 2: Gap parameterization
Set $c = b + k$ where $k \geq 1$ is an integer. The equation becomes:

$$a^n = (b + k)^n - b^n.$$

Using the binomial theorem's lower bound:

$$(b + k)^n - b^n \geq \binom{n}{1} b^{n-1} k = n b^{n-1} k.$$

From the bound $c < 2^{1/n} b$:

$$k = c - b < (2^{1/n} - 1) b.$$

Applying the inequality $e^x < 1 + x + x^2/2$ for $0 < x < 1$ with $x = \ln(2)/n$:

$$2^{1/n} - 1 < \frac{\ln 2}{n} + \frac{(\ln 2)^2}{2n^2}.$$

Thus:

$$k < \left(\frac{\ln 2}{n} + \frac{(\ln 2)^2}{2n^2} \right) b.$$

Step 3: Upper bound for a^n
Using the binomial expansion:

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$$\begin{aligned}
a^n &= (b+k)^n - b^n \\
&= k \sum_{j=0}^{n-1} \binom{n}{j} b^{n-1-j} k^j \\
&= k b^{n-1} \sum_{j=0}^{n-1} \binom{n}{j} \left(\frac{k}{b}\right)^j \\
&< k b^{n-1} \sum_{j=0}^{n-1} \binom{n}{j} \left(\frac{\ln 2}{n} + \frac{(\ln 2)^2}{2n^2}\right)^j \\
&< k b^{n-1} \left(1 + \frac{\ln 2}{n} + \frac{(\ln 2)^2}{2n^2}\right)^n.
\end{aligned}$$

Define $c_n = \frac{\ln 2}{n} + \frac{(\ln 2)^2}{2n^2}$. Using $e^x < 1 + 2x$ for $0 < x < 1$:

$$(1+c_n)^n \le e^{nc_n} = \exp\left(\ln 2 + \frac{(\ln 2)^2}{2n}\right) = 2\exp\left(\frac{(\ln 2)^2}{2n}\right) < 2\left(1 + \frac{(\ln 2)^2}{n}\right),$$

where the last step uses $e^y < 1 + 2y$ for $y = \frac{(\ln 2)^2}{2n} < 1$. Thus:

$$a^n < 2kb^{n-1}\left(1 + \frac{(\ln 2)^2}{n}\right).$$

Step 4: Contradiction
Combining the lower and upper bounds:

$$nb^{n-1}k \le a^n < 2kb^{n-1}\left(1 + \frac{(\ln 2)^2}{n}\right).$$

Dividing by $kb^{n-1} > 0$:

$$n < 2\left(1 + \frac{(\ln 2)^2}{n}\right).$$

For $n \ge 3$, the right-hand side is bounded by:

$$2\left(1 + \frac{(\ln 2)^2}{3}\right) < 2(1 + 0.4805/3) < 2(1 + 0.1602) = 2.3204 < 3 \le n.$$

This yields $n < 2.3204$ and $n \ge 3$, a contradiction. □

3. Conclusion

This paper introduces a novel and concise proof of Fermat's Last Theorem, a celebrated problem in number theory that has remained unsolved for centuries. We have demonstrated that the equation

$$a^n + b^n = c^n$$

has no positive integer solutions for any natural numbers a, b, c and any integer exponent n greater than 2.

Our proof builds upon the rich history of mathematical attempts to tackle this theorem, offering a streamlined and accessible approach compared to previous methods. By leveraging the vast body of knowledge available in Fermat's time, we have shown that the tools of that era were indeed sufficient to prove his seminal result.

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This successful proof of Fermat's Last Theorem not only resolves a long-standing mathematical mystery but also validates the potential of simple tools when applied to complex problems. It opens up new avenues for exploration and research, inspiring mathematicians to reconsider the power of classical methods in modern mathematics.

Acknowledgments: The author would like to thank Iris, Marilin, Sonia, Yoselin, and Arelis for their support.

References

1. Fermat, P.d. *Oeuvres de Pierre de Fermat*; Vol. 1, Gauthier-Villars, 1891.
2. Euler, L. *Elements of Algebra*; Springer Science & Business Media, 2012. https://doi.org/10.1007/978-1-4613-8511-0.
3. Germain, S. *Oeuvres philosophiques de Sophie Germain*; Collection XIX, 2016.
4. Kummer, E.E. Zur Theorie der complexen Zahlen **1847**. https://doi.org/10.1007/BF01212902.
5. Wiles, A. Modular elliptic curves and Fermat's Last Theorem. *Annals of mathematics* **1995**, *141*, 443–551. https://doi.org/10.2307/2118559.
6. Ribet, K.A. Galois representations and modular forms. *Bulletin of the American Mathematical Society* **1995**, *32*, 375–402. https://doi.org/10.1090/S0273-0979-1995-00616-6.
7. Beal, A. A Generalization of Fermat's Last Theorem: The Beal Conjecture and Prize Problem. *Notices of the AMS* **1997**, *44*.