

# On Variable Speed of Light and Relativity of Constantaneity

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## Abstract

An axiomatic theory is proposed that reconciles the existence of an absolute scale for time (Planck time) and special relativity. According to this theory speed of light  $c$  becomes a variable which is proposed to be taken as the fifth dimension.

**Keywords**— relativity of constantaneity, variable speed of light

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## 1 Introduction

Although proposed in the same year of *annus mirabilis* together with the Photoelectric paper which fundamentally used  $h$ , Special Theory of Relativity was conceived and consolidated without regard to Planck's constant which is the reason it is usually considered a 'classical' theory despite being a pillar of 'modern physics'. In hindsight, observing all the confusion

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and inelegant approaches like *Doubly Special Relativity (DSR)*[1, 2, 3] and *Variable Speed of Light (VSL)* [4] theories, the author maintains that it was a wise choice of Einstein not to get Special Relativity contaminated by quantum mechanics. Orthodox quantum theory is still an extremely vague *algorithm* –not a *theory*– with many elements which even do not have clear definitions. Any attempt of basing our picture of *reality* and ontology based on a temporary incomplete theory is at best imprudent, if not rash. The bare minimum that is sure to stay in any theory of quantum mechanics is the Planck constant, thus  $\hbar$  is the *only* safe ground that one must base upon any theory which is probable to have deep ontologic consequences. It is in this light that in this paper we look at a variable speed of light from a fresh perspective that is entirely novel to the best of our knowledge. To this purpose we begin a review of special relativity that might reflect author's personal reading: Before Maxwell all velocities were relative (Galilean invariance), but with Fizeau and Foucault's experiments and Maxwell's theory, for the first time in history we found a *constant*  $c$  that was a speed. This immediately contradicted our previous idea that *all* velocities were relative and the climax of attempts of reconciliation was Einstein's special relativity, according to which as a result of this reconciliation *one has to add the only invariant parameter of the previous theory of Galilean transformations to the coordinates, viz. time has to become the fourth dimension*. From this perspective it is easy to find our way to a theory which considers the kinematical consequences of the Planck constant. Now we are in an analogous position in which Einstein found himself: According to the previous theory i.e. special relativity, all times are relative on one hand, *but* on the other hand as a consequence of existence of the Planck constant we now have a constant that is a time

$$t_P = \sqrt{\frac{\hbar G}{c^5}}.$$

This again immediately runs into contradiction with a result of special relativity that 'all times are relative' as we have a constant of time which is not relative. It would be a naïve and rash step to infer from this the existence of any notion of absolute time as some have done[5] when it is possible to have a theory that keeps special relativity and reconcile it with the existence of Planck time. The same logic that drives 'cosmic time' could result to infer the absoluteness of all velocities from the observation that there is a constant of nature  $c$  that is a velocity. The problem being analogous<sup>1</sup> it would not be surprising that the solution should be analogous: Add the only invariant parameter of the previous theory (Lorentz transformations) to the coordinates. The only invariant parameter of Lorentz transformations being  $c$  this means that,  **$c$  has to become the fifth dimension**. This is the first element of my proposal. To arrive at the second, we need to revise the formal logical structure of Einstein's *Zur Elektrodynamik bewegter Körper*. Adding  $c$  as the variable of the fifth dimension inevitably takes us 'outside' of the Minkowski space. This sound

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<sup>1</sup>The analogy is so strong that if you replace 'time' with 'speed of light' everywhere in Einstein's *Zur Elektrodynamik bewegter Körper* you will get meaningful questions/statements most of the time!

like a heresy as for the academic mindset Minkowski space *is* the (local) ‘reality’; but if we are to reconcile special relativity and existence of a fundamental constant of time, that view is nothing but a preconception; same way that going beyond the Euclidean space was a heresy –even for Einstein himself at the beginning–. We name this new space the *Dicke space*<sup>2</sup> which can be pictured as

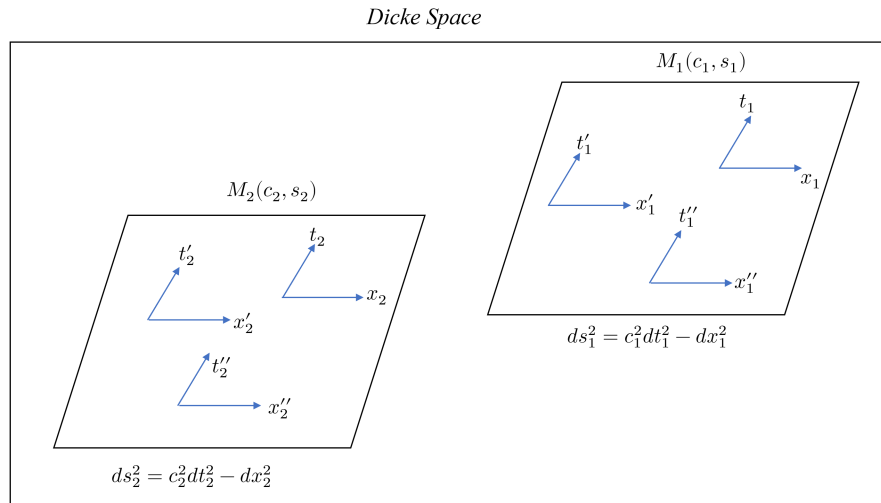


Figure 1: *The Kant space can be thought of as a collection of Minkowski spaces  $M_i$  each with their own speed of light  $c_i$  hence their own line element  $s_i$ . In the limit of a continuous  $c$  we can think of it as a coordinate that takes us from one Minkowski space to another.*

The crucial point that DSR fails to consider is whether its first principle (equivalence of inertial frames) is untouched by existence of Planck time. That principle relying on inertial frames as those with relative *constant velocity* is a remnant of Galilean relativity and is not appropriate for founding a new fundamental generalisation of special relativity. We maintain that special relativity changes the notion of inertial frames themselves: we can use the second principle of special relativity to *define* inertial frames as those which agree on a *constant  $c$* . To mathematically state this definition we need a preliminary.

## 2 Relativity of *Constantaneity*

The starting point of Einstein was the notion of *relativity of simultaneity* to which he arrived via synchronization of clocks by light signals. No such analogue of light signals is known to us that could play the same

<sup>2</sup>In honour of Robert Henry Dicke who was one of the first people to consider a variable speed of light after the advent of special relativity.

role in the Dicke space that light signals play in the Minkowski space. Unlike Einstein that had Maxwell's light signals and a complete theory of Electromagnetism to play with, we are like someone trying to construct special relativity long before Maxwell –but after Newton– knowing only that there is a constant of nature with dimensions of velocity. Standing on shoulders of Einstein we know however that interestingly such person *can* find special relativity; similar for us. Being short of a theory of quantum gravity we cannot think of fancy thought experiments, but can only think formally: Einstein's starting point was

$$\frac{2AB}{t' - t} = c, \quad (1)$$

which can be stated infinitesimally as

$$\frac{d\mathbf{x}}{dt} = c,$$

which expresses the null paths of Minkowski space. From this one can arrive at the Minkowski metric by measuring how much two observers fail to be Einstein-simultaneous, viz.

$$\begin{aligned} d\mathbf{x} &= cdt \\ \Rightarrow (cdt)^2 - d\mathbf{x}^2 &=: ds^2; \end{aligned}$$

Let us now take a blind formal perspective: The numerator of (1) is the Euclidean distance which after Einstein and Minkowski should become the Minkowskian distance  $ds$ ; the denominator is the variable that is about to become the new dimension which we earlier proposed to be  $c$ , therefore

**Definition 1** (Constantaneity).

$$\frac{2MM'}{c' - c} = t_P \quad (2)$$

where  $M$  and  $M'$  are two points in the Minkowski space, and  $t_P$  is the Planck time. Infinitesimally thus

$$\boxed{\frac{ds}{dc} = t_P} \quad (3)$$

The quantity  $ds/dc$  is the key that is missed by DSR and VSL. Equation (3) defines the null paths of the Dicke space, which we name as *Einstein signals*. This new quantity  $ds/dc$  deserves a name:

**Definition 2** (Kant Time<sup>3</sup>).

$$\boxed{u := \frac{ds}{dc}} \quad (4)$$

By measuring how two observers fail to be constantaneous we arrive at the metric of the Dicke space as

$$\boxed{dw^2 = t_P^2 dc^2 - ds^2} = t_P^2 dc^2 - c^2 dt^2 + d\mathbf{x}^2. \quad (5)$$

We are now in a position to mathematically express the notion of inertial frames proposed earlier

<sup>3</sup>In honour of Immanuel Kant who devised of an *epistemological absolute time*.

**Definition 3** (Inertial Frame of Reference). *Two observers (in the Dicke space) are called to be inertial if they are related by a constant Kant time, viz.*

$$\begin{aligned}\frac{d^2 s}{dc^2} &= 0 \\ \Rightarrow s &= uc\end{aligned}$$

In other words  $s = uc$  creates an equivalence class between all observers that agree on a particular constant speed of light. Now that we have corrected all the mistakes of DSR in its starting principles, we can proceed to axiomise our proposal.

### 3 Principles

**Principle 1** (Principle of Relativity). *The laws of physics must have the same form in all inertial frames of reference.*

**Principle 2** (Constancy of Kant time of Einstein signals). *In vacuum all inertial frames of reference agree on a constant Kant time for Einstein signals, the constant being Planck time  $t_P$ .*

If we define

$$d\sigma = dw/t_P$$

we can write (5) as

$$t_P^2 d\sigma^2 = t_P^2 dc^2 - c^2 dt^2 + d\mathbf{x}^2,$$

therefore

$$\left(\frac{t_P d\sigma}{t_P dc}\right)^2 = \left(\frac{d\sigma}{dc}\right)^2 = 1 - \left(\frac{cdt}{t_P dc}\right)^2 + \left(\frac{d\mathbf{x}}{t_P dc}\right)^2,$$

using chain rule we have

$$\frac{dt}{dc} = \frac{dt}{ds} \frac{ds}{dc} = u \frac{dt}{d(c\tau)} = \frac{\gamma}{c} u$$

and

$$\frac{d\mathbf{x}}{dc} = \frac{d\mathbf{x}}{ds} \frac{ds}{dc} = u \frac{d\mathbf{x}}{d(c\tau)} = \frac{\gamma}{c} u \mathbf{v},$$

hence

$$\boxed{\zeta := \frac{d\sigma}{dc} = \sqrt{1 - u^2 \gamma^2 \left(\frac{1}{t_P^2} - \frac{\mathbf{v}^2}{l_P^2}\right)}} = \sqrt{1 - u^2 \frac{\frac{1}{t_P^2} - \frac{\mathbf{v}^2}{l_P^2}}{1 - \frac{\mathbf{v}^2}{c^2}}}. \quad (6)$$

*This is the way to do ‘doubly’ special relativity.*

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