On the Inverse Fourier Transform of the Planck-Einstein law

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Abstract

After proposing a natural metric for the space in which particles spin which implements the principle of maximum frequency, $E = hf$ is generalised and its inverse Fourier transform is calculated.

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In [1] I proposed the notion of *angles-time space* and the *principle of maximum frequency* based on the fundamental guiding principle that the so-called wave-particle duality is a purely classical result of spin motion of particles and argued that the Planck–Einstein relation $E = \hbar\omega$ is nothing but $E = I\omega^2$ of classical mechanics with a change of dress. To recap, our starting point is the following comparison

$$E = \frac{1}{2}m\mathbf{v}^2 \quad \text{versus,} \quad E = \frac{1}{2}I\boldsymbol{\omega}^2$$

By analogy between $\mathbf{v}$ and $\boldsymbol{\omega}$, *if there is a maximum for* $\boldsymbol{v}$ *then there must also be a maximum for* $\boldsymbol{\omega}$. This analogy leads us to the following

Principle 1 (Principle of Maximum Angular Frequency)**.** *All angular frequencies of rotation are bounded above by* ω_P *where*

$$\omega_P := 2\pi f_P = 2\pi\sqrt{\frac{c^5}{\hbar G}}$$

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As pointed earlier, contrary to the mainstream view that sees spin of elementary particles unrelated to any kind of rotational spin, I conjecture that it is the spin (rotational) motion of elementary particles that creates de Broglie matterwaves hence we must demand $E = \frac{1}{2}I\boldsymbol{\omega}^2$ to yield $E = \hbar\omega$. To that end *assuming that the spin (rotational) motion of an elementary particle is completely independent from its translational motion*, we conclude that the spin motions must occur in an ontologically distinct space which I dubbed as the space of *angles-time*. The distances in this angles-time space are measured by the following metric[1]

$$d\Theta^2 = \omega_P^2 dt^2 \pm d\boldsymbol{\theta}^2, \tag{1}$$

which naturally implements the principle of maximum frequency. Regarding the doubt in the sign of metric, it is something that must be singled out by experiments therefore I shall consider both cases.
I now prove that the principle of maximum frequency solves one of the fundational problems of quantum mechanics: Taking Planck-Einstein relation $E = \hbar\omega$ as a fundamental law of nature and abiding by the principle that **a fundamental law of nature must not depend on our choice of basis for the function space (Fourier basis)**, we expect the Inverse Fourier Transform of $E = \hbar\omega$ to yield the energy of electromagnetic field as a function of time, viz.

$$E(t) = \frac{\hbar}{2\pi}\int_{-\infty}^{\infty} e^{i\omega t}\omega \, d\omega,$$

but this integral is wildly divergent, and I maintain that *this is the root of the problem of renormalisation in QFT*. As a result of the principle of maximum frequency the rotational energy of a particle is given by

$$E = \frac{I\omega_P^2}{\sqrt{1-(\omega/\omega_P)^2}} \tag{2}$$

or

$$E = -\frac{I\omega_P^2}{\sqrt{1+(\omega/\omega_P)^2}} \tag{3}$$

depending on the sign of the metric (1). By analogy with the gamma factor of special relativity we expect the $\pm I\omega_P^2$ to yield the inertial energy of rotation but that would only be the case if I was a constant, but it turns out that if we are to recover $E = \hbar\omega$ from (2) or (3), I is not a constant and depends on the frequency of rotation. To see this, observe that the second term in the Taylor expansions of (2) and (3) is

$$\frac{1}{2}I\omega^2$$

showing the expected compatibility with classical mechanics. But to get $E = \hbar\omega$ we need to recall the following familiar relation between I and angular momentum S

$$S = I\omega$$

[1] Another motivation for introducing a whole new independent space is to get close to explaining how particles which are separeted by space-like distances can still be correlated (quantum entanglement), for a new space with a new metric means that this space has a new independent notion of *locality*, independent of the locality in spacetime.

thus

$$I = \frac{S}{\omega}$$

which yields

$$E = \frac{1}{2} S\omega \tag{4}$$

which can be interpreted as meaning that wave-particle duality is a solely classical fact and it is in the quantisation of spin that quantum mechanics enters: to get $E = \hbar\omega$ we now only need to let

$$S = 2\hbar$$

in (4). This $S = 2\hbar$ is nothing but an 'old' quantum-mechanical quantisation rule. Consequently

$$I = \frac{2\hbar}{\omega} \tag{5}$$

implies that photons are unlike any rigid body we know in classical mechanics, as *the moment of inertia of photons depends on the angular frequency of their spin.* Putting all these considerations together, complete exact form of Planck–Einstein relation is given by

$$\boxed{E(\omega) = \frac{2\hbar\omega_P^2}{\omega}\left(\frac{1}{\sqrt{1-(\omega/\omega_P)^2}} - 1\right)} \tag{6}$$

or

$$\boxed{E(\omega) = \frac{2\hbar\omega_P^2}{\omega}\left(1 - \frac{1}{\sqrt{1+(\omega/\omega_P)^2}}\right)} \tag{7}$$

Note that by construction $E = \hbar\omega$ is an approximation to the above equation, by being the first term in the Taylor expansion of the angles-time space factor. Unlike $E = \hbar\omega$ however, my proposed generalisation of Planck–Einstein relation is not plagued by infinities and it is perfectly possible to take its inverse Fourier transform. To this purpose, recall that

$$\mathcal{F}_\omega^{-1}[\frac{1}{\sqrt{1+(\omega/\omega_P)^2}}] = \omega_P\sqrt{\frac{2}{\pi}}K_0\big(\omega_P t\big) \tag{8}$$

where $K_0(t)$ is the zeroth order modified Bessel function of the second kind. And

$$\mathcal{F}_\omega^{-1}[\frac{1}{\sqrt{1-(\omega/\omega_P)^2}}] = -i\omega_P\sqrt{\frac{2}{\pi}}K_0\big(-i\omega_P t\big). \tag{9}$$

Now observe that

$$i\omega E(\omega) = 2i\hbar\omega_P^2\left(1 - \frac{1}{\sqrt{1+(\omega/\omega_P)^2}}\right),$$

and

$$i\omega E(\omega) = 2i\hbar\omega_P^2\left(\frac{1}{\sqrt{1-(\omega/\omega_P)^2}} - 1\right),$$

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which is

$$\mathcal{F}[\frac{d}{dt}E(t)] = 2i\hbar\omega_P^2\mathcal{F}[\delta(t) - \omega_P\sqrt{\frac{2}{\pi}}K_0\big(\omega_P t\big)],$$

and

$$\mathcal{F}[\frac{d}{dt}E(t)] = -2i\hbar\omega_P^2\mathcal{F}[i\omega_P\sqrt{\frac{2}{\pi}}K_0\big(-i\omega_P t\big) + \delta(t)],$$

therefore

$$\boxed{E(t) = 2i\hbar\omega_P^2\left(H(t) - \omega_P\sqrt{\frac{2}{\pi}}\int^t K_0\big(\omega_P\tau\big)d\tau\right)} \tag{10}$$

where $H(t)$ is the Heaviside step function; or

$$\boxed{E(t) = -2i\hbar\omega_P^2\left(i\omega_P\sqrt{\frac{2}{\pi}}\int^t K_0\big(-i\omega_P\tau\big)d\tau + H(t)\right)} \tag{11}$$

References

[1] Alireza Jamali. Towards an einsteinian quantum mechanics (preprint). *viXra:2103.0006*, 2021.