Galaxy Formation, Evolution and Rotation as a 4D Relativistic Cloud-World Embedded in a 4D Conformal Bulk

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Abstract: The observation of the S-star orbits has established the existence of a supermassive compact object at the galactic centre. However, the G2 gas cloud orbit and absence of observations on event horizon-scale distances have challenged the central black hole model. In addition, the Planck Legacy 2018 (PL18) release has preferred a positively curved early Universe with a confidence level greater than 99%. In this study, the formation of a galaxy from the collapse of a supermassive gas cloud in the early Universe is modelled based on interaction field equations as a 4D relativistic cloud-world that flows and spins through a 4D conformal bulk of a primordial positive curvature considering the preference of the PL18 release. Owing to the curved background, this scenario of galaxy formation reveals that the core of the galaxy undergoes a forced vortex formation with a central event horizon leading to opposite vortices (traversable wormholes) that spatially shrink through evolving in the conformal time. It indicates that the galaxy and its core are formed at the same process where the surrounding gas clouds form the spiral arms due to the frame-dragging induced by the fast-rotating core. Accordingly, the G2 gas cloud that only faced the drag effects could be explained if its orbit is around one of the traversable wormholes but at a distance from the central event horizon. Further, the simulation of the cloud-world flow through a positively curved early bulk demonstrates the fast orbital speed of outer stars owing to external fields exerted on galaxies as they have travelled through conformally curved spacetimes. These findings could elucidate the fast orbital speed of outer stars in galaxies while the formation of a galaxy and its core simultaneously could explain the supermassive compact galaxy core growth to a mass of ~10^9 $M_\odot$ at just 6% of the current Universe age.

Keywords: Galaxy Formation; Conformal Spacetime; Brane-World Modified Gravity.

1. Introduction

The Deep Extragalactic Evolutionary Probe 2 Survey of a large sample of disk galaxies found that the motions of these galaxies were steadily getting in order with their rotation velocity increasing over the last eight billion years [1,2]. In addition, galactic rotation curves were found to be influenced by external fields supporting modified gravity theories beyond General Relativity (GR) [3]. Further, the Dark Energy Survey Collaboration has recently released the largest maps of galaxy distribution and shapes, which showed that dark matter (or dark gravity) is more distributed than that predicted by GR [4,5]. On the other hand, some studies reported that some galaxies are missing dark matter [6–8].

Recently, the Planck Legacy 2018 (PL18) release has confirmed the existence of an enhanced lensing amplitude in the cosmic microwave background power spectra, which prefers a positively curved early Universe with a confidence level greater than 99% [9,10]. Additionally, the observations of the G2 gas cloud orbit around the galactic centre has challenged the supermassive black hole model due to the absence of observations on the event horizon-scale distances [11,12].

By considering these observations and the PL18's preferred primordial curvature, this study presents a new galaxy formation scenario utilizing interaction field equations in which celestial objects are considered as 4D relativistic cloud-worlds that flow and spin through a 4D conformal bulk of a primordial curvature.

2. Interaction Field Equations

The PL18 release has preferred a positively curved early Universe, that is, is a sign of a primordial background curvature or a curved bulk. To incorporate the bulk curvature and its evolution over the conformal time, a modulus of spacetime deformation, E_D in terms of energy density, is introduced based on the theory of elasticity [13]. The modulus can be expressed in terms of the resistance of the bulk to the localized curvature induced by celestial objects using Einstein field equations or in terms of the field strength of the bulk using the Lagrangian formulation of the energy density existing in the bulk as a manifestation of the vacuum energy density as follows

$$E_D = \frac{T_{\mu\nu} - T g_{\mu\nu}/2}{R_{\mu\nu}/\mathcal{R}} = \frac{-\mathcal{F}_{\lambda\rho}\mathcal{F}^{\lambda\rho}}{4\mu_0} \tag{1}$$

where the stress is signified by the stress-energy tensor $T_{\mu\nu}$ of trace T while the strain is signified by the Ricci curvature tensor $R_{\mu\nu}$ as the change in the curvature divided by the scalar of the bulk curvature $\mathcal{R}$ as the background or conformal curvature. $\mathcal{F}_{\lambda\rho}$ is the field strength tensor and μ_0 is vacuum permeability. By incorporating the bulk influence, the Einstein–Hilbert action can be extended to

$$S = E_D \int_C \left[\frac{R}{\mathcal{R}} + \frac{L}{\mathcal{L}} \right] \sqrt{-g}\, d^4\rho \tag{2}$$

where R is the Ricci scalar curvature representing the localized curvature induced into the bulk by a celestial object that is regarded as a 4D relativistic cloud-world of metric $g_{\mu\nu}$ and Lagrangian density L. $\mathcal{R}$ represents the scalar curvature of the 4D bulk of metric $\tilde{g}_{\mu\nu}$ while $\mathcal{L}$ is the Lagrangian density of the bulk as an indication of its internal stresses and momenta reflecting its curvature. Since the bulk modulus, E_D, is constant with regards the cloud-world action under constant vacuum energy density condition and by considering the bulk expansion over the conformal time owing to the Universe expansion and its implication on the field strength of the bulk, a dual-action concerning the conservation of energy on global (bulk) and local (cloud-world) scales can be introduced as follows

$$S = \int_B \left[\frac{-\mathcal{F}_{\lambda\rho}\tilde{g}^{\lambda\gamma}\mathcal{F}_{\gamma\alpha}\tilde{g}^{\rho\alpha}}{4\mu_0} \right] \sqrt{-\tilde{g}} \int_C \left[\frac{R_{\mu\nu}g^{\mu\nu}}{\mathcal{R}_{\mu\nu}\tilde{g}^{\mu\nu}} + \frac{L_{\mu\nu}g^{\mu\nu}}{\mathcal{L}_{\mu\nu}\tilde{g}^{\mu\nu}} \right] \sqrt{-g}\, d^4\rho\, d^4\sigma \tag{3}$$

where the relationship of the conformal bulk metric $\tilde{g}_{\mu\nu}$ with the embedded cloud-world metric $g_{\mu\nu}$ can be characterized by Weyl's conformal transformation as $\tilde{g}_{\mu\nu} = g_{\mu\nu}\Omega^2$, here Ω^2 is a conformal function [14]. The global-local action should hold for any variation as

$$\delta S = \int_B \left[\begin{array}{c} \frac{-\delta(\mathcal{F}_{\lambda\rho}\tilde{g}^{\lambda\gamma}\mathcal{F}_{\gamma\alpha}\tilde{g}^{\rho\alpha})\sqrt{-\tilde{g}}}{4\mu_0} \\[2mm] -\frac{\mathcal{F}_{\lambda\rho}\tilde{g}^{\lambda\gamma}\mathcal{F}_{\gamma\alpha}\tilde{g}^{\rho\alpha}\delta\sqrt{-\tilde{g}}}{4\mu_0} \end{array} \right] \int_C \left[\begin{array}{c} \frac{\delta(R_{\mu\nu}g^{\mu\nu})\sqrt{-g}}{\mathcal{R}_{\mu\nu}\tilde{g}^{\mu\nu}} - \frac{\delta(\mathcal{R}_{\mu\nu}\tilde{g}^{\mu\nu})R_{\mu\nu}g^{\mu\nu}\sqrt{-g}}{(\mathcal{R}_{\mu\nu}\tilde{g}^{\mu\nu})^2} + \frac{R_{\mu\nu}g^{\mu\nu}\delta\sqrt{-g}}{\mathcal{R}_{\mu\nu}\tilde{g}^{\mu\nu}} \\[2mm] + \frac{\delta(L_{\mu\nu}g^{\mu\nu})\sqrt{-g}}{\mathcal{L}_{\mu\nu}\tilde{g}^{\mu\nu}} - \frac{\delta(\mathcal{L}_{\mu\nu}\tilde{g}^{\mu\nu})L_{\mu\nu}g^{\mu\nu}\sqrt{-g}}{(\mathcal{L}_{\mu\nu}\tilde{g}^{\mu\nu})^2} + \frac{L_{\mu\nu}g^{\mu\nu}\delta\sqrt{-g}}{\mathcal{L}_{\mu\nu}\tilde{g}^{\mu\nu}} \end{array} \right] d^4\rho\, d^4\sigma \tag{4}$$

By utilizing Jacobi's formula, $\delta\sqrt{-g} = -\sqrt{-g}\, g_{\mu\nu}\delta g^{\mu\nu}/2$ [15]. Hence, the variation is

$$\delta S = \int_B \left[\begin{array}{c} -\frac{\mathcal{F}_{\lambda\rho}\mathcal{F}_\gamma^\rho\delta\tilde{g}^{\lambda\gamma} + \delta\mathcal{F}_{\lambda\rho}\mathcal{F}^{\lambda\rho}}{2\mu_0} \\[2mm] +\mathcal{F}_{\lambda\rho}\tilde{g}^{\lambda\gamma}\mathcal{F}_{\gamma\alpha}\tilde{g}^{\rho\alpha}\frac{\tilde{g}_{\mu\nu}\delta\tilde{g}^{\mu\nu}}{8\mu_0} \end{array} \right] \sqrt{-\tilde{g}} \int_C \left[\begin{array}{c} \frac{R_{\mu\nu}\delta g^{\mu\nu} + g^{\mu\nu}\delta R_{\mu\nu}}{\mathcal{R}} - \frac{\mathcal{R}_{\mu\nu}\delta\tilde{g}^{\mu\nu} + \tilde{g}^{\mu\nu}\delta\mathcal{R}_{\mu\nu}}{\mathcal{R}^2}R - \frac{g_{\mu\nu}\delta g^{\mu\nu}}{2\mathcal{R}}R \\[2mm] + \frac{L_{\mu\nu}\delta g^{\mu\nu} + g^{\mu\nu}\delta L_{\mu\nu}}{\mathcal{L}} - \frac{\mathcal{L}_{\mu\nu}\delta\tilde{g}^{\mu\nu} + \tilde{g}^{\mu\nu}\delta\mathcal{L}_{\mu\nu}}{\mathcal{L}^2}L - \frac{g_{\mu\nu}\delta g^{\mu\nu}}{2\mathcal{L}}L \end{array} \right] \sqrt{-g}\, d^4\rho\, d^4\sigma \tag{5}$$

where the electromagnetic and Lagrangian densities are considered on the boundaries.

By considering the cloud-world's boundary term: $\int_c g^{\mu\nu}\delta R_{\mu\nu}\sqrt{-g}\, d^4\sigma/\mathcal{R}$, the variation in the Ricci curvature tensor δR_{uv} can be expressed in terms of the covariant derivative of the difference between two Levi-Civita connections, the Palatini identity: $\delta R_{\mu\nu} = \nabla_\rho(\delta\Gamma^\rho_{\nu\mu}) - \nabla_\nu(\delta\Gamma^\rho_{\rho\mu})$, where the variation with respect to the inverse metric $g^{\mu\nu}$ can be obtained by using the metric compatibility of the covariant derivative, $\nabla_\rho g^{\mu\nu} = 0$ [15], as $g^{\mu\nu}\delta R_{\mu\nu} = \nabla_\rho(g^{\mu\nu}\delta\Gamma^\rho_{\nu\mu} - g^{\mu\rho}\delta\Gamma^\sigma_{\sigma\mu})$. Consequently, the cloud-world's boundary term as a total derivative for any tensor density can be transformed based on Stokes' theorem with renaming the dummy indices as follows

$$\int_C \left[\frac{g^{\mu\nu}\delta R_{\mu\nu}}{\mathcal{R}}\right]\sqrt{-g}\, d^4\rho = \frac{1}{\mathcal{R}}\int_C \left[\nabla_\rho(g^{\mu\nu}\delta\Gamma^\rho_{\nu\mu} - g^{\mu\rho}\delta\Gamma^\sigma_{\sigma\mu})\right]\sqrt{-g}\, d^4\rho$$

$$= \frac{1}{\mathcal{R}}\int_C \left[\nabla_\mu H^\mu\right]\sqrt{-g}\, d^4\rho = \frac{\epsilon}{\mathcal{R}}\int_{\partial C} [K]\sqrt{|q|}\, d^3\varrho \tag{6}$$

where the bulk scalar curvature, $\mathcal{R}$, is left outside the integral transformation as it only acts as a scalar to the transformation. The same transformations are applied to the bulk and Lagrangian boundary terms and the electromagnetic tensor variation as it represents a boundary term. Thus, the transformed boundary action, S_b, along with a transformed global boundary term are

$$S_b = \int_{\partial B}\left[\frac{\epsilon J_\rho}{2}\right]\sqrt{-\tilde{q}}\left(\frac{\epsilon}{\mathcal{R}}\int_{\partial C}[K]\sqrt{|q|} - \frac{R\epsilon}{\mathcal{R}^2}\int_{\partial C}[\mathcal{K}]\sqrt{|\tilde{q}|} + \frac{\epsilon}{\mathcal{L}}\int_{\partial C}[l]\sqrt{|q|} - \frac{L\epsilon}{\mathcal{L}^2}\int_{\partial C}[\ell]\sqrt{|\tilde{q}|}\right)d^3\varrho\, d^3\varsigma \tag{7}$$

where K and $\mathcal{K}$ are the traces of the cloud-world and the bulk extrinsic curvatures, l and ℓ are the extrinsic traces of the Lagrangian density on the cloud-world and the bulk boundaries, q and $\tilde{q}$ are the determinants of their induced metrics respectively, and ϵ equals 1 when the normal $\hat{n}_u$ is a spacelike entity and equals -1 when it is a timelike entity. J_ρ is the four current. The boundary action should hold for any variation and by considering the transformed cloud-world's boundary term, the variation in the term yields

$$\frac{\epsilon}{\mathcal{R}}\int_{\partial C}\left[K_{\mu\nu}\delta q^{\mu\nu} + q^{\mu\nu}\delta K_{\mu\nu} + K\frac{\delta\sqrt{|q|}}{\sqrt{|q|}}\right]\sqrt{|q|}\, d^3\varrho \tag{8}$$

where $K = K_{\mu\nu}q^{\mu\nu}$. By utilising Jacobi's formula for the determinant differentiation; thus, $\delta\sqrt{|q|} = -\sqrt{|q|}\, q_{\mu\nu}\delta q^{\mu\nu}/2$ and by utilising the variation in the metric times the inverse metric, $q^{\mu\nu}q_{\mu\nu} = \delta^\mu_\nu$ as $q^{\mu\nu} = -q_{\mu\nu}\delta q^{\mu\nu}/\delta q_{\mu\nu}$; thus, the boundary term is

$$\frac{\epsilon}{\mathcal{R}}\int_{\partial C}\left[K_{\mu\nu}\delta q^{\mu\nu} - \frac{1}{2}K\left(q_{\mu\nu}\delta q^{\mu\nu} + 2q_{\mu\nu}\frac{\delta K_{\mu\nu}}{\delta q_{\mu\nu}K}\delta q^{\mu\nu}\right)\right]\sqrt{|q|}\, d^3\varrho \tag{9}$$

here $\delta K_{\mu\nu}/\delta q_{\mu\nu}K = (\delta K_{\mu\nu}/K_{\mu\nu})(q_{\mu\nu}/\delta q_{\mu\nu}) = \delta\ln K_{\mu\nu}/\delta\ln q_{\mu\nu}$ resembles the Ricci flow in a normalized form reflecting the conformal distortion in the boundary over the conformal time, which can be expressed as a positive function Ω^2 according to Weyl's conformal transformation as $\tilde{q}_{\mu\nu} = q_{\mu\nu}\Omega^2$ [16]. Thus, the boundary term is

$$\frac{\epsilon}{\mathcal{R}}\int_{\partial C}\left[K_{\mu\nu}\delta q^{\mu\nu} - \frac{1}{2}K\hat{q}_{\mu\nu}\delta q^{\mu\nu}\right]\sqrt{|q|}\, d^3\varrho \tag{10}$$

where $\hat{q}_{\mu\nu} = q_{\mu\nu} + 2\tilde{q}_{\mu\nu}$ denoting the conformally transformed induced metric on the cloud-world boundary. The same is applied for the bulk and Lagrangian boundary terms.

Thus, the variation in the whole action with renaming the dummy indices is

$$
\delta S = \left(- \int_B \left[\frac{1}{2\mu_0} \left(\mathcal{F}_{\mu\lambda}\mathcal{F}_\nu^\lambda - \frac{\mathcal{F}_{\lambda\rho}\mathcal{F}^{\lambda\rho}}{4} \tilde{g}_{\mu\nu} \right) \right] \delta\tilde{g}^{\mu\nu}\sqrt{-\tilde{g}}\, d^4\sigma \ - \int_{\partial B} \left[\frac{\epsilon}{2} \delta J_\nu / \delta\tilde{q}^{\mu\nu} \right] \delta\tilde{q}^{\mu\nu}\sqrt{|\tilde{q}|}\, d^3\varsigma \right)
$$

$$
\left(\int_C \left[\frac{R_{\mu\nu}}{\mathcal{R}} - \frac{\mathcal{R}_{\mu\nu}}{\mathcal{R}^2}R - \frac{R}{2\mathcal{R}}g_{\mu\nu} + \frac{L_{\mu\nu}}{\mathcal{L}} - \frac{\mathcal{L}_{\mu\nu}}{\mathcal{L}^2}L - \frac{L}{2\mathcal{L}}g_{\mu\nu} \right] \delta g^{\mu\nu}\sqrt{-g}\, d^4\rho \right.
$$

$$
+ \int_{\partial C} \left[\frac{\epsilon}{\mathcal{R}}\left(K_{\mu\nu} - \frac{1}{2}K\hat{q}_{\mu\nu} \right) + \frac{\epsilon}{\mathcal{L}}\left(l_{\mu\nu} - \frac{1}{2}l\hat{q}_{\mu\nu} \right) \right] \delta q^{\mu\nu}\sqrt{|q|}\, d^3\varrho \tag{11}
$$

$$
\left. - \int_{\partial C} \left[\frac{R\epsilon}{\mathcal{R}^2}\left(\mathcal{K}_{\mu\nu} - \frac{1}{2}\mathcal{K}\hat{q}_{\mu\nu} \right) + \frac{L\epsilon}{\mathcal{L}^2}\left(\ell_{\mu\nu} - \frac{1}{2}\ell\hat{q}_{\mu\nu} \right) \right] \delta q^{\mu\nu}\sqrt{|q|}\, d^3\varrho \right)
$$

By encapsulating the new Lagrangian terms on boundaries into an extended stress-energy tensor defined as $\hat{T}_{\mu\nu} := (2L_{\mu\nu} - L\hat{g}_{\mu\nu}) - (2l_{\mu\nu} - l\hat{q}_{\mu\nu}) + (2\ell_{\mu\nu} - \ell\hat{q}_{\mu\nu})L/\mathcal{L}$ that counts for the energy density and flux of the cloud-world, $L_{\mu\nu}$, and electromagnetic energy flux from its boundary, $l_{\mu\nu}$, over the conformal time where $\hat{g}_{\mu\nu} = g_{\mu\nu} + 2\tilde{g}_{\mu\nu}$ is the conformally transformed metric since $\tilde{g}_{\mu\nu} = \mathcal{L}_{\mu\nu}/\mathcal{L} = \mathcal{L}_{\mu\nu}/\mathcal{L}_{\mu\nu}\tilde{g}^{\mu\nu}$ while the Lagrangian density on bulk's boundary, $\ell_{\mu\nu}$, has $L/\mathcal{L}$ factor, i.e. it is only significant when the cloud-world has a high Lagrangian density such as black holes where the entire contribution belongs to the boundary terms when finding the black hole entropy [17,18]. The outcomes of the global action resembled an extended electromagnetic stress-energy tensor as $\mathcal{T}_{\mu\nu} := (\mathcal{F}_{\mu\lambda}\mathcal{F}_\nu^\lambda - \mathcal{F}_{\lambda\rho}\mathcal{F}^{\lambda\rho}\tilde{g}_{\mu\nu}/4)/\mu_0) + \delta J_\nu/\delta\tilde{q}^{\mu\nu}$ denoting energy density exists in the bulk as the vacuum energy density in addition to the variation of the four current with regards bulk's boundary variation over conformal time. From Equations (1), (2) and (11), $\mathcal{T}_{\mu\nu} := E_D = \mathcal{L} = \mathcal{R}c^4/8\pi G_t$ is proportional to the fourth power of the speed of light that in turn is directly proportional to the frequency, which can be in accordance with frequency cut-off predictions of vacuum energy density in the quantum field theory [19,20]. By choosing ϵ as a timelike entity and applying the principle of stationary action yields

$$
\frac{R_{\mu\nu}}{\mathcal{R}} - \frac{1}{2}\frac{R}{\mathcal{R}}g_{\mu\nu} - \frac{\mathcal{R}_{\mu\nu}}{\mathcal{R}^2}R + \frac{R\left(\mathcal{K}_{\mu\nu} - \frac{1}{2}\mathcal{K}\hat{q}_{\mu\nu} \right) - \mathcal{R}\left(K_{\mu\nu} - \frac{1}{2}K\hat{q}_{\mu\nu} \right)}{\mathcal{R}^2} = \frac{\hat{T}_{\mu\nu}}{\mathcal{T}_{\mu\nu}} \tag{12}
$$

The interaction field equations can be interpreted as indicating that the cloud-world's induced curvature over the bulk's pre-existing curvature equals the ratio of the cloud-world's imposed energy density and its flux to the bulk's vacuum energy density and its flux through the expanding/contracting Universe. By utilizing Equations (1) and (11) that state $\mathcal{T}_{\mu\nu} := E_D = \mathcal{R}c^4/8\pi G_t$, the field equations can be simplified to

$$
R_{\mu\nu} - \frac{1}{2}R\hat{g}_{\mu\nu} + \frac{R\left(\mathcal{K}_{\mu\nu} - \frac{1}{2}\mathcal{K}\hat{q}_{\mu\nu} \right) - \mathcal{R}\left(K_{\mu\nu} - \frac{1}{2}K\hat{q}_{\mu\nu} \right)}{\mathcal{R}} = \frac{8\pi G_t}{c^4}\hat{T}_{\mu\nu} \tag{13}
$$

where $\hat{g}_{\mu\nu} = g_{\mu\nu} + 2\tilde{g}_{\mu\nu}$ is the conformally transformed metric counting for both cloud-world, $g_{\mu\nu}$, and bulk, $\tilde{g}_{\mu\nu} = \mathcal{R}_{\mu\nu}/\mathcal{R} = \mathcal{R}_{\mu\nu}/\mathcal{R}_{\mu\nu}\tilde{g}^{\mu\nu}$, metrics whereas Einstein spaces are a subclass of conformal spaces [14]. The evolution in G_t reflects the field strength evolution with Universe expansion and it can accommodate the bulk curvature evolution over the conformal time against constant G for a special flat spacetime case. The new boundary term is only significant at high energies such as within black holes [17]. These field equations could remove the singularities and satisfy a conformal invariance theory.

3. Galaxy Formation, Evolution and Rotation

The entire contribution comes from the boundary term when calculating the black hole entropy using the semiclassical approach [17,18]. Applying this concept and by re-arranging the field equations for this setting as

$$R_{\mu\nu} - \frac{1}{2}R\hat{g}_{\mu\nu} = \frac{8\pi G_t}{c^4}\hat{T}_{\mu\nu} - \frac{R\left(\mathcal{K}_{\mu\nu} - \frac{1}{2}\mathcal{K}\hat{q}_{\mu\nu}\right) - \mathcal{R}\left(K_{\mu\nu} - \frac{1}{2}K\hat{q}_{\mu\nu}\right)}{\mathcal{R}} = 0 \qquad (14)$$

The field equations can describe the interaction between a 4D relativistic cloud-world of intrinsic $R_{\mu\nu}$ and extrinsic $K_{\mu\nu}$ curvatures with a stress-energy $\hat{T}_{\mu\nu}$ and the 4D bulk of intrinsic $\mathcal{R}_{\mu\nu}$ and extrinsic $\mathcal{K}_{\mu\nu}$ curvatures with a stress-energy $\mathcal{T}_{\mu\nu}$. From Equation (14), the field equations yield

$$R_{\mu\nu} = \frac{1}{2}R\hat{g}_{\mu\nu} = \frac{1}{2}Rg_{\mu\nu} + \frac{\mathcal{R}_{\mu\nu}}{\mathcal{R}}R = \frac{1}{2}R(g_{\mu\nu} + 2\tilde{g}_{\mu\nu}) = \frac{1}{2}Rg_{\mu\nu}(1 + 2\Omega^2) = 0 \qquad (15)$$

where $\hat{g}_{\mu\nu} = g_{\mu\nu} + 2\tilde{g}_{\mu\nu}$ and $\tilde{g}_{\mu\nu} = \mathcal{R}_{\mu\nu}/\mathcal{R} = \mathcal{R}_{\mu\nu}/\mathcal{R}_{\mu\nu}\tilde{g}^{\mu\nu}$ is the conformal bulk metric, which can be expressed as proportional to cloud-world metric $g_{\mu\nu}$ as $\tilde{g}_{\mu\nu} = g_{\mu\nu}\Omega^2$ by utilizing Ω^2, the conformal transformation function. The conformally transformed metric $\hat{g}_{\mu\nu} = g_{\mu\nu}(1 + 2\Omega^2)$ can be expressed as

$$ds^2 = -A(r)(1 + 2\Omega^2(r,\tau))c^2dt^2 + S^2\left(B(r)(1 + 2\Omega^2(r,\mathbf{r}))dr^2 + r^2d\theta^2 + r^2sin^2\theta\, d\phi^2\right)(16)$$

where A and B are functions of the cloud-world radius r, while the conformal function Ω^2 is a function of the bulk radius of curvature τ and it can be influenced by the cloud-world radius. S^2 is a dimensionless conformal scale factor. By performing the coordinate transformation as follows

$$ds^2 = -\left(A(\lambda) + 2A(\lambda)\Omega^2(\lambda,\tau)\right)c^2dt^2 + \left(B(\lambda) + 2B(\lambda)\Omega^2(\lambda,\mathbf{r})\right)d\lambda^2 + \lambda^2d\theta^2 + \lambda^2sin^2\theta\, d\phi^2(17)$$

The Christoffel symbols of this metric are

$$\Gamma^1_{00} = \frac{\dot{A}(1 + 2\Omega^2) + 4A\dot{\Omega}}{2(B + 2B\Omega^2)}, \qquad \Gamma^0_{01} = \frac{\dot{A}(1 + 2\Omega^2) + 4A\dot{\Omega}}{2(A + 2A\Omega^2)}, \qquad \Gamma^1_{11} = \frac{\dot{B}(1 + 2\Omega^2) + 4B\dot{\Omega}}{2(B + 2B\Omega^2)}$$

$$\Gamma^1_{22} = \frac{-\lambda}{(B + 2B\Omega^2)}, \qquad \Gamma^1_{33} = \frac{-\lambda sin^2\theta}{(B + 2B\Omega^2)}, \qquad \Gamma^2_{21} = \Gamma^2_{12} = \frac{1}{\lambda} \qquad (18)$$

$$\Gamma^2_{33} = -sin\theta\, cos\theta, \qquad \Gamma^3_{32} = \Gamma^3_{23} = \frac{cos\theta}{sin\theta}$$

The Ricci tensor components are

$$R_{tt} = -\frac{\frac{d}{d\lambda}\left(\dot{A}\left(1 + 2\Omega^2\right) + 4A\dot{\Omega}\right)}{2\left(B + 2B\Omega^2\right)} - \frac{\dot{A}\left(1 + 2\Omega^2\right) + 4A\dot{\Omega}}{2\left(B + 2B\Omega^2\right)}\frac{\dot{B}\left(1 + 2\Omega^2\right) + 4B\dot{\Omega}}{2\left(B + 2B\Omega^2\right)}$$

$$+ \frac{\dot{A}\left(1 + 2\Omega^2\right) + 4A\dot{\Omega}}{2\left(B + 2B\Omega^2\right)}\frac{\dot{A}\left(1 + 2\Omega^2\right) + 4A\dot{\Omega}}{2\left(A + 2A\Omega^2\right)} - \frac{1}{\lambda}\frac{\dot{A}\left(1 + 2\Omega^2\right) + 4A\dot{\Omega}}{2\left(B + 2B\Omega^2\right)}$$

$$(19)$$

$$R_{tt} = -\frac{\ddot{A}\left(1 + 2\Omega^2 + 4\dot{\Omega}\right) + 4A\ddot{\Omega} + 4\dot{\Omega}\,\dot{A}}{2\left(B + 2B\Omega^2\right)} + \frac{\left(\dot{A}\left(1 + 2\Omega^2\right) + 4A\dot{\Omega}\right)\left(\dot{B}\left(1 + 2\Omega^2\right) + 4B\dot{\Omega}\right)}{4\left(B + 2B\Omega^2\right)^2}$$

$$+ \frac{\left(\dot{A}\left(1 + 2\Omega^2\right) + 4A\dot{\Omega}\right)^2}{4\left(A + 2A\Omega^2\right)\left(B + 2B\Omega^2\right)} - \frac{1}{\lambda}\frac{\dot{A}\left(1 + 2\Omega^2\right) + 4A\dot{\Omega}}{\left(B + 2B\Omega^2\right)} \tag{20}$$

$$R_{rr} = \frac{1}{2}\left(\frac{\ddot{A}\left(1 + 2\Omega^2 + 4\dot{\Omega}\right) + 4A\ddot{\Omega} + 4\dot{\Omega}\,\dot{A}}{\left(A + 2A\Omega^2\right)} - \frac{\left(\dot{A}\left(1 + 2\Omega^2\right) + 4A\dot{\Omega}\right)^2}{2\left(A + 2A\Omega^2\right)^2}\right)$$

$$- \frac{\left(\dot{A}\left(1 + 2\Omega^2\right) + 4A\dot{\Omega}\right)\left(\dot{B}\left(1 + 2\Omega^2\right) + 4B\dot{\Omega}\right)}{4\left(A + 2A\Omega^2\right)\left(B + 2B\Omega^2\right)} - \frac{1}{\lambda}\frac{\dot{B}\left(1 + 2\Omega^2\right) + 4B\dot{\Omega}}{B + 2B\Omega^2} \tag{21}$$

$$R_{\theta\theta} = \frac{1}{\left(B + 2B\Omega^2\right)} - \frac{\lambda}{2\left(B + 2B\Omega^2\right)}\left(\frac{\dot{B}\left(1 + 2\Omega^2\right) + 4B\dot{\Omega}}{\left(B + 2B\Omega^2\right)} - \frac{\dot{A}\left(1 + 2\Omega^2\right) + 4A\dot{\Omega}}{\left(A + 2A\Omega^2\right)}\right) - 1 \tag{22}$$

$$R_{\phi\phi} = \frac{sin^2\theta}{\left(B + 2B\Omega^2\right)} - \frac{\lambda sin^2\theta}{2\left(B + 2B\Omega^2\right)}\left(\frac{\dot{B}\left(1 + 2\Omega^2\right) + 4B\dot{\Omega}}{\left(B + 2B\Omega^2\right)} - \frac{\dot{A}\left(1 + 2\Omega^2\right) + 4A\dot{\Omega}}{\left(A + 2A\Omega^2\right)}\right) - sin^2\theta \tag{23}$$

Using Ricci tensor components gives

$$\left(\dot{A}(1 + 2\Omega^2) + 4A\dot{\Omega}\right)(B + 2B\Omega^2) + (A + 2A\Omega^2)\left(\dot{B}(1 + 2\Omega^2) + 4B\dot{\Omega}\right) = 0 \tag{24}$$

Equation (24) gives

$$B + 2B\Omega^2 = \frac{k}{A + 2A\Omega^2} \tag{25}$$

where $k = (1 + 2\Omega^2)^2$ for the conformally transformed metric by considering the bulk curvature. By applying the weak-field limit: $\hat{g}_{\mu\nu} \approx \eta_{\mu\nu} + \hat{h}_{\mu\nu}$, as follows

$$\Gamma^i_{tt} = \frac{1}{2}\int \partial_i \hat{h}_{tt} = \frac{1}{c^2}\int \partial_i \varphi \tag{26}$$

where φ is the Newtonian gravitational potential. By integrating both sides

$$\hat{g}_{tt} = -A(1 + 2\Omega^2) = -\left(\eta_{tt} + \frac{2\varphi_c}{c^2} + \frac{2\varphi_b}{c^2}\right) \tag{27}$$

where $\varphi_c = -GM/\lambda$ is the gravitational potential of the cloud-world's spherical mass and φ_b that arises from the integration can be interpreted as the gravitational potential resulting from the bulk curvature, which can be expressed, using the same Newtonian analogue, in terms of the mass of the early Universe plasma of preferred positive curvature, M_p, and the bulk curvature radius $\mathfrak{r}$ as $\varphi_b = -G_p M_p/\mathfrak{r}$. The metric should yield only the gravitational potential of the cloud-world when there is no bulk curvature ($\Omega^2 = 0$) and ($\varphi_b = 0$); hence, $A = (1 + 2\varphi_c/c^2)$; consequently, the conformal function is $\Omega^2 = \varphi_b/Ac^2$.

By performing the coordinate retransformation and combining Equations (25 - 27) yield

$$\Omega^2 = -\frac{G_p M_p}{\tau c^2}\left(1 - \frac{2GM}{rc^2}\right)^{-1}, \qquad A = 1 - \frac{2GM}{rc^2}, \qquad B = \left(1 - \frac{2GM}{rc^2}\right)^{-1} \qquad (28)$$

where the conformal function Ω^2 relies on the gravitational potential of the bulk while its influence is inversely proportional to cloud-world potential. In the case of PI18's preferred early Universe positive curvature, the gravitational potential of the bulk can be expressed in terms of the early Universe plasma of mass, M_p, and τ denoting the radius of curvature of the bulk, where the bulk's potential decreases with the Universe expansion and vanishes in the flat spacetime background ($\tau \to \infty$). The minus sign of Ω^2 reveals a spatial shrinking through evolving in the conformal time, which agrees with the vortex model that can occur due to the high-speed spinning. By substituting Equations (28) to Equation (16), the conformally metric $\hat{g}_{\mu\nu} = g_{\mu\nu} + 2\tilde{g}_{\mu\nu} = g_{\mu\nu}(1 + 2\Omega^2)$ is

$$ds^2 = \left(1 - \frac{r_s}{r} - \frac{\tau_p}{\tau}\right)\left(-c^2 dt^2 + S^2\left(\frac{dr^2}{1 + \frac{r_s^2}{r^2} - 2\frac{r_s}{r}} + \frac{r^2 d\theta^2 + r^2 \sin^2\theta d\phi^2}{1 - \frac{r_s}{r} - \frac{\tau_p}{\tau}}\right)\right) \qquad (29)$$

This metric reduces to the Schwarzschild metric in a flat background ($\tau \to \infty$), where $\mathfrak{r}$ is the background or bulk curvature radius, r_s is Schwarzschild radius, $\tau_p = 2G_p M_p/c^2$ and S^2 is a dimensionless spatial scale factor. The denominator of the radial dimension can be interpreted as an intrinsic curvature term where the metric on the radial and two-sphere is warped by the bulk and cloud-world radii. The metric can be visualized through evolving in the conformal time using Flamm's approach as

$$w(\mathfrak{r}, r) = \mp \int \frac{\sqrt{\left(\frac{r_s}{r} - \frac{r_s^2}{r^2} - \frac{\tau_p}{\tau}\right)}}{\left(1 - \frac{r_s}{r}\right)} dr = \mp \sqrt{r_s(r - r_s) - \tau_p \frac{r^2}{\tau} + T} + C \qquad (30)$$

where C is a constant and T denotes less significant terms. By choosing an appropriate C, Figure 1 shows the proper distances, and their corresponding proper areas are increasing as they are evolving in the conformal time while the radius decreases
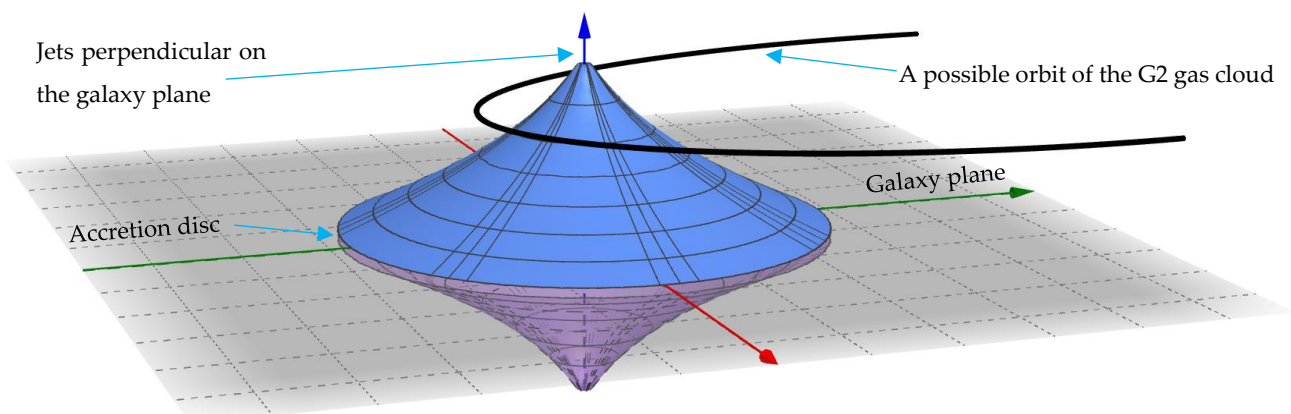
Figure 1. The metric of the supermassive compact object as a central event horizon leading to opposite vortices with two jets that are perpendicular on the galaxy plane

The increase in the proper distances and their corresponding proper areas as the radius decreases is also agreed with the vortex model as shown in Figure 2b while the positive and negative solutions of Equation (30) indicate the evolution of the vortex in opposite directions, i.e., forming a dual vortex perpendicular on the galaxy plane.

Figure 2 shows the evolution of the 4D cloud-world through its travel and spin in the conformal space-time of the 4D bulk.
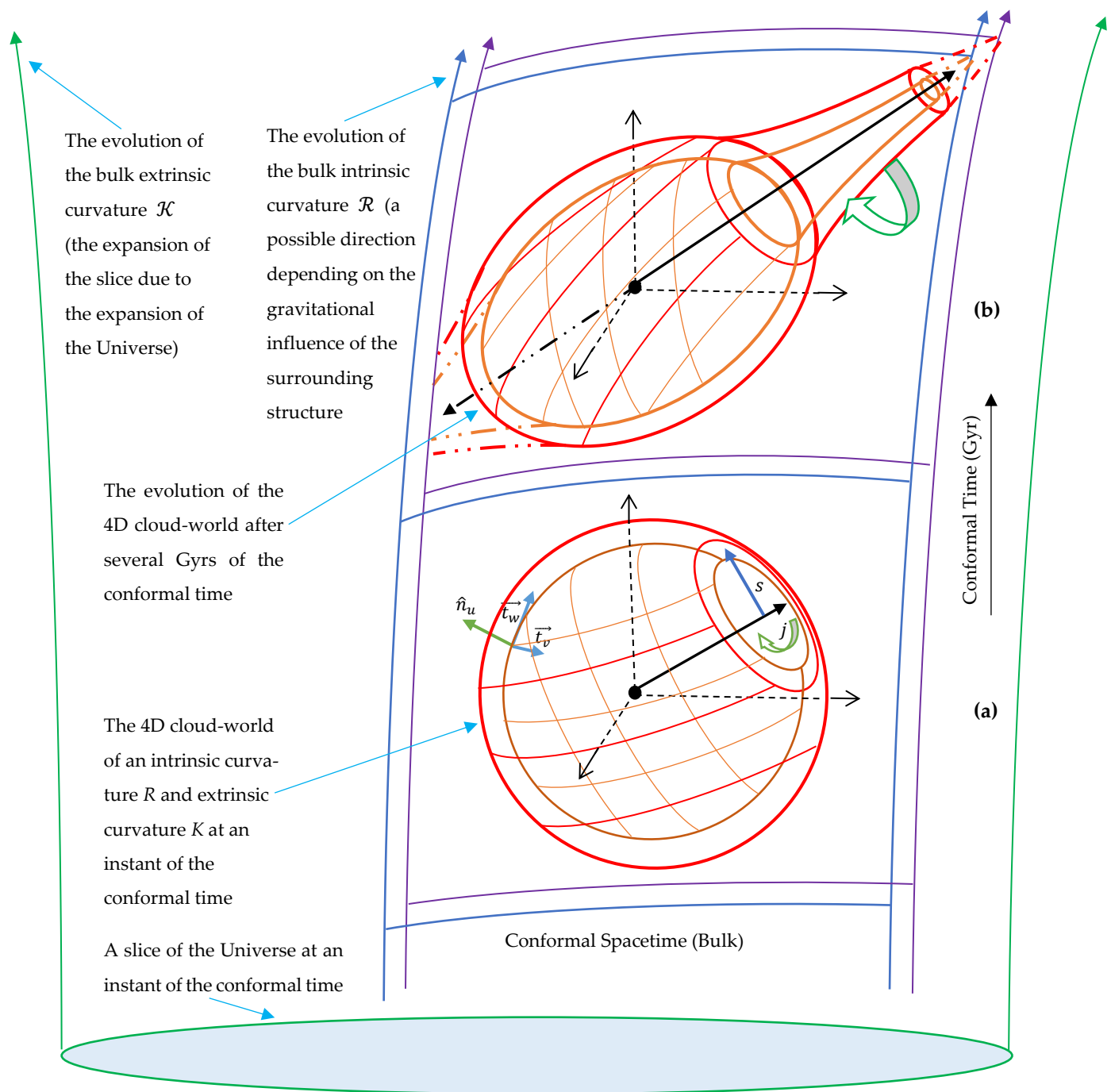
Figure 2. The hypersphere of a compact core of a galaxy (the red-orange 4D cloud-world) along with its travel and spin through the conformal spacetime (the blue-purple 4D bulk representing the bulk of distinctive curvature evolving over the conformal time.

The G2 cloud has only faced drag forces [11,12], where orbiting a wormhole could explain its observations. In addition, the observations of the superluminal motion in the x-ray jet of M87 [21] could be travelled through these traversable wormholes.

To evaluate the influence of the spinning and the curvature of the bulk on the core of the galaxy and the surrounding gas clouds (the spiral arms), a fluid simulation was performed based on Newtonian dynamics by using the Fluid Pressure and Flow software [22]. In this simulation, the fluid was deemed to represent the spacetime continuum throughout incrementally flattening curvature paths representing conformal curvature evolution to analyze the external momenta exerted on objects flowing throughout the incrementally flattening curvatures. The momenta yielded by the fluid simulation were used to simulate a spiral galaxy as a forced vortex as shown in Figure 3.
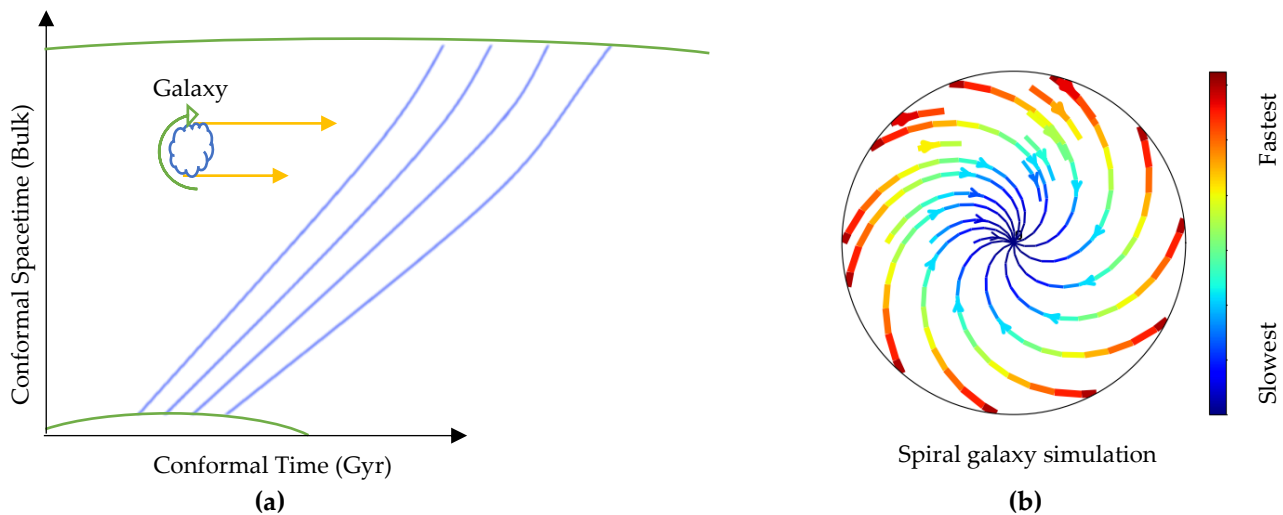
Figure 3. (**a**) External fields exerted on a galaxy due to the spacetime conformal curvature evolution. Green curves represent the curvature of spacetime worldlines. Blue curves represent the simulated spacetime continuum. (**b**) Simulation of spiral galaxy rotation. Blue represents the slowest tangential speeds and red represents the fastest speeds.

The simulation shows that the tangential speeds of the outer parts of the spiral galaxy are rotating faster in comparison with the rotational speeds of the inner parts, which resembles observations of galaxy rotation except the simulation used an ideal fluid.

4. Conclusions and Future Works

In this study, interaction field equations are derived in which the curvature of the background or the 4D conformal bulk evolves over the conformal time based on the PL18 recent release which has preferred a positively curved early Universe with a confidence level higher than 99%. Throughout this bulk, 4D relativistic cloud-worlds flow and spin.

Owing to the curved background, the findings of galaxy formation showed that the core of the galaxy undergoes a forced vortex formation with a central event horizon leading to opposite traversable wormholes that are spatially shrinking through evolving in the conformal time. It revealed that the galaxy and its core form at the same process, while the gas clouds outside the core can form the spiral arms owning to the fast-rotating core that induces the frame-dragging. The formation of the galaxy and its core at the same process can explain the formation of supermassive compact galaxy cores with a mass of ~$10^9\ M_\odot$ at just 6% of the current Universe age and could solve the black hole hierarchy problem. Orbiting a wormhole could explain the observation of the G2 cloud which only faced the drag effects. In addition, the observation of the superluminal motion in the x-ray jet of M87 could be travelling through these traversable wormholes. Further, this scenario can explain the fast orbital speed of outer stars due to travelling in curved 4D conformal bulk. The derived field equations should be applied to atoms and electron clouds as that with galaxies, which can be investigated in future, where a brief is presented in Appendix 1.

Conflicts of Interest: The author declares no conflict of interest.

Appendix 1.

In this section, the action in Equation (3) is expanded to investigate the interaction of a quantum field (for example an electron field) with the electromagnetic fields under the influence of the vacuum energy's field strength that in turn is dependent on the induced curvature by the presence of the conformal gravitational fields. It could be possible to expand the action using a similar procedure for a quantum field or cloud which is confined in the boundary of the cloud-world as follows

$$S = \int_B \left[\frac{-\mathcal{F}_{\lambda\rho} \mathcal{F}^{\lambda\rho}}{4\mu_0} \right] \sqrt{-\tilde{g}} \int_C \left[\frac{R_{\mu\nu} g^{\mu\nu}}{\mathcal{R}_{\mu\nu} \tilde{g}^{\mu\nu}} \right] \sqrt{-g} \int_Q \left[\frac{D}{H} + \frac{F_{\lambda\rho} F^{\lambda\rho}}{\hat{\mathcal{F}}_{\lambda\rho} \hat{\mathcal{F}}^{\lambda\rho}} \right] \sqrt{-q} \; d^4\alpha \; d^4\rho \; d^4\sigma \qquad (A.1)$$

where $\hat{\mathcal{F}}_{\lambda\rho} \hat{\mathcal{F}}^{\lambda\rho}$ is vacuum energy's field strength that accounts for the contributions of gravitational fields from the bulk field strength, $\mathcal{F}_{\lambda\rho} \mathcal{F}^{\lambda\rho}$, and the stress-energy from the embedded cloud-worlds. $F_{\lambda\rho} F^{\lambda\rho}$ is the applied electromagnetic field, H represents the trace of the energy-momentum tensor of the vacuum energy, which could be expressed as the Klein–Gordon field Lagrangian, D is the trace of the energy-momentum tensor of the quantum field Lagrangian and q is the determinant of the field. The variation of the action gives

$$\delta S = \int_B [\mathcal{T}_{\mu\nu}] \delta \tilde{g}^{\mu\nu} \sqrt{-\tilde{g}} \int_C \left[\frac{G_{\mu\nu}}{\mathcal{R}} \right] \delta g^{\mu\nu} \sqrt{-g} \int_Q \left[\begin{array}{c} \dfrac{\delta D}{H} - \dfrac{D \delta H}{H^2} - \dfrac{q_{\mu\nu} \delta q^{\mu\nu}}{2H} D + \dfrac{\delta F_{\lambda\rho} F^{\lambda\rho}}{\hat{\mathcal{F}}_{\lambda\rho} \hat{\mathcal{F}}^{\lambda\rho}} - \\ \dfrac{\delta \hat{\mathcal{F}}_{\lambda\rho} \hat{\mathcal{F}}^{\lambda\rho}}{\left(\hat{\mathcal{F}}_{\lambda\rho} \hat{\mathcal{F}}^{\lambda\rho} \right)^2} F_{\lambda\rho} F^{\lambda\rho} - \dfrac{q_{\mu\nu} \delta q^{\mu\nu}}{2\hat{\mathcal{F}}_{\lambda\rho} \hat{\mathcal{F}}^{\lambda\rho}} F_{\lambda\rho} F^{\lambda\rho} \end{array} \right] \sqrt{-q} \; d^4\rho \; d^4\sigma \; d^4\alpha \quad (A.2)$$

By applying the principle of stationary action:

$$\frac{\delta D}{H} - \frac{\delta H}{H^2} D = \frac{\mathcal{T}_{\mu\nu} - F_{\lambda\rho} F^{\lambda\rho}}{\hat{\mathcal{T}}_{\mu\nu}} \qquad (A.3)$$

where $\hat{\mathcal{T}}_{\mu\nu}$ is the electromagnetic stress-energy tensor of the vacuum energy's field strength based on the contributions of gravitational fields from the bulk and embedded cloud-worlds. $\mathcal{T}_{\mu\nu}$ is the applied electromagnetic stress-energy tensor of field strength $F_{\lambda\rho}$. By utilizing Equations (1), the field equations can be simplified to

$$\delta D - D \delta \ln H = \frac{g_x}{C} \left(\mathcal{T}_{\mu\nu} - F_{\lambda\rho} F^{\lambda\rho} \right) \qquad (A.4)$$

where C is a constant and g_x is the gravity of the cloud-world as a function of the distance from its centre. Equations (A.4) predict a gravitational influence on the quantum field δD and its Lagrangian D.

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